
Bayesian Compressive Sensing in Multi-Resolution Contrast Source Estimation

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1 Inhomogeneous Square Object, $\ell = 1.5\lambda$

Test Case Description

Direct solver:

- Side of the investigation domain: $L = 6.0\lambda$
- Cubic domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Number of cells for the direct solver: $D = 1600$ (discretization = $\lambda/10$)

Investigation domain:

- Cubic domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- Number of cells for the inversion:
 - First Step IMSA: $N^{(1)} = 100$ (discretization = $\lambda/10$)
 - Following Steps IMSA: $N^{(i)}$ not fixed, defined according to the estimated $RoI \mathcal{D}^{(i)}$

Measurement domain:

- Total number of measurements: $M = 60$
- Measurement points placed on circles of radius $\rho = 4.5\lambda$

Sources:

- Plane waves
- Number of views: $V = 60$; $\theta_{inc}^v = 0 + (v - 1) \times (360/V)$
- Amplitude: $A = 1.0$
- Frequency: $F = 300$ MHz ($\lambda = 1$)

Background:

- $\varepsilon_r = 1.0$
- $\sigma = 0$ [S/m]

Scatterer

- Inhomogeneous square object, $\ell = 1.5\lambda$
- $\varepsilon_r^{(1)} \in \{1.20, 1.60, 2.00\}$ (internal circle)
- $\varepsilon_r^{(2)} = \frac{\varepsilon_r^{(1)}}{2}$ (central circle)
- $\varepsilon_r^{(3)} = \frac{\varepsilon_r^{(1)}}{4}$ (external circle)
- $\sigma = 0$ [S/m]

1.0.1 Inhomogeneous Square Object, $\ell = 1.5\lambda$, $\tau^{(1)} = 0.20$ - IMSA-BCS vs. BARE-BCS reconstructed profiles

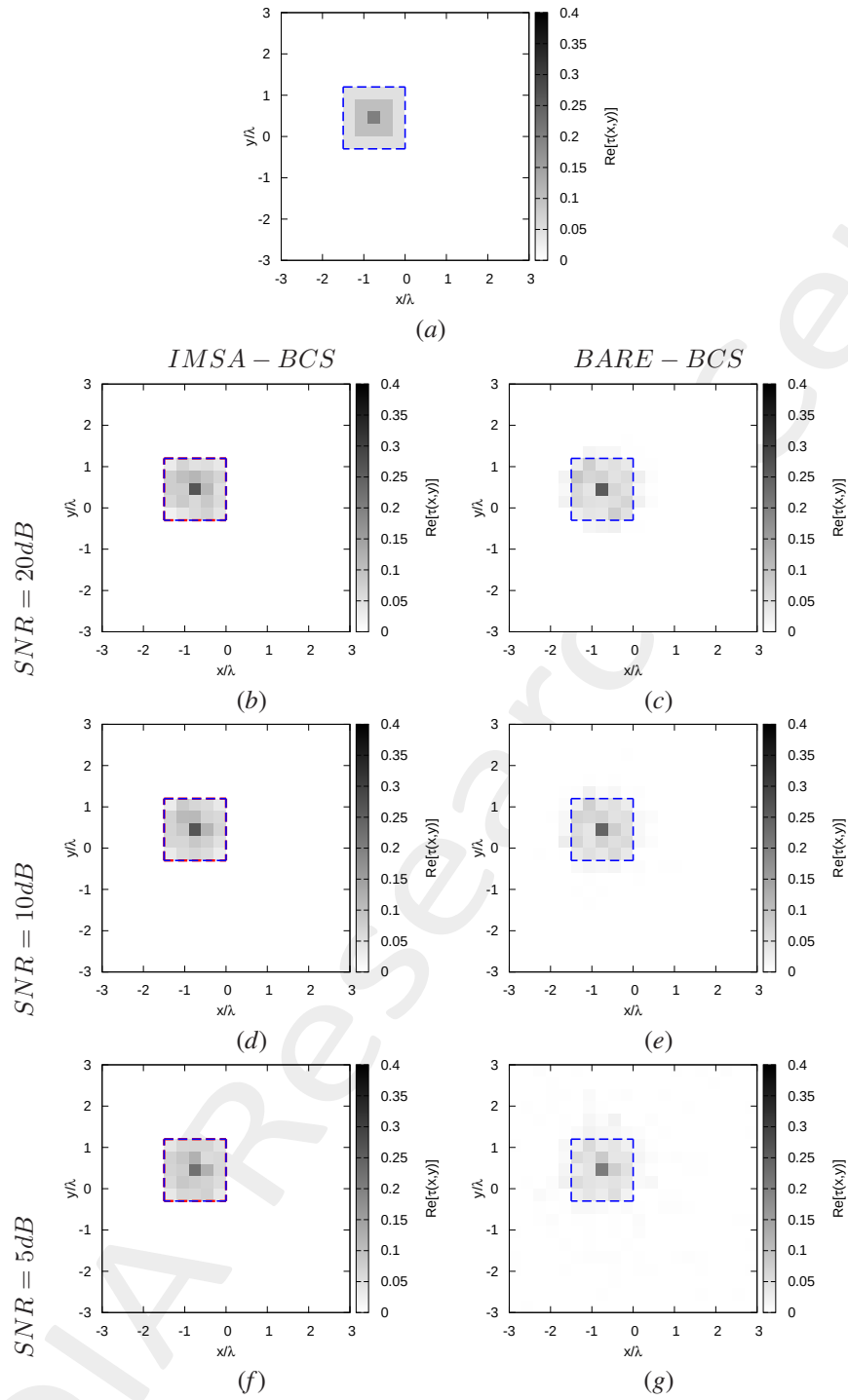


Figure 1: *Inhomogeneous Square Object*, $\ell = 1.5\lambda$, $\tau^{(1)} = 0.20$ - IMSA-BCS vs. BARE-BCS - (a) Actual profile, (b)(d)(f) IMSA-BCS and BARE-BCS reconstructed profiles for (b)(c) SNR = 20 [dB], (d)(e) SNR = 10 [dB] and (f)(g) SNR = 5 [dB].

	$SNR = 50dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	1.25×10^{-3}	2.42×10^{-3}
ξ_{int}	1.82×10^{-2}	2.45×10^{-2}
ξ_{ext}	0.00×10^{-1}	7.50×10^{-4}
	$SNR = 20dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	1.10×10^{-3}	2.30×10^{-3}
ξ_{int}	1.52×10^{-2}	2.40×10^{-2}
ξ_{ext}	0.00×10^{-1}	7.06×10^{-4}
	$SNR = 10dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	1.14×10^{-3}	2.47×10^{-3}
ξ_{int}	1.68×10^{-2}	2.07×10^{-2}
ξ_{ext}	0.00×10^{-1}	9.79×10^{-4}
	$SNR = 5dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	1.07×10^{-3}	3.61×10^{-3}
ξ_{int}	1.59×10^{-2}	2.25×10^{-2}
ξ_{ext}	0.00×10^{-1}	1.86×10^{-3}

Table I: *Inhomogeneous Square Object*, $\ell = 1.5\lambda$, $\tau^{(1)} = 0.20$ - *IMSA-BCS* vs. *BARE-BCS* - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

1.0.2 Inhomogeneous Square Object, $\ell = 1.5\lambda$, $\tau^{(1)} = 0.60$ - IMSA-BCS vs. BARE-BCS reconstructed profiles

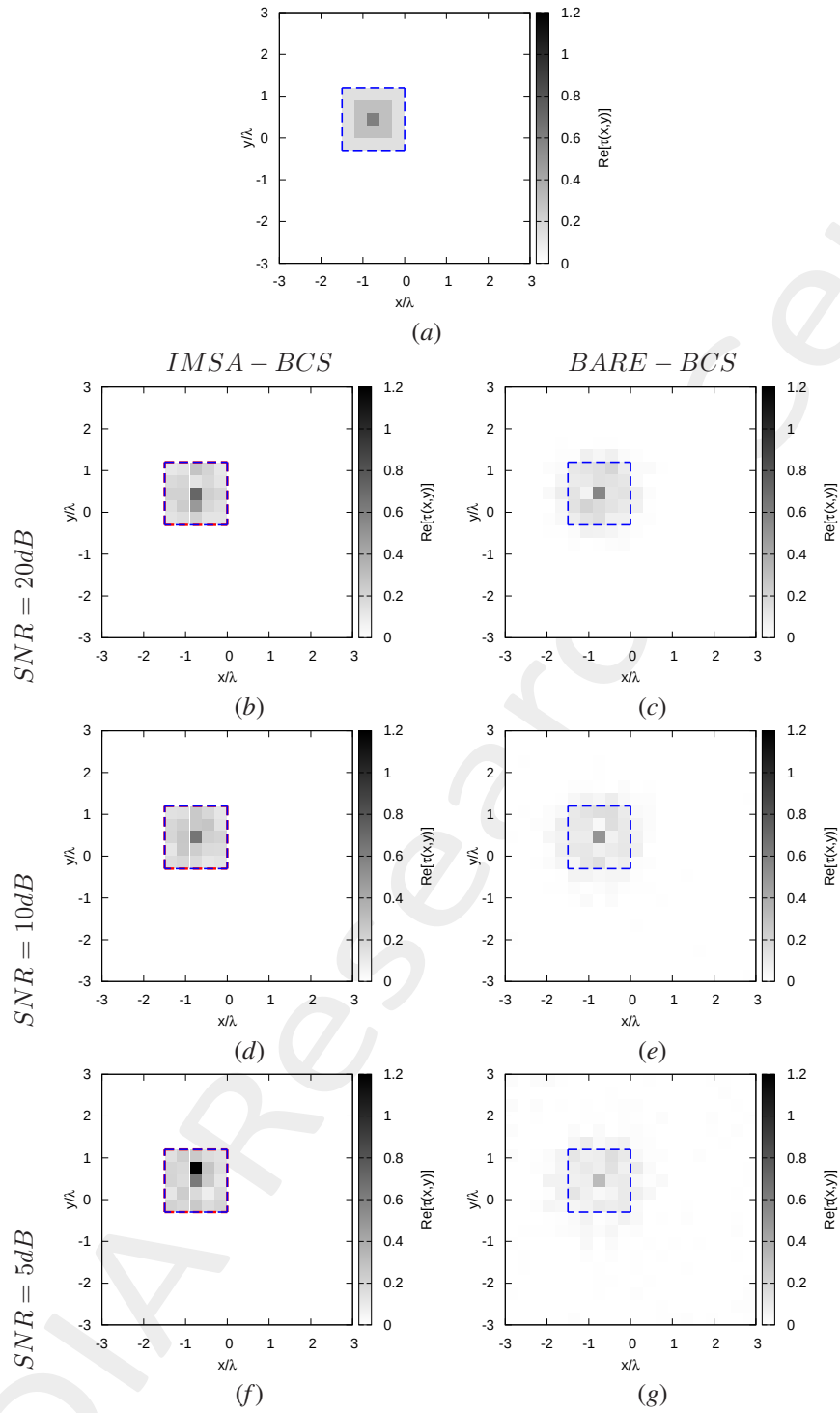


Figure 2: Inhomogeneous Square Object, $\ell = 1.5\lambda$, $\tau^{(1)} = 0.60$ - IMSA-BCS vs. BARE-BCS - (a) Actual profile, (b)(d)(f) IMSA-BCS and BARE-BCS reconstructed profiles for (b)(c) SNR = 20 [dB], (d)(e) SNR = 10 [dB] and (f)(g) SNR = 5 [dB].

	$SNR = 50dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	3.32×10^{-3}	7.98×10^{-3}
ξ_{int}	4.14×10^{-2}	5.60×10^{-2}
ξ_{ext}	0.00×10^{-1}	3.16×10^{-3}
	$SNR = 20dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	3.81×10^{-3}	8.67×10^{-3}
ξ_{int}	5.05×10^{-2}	6.12×10^{-2}
ξ_{ext}	0.00×10^{-1}	3.42×10^{-3}
	$SNR = 10dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	2.77×10^{-3}	1.21×10^{-2}
ξ_{int}	3.40×10^{-2}	7.70×10^{-2}
ξ_{ext}	0.00×10^{-1}	5.86×10^{-3}
	$SNR = 5dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	1.06×10^{-2}	1.60×10^{-2}
ξ_{int}	1.38×10^{-1}	9.21×10^{-2}
ξ_{ext}	0.00×10^{-1}	8.18×10^{-3}

Table II: *Inhomogeneous Square Object*, $\ell = 1.5\lambda$, $\tau^{(1)} = 0.60$ - *IMSA-BCS* vs. *BARE-BCS* - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

1.0.3 Inhomogeneous Square Object, $\ell = 1.5\lambda$, $\tau^{(1)} = 1.00$ - IMSA-BCS vs. BARE-BCS reconstructed profiles

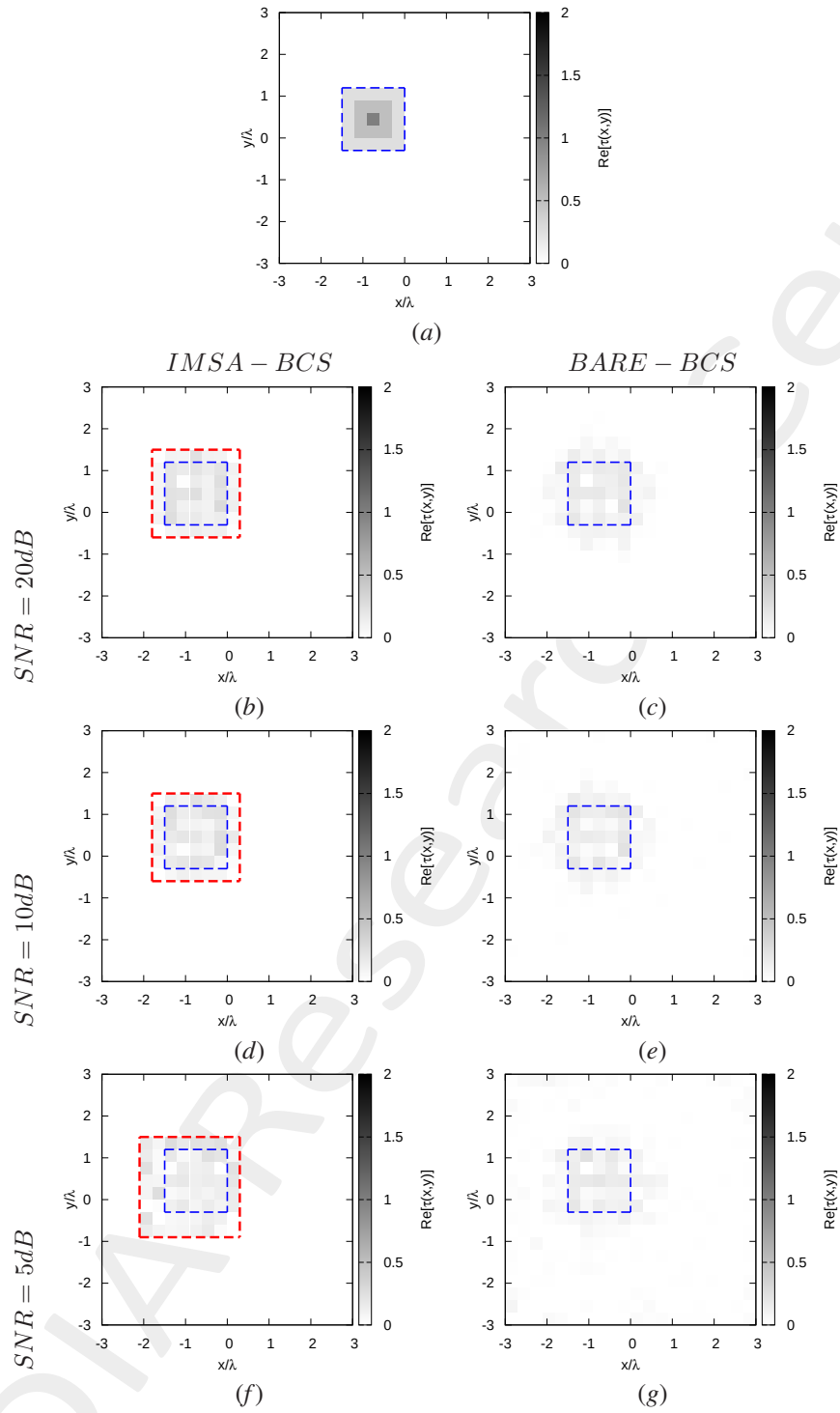


Figure 3: Inhomogeneous Square Object, $\ell = 1.5\lambda$, $\tau^{(1)} = 1.00$ - IMSA-BCS vs. BARE-BCS - (a) Actual profile, (b)(d)(f) IMSA-BCS and BARE-BCS reconstructed profiles for (b)(c) $\text{SNR} = 20$ [dB], (d)(e) $\text{SNR} = 10$ [dB] and (f)(g) $\text{SNR} = 5$ [dB].

	$SNR = 50dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	1.47×10^{-2}	1.82×10^{-2}
ξ_{int}	1.24×10^{-1}	1.51×10^{-1}
ξ_{ext}	5.63×10^{-3}	7.71×10^{-3}
	$SNR = 20dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	1.50×10^{-2}	1.92×10^{-2}
ξ_{int}	1.26×10^{-1}	1.53×10^{-1}
ξ_{ext}	5.51×10^{-3}	8.20×10^{-3}
	$SNR = 10dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	1.55×10^{-2}	2.23×10^{-2}
ξ_{int}	1.33×10^{-1}	1.56×10^{-1}
ξ_{ext}	5.45×10^{-3}	9.84×10^{-3}
	$SNR = 5dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	2.63×10^{-2}	3.12×10^{-2}
ξ_{int}	1.63×10^{-1}	1.63×10^{-1}
ξ_{ext}	1.39×10^{-2}	1.57×10^{-2}

Table III: *Inhomogeneous Square Object*, $\ell = 1.5\lambda$, $\tau^{(1)} = 1.00$ - *IMSA-BCS* vs. *BARE-BCS* - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

1.1 Punctured Rectangle

Test Case Description

Direct solver:

- Side of the investigation domain: $L = 6.0\lambda$
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 - Following Steps IMSA: $N^{(i)}$ not fixed, defined according to the estimated $RoI \mathcal{D}^{(i)}$

Measurement domain:

- Total number of measurements: $M = 60$
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Sources:

- Plane waves
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- Amplitude: $A = 1.0$
- Frequency: $F = 300$ MHz ($\lambda = 1$)

Background:

- $\varepsilon_r = 1.0$
- $\sigma = 0$ [S/m]

Scatterer

- Punctured Rectangle
- $\varepsilon_r \in \{1.05, 1.10, 1.20, 2.00\}$
- $\sigma = 0$ [S/m]

1.1.1 Punctured Rectangle, $\tau = 0.10$ - IMSA-BCS vs. BARE-BCS reconstructed profiles

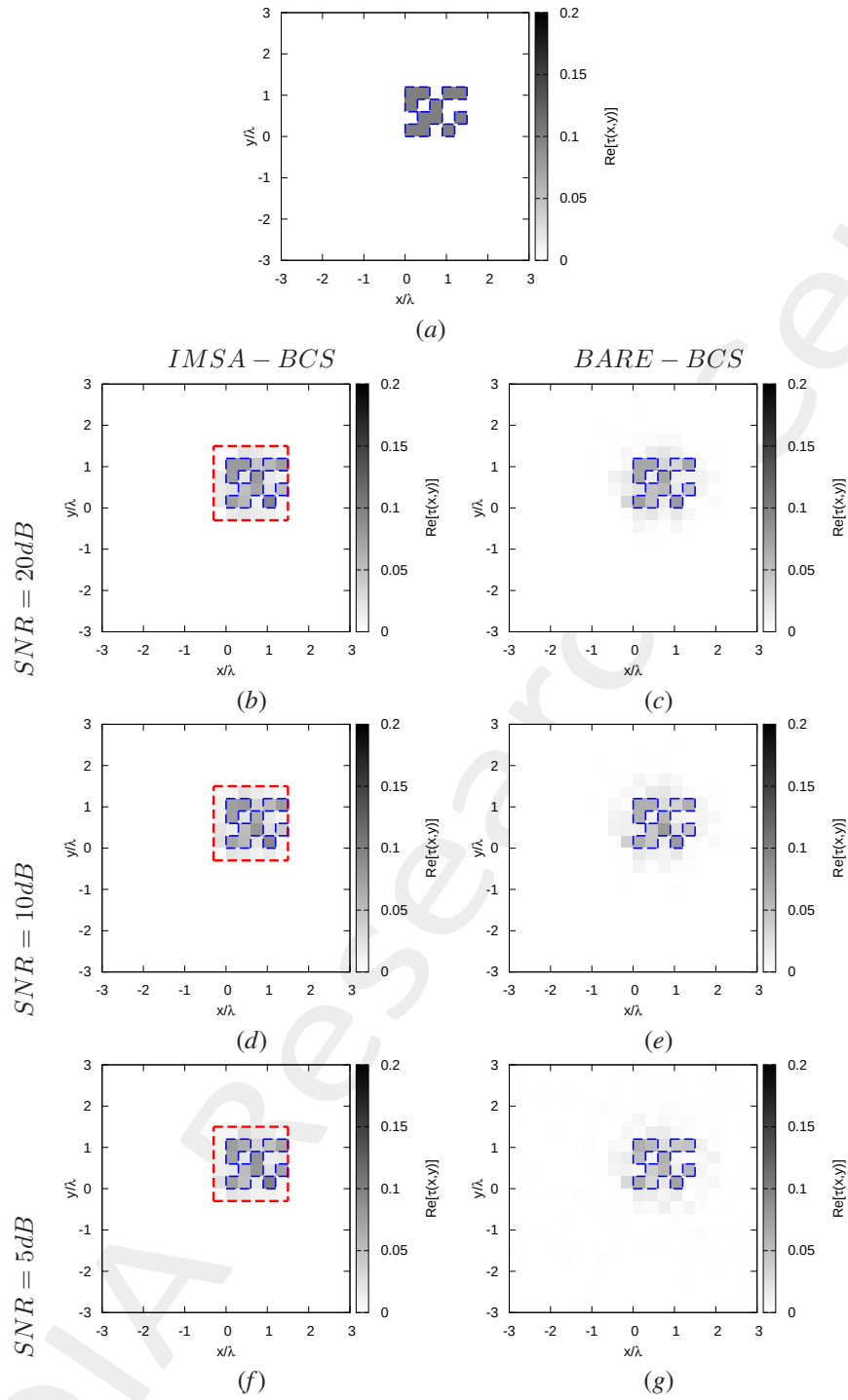


Figure 4: Punctured Rectangle, $\tau = 0.10$ - IMSA-BCS vs. BARE-BCS - (a) Actual profile, (b)(d)(f) IMSA-BCS and BARE-BCS reconstructed profiles for (b)(c) SNR = 20 [dB], (d)(e) SNR = 10 [dB] and (f)(g) SNR = 5 [dB].

	$SNR = 50dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	1.56×10^{-3}	2.19×10^{-3}
ξ_{int}	2.44×10^{-2}	3.56×10^{-2}
ξ_{ext}	8.09×10^{-4}	1.12×10^{-3}
	$SNR = 20dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	1.73×10^{-3}	2.20×10^{-3}
ξ_{int}	2.72×10^{-2}	3.54×10^{-2}
ξ_{ext}	9.17×10^{-4}	1.13×10^{-3}
	$SNR = 10dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	1.84×10^{-3}	2.41×10^{-3}
ξ_{int}	2.88×10^{-2}	3.80×10^{-2}
ξ_{ext}	9.77×10^{-4}	1.23×10^{-3}
	$SNR = 5dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	1.80×10^{-3}	3.03×10^{-3}
ξ_{int}	2.70×10^{-2}	4.26×10^{-2}
ξ_{ext}	9.71×10^{-4}	1.57×10^{-3}

Table IV: *Punctured Rectangle*, $\tau = 0.10$ - *IMSA-BCS* vs. *BARE-BCS* - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

1.1.2 Punctured Rectangle, $\tau = 0.20$ - IMSA-BCS vs. BARE-BCS reconstructed profiles

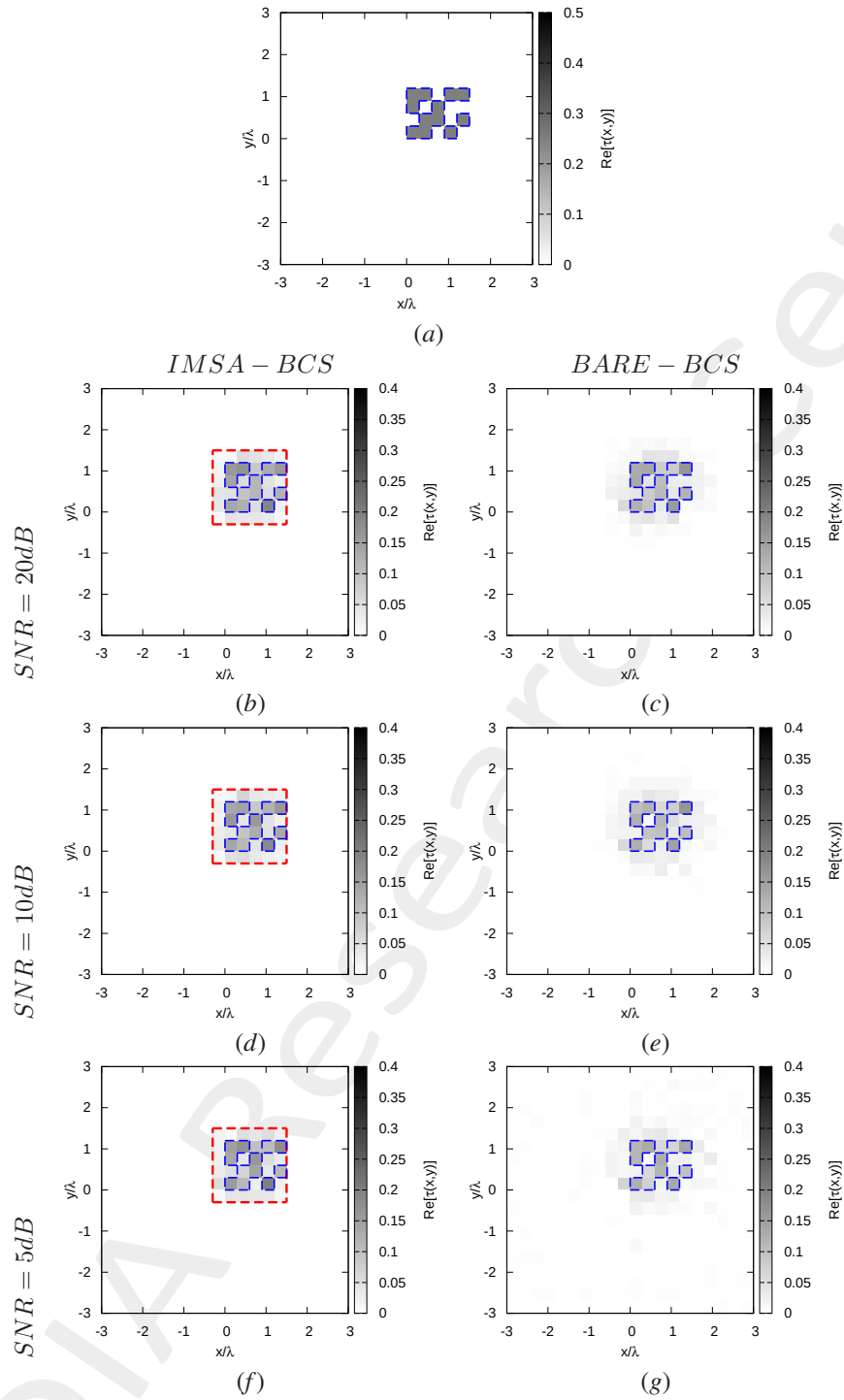


Figure 5: Punctured Rectangle, $\tau = 0.20$ - IMSA-BCS vs. BARE-BCS - (a) Actual profile, (b)(d)(f) IMSA-BCS and BARE-BCS reconstructed profiles for (b)(c) $\text{SNR} = 20$ [dB], (d)(e) $\text{SNR} = 10$ [dB] and (f)(g) $\text{SNR} = 5$ [dB].

	$SNR = 50dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	3.81×10^{-3}	4.61×10^{-3}
ξ_{int}	5.15×10^{-2}	6.38×10^{-2}
ξ_{ext}	2.25×10^{-3}	2.59×10^{-3}
	$SNR = 20dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	3.83×10^{-3}	4.81×10^{-3}
ξ_{int}	5.19×10^{-2}	6.68×10^{-2}
ξ_{ext}	2.21×10^{-3}	2.71×10^{-3}
	$SNR = 10dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	3.73×10^{-3}	5.78×10^{-3}
ξ_{int}	5.08×10^{-2}	7.47×10^{-2}
ξ_{ext}	2.18×10^{-3}	3.24×10^{-3}
	$SNR = 5dB$	
	$IMSA - BCS$	$BARE - BCS$
ξ_{tot}	3.81×10^{-3}	6.84×10^{-3}
ξ_{int}	4.94×10^{-2}	7.63×10^{-2}
ξ_{ext}	2.25×10^{-3}	3.79×10^{-3}

Table V: *Punctured Rectangle*, $\tau = 0.20$ - *IMSA-BCS* vs. *BARE-BCS* - Reconstruction errors: total (ξ_{tot}), internal (ξ_{int}) and external (ξ_{ext}) errors.

More information on the topics of this document can be found in the following list of references.

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