Investigation of Tolerance in Monopulse Linear Antenna Arrays with Interval Arithmetic

N. Anselmi, P. Rocca, and A. Massa

2024/12/13

Contents

1 Patterns Features Comparisons - Tolerance Over Common Elements

SLL:

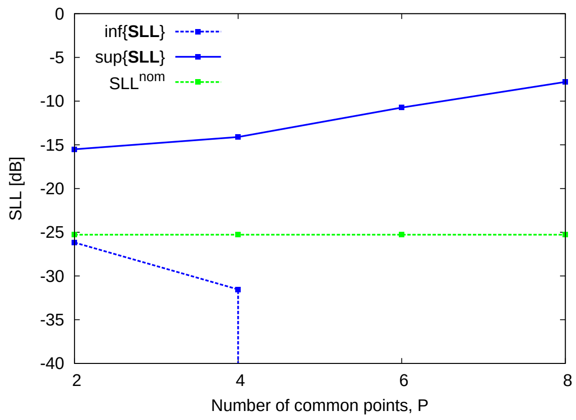

Figure 1. Sum Pattern **SLL** vs P

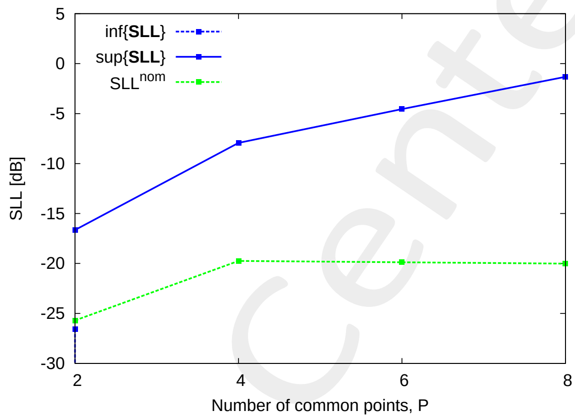

Figure 2. Difference Pattern **SLL** vs P

P	2	4	6	8
nominal	-25.28	-25.28	-25.28	-25.28
inf	-26.17	-31.54	$-\infty$	$-\infty$
sup	-15.51	-14.1	-10.7	-7.8

Table 1. Sum Pattern **SLL** values

P	2	4	6	8
nominal	-25.7	-19.7	-19.9	-20.0
inf	-25.56	$-\infty$	$-\infty$	$-\infty$
sup	-16.65	-7.9	-4.6	-1.3

Table 2. Difference Pattern **SLL** values

BW:

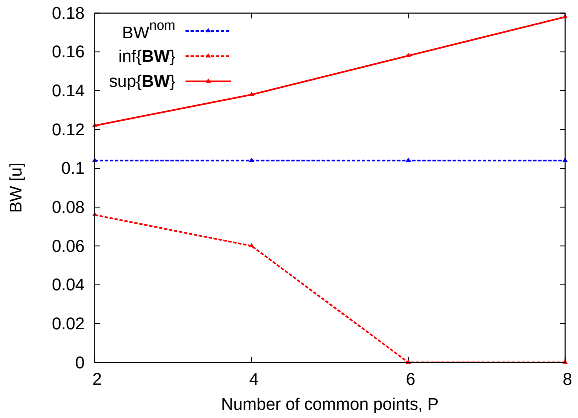

Figure 3. Sum Pattern $\mathbf{BW}$ vs P

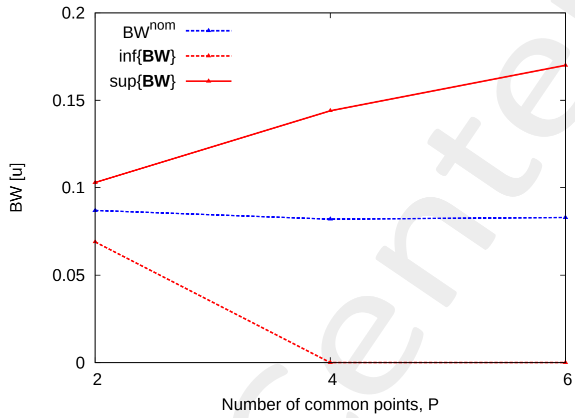

Figure 4. Difference Pattern $\mathbf{BW}$ vs P

P	2	4	6	8
nominal	0.104	0.104	0.104	0.104
inf	0.076	0.06	0.0	0.0
sup	0.122	0.138	0.158	0.178

Table 3. Sum Pattern $\mathbf{BW}$ values

P	2	4	6	8
nominal	0.087	0.082	0.083	0.083
inf	0.069	0.0	0.0	0.0
sup	0.103	0.144	0.17	4.0

Table 4. Difference Pattern $\mathbf{BW}$ vs P

Directivity / Slope:

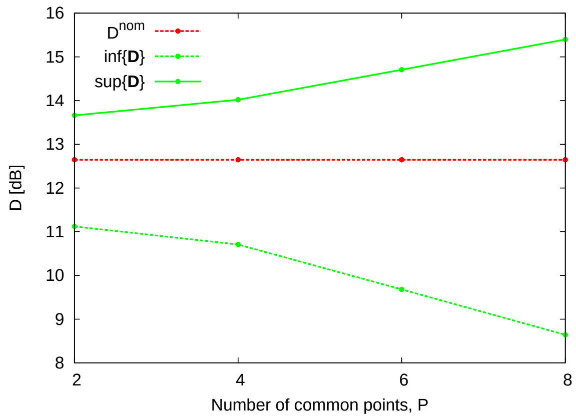

Figure 5. Sum Pattern **D** vs P

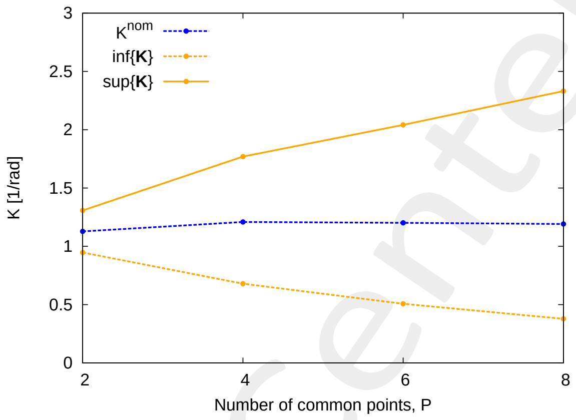

Figure 6. Difference Pattern **K** vs P

P	2	4	6	8
nominal	12.65	12.65	12.65	12.65
inf	11.12	10.71	9.68	8.64
sup	13.66	14.02	14.71	15.4

Table 5. Sum Pattern **D** values

P	2	4	6	8
nominal	1.1281	1.2084	1.2011	1.1912
inf	0.9469	0.6795	0.5069	0.3776
sup	1.3077	1.7695	2.0416	2.3317

Table 6. Difference Pattern **K** values

Pattern Tolerance:

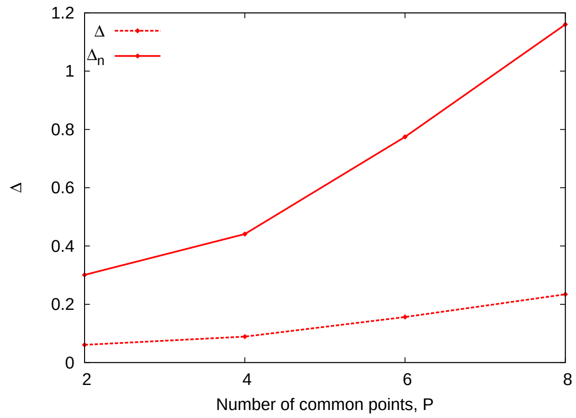

Figure 8. Sum Pattern Δ and Δ_n vs P

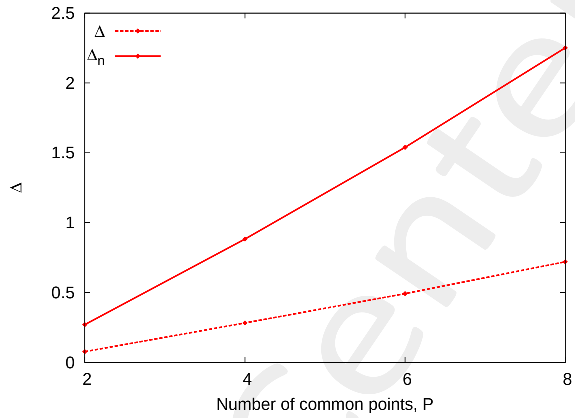

Figure 9. Difference Pattern Δ and Δ_n vs P

P	2	4	6	8
Δ	0.0607	0.089	0.1564	0.2342
Δ_n	0.3008	0.441	0.7746	1.1603

Table 8. Sum Pattern Δ and Δ_n values

P	2	4	6	8
Δ	0.0772	0.2825	0.4919	0.72
Δ_n	0.2703	0.8826	1.5388	2.251

Table 9. Difference Pattern Δ and Δ_n values

Pareto Fronts - Tolerance Over Common Elements

In the following figures the quantity $\phi_{\max}$ versus the performances descriptors are plotted.

SLL:

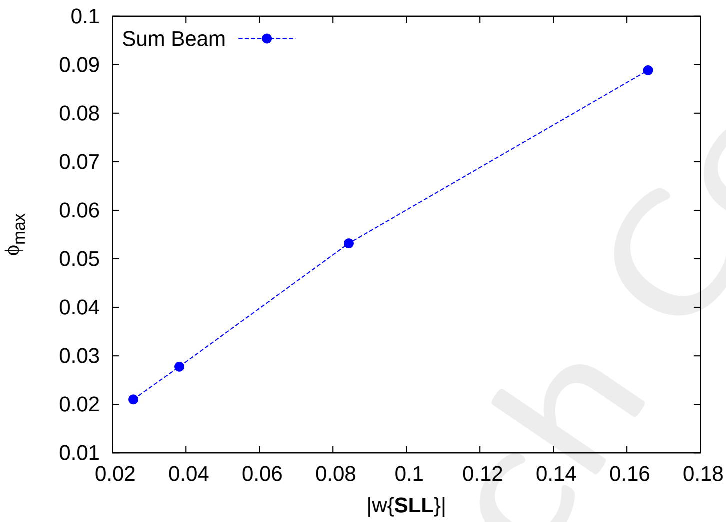

Figure 10. Sum Pattern $|w\{\mathbf{SLL}\}|$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.021	0.0278	0.0532	0.0889
$\lvert w\{\mathbf{SLL}\}\rvert$	0.0257	0.0382	0.0843	0.1658

Table 10. Sum Pattern $|w\{\mathbf{SLL}\}|$ and $\phi_{\max}$ values.

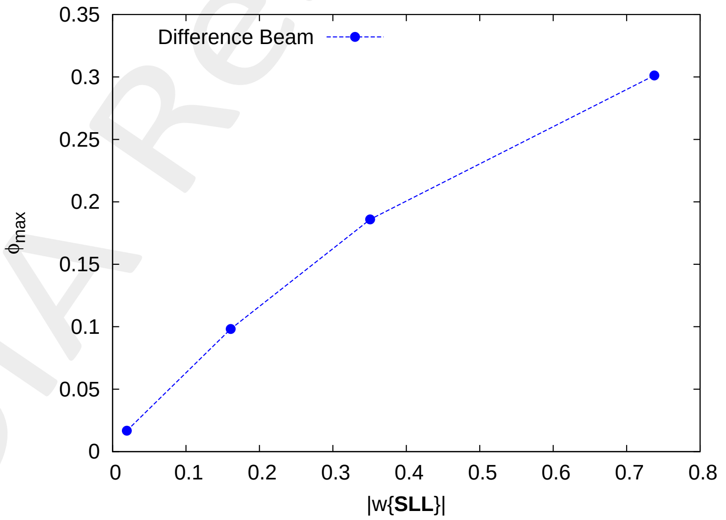

Figure 11. Difference Pattern $|w\{\mathbf{SLL}\}|$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0167	0.0982	0.1859	0.3012
$\lvert w\{\mathbf{SLL}\}\rvert$	0.0194	0.1609	0.3506	0.7378

Table 11. Difference Pattern $|w\{\mathbf{SLL}\}|$ and $\phi_{\max}$ values.

BW:

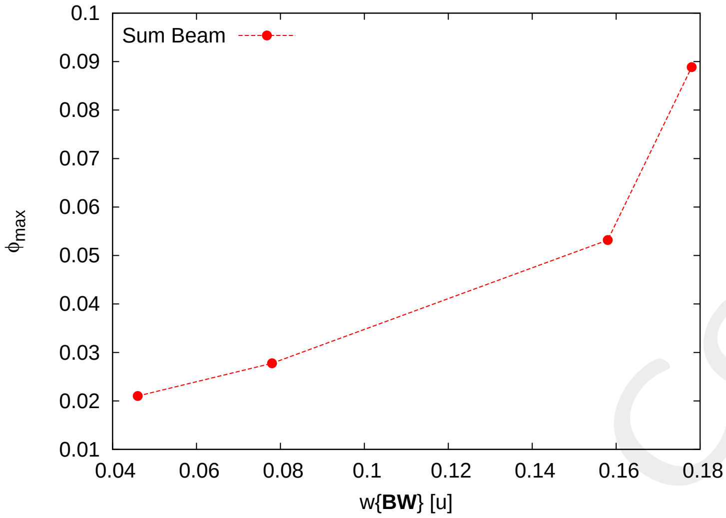

Figure 12. Sum Pattern $w\{\mathbf{BW}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.021	0.0278	0.0532	0.0889
$w\{\mathbf{BW}\}$	0.046	0.078	0.158	0.178

Table 12. Sum Pattern $w\{\mathbf{BW}\}$ and $\phi_{\max}$ values.

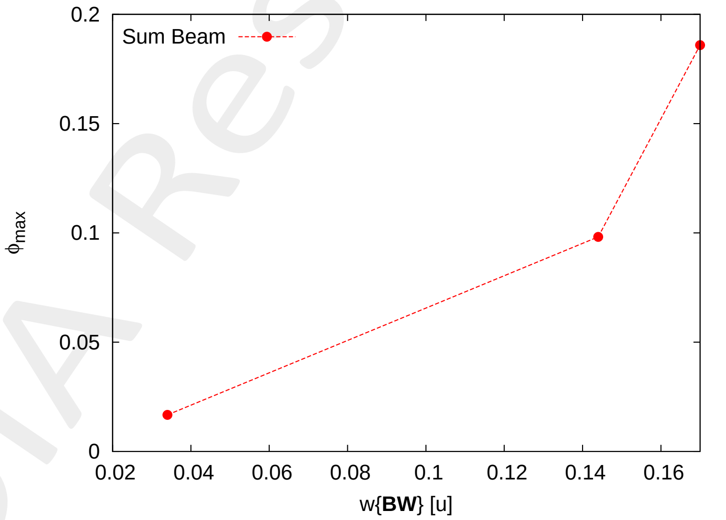

Figure 13. Difference Pattern $w\{\mathbf{BW}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0167	0.0982	0.1859	0.3012
$w\{\mathbf{BW}\}$	0.0434	0.144	0.17	4.0

Table 13. Difference Pattern $w\{\mathbf{BW}\}$ and $\phi_{\max}$ values.

Directivity:

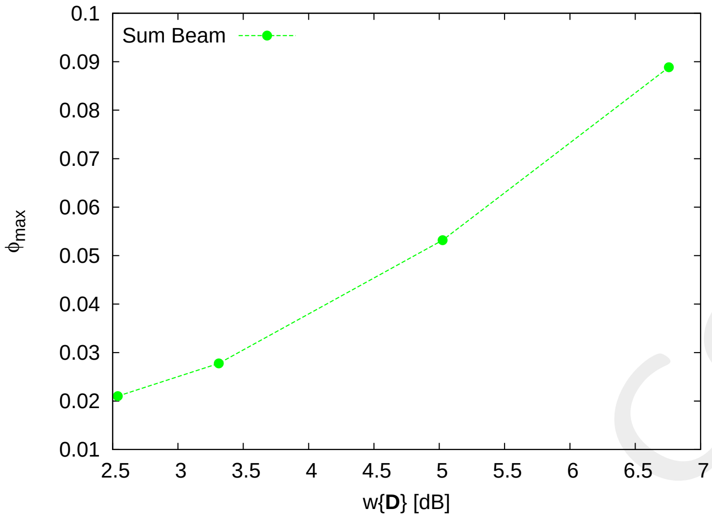

Figure 14. Sum Pattern $w\{\mathbf{D}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.021	0.0278	0.0532	0.0889
$w\{\mathbf{D}\}$	2.54	3.31	5.03	6.76

Table 14. Sum Pattern $w\{\mathbf{D}\}$ and $\phi_{\max}$ values.

Slope:

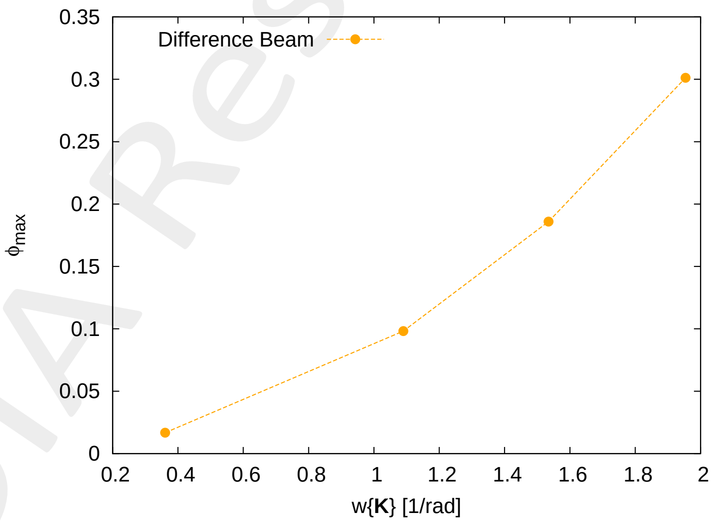

Figure 15. Difference Pattern $w\{\mathbf{K}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0167	0.0982	0.1859	0.3012
$w\{\mathbf{K}\}$	0.36	1.09	1.53	1.95

Table 15. Difference Pattern $w\{\mathbf{K}\}$ and $\phi_{\max}$ values.

2 Patterns Features Comparisons - Tolerance Over Not-Common Elements

SLL:

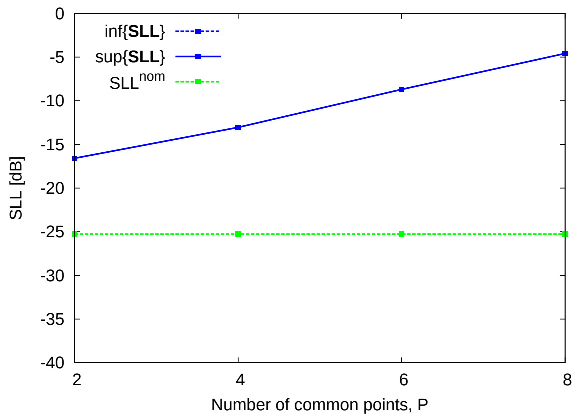

Figure 1. Sum Pattern **SLL** vs P

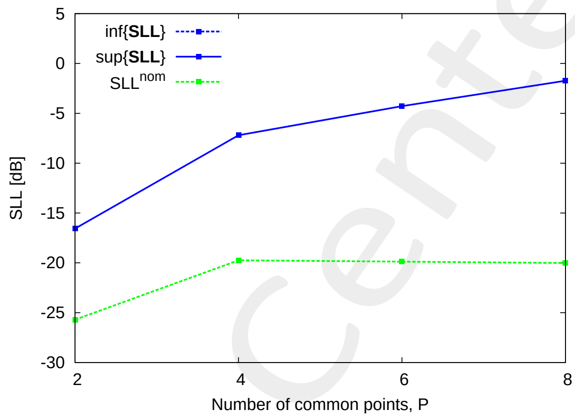

Figure 2. Difference Pattern **SLL** vs P

P	2	4	6	8
nominal	-25.28	-25.28	-25.28	-25.28
inf	-52.95	$-\infty$	$-\infty$	$-\infty$
sup	-16.581	-13.04	-8.7	-4.59

Table 1. Sum Pattern **SLL** values

P	2	4	6	8
nominal	-25.7	-19.7	-19.9	-20.0
inf	-48.53	$-\infty$	$-\infty$	$-\infty$
sup	-16.56	-7.19	-4.28	-1.73

Table 2. Difference Pattern **SLL** values

BW:

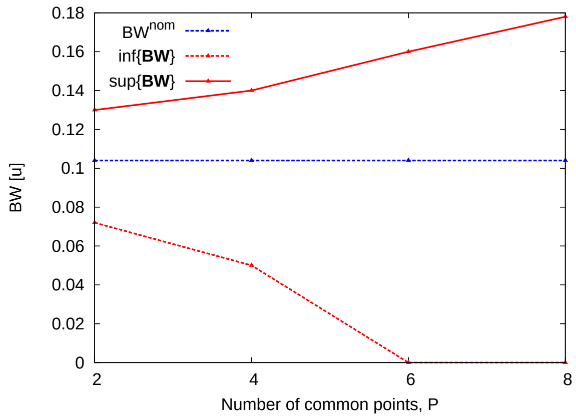

Figure 3. Sum Pattern $\mathbf{BW}$ vs P

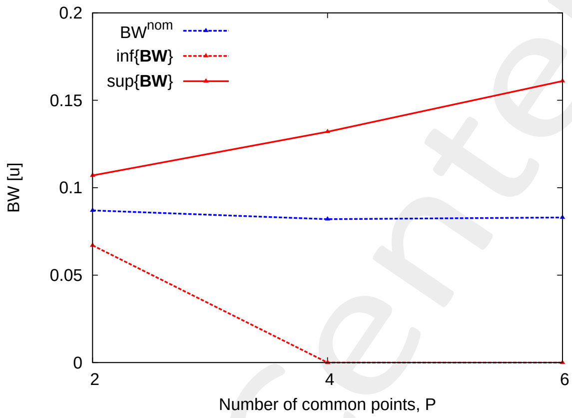

Figure 4. Difference Pattern $\mathbf{BW}$ vs P

P	2	4	6	8
nominal	0.104	0.104	0.104	0.104
inf	0.072	0.05	0.0	0.0
sup	0.13	0.14	0.16	0.178

Table 3. Sum Pattern $\mathbf{BW}$ values

P	2	4	6	8
nominal	0.087	0.082	0.083	0.083
inf	0.067	0.0	0.0	0.0
sup	0.107	0.132	0.161	0.428

Table 4. Difference Pattern $\mathbf{BW}$ vs P

Directivity / Slope:

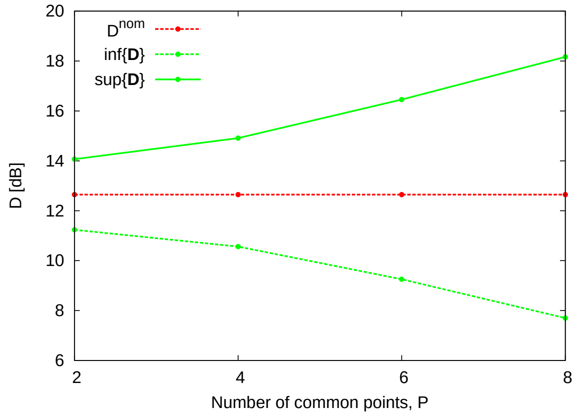

Figure 5. Sum Pattern **D** vs P

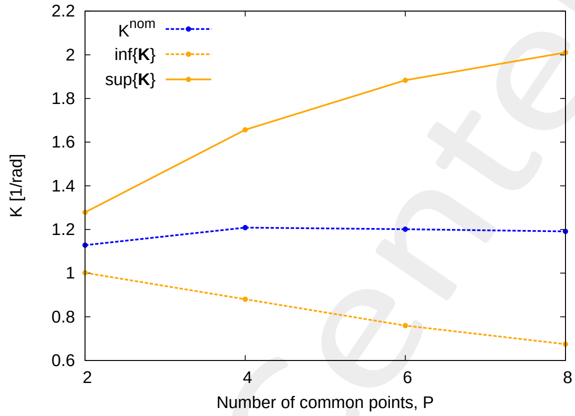

Figure 6. Difference Pattern **K** vs P

P	2	4	6	8
nominal	12.65	12.65	12.65	12.65
inf	11.23	10.56	9.25	7.7
sup	14.07	14.91	16.46	18.17

Table 5. Sum Pattern **D** values

P	2	4	6	8
nominal	1.1281	1.2084	1.2011	1.1912
inf	1.0016	0.8803	0.7597	0.6745
sup	1.2786	1.6566	1.8833	2.0105

Table 6. Difference Pattern **K** values

Pattern Tolerance:

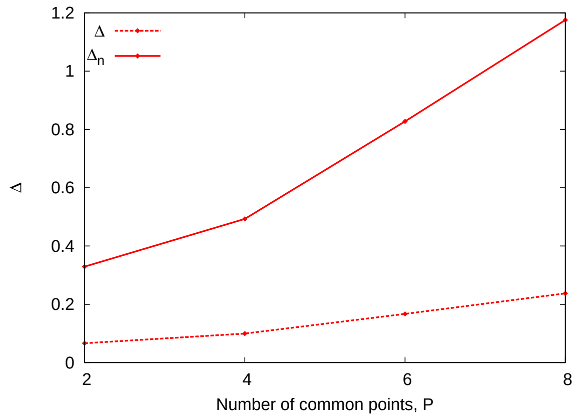

Figure 8. Sum Pattern Δ and Δ_n vs P

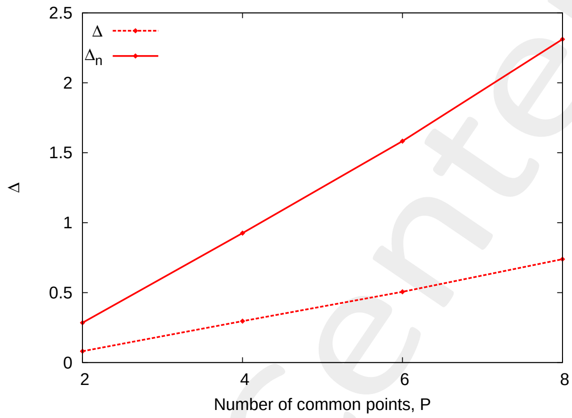

Figure 9. Difference Pattern Δ and Δ_n vs P

P	2	4	6	8
Δ	0.0664	0.0995	0.1671	0.2373
Δ_n	0.3291	0.4931	0.8275	1.1754

Table 8. Sum Pattern Δ and Δ_n values

P	2	4	6	8
Δ	0.0813	0.2962	0.506	0.7393
Δ_n	0.2846	0.9254	1.5831	2.3112

Table 9. Difference Pattern Δ and Δ_n values

Pareto Fronts - Tolerance Over Not-Common Elements

In the following figures the quantity $\phi_{\max}$ versus the performances descriptors are plotted.

SLL:

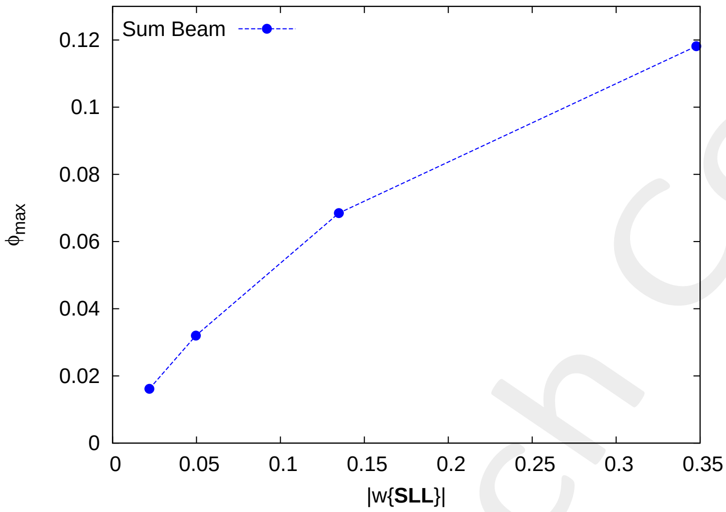

Figure 10. Sum Pattern $|w\{\mathbf{SLL}\}|$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0162	0.032	0.0685	0.1182
$\lvert w\{\mathbf{SLL}\}\rvert$	0.0219	0.0496	0.1348	0.3478

Table 10. Sum Pattern $|w\{\mathbf{SLL}\}|$ and $\phi_{\max}$ values.

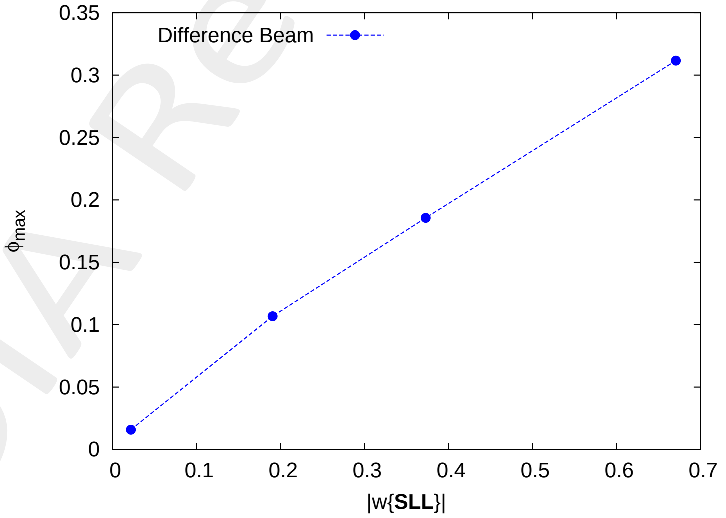

Figure 11. Difference Pattern $|w\{\mathbf{SLL}\}|$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0158	0.1068	0.1856	0.3116
$\lvert w\{\mathbf{SLL}\}\rvert$	0.0221	0.1907	0.373	0.6709

Table 11. Difference Pattern $|w\{\mathbf{SLL}\}|$ and $\phi_{\max}$ values.

BW:

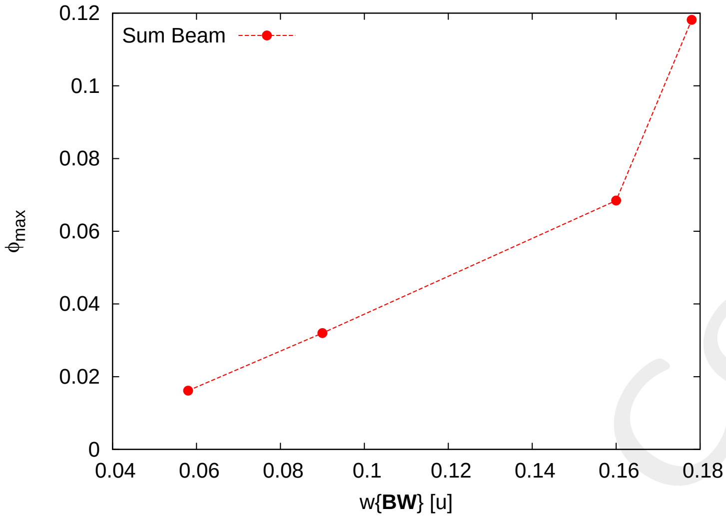

Figure 12. Sum Pattern $w\{\mathbf{BW}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0162	0.032	0.0685	0.1182
$w\{\mathbf{BW}\}$	0.058	0.09	0.16	0.178

Table 12. Sum Pattern $w\{\mathbf{BW}\}$ and $\phi_{\max}$ values.

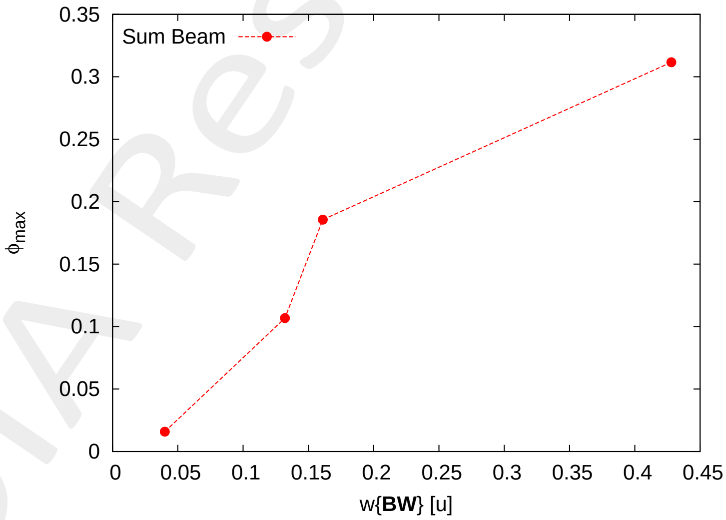

Figure 13. Difference Pattern $w\{\mathbf{BW}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0158	0.1068	0.1856	0.3116
$w\{\mathbf{BW}\}$	0.04	0.132	0.161	0.428

Table 13. Difference Pattern $w\{\mathbf{BW}\}$ and $\phi_{\max}$ values.

Directivity:

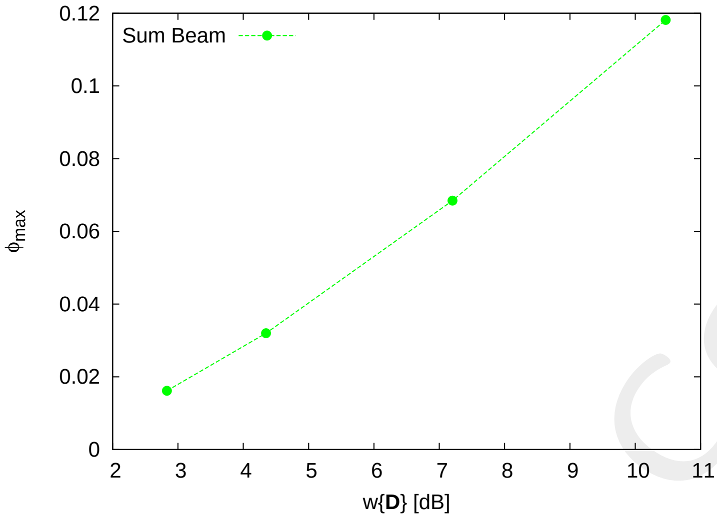

Figure 14. Sum Pattern $w\{\mathbf{D}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0162	0.032	0.0685	0.1182
$w\{\mathbf{D}\}$	2.83	4.35	7.21	10.47

Table 14. Sum Pattern $w\{\mathbf{D}\}$ and $\phi_{\max}$ values.

Slope:

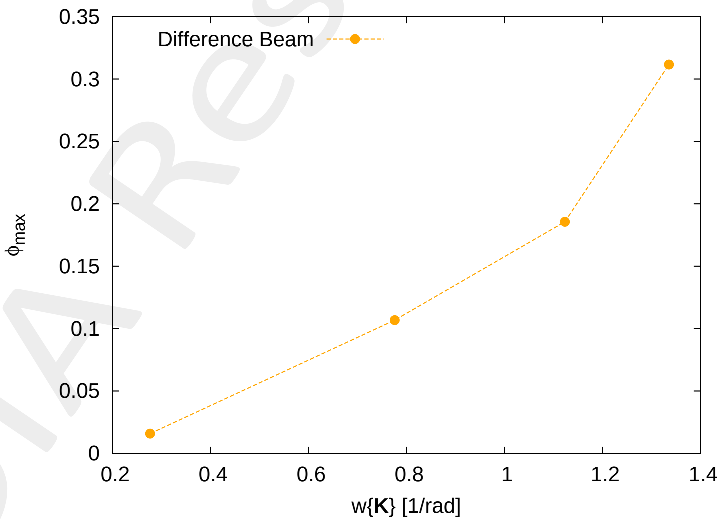

Figure 15. Difference Pattern $w\{\mathbf{K}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0158	0.1068	0.1856	0.3116
$w\{\mathbf{K}\}$	0.26	0.78	1.12	1.34

Table 15. Difference Pattern $w\{\mathbf{K}\}$ and $\phi_{\max}$ values.

3 Resume

In the following pictures the bounds of the antenna arrays descriptors are compared, considering the amplitude tolerance over the common elements (es. $\sup\{\mathbf{SLL}\}$) and over the not-common elements (es. $\sup\{\mathbf{SLL}\}*$).

SLL:

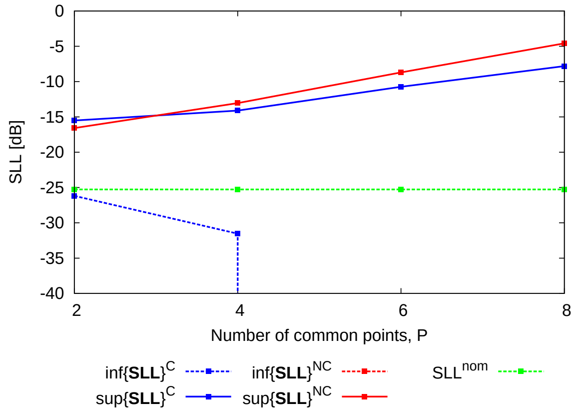

Figure 1. Sum Pattern $\mathbf{SLL}$ vs P

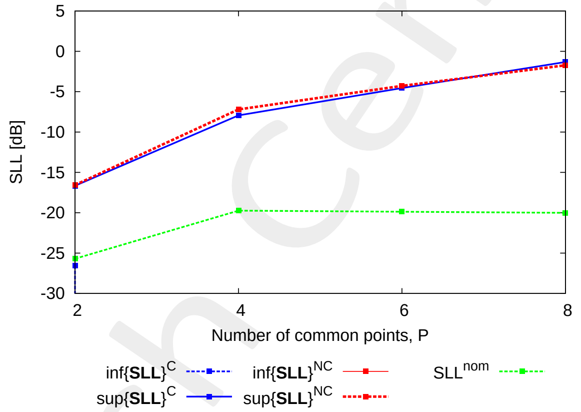

Figure 2. Difference Pattern $\mathbf{SLL}$ vs P

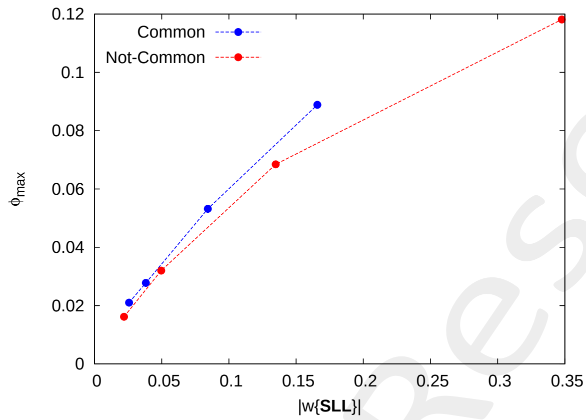

Figure 3. Sum Pattern $|w\{\mathbf{SLL}\}|$ vs $\phi_{\max}$.

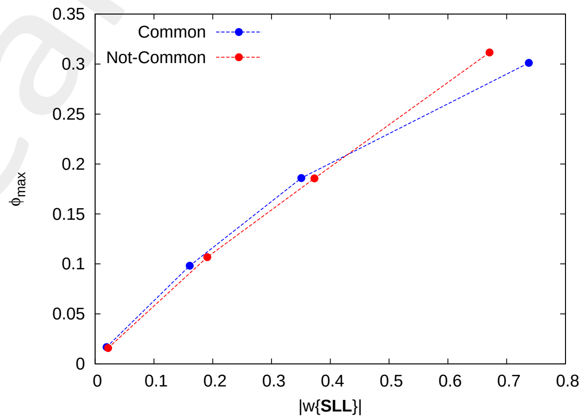

Figure 4. Difference Pattern $|w\{\mathbf{SLL}\}|$ vs $\phi_{\max}$.

BW:

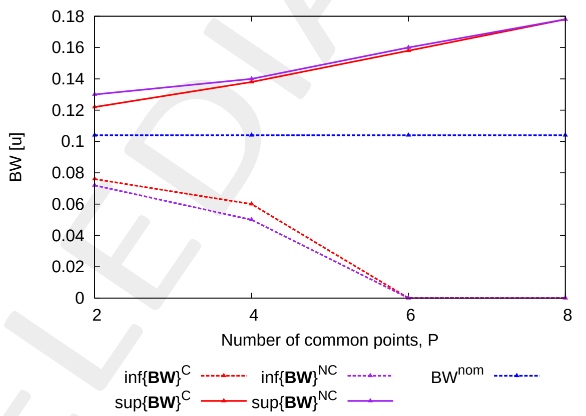

Figure 5. Sum Pattern $\mathbf{BW}$ vs P

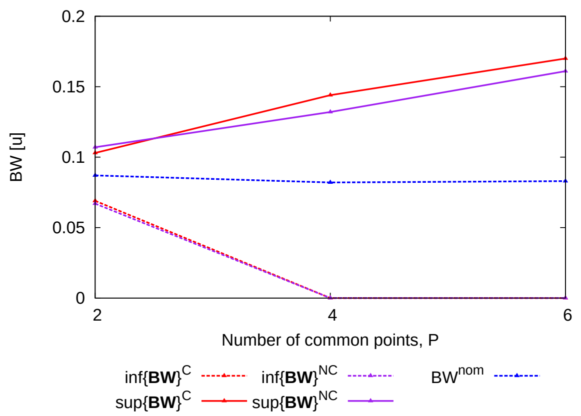

Figure 6. Difference Pattern $\mathbf{BW}$ vs P

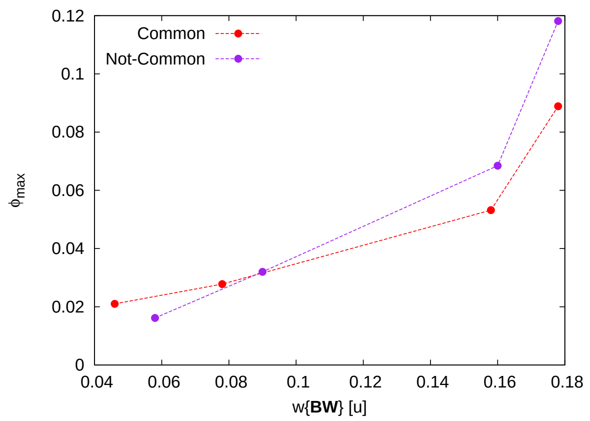

Figure 7. Sum Pattern $w\{\mathbf{BW}\}$ vs $\phi_{\max}$.

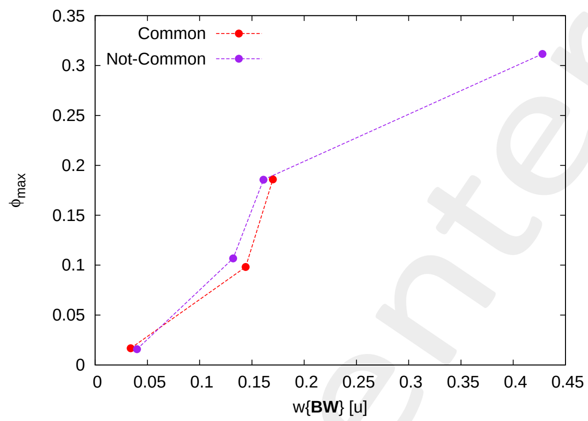

Figure 8. Difference Pattern $w\{\mathbf{BW}\}$ vs $\phi_{\max}$.

Directivity / Slope:

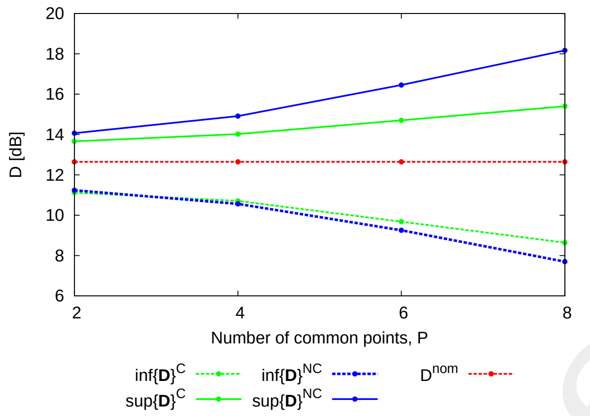

Figure 9. Sum Pattern $\mathbf{D}$ vs P

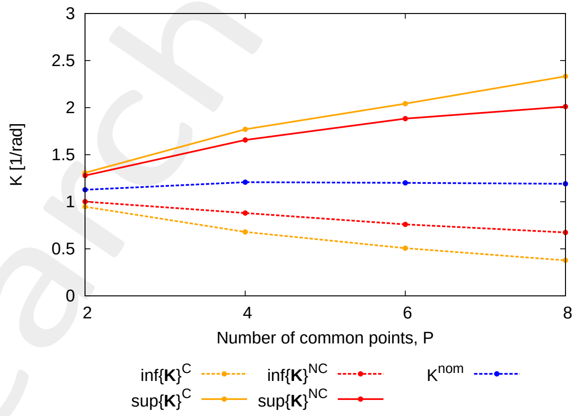

Figure 10. Difference Pattern $\mathbf{K}$ vs P

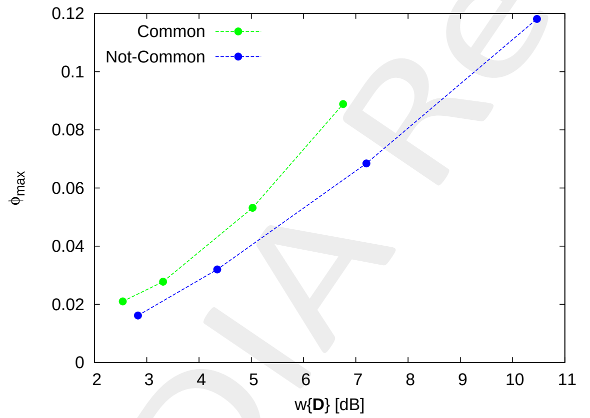

Figure 11. Sum Pattern $w\{\mathbf{D}\}$ vs $\phi_{\max}$.

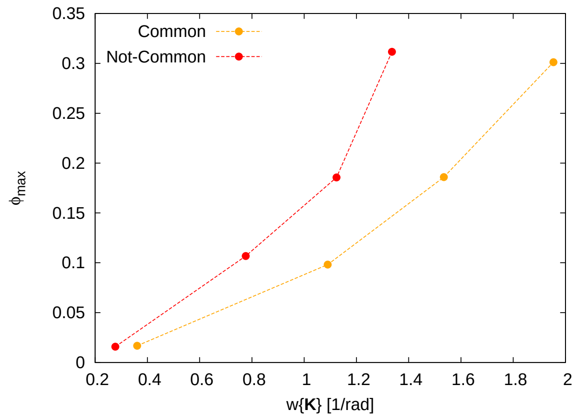

Figure 12. Difference Pattern $w\{\mathbf{K}\}$ vs $\phi_{\max}$.

Pattern Tolerance:

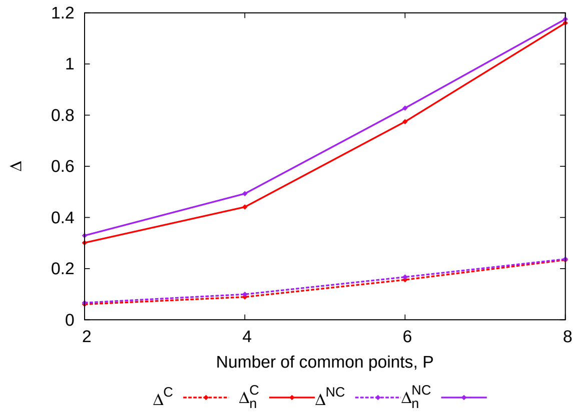

Figure 13. Sum Pattern Δ and Δ_n vs P

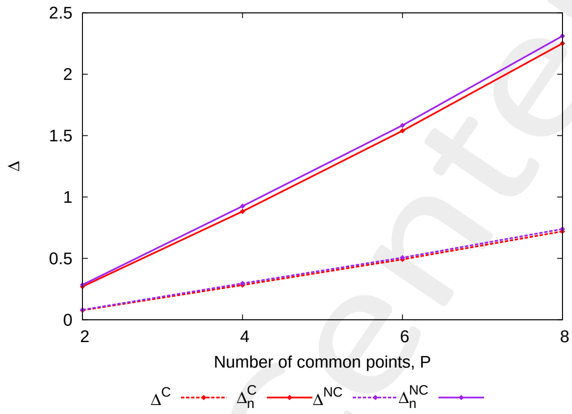

Figure 14. Difference Pattern Δ and Δ_n vs P

More information on the topics of this document can be found in the following list of references.

References

[1] G. Ding, N. Anselmi, W. Xu, P. Li, and P. Rocca, "Interval-bounded optimal power pattern synthesis of array antenna excitations robust to mutual coupling," *IEEE Antennas Wireless Propag. Lett.*, vol. 22, no. 11, pp. 2725-2729, Nov. 2023 (DOI: 10.1109/LAWP.2023.3291428).

[2] N. Anselmi, P. Rocca, and A. Massa, "Tolerance analysis of reconfigurable monopulse linear antenna arrays through interval arithmetic," *J. Electromagn. Waves Appl. J*, pp. 1066-1081, 2023 (DOI: 10.1080/09205071.2023.2224080).

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