A Study on Tolerance in Reconfigurable Linear Antenna Arrays via Interval Arithmetic

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Contents

1 Common / Not-common Faulty Elements Robustness Analysis

1.1 Patterns Features Analysis - Tolerance Over Common Elements

SLL:

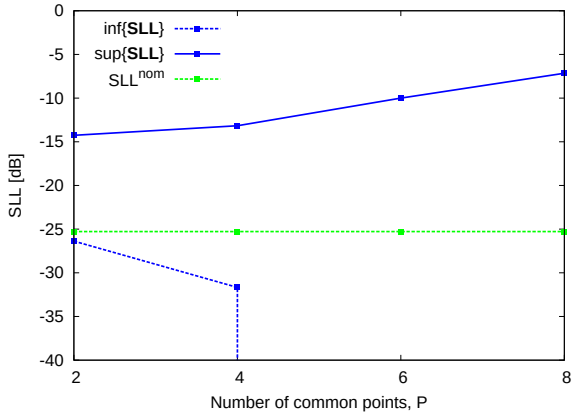

Figure 1. Sum Pattern **SLL** vs P

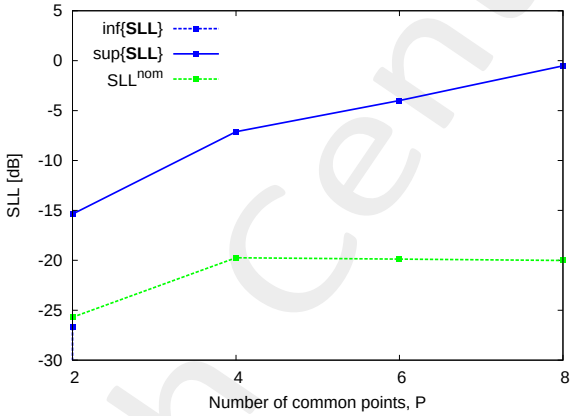

Figure 2. Difference Pattern **SLL** vs P

P	2	4	6	8
nominal	-25.28	-25.28	-25.28	-25.28
inf	-26.37	-31.68	$-\infty$	$-\infty$
sup	-14.26	-13.16	-10.0	-7.15

Table 1. Sum Pattern **SLL** values

P	2	4	6	8
nominal	-25.7	-19.7	-19.8	-20.0
inf	-26.7	$-\infty$	$-\infty$	$-\infty$
sup	-15.4	-7.1	-4.0	-0.52

Table 2. Difference Pattern **SLL** values

BW:

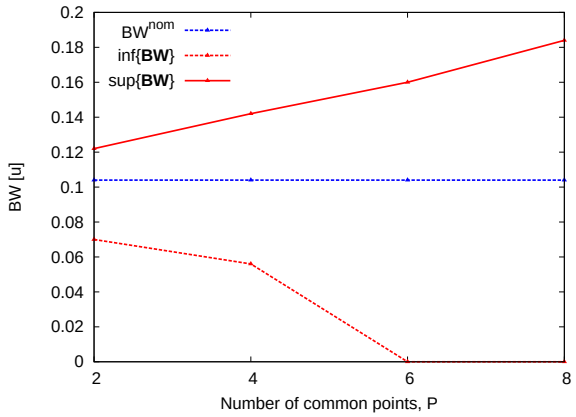

Figure 3. Sum Pattern $\mathbf{BW}$ vs P

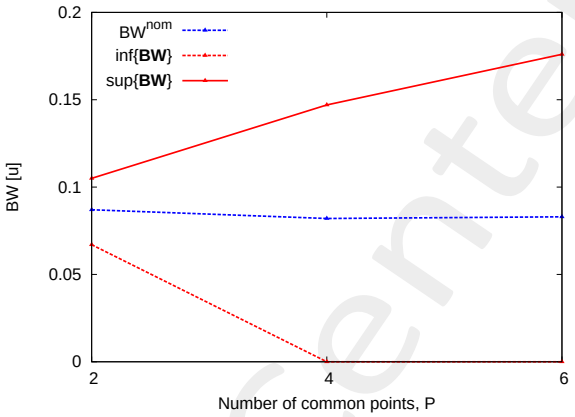

Figure 4. Difference Pattern $\mathbf{BW}$ vs P

P	2	4	6	8
nominal	0.104	0.104	0.104	0.104
inf	0.07	0.056	0.0	0.0
sup	0.122	0.142	0.16	0.184

Table 3. Sum Pattern $\mathbf{BW}$ values

P	2	4	6	8
nominal	0.087	0.082	0.083	0.083
inf	0.067	0.0	0.0	0.0
sup	0.105	0.147	0.176	4.0

Table 4. Difference Pattern $\mathbf{BW}$ vs P

Directivity / Slope:

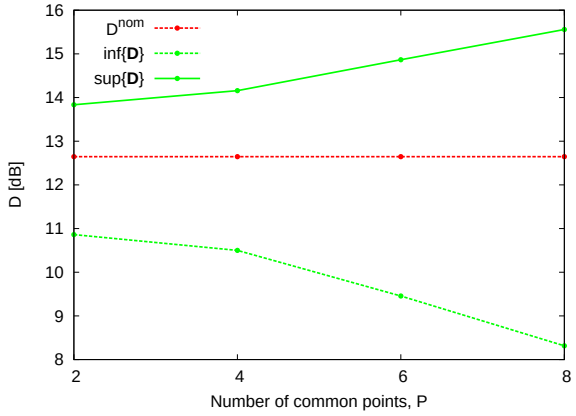

Figure 5. Sum Pattern $\mathbf{D}$ vs P

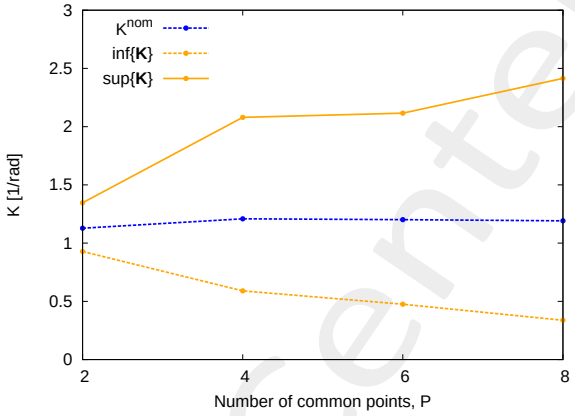

Figure 6. Difference Pattern $\mathbf{K}$ vs P

P	2	4	6	8
nominal	12.65	12.65	12.65	12.65
inf	10.86	10.5	9.46	8.31
sup	13.83	14.16	14.87	15.56

Table 5. Sum Pattern $\mathbf{D}$ values

P	2	4	6	8
nominal	1.1281	1.2084	1.2011	1.1912
inf	0.9282	0.59	0.4753	0.3378
sup	1.3464	2.08	2.1159	2.4143

Table 6. Difference Pattern $\mathbf{K}$ values

Pattern Tolerance:

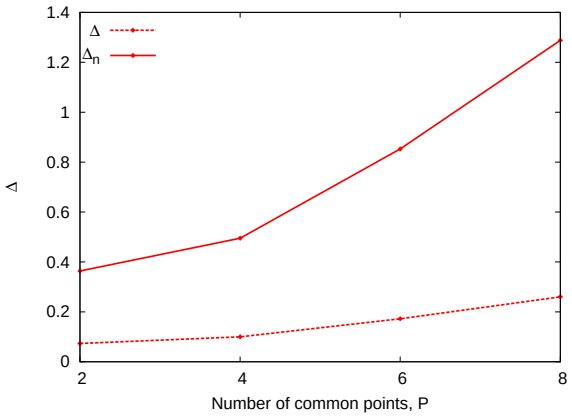

Figure 8. Sum Pattern Δ and Δ_n vs P

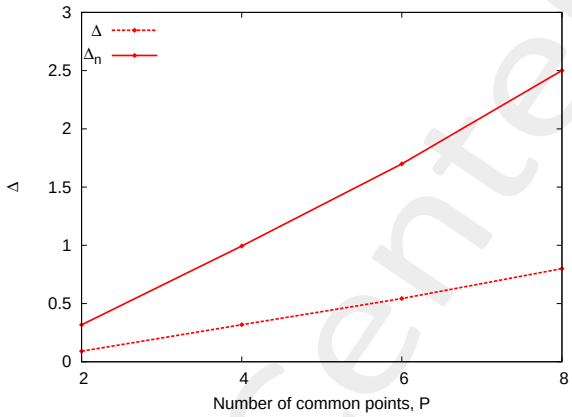

Figure 9. Difference Pattern Δ and Δ_n vs P

P	2	4	6	8
Δ	0.0734	0.1	0.1722	0.26
Δ_n	0.3638	0.4951	0.8529	1.288

Table 8. Sum Pattern Δ and Δ_n values

P	2	4	6	8
Δ	0.0907	0.3180	0.5431	0.7999
Δ_n	0.3173	1.9936	1.6989	2.5009

Table 9. Difference Pattern Δ and Δ_n values

Pareto Fronts - Tolerance Over Common Elements

In the following figures the quantity $\phi_{\max}$ versus the performances descriptors are plotted.

SLL:

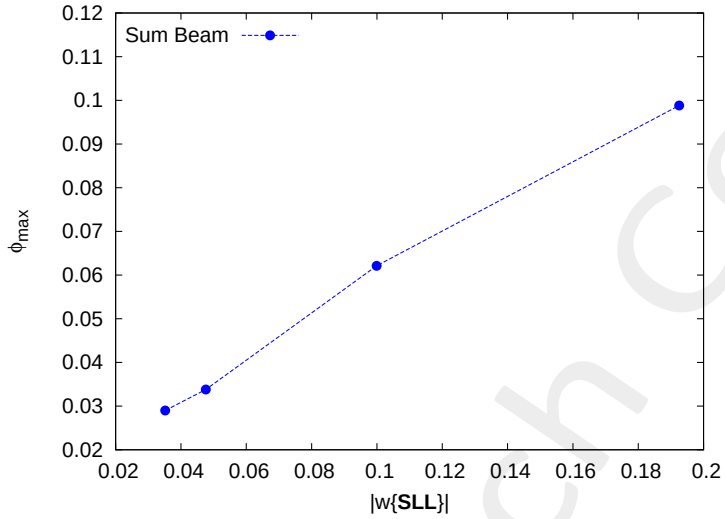

Figure 10. Sum Pattern $|w\{\mathbf{SLL}\}|$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0141	0.0338	0.0621	0.0988
$\lvert w\{\mathbf{SLL}\}\rvert$	0.0352	0.0476	0.0999	0.1926

Table 10. Sum Pattern $|\sup\{\mathbf{SLL}\}|$ and $\phi_{\max}$ values.

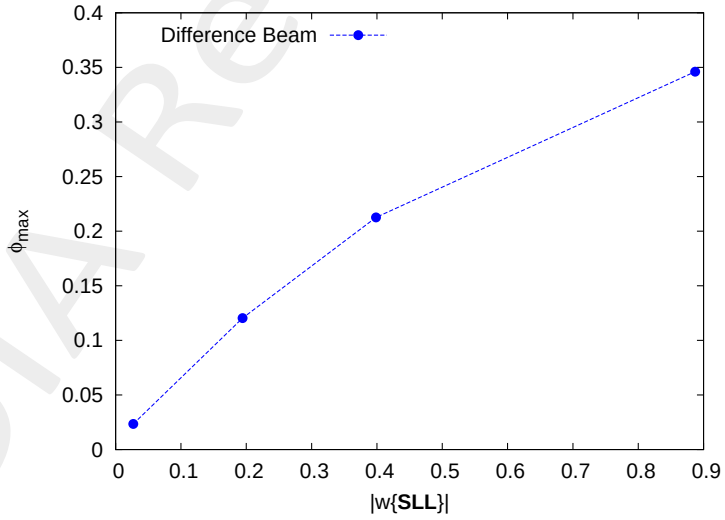

Figure 11. Difference Pattern $|\sup\{\mathbf{SLL}\}|$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0541	0.1654	0.2126	0.346
$\lvert\sup\{\mathbf{SLL}\}\rvert$	0.02693	0.1944	0.3985	0.8872

Table 11. Difference Pattern $|\sup\{\mathbf{SLL}\}|$ and $\phi_{\max}$ values.

BW:

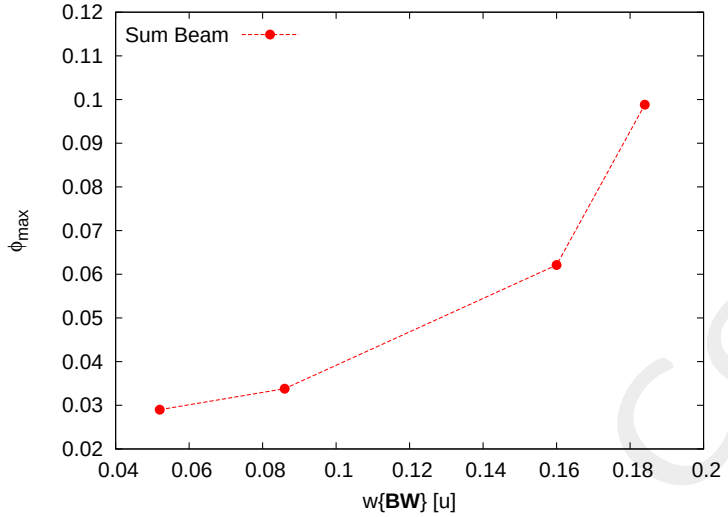

Figure 12. Sum Pattern $w\{\mathbf{BW}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0141	0.0338	0.0621	0.0988
$w\{\mathbf{BW}\}$	0.052	0.086	0.16	0.184

Table 12. Sum Pattern $w\{\mathbf{BW}\}$ and $\phi_{\max}$ values.

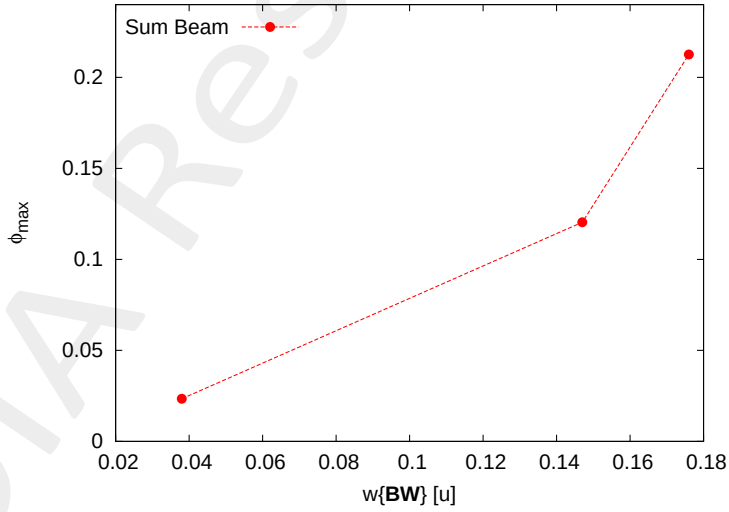

Figure 13. Difference Pattern $w\{\mathbf{BW}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0541	0.1654	0.2126	0.346
$w\{\mathbf{BW}\}$	0.38	0.147	0.176	4.0

Table 13. Difference Pattern $w\{\mathbf{BW}\}$ and $\phi_{\max}$ values.

Directivity:

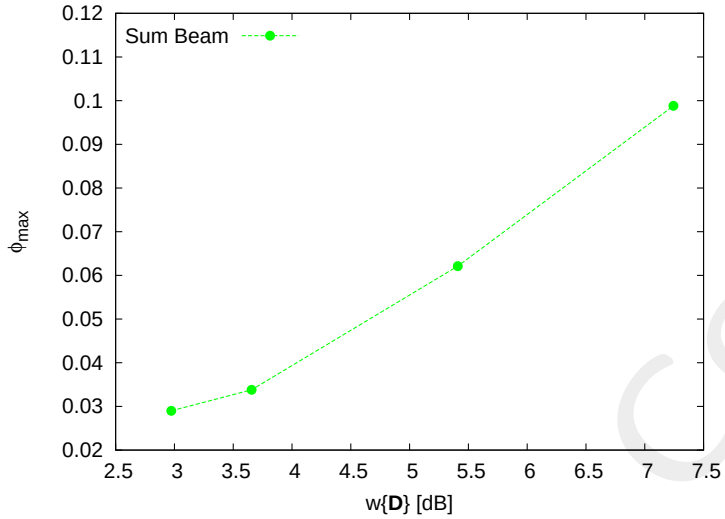

Figure 14. Sum Pattern $w\{\mathbf{D}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0141	0.0338	0.0621	0.0988
$w\{\mathbf{D}\}$	2.9735	3.6563	5.41	7.2435

Table 14. Sum Pattern $w\{\mathbf{D}\}$ and $\phi_{\max}$ values.

Slope:

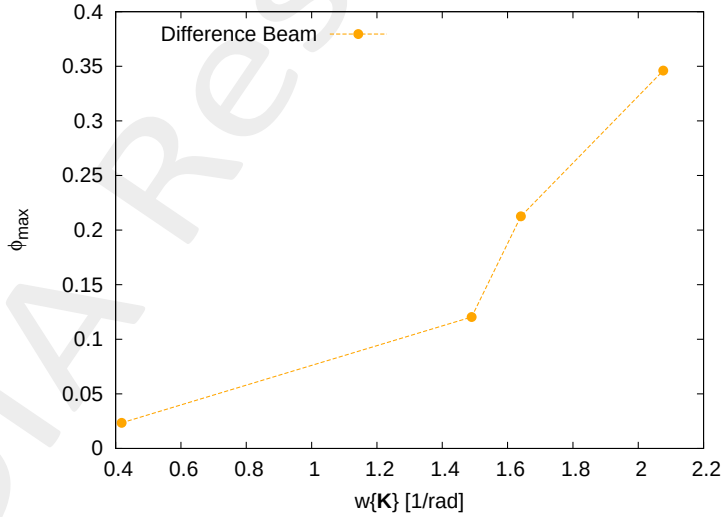

Figure 15. Difference Pattern $w\{\mathbf{K}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0541	0.1654	0.2126	0.346
$w\{\mathbf{K}\}$	0.4182	1.49	1.6406	2.0765

Table 15. Difference Pattern $w\{\mathbf{K}\}$ and $\phi_{\max}$ values.

1.2 Patterns Features Analysis - Tolerance Over Not-Common Elements

SLL:

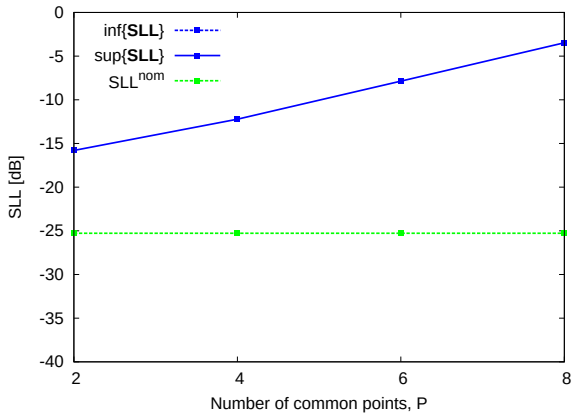

Figure 1. Sum Pattern **SLL** vs P

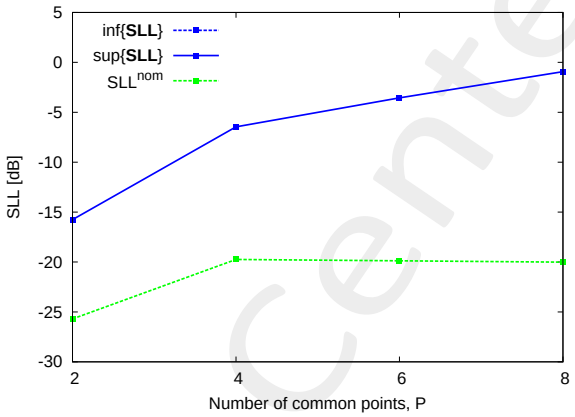

Figure 2. Difference Pattern **SLL** vs P

P	2	4	6	8
nominal	-25.28	-25.28	-25.28	-25.28
inf	$-\infty$	$-\infty$	$-\infty$	$-\infty$
sup	-15.79	-12.22	-7.87	-3.47

Table 1. Sum Pattern **SLL** values

P	2	4	6	8
nominal	-25.7	-19.7	-19.9	-20.0
inf	$-\infty$	$-\infty$	$-\infty$	$-\infty$
sup	-15.8	-6.5	-3.6	-0.9

Table 2. Difference Pattern **SLL** values

BW:

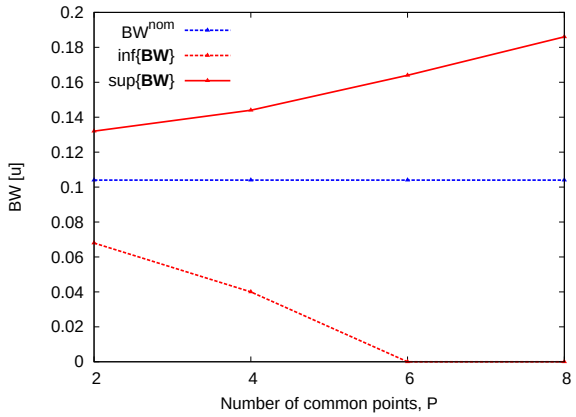

Figure 3. Sum Pattern $\mathbf{BW}$ vs P

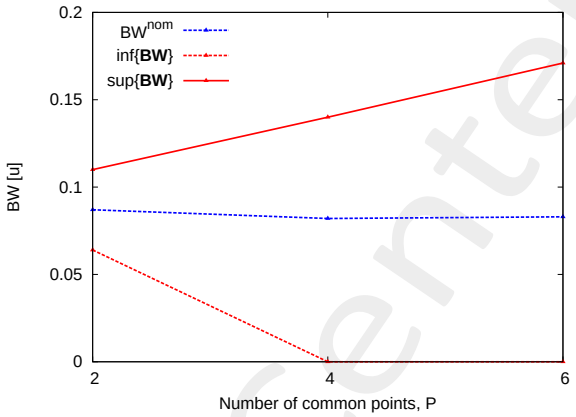

Figure 4. Difference Pattern $\mathbf{BW}$ vs P

P	2	4	6	8
nominal	0.104	0.104	0.104	0.104
inf	0.068	0.04	0.0	0.0
sup	0.132	0.144	0.164	0.186

Table 3. Sum Pattern $\mathbf{BW}$ values

P	2	4	6	8
nominal	0.087	0.082	0.083	0.083
inf	0.064	0.0	0.0	0.0
sup	0.11	0.14	0.171	4.0

Table 4. Difference Pattern $\mathbf{BW}$ vs P

Directivity / Slope:

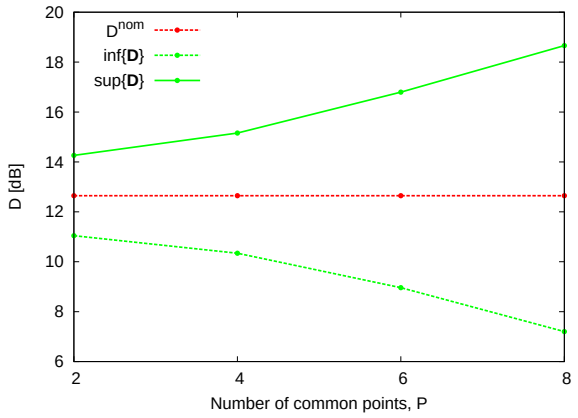

Figure 5. Sum Pattern $\mathbf{D}$ vs P

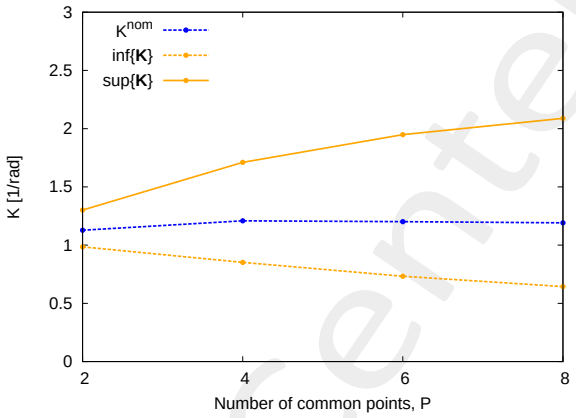

Figure 6. Difference Pattern $\mathbf{K}$ vs P

P	2	4	6	8
nominal	12.65	12.65	12.65	12.65
inf	11.05	10.34	8.96	7.2
sup	14.26	15.16	16.8	18.66

Table 5. Sum Pattern $\mathbf{D}$ values

P	2	4	6	8
nominal	1.1281	1.2084	1.2011	1.1912
inf	0.9854	0.852	0.7327	0.644
sup	1.3007	1.7103	1.9489	2.0886

Table 6. Difference Pattern $\mathbf{K}$ values

Pattern Tolerance:

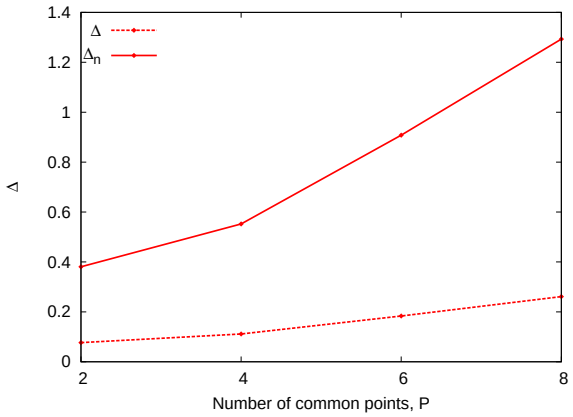

Figure 8. Sum Pattern Δ and Δ_n vs P

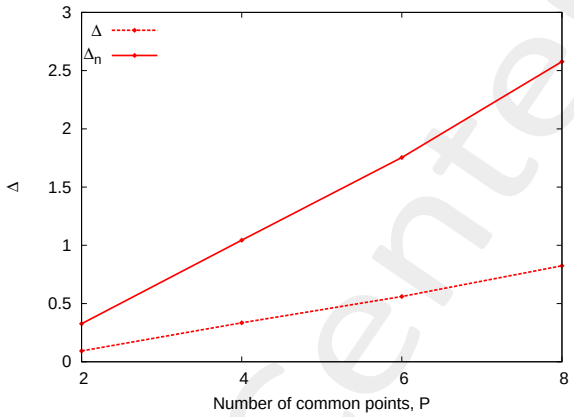

Figure 9. Difference Pattern Δ and Δ_n vs P

P	2	4	6	8
Δ	0.0768	0.1115	0.1833	0.261
Δ_n	0.3806	0.5524	0.9079	1.2929

Table 8. Sum Pattern Δ and Δ_n values

P	2	4	6	8
Δ	0.0934	0.3343	0.5606	0.8244
Δ_n	0.3268	1.0445	1.7536	2.5772

Table 9. Difference Pattern Δ and Δ_n values

Pareto Fronts - Tolerance Over Not-Common Elements

In the following figures the quantity $\phi_{\max}$ defined versus the performances descriptors are plotted.

SLL:

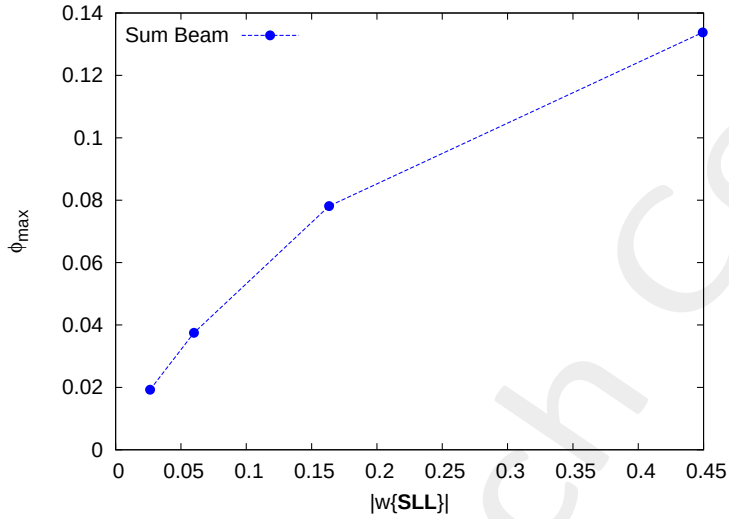

Figure 10. Sum Pattern $|w\{\mathbf{SLL}\}|$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0192	0.0375	0.07811	0.1338
$\lvert w\{\mathbf{SLL}\}\rvert$	0.0263	0.0599	0.1634	0.4494

Table 10. Sum Pattern $|w\{\mathbf{SLL}\}|$ and $\phi_{\max}$ values.

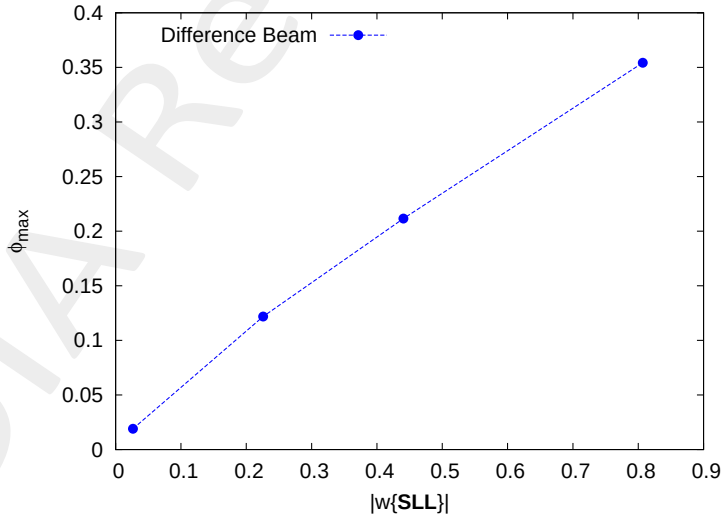

Figure 11. Difference Pattern $|w\{\mathbf{SLL}\}|$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.019	0.1219	0.2116	0.3542
$\lvert w\{\mathbf{SLL}\}\rvert$	0.0266	0.2259	0.4406	0.8069

Table 11. Difference Pattern $|w\{\mathbf{SLL}\}|$ and $\phi_{\max}$ values.

BW:

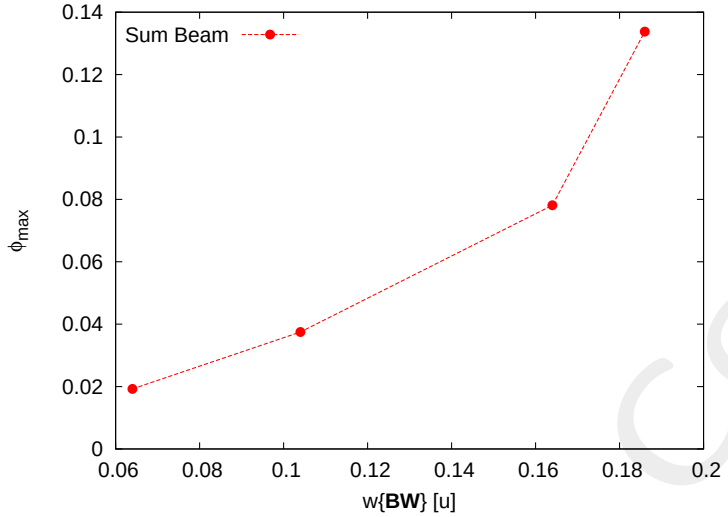

Figure 12. Sum Pattern $w\{\mathbf{BW}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0192	0.0375	0.07811	0.1338
$w\{\mathbf{BW}\}$	0.064	0.104	0.164	0.186

Table 12. Sum Pattern $w\{\mathbf{BW}\}$ and $\phi_{\max}$ values.

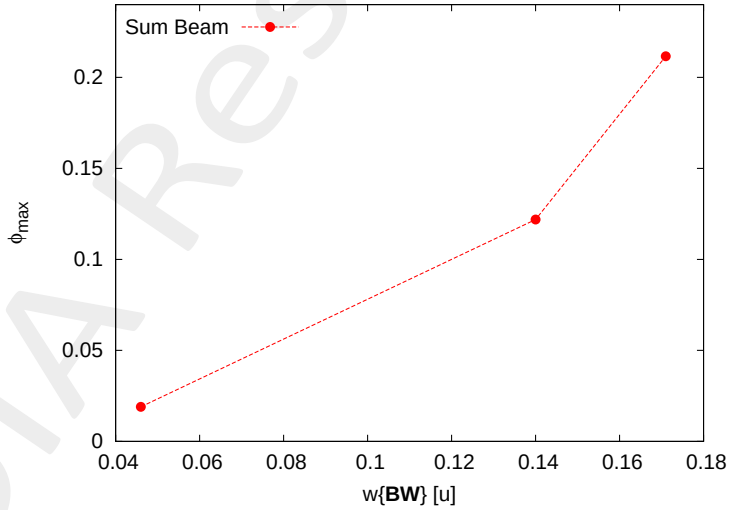

Figure 13. Difference Pattern $w\{\mathbf{BW}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.019	0.1219	0.2116	0.3542
$w\{\mathbf{BW}\}$	0.046	0.14	0.171	4.0

Table 13. Difference Pattern $w\{\mathbf{BW}\}$ and $\phi_{\max}$ values.

Directivity:

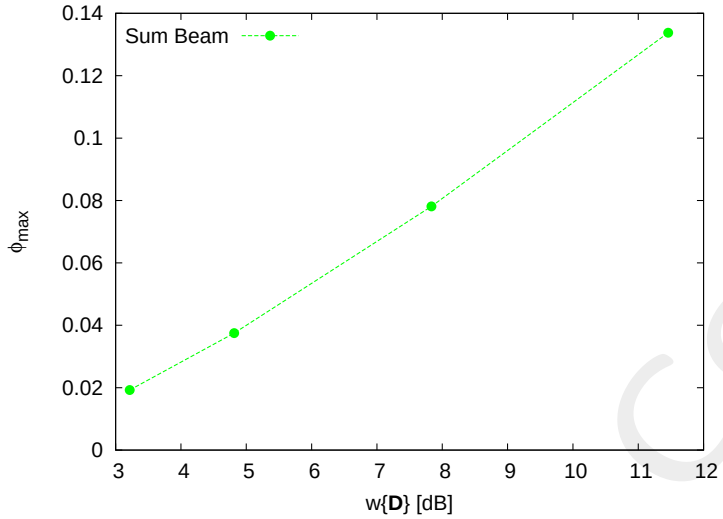

Figure 14. Sum Pattern $w\{\mathbf{D}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.0192	0.0375	0.07811	0.1338
$w\{\mathbf{D}\}$	3.2149	4.815	7.8342	11.4591

Table 14. Sum Pattern $w\{\mathbf{D}\}$ and $\phi_{\max}$ values.

Slope:

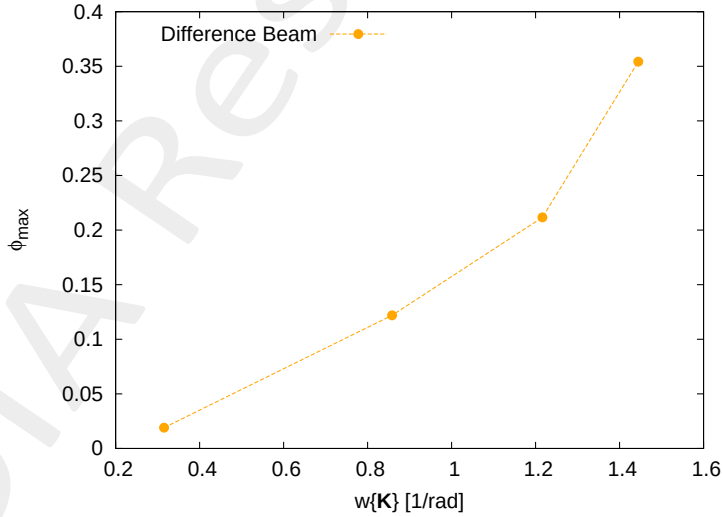

Figure 15. Difference Pattern $w\{\mathbf{K}\}$ vs $\phi_{\max}$.

P	2	4	6	8
$\phi_{\max}$	0.019	0.1219	0.2116	0.3542
$w\{\mathbf{K}\}$	0.3153	0.8583	1.2162	1.4446

Table 15. Difference Pattern $w\{\mathbf{K}\}$ and $\phi_{\max}$ values.

Comparison of previous results: tolerances over common/not-common elements

In the following pictures the bounds of the antenna arrays descriptors are compared, considering the amplitude tolerance over the common elements (es. $\sup\{\mathbf{SLL}\}^{C}$) and over the not-common elements (es. $\sup\{\mathbf{SLL}\}^{NC}$).

SLL:

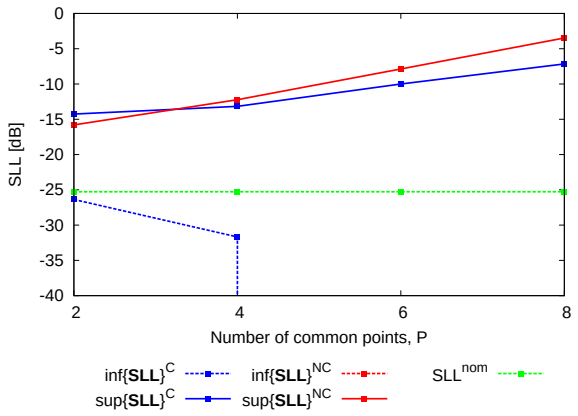

Figure 1. Sum Pattern **SLL** vs P

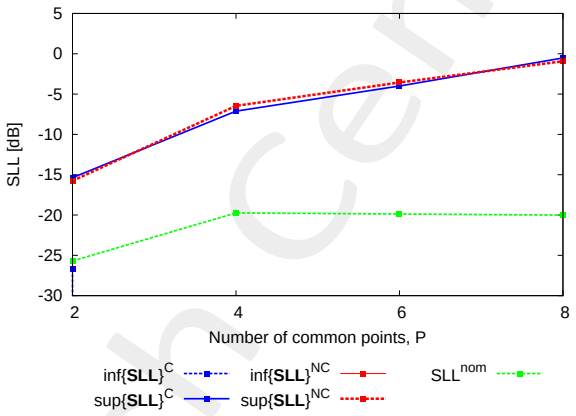

Figure 2. Difference Pattern **SLL** vs P

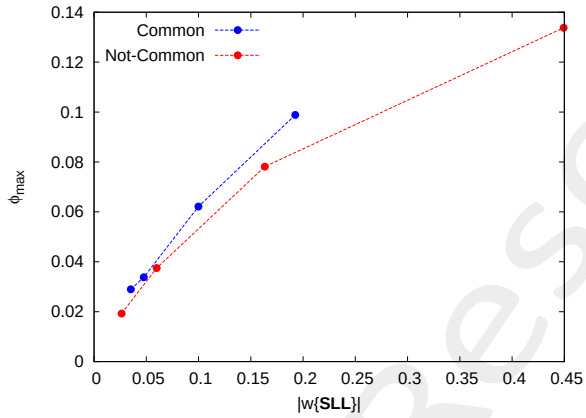

Figure 3. Sum Pattern $|w\{\mathbf{SLL}\}|$ vs $\phi_{\max}$.

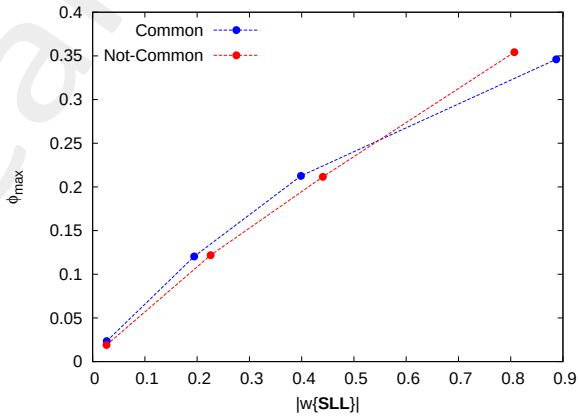

Figure 4. Difference Pattern $|w\{\mathbf{SLL}\}|$ vs $\phi_{\max}$.

BW:

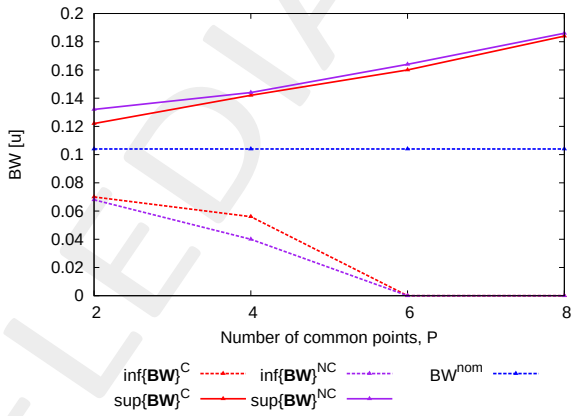

Figure 5. Sum Pattern **BW** vs P

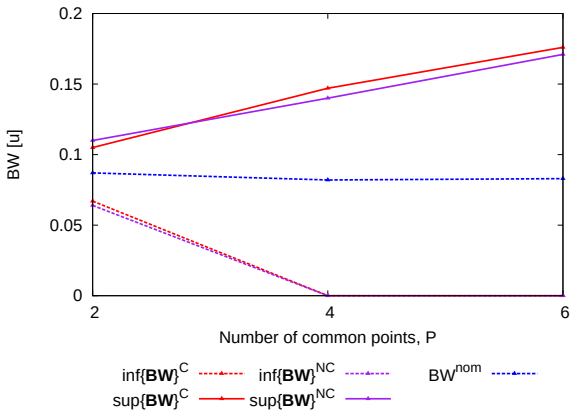

Figure 6. Difference Pattern **BW** vs P

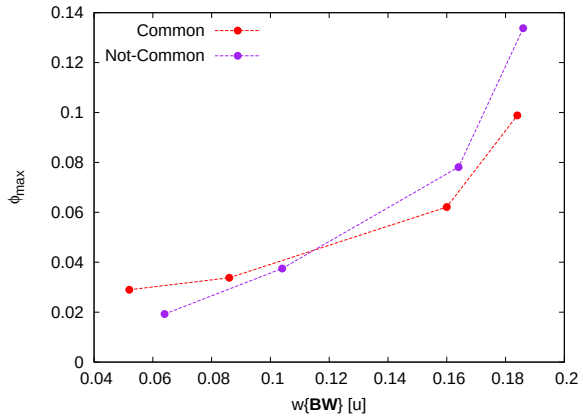

Figure 7. Sum Pattern $w\{\mathbf{BW}\}$ vs $\phi_{\max}$.

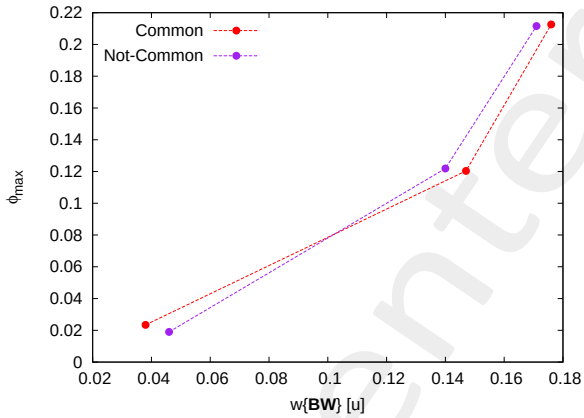

Figure 8. Difference Pattern $w\{\mathbf{BW}\}$ vs $\phi_{\max}$.

Directivity / Slope:

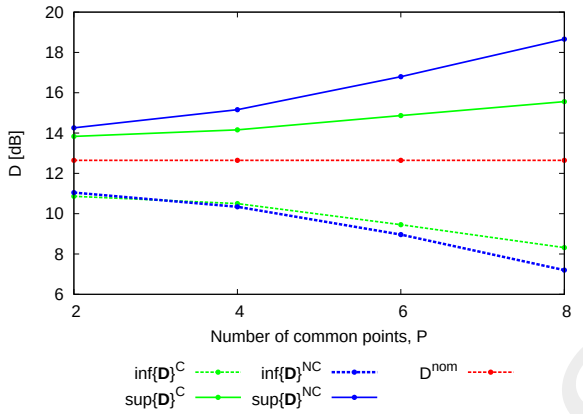

Figure 9. Sum Pattern $\mathbf{D}$ vs P

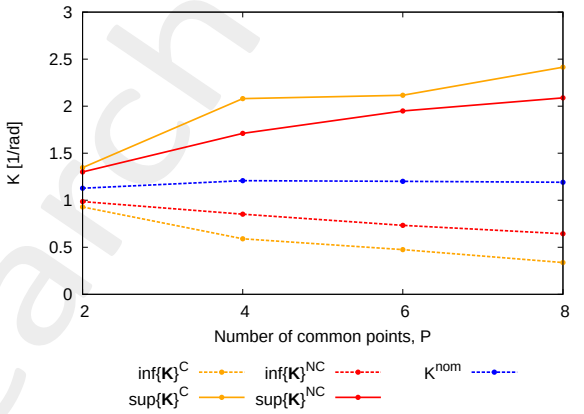

Figure 10. Difference Pattern $\mathbf{K}$ vs P

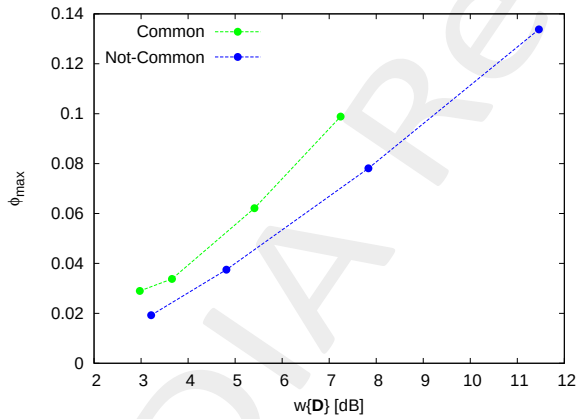

Figure 11. Sum Pattern $w\{\mathbf{D}\}$ vs $\phi_{\max}$.

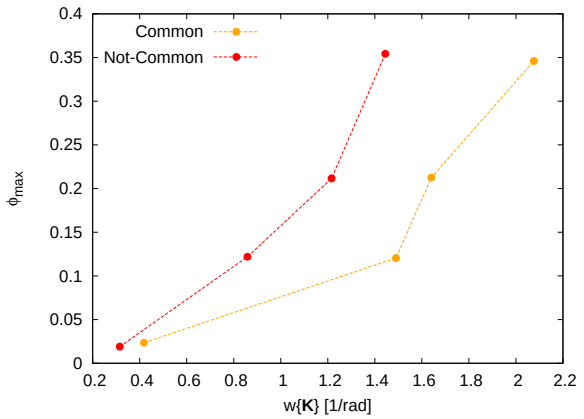

Figure 12. Difference Pattern $w\{\mathbf{K}\}$ vs $\phi_{\max}$.

Pattern Tolerance:

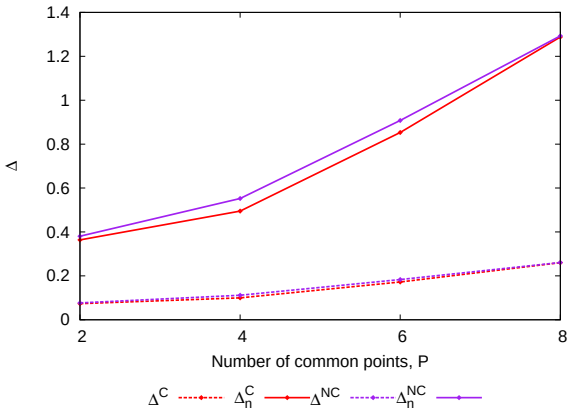

Figure 13. Sum Pattern Δ and Δ_n vs P

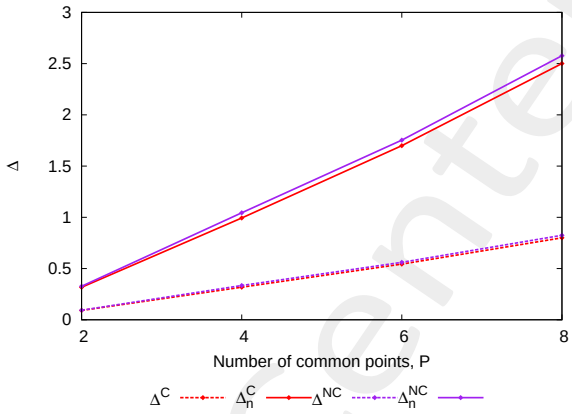

Figure 14. Difference Pattern Δ and Δ_n vs P

More information on the topics of this document can be found in the following list of references.

References

[1] G. Ding, N. Anselmi, W. Xu, P. Li, and P. Rocca, "Interval-bounded optimal power pattern synthesis of array antenna excitations robust to mutual coupling," *IEEE Antennas Wireless Propag. Lett.*, vol. 22, no. 11, pp. 2725-2729, Nov. 2023.

[2] N. Anselmi, P. Rocca, and A. Massa, "Tolerance analysis of reconfigurable monopulse linear antenna arrays through interval arithmetic," *J. Electromagn. Waves Appl. J*, pp. 1066-1081, 2023.

[3] P. Rocca, N. Anselmi, A. Benoni, and A. Massa, "Probabilistic interval analysis for the analytic prediction of the pattern tolerance distribution in linear phased arrays with random excitation errors," *IEEE Trans. Antennas Propag.*, vol. 68, no. 2, pp. 7866-7878, Dec. 2020.

[4] L. Tenuti, N. Anselmi, P. Rocca, M. Salucci, and A. Massa, "Minkowski sum method for planar arrays sensitivity analysis with uncertain-but-bounded excitation tolerances" *IEEE Trans. Antennas Propag.*, vol. 65, no. 1, pp. 167-177, Jan. 2017.

[5] N. Anselmi, P. Rocca, M. Salucci, and A. Massa, "Optimization of excitation tolerances for robust beamforming in linear arrays," *IET Microwaves, Antennas & Propagation*, vol. 10, no. 2, pp. 208-214, 2016.

[6] N. Anselmi, P. Rocca, M. Salucci, and A. Massa, "Power pattern sensitivity to calibration errors and mutual coupling in linear arrays through circular interval arithmetics," *Sensors*, vol. 16, no. 6 (791), pp. 1-14, 2016.

[7] L. Poli, P. Rocca, N. Anselmi, and A. Massa, "Dealing with uncertainties on phase weighting of linear antenna arrays by means of interval-based tolerance analysis," *IEEE Trans. Antennas Propag.*, vol. 63, no. 7, pp. 3299-3234, Jul. 2015.

[8] P. Rocca, N. Anselmi, and A. Massa, "Optimal synthesis of robust array configurations exploiting interval analysis and convex optimization," *IEEE Trans. Antennas Propag.*, vol. 62, no. 7, pp. 3603-3612, July 2014.

[9] T. Moriyama, L. Poli, N. Anselmi, M. Salucci, and P. Rocca, "Real array pattern tolerances from amplitude excitation errors," *IEICE Electronics Express*, vol. 11, no. 17, pp. 1-8, Sep. 2014.

[10] N. Anselmi, L. Manica, P. Rocca, and A. Massa, "Tolerance analysis of antenna arrays through interval arithmetic," *IEEE Trans. Antennas Propag.*, vol. 61, no. 11, pp. 5496-5507, Nov. 2013.

[11] L. Manica, N. Anselmi, P. Rocca, and A. Massa, "Robust mask-constrained linear array synthesis through an interval-based particle swarm optimisation," *IET Microwaves, Antennas and Propagation*, vol. 7, no. 12, pp. 976-984, Sep. 2013.

[12] P. Rocca, L. Manica, N. Anselmi, and A. Massa, "Analysis of the pattern tolerances in linear arrays with arbitrary amplitude errors," *IEEE Antennas Wireless Propag. Lett.*, vol. 12, pp. 639-642, 2013.

[13] L. Manica, P. Rocca, N. Anselmi, and A. Massa, "On the synthesis of reliable linear arrays through interval arithmetic," *IEEE International Symposium on Antennas Propag. (APS/URSI 2013)*, *Orlando, Florida*, *USA*, Jul. 7-12, 2013.

[14] L. Manica, P. Rocca, G. Oliveri, and A. Massa, "Designing radiating systems through interval analysis tools," *IEEE International Symposium on Antennas Propag. (APS/URSI 2013), Orlando, Florida, USA*, Jul. 7-12, 2013.

[15] M. Carlin, N. Anselmi, L. Manica, P. Rocca, and A. Massa, "Exploiting interval arithmetic for predicting real arrays performances - The linear case," *IEEE International Symposium on Antennas Propag. (APS/URSI 2013), Orlando, Florida, USA*, Jul. 7-12, 2013.

[16] N. Anselmi, M. Salucci, P. Rocca, and A. Massa, "Generalised interval-based analysis tool for pattern distortions in reflector antennas with bump-like surface deformations," *IET Microwaves, Antennas & Propagation*, vol. 10, no. 9, p. 909-916, June 2016.

[17] P. Rocca, L. Poli, N. Anselmi, M. Salucci, and A. Massa, "Predicting antenna pattern degradations in microstrip reflectarrays through interval arithmetic," *IET Microwaves, Antennas & Propagation*, vol. 10, no. 8, pp. 817-826, May 2016.

[18] P. Rocca, L. Manica, and A. Massa, "Interval-based analysis of pattern distortions in reflector antennas with bump-like surface deformations," *IET Microwaves, Antennas and Propagation*, vol. 8, no. 15, pp. 1277-1285, Dec. 2014.

[19] P. Rocca, N. Anselmi, and A. Massa, "Interval Arithmetic for pattern tolerance analysis of parabolic reflectors," *IEEE Trans. Antennas Propag.*, vol. 62, no. 10, pp. 4952-4960, Oct. 2014.