
Surrogate Assisted Radome Design: Preliminary Analysis

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1 Glossary and Definitions

1.1 Parameters of the optimization

1.1.1 General Parameters

- K : Dimension of the input space (i.e., the number of variables);
- P : Population size;
- $\Phi(\mathbf{x}_p^{(i)})$: real fitness associated to individual p at iteration i -th;
- $\tilde{\Phi}(\mathbf{x}_p^{(i)})$: predicted fitness associated to individual p at iteration i -th;

1.1.2 SADE Parameters

- Q : DE scaling factor;
- τ : Dimension of the training set built at each *SADE* iteration;
- i : Iteration index;
- $D^{(i)}$: Dimension of the database of simulated individuals at iteration i -th;
- $s^2 \{ \tilde{\Phi}(\mathbf{x}_p^{(i)}) \}$: Kriging prediction uncertainty (*MSE*) associated to prediction $\tilde{\Phi}(\mathbf{x}_p^{(i)})$;
- ω : “*exploration*” parameter for the Lower Confidence Bound (*LCB*) pre-screening method;
- $Y(\cdot)$: Cumulative distribution function of the standard normal distribution

$$Y(x) = \frac{1}{2} \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) + \frac{1}{2} \quad (1)$$

where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (2)$$

- $y(\cdot)$: Probability density function of the standard normal distribution

$$y(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad (3)$$

1.2 Prediction errors definition

Let be defined the following quantities:

- \hat{y}_m : predicted value associated to the m -th test sample;
- y_m : true value associated to the m -th test sample;
- M : number of test samples (size of the test set);
- \bar{y} : average of the true values associated to M test samples.

The following error metrics are defined and used hereinafter to evaluate the accuracy of a given predictor:

1. Mean Absolute Error (MAE)

$$MAE = \frac{1}{M} \sum_{m=1}^M |\hat{y}_m - y_m|$$

2. Normalized Mean Error (NME)

$$NME = \frac{1}{M} \sum_{m=1}^M \frac{|\hat{y}_m - y_m|}{|y_m|}$$

3. Mean Squared Error (MSE)⁽¹⁾

$$MSE = \frac{1}{M} \sum_{m=1}^M (\hat{y}_m - y_m)^2$$

4. Root Mean Squared Error (RMSE)

$$RMSE = \sqrt{MSE}$$

5. Normalized Root Mean Squared Error (NRMSE)

$$NRMSE = \sqrt{\frac{\sum_{m=1}^M (\hat{y}_m - y_m)^2}{\sum_{m=1}^M (y_m)^2}}$$

6. Normalized Maximum Error (NEMAX)

$$NEMAX = \frac{\max_{m=1,\dots,M} \{|\hat{y}_m - y_m|\}}{\sqrt{\frac{1}{M} \sum_{m=1}^M (y_m - \bar{y})^2}}$$

⁽¹⁾**NOTE:** This doesn't correspond to the definition of the Kriging MSE (prediction uncertainty)!

7. Matching Error (ME)

$$ME = \frac{\sum_{m=1}^M |\hat{y}_m - y_m|^2}{\sum_{m=1}^M |y_m|^2}$$

8. Variance-Normalized Mean Squared Error ($VNMSE$)

$$VNMSE = \frac{MSE}{Var(\{y_m\})} = \frac{\frac{1}{M} \sum_{m=1}^M (\hat{y}_m - y_m)^2}{\frac{1}{M} \sum_{m=1}^M (y_m - \bar{y})^2} = \frac{\sum_{m=1}^M (\hat{y}_m - y_m)^2}{\sum_{m=1}^M (y_m - \bar{y})^2}$$

1.3 Time saving definition

Let us assume that our goal is to evaluate a given cost function in S user-defined samples. We can proceed in two ways:

1. Evaluate the S samples by using a full wave simulator.
2. Evaluate the S samples by using a predictor. In this case, we will have
 - (a) $N < S$ simulations using the full wave simulator, to build the training set;
 - (b) $M = (S - N)$ evaluations using the trained predictor.

The time saving using the predictor instead of computing all the S samples with the simulator is defined as

$$\Delta t^{saving}(N) = \left(\frac{\Delta t_{tot}^{sim} - \Delta t_{tot}^{pred}(N)}{\Delta t_{tot}^{sim}} \right) \quad (4)$$

where

- $\Delta t_{tot}^{sim} = S \times \Delta t^{sim}$: time required to simulate S samples using the e.m. simulator;
- Δt^{sim} is the average time required to perform a single e.m. simulation;
- $\Delta t_{tot}^{pred}(N) = (N \times \Delta t^{sim}) + \Delta t^{train}(N) + M \times \Delta t^{test}(N)$: time required if the predictor is used to estimate the value of the cost function in $M = (S - N)$ samples;
 - $N \times \Delta t^{sim}$: time required to generate the training set for the predictor made of N training samples;
 - $\Delta t^{train}(N)$: time required to train the predictor using N training samples;
 - $\Delta t^{test}(N)$: time required by the predictor to predict a single function value when trained with N training samples.

1.3.1 Incremental training generation with random selection from S samples

1. Randomly select N_1 training samples from the S available input-output pairs (e.g., an *LHS* training set);
2. Train a predictor using the selected training samples;
3. Predict the remaining $M_1 = (S - N_1)$ samples and compute the associate prediction error;
4. Randomly select new ΔN samples from S and add them to the previous training set ($N_2 = N_1 + \Delta N$).
The test set now contains the remaining $M_2 = (S - N_2)$ sites;
5. Go to 2, iterate until the selected maximum training dimension is considered to train the predictor (note that $N_{max} < S$, in order to have some test samples on which evaluate the prediction accuracy).

1.4 Ogive radome shape

In the following report, the 3D ogive radome surface is modeled by using the superspheroidal surface for the unified description of a wide range of airborne radome shapes. The surface of a superspheroidal body is given by the equation

$$x^2 + y^2 = \left(\frac{D}{2L}\right)^2 (L^\nu - z^\nu)^{2/\nu}, \quad z \in [0, L] \quad (5)$$

where

- D : base diameter of the radome;
- L : length of the radome;
- ν : parameter associated to the curvature of the radome ($\nu \in [1, 2]$);

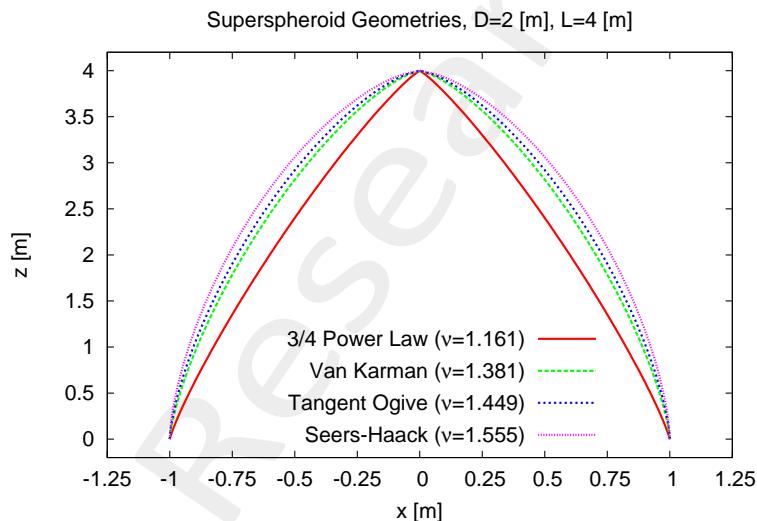


Figure 1: Plot of the ogive shapes.

1.4.1 Multilayer radome

The volume Ω_n associated to the n -th layer with thickness t_n of a N layers ogive radome is defined as

$$\Omega_n = \Omega'_{n-1} \setminus \Omega'_n, \quad n = 1, \dots, N \quad (6)$$

where

$$\Omega'_n = \left\{ (x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 \leq \left(\frac{D_n}{2L_n}\right)^2 (L_n^\nu - z^\nu)^{2/\nu}, \quad z \in [0, L_n] \right\}, \quad n = 0, \dots, N \quad (7)$$

and

$$D_n = \begin{cases} D & \text{if } n = 0 \\ D - 2 \sum_{n=1}^N t_n & \text{if } n > 0 \end{cases}, \quad n = 0, \dots, N \quad (8)$$

$$L_n = \begin{cases} L & \text{if } n = 0 \\ L - \sum_{n=1}^N t_n & \text{if } n > 0 \end{cases}, \quad n = 0, \dots, N \quad (9)$$

The operand \ denotes the subtraction of the volume Ω'_n from the volume Ω'_{n-1} .

1.4.2 Spline-Based Radome Thickness Profile

The external surface of the radome is modeled by means of the Equation (5), while the thickness profile of the structure is modeled with a spline, as shown in Figure (2).

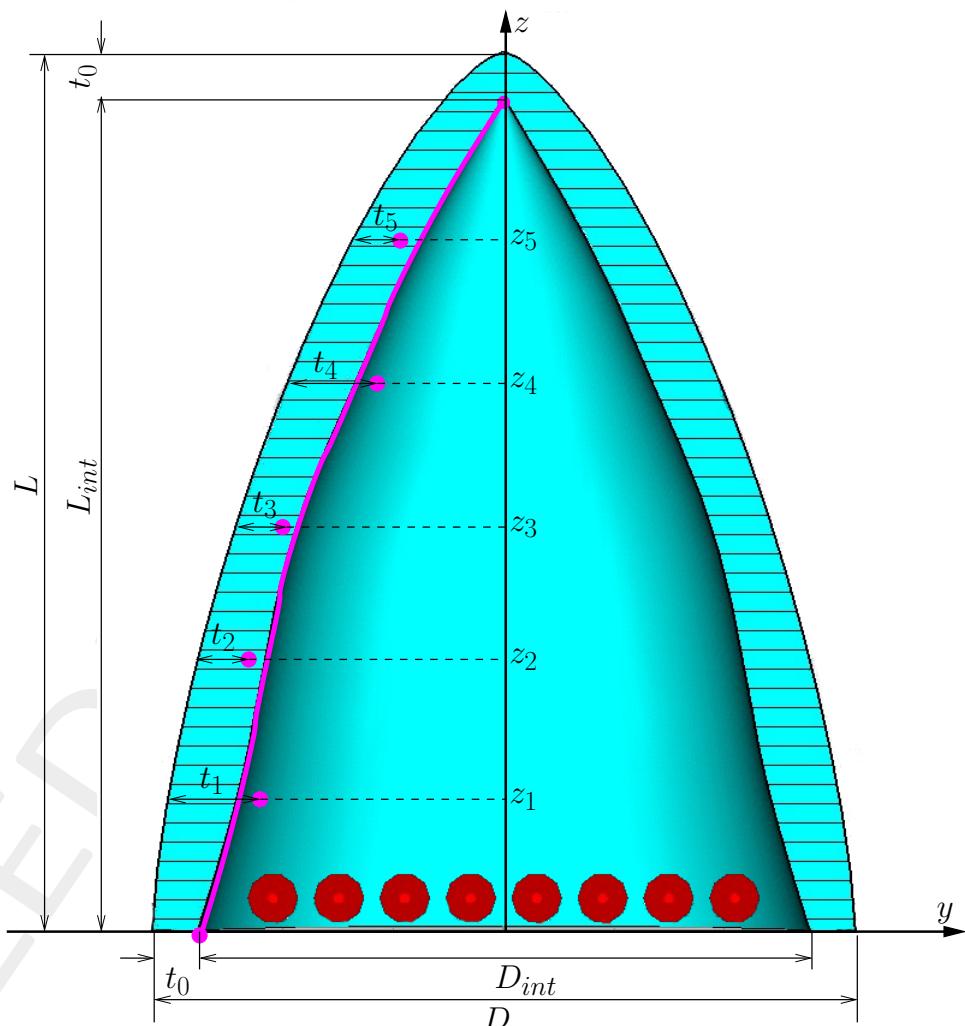


Figure 2: Plot of the ogive-shaped radome with spline thickness profile.

1.4.3 Quadratic Bezier curve: Definition

Let be given

- Q : number of control points;
- $(P_x^{(q)}, P_y^{(q)})$: coordinates of the q -th control point;

A Bezier quadratic spline interpolating the Q control points is built as follows

1. Compute the coordinates of the Q virtual points. The $q - th$ virtual point is located in the middle of the line between the two control points q and $q + 1$, and its coordinates are given by

$$V_x^{(q)} = \frac{P_x^{(q)} + P_x^{(q+1)}}{2}$$

$$V_y^{(q)} = \frac{P_y^{(q)} + P_y^{(q+1)}}{2}$$

2. For each control point ($q = 1, \dots, Q$) draw a quadratic curve by considering its two adjacent virtual points.

The curve for the control point q is defined as

$$B_x^{(q)}(t) = (1 - t^2) V_x^{(q)} + 2t(1 - t) P_x^{(q)} + t^2 V_x^{(q+1)}$$

$$B_y^{(q)}(t) = (1 - t^2) V_y^{(q)} + 2t(1 - t) P_y^{(q)} + t^2 V_y^{(q+1)}$$

for $t \in [0, 1]$.

3. Join the q segments to form the full interpolating spline curve.

Example

The following figure shows the computed spline for $Q = 5$ control points defined in table I. Note that the spline curve doesn't pass through any of the selected control points.

Index	Control Points $(P_x^{(q)}, P_y^{(q)})$	Virtual Points $(V_x^{(q)}, V_y^{(q)})$
1	(3, 5)	(4, 4)
2	(5, 3)	(4.5, 2)
3	(4, 1)	(3.5, 2)
4	(3, 3)	(2, 3.5)
5	(1, 4)	(2, 4.5)

Table I: Location of the control points and of the virtual points.

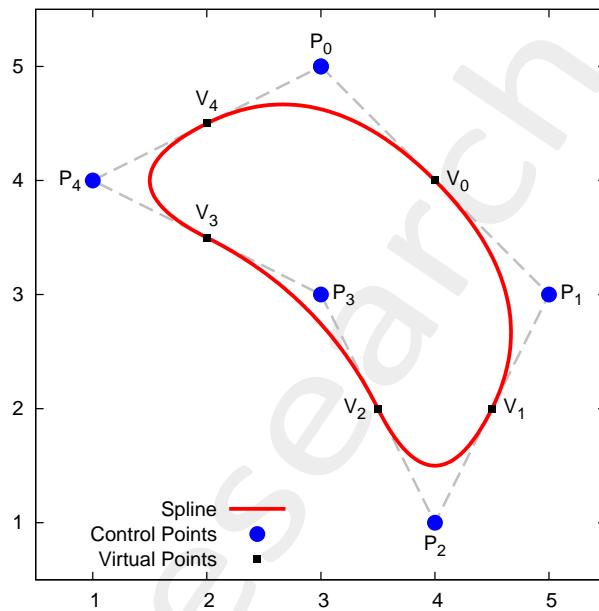


Figure 3: Plot of a quadratic spline passing through the selected control points.

2 Radome effect analysis

This aim of this section is the validation of the implemented *FEKO* radome models.

2.1 Spherical radome

2.1.1 Single layer structure (comparison with Arvas.1990)

The following section reports the results obtained by simulating the spherical radome.

Simulation Parameters

- Frequency: $f = 3.00 \times 10^8$ [Hz]
- Free space wavelength: $\lambda = 1.0$ [m]
- EM Solver: *FEM* (Finite Element Method)

Radome Parameters

- Spherical radome centered in $(x, y, z) = (0, 0, 0)$
- Number of layer: $N = 1$
- Radome radius: $R \in \{0.5, 0.9\}$ [m] = $\{0.5, 0.9\} \lambda$
- Radome thickness: $t = 0.1$ [m] = $\frac{\lambda}{10}$
- Radome relative permittivity: $\varepsilon \in \{1.0, 2.5, 4.0\}$

Antenna Parameters

- Short dipole centered in $(x, y, z) = (0, 0, 0)$ and directed along \hat{z}
- Dipole length: $l_d = 8.00 \times 10^{-2}$ [m] $\simeq \frac{\lambda}{12}$

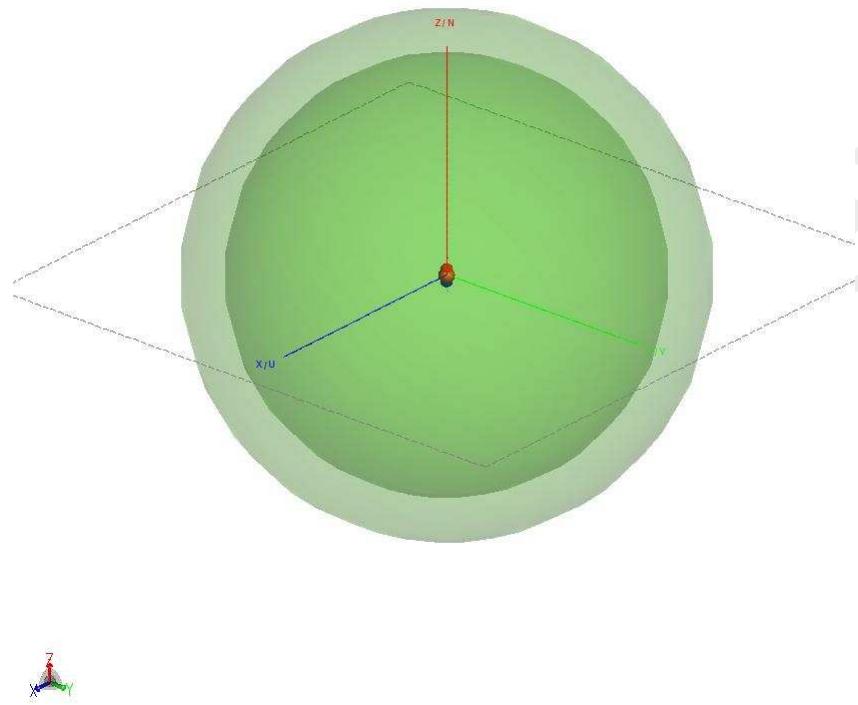


Figure 4: (*Single-layer spherical radome*) – Geometry of the radome.

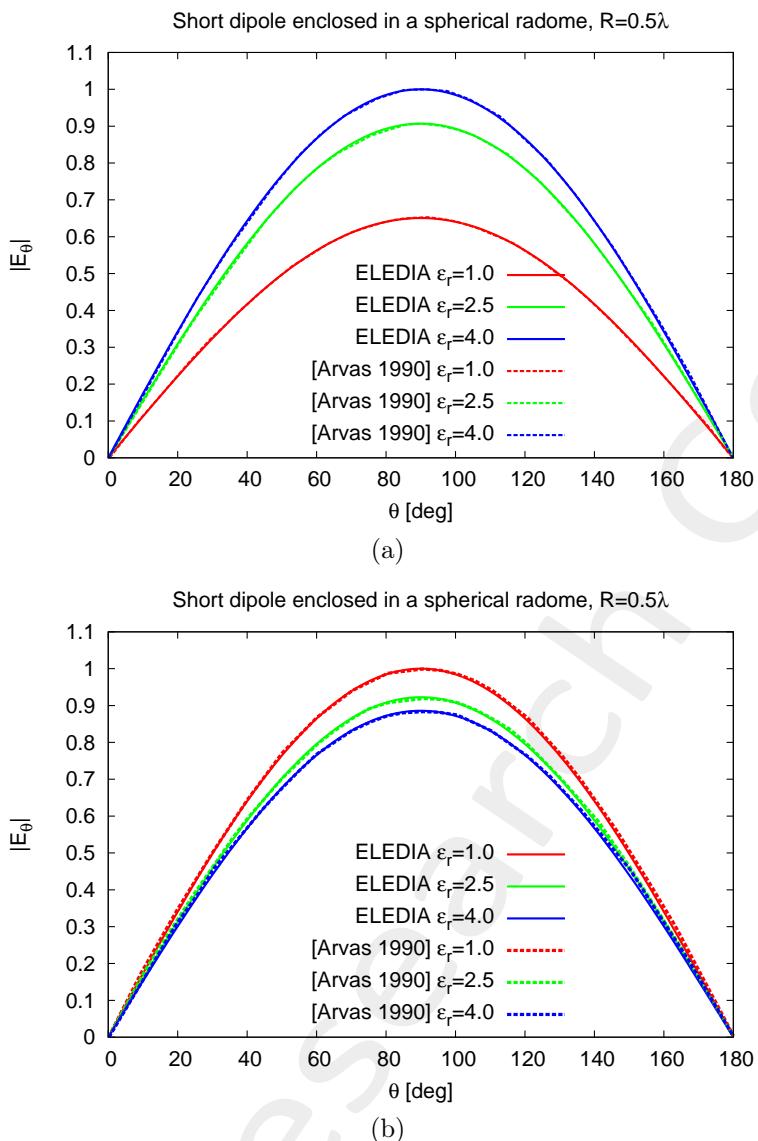


Figure 5: (*Single-layer spherical radome*) – Plot of the radiated field of the antenna in free-space and in presence of a radome with radius (a) $R = 0.5 \lambda$ and (b) $R = 0.9 \lambda$.

2.2 Ogive radome

2.2.1 Single layer structure (comparison with Zhao.2005)

The following section reports the results obtained by simulating the ogive radome.

Simulation Parameters

- Frequency: $f = 2.00 \times 10^8$ [Hz]
- Free space wavelength: $\lambda = 1.5$ [m]
- EM Solver: *FEM* (Finite Element Method)

Radome Parameters

- Number of layer: $N = 1$
- Radome length: $L = 1.75$ [m] $\simeq 1.17\lambda$
- Radome base diameter: $D = 1.6$ [m] $\simeq 1.07\lambda$
- Curvature type: $\nu = 1.449$ (tangent ogive)
- Radome thickness: $t = 7.00 \times 10^{-2}$ [m] $\simeq \frac{\lambda}{21}$
- Radome relative permittivity: $\epsilon = 2 - j$

Antenna Parameters

- Short dipole centered in $(x, y, z) = (0, 0, 0)$ and directed along \hat{y}
- Dipole length: $l_d = 0.1$ [m] $\simeq \frac{\lambda}{15}$

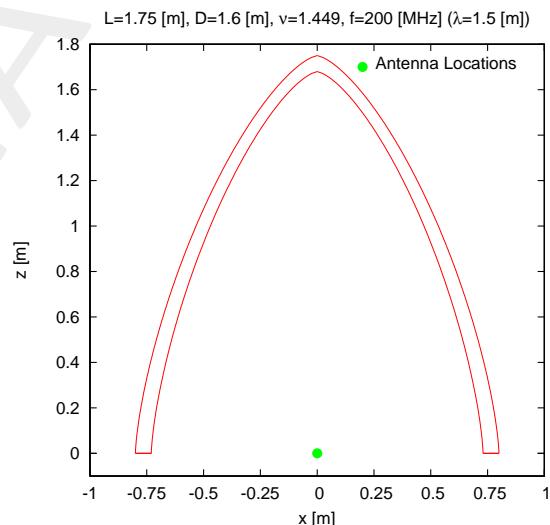


Figure 6: (*Single-layer ogive radome*) – Geometry of the radome.

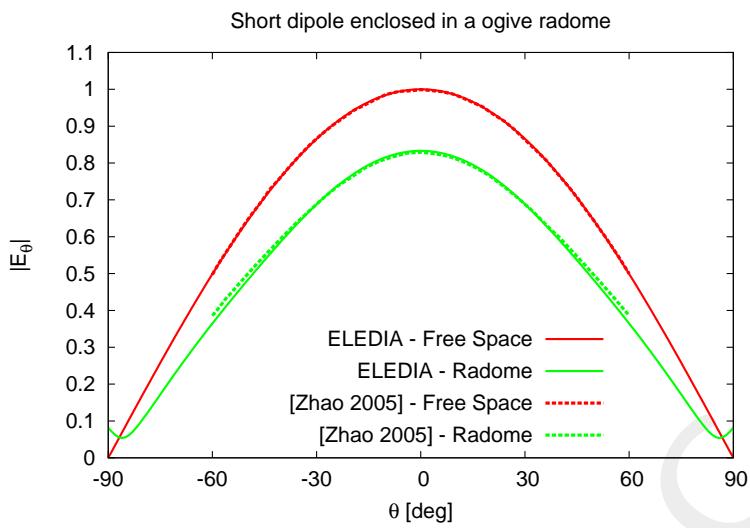


Figure 7: (single-layer ogive radome) – Plot of the radiated field of the antenna in free-space and in presence of the radome.

2.2.2 2-layers structure

Simulation Parameters

- Frequency: $f = 2.00 \times 10^8$ [Hz]
- Free space wavelength: $\lambda = 1.5$ [m]
- EM Solver: FEM (Finite Element Method)

Radome Parameters

- Number of layer: $N = 2$
- Radome length: $L = 1.75$ [m] $\simeq 1.17\lambda$
- Radome base diameter: $D = 1.6$ [m] $\simeq 1.07\lambda$
- Curvature type: $\nu = 1.449$ (tangent ogive)
- Radome thickness: $t = 7.16 \times 10^{-2}$ [m] $\simeq \frac{\lambda}{21}$
 - $t_1 = 3.16 \times 10^{-2}$ [m]
 - $t_2 = 4.00 \times 10^{-2}$ [m]
- Radome relative permittivity:
 - $\epsilon_1 = 3.72$
 - $\epsilon_2 = 4.22$

Antenna Parameters

- Dipole centered in $(x, y, z) = (0, 0, 0)$ and directed along \hat{y}
- Dipole length: $l_d = 0.75$ [m] $= \frac{\lambda}{2}$

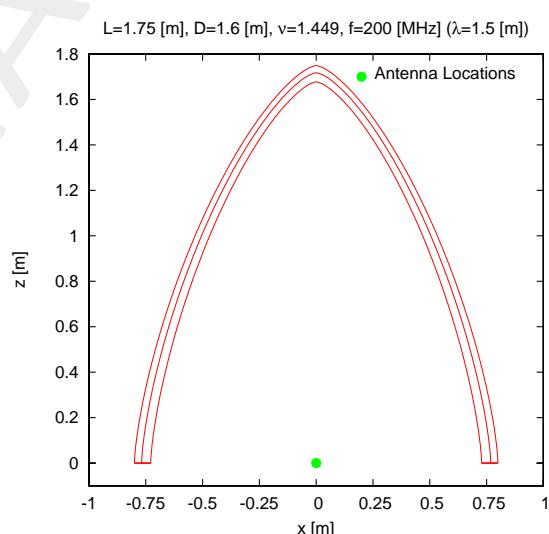


Figure 8: (2-layer ogive radome) – Geometry of the radome.

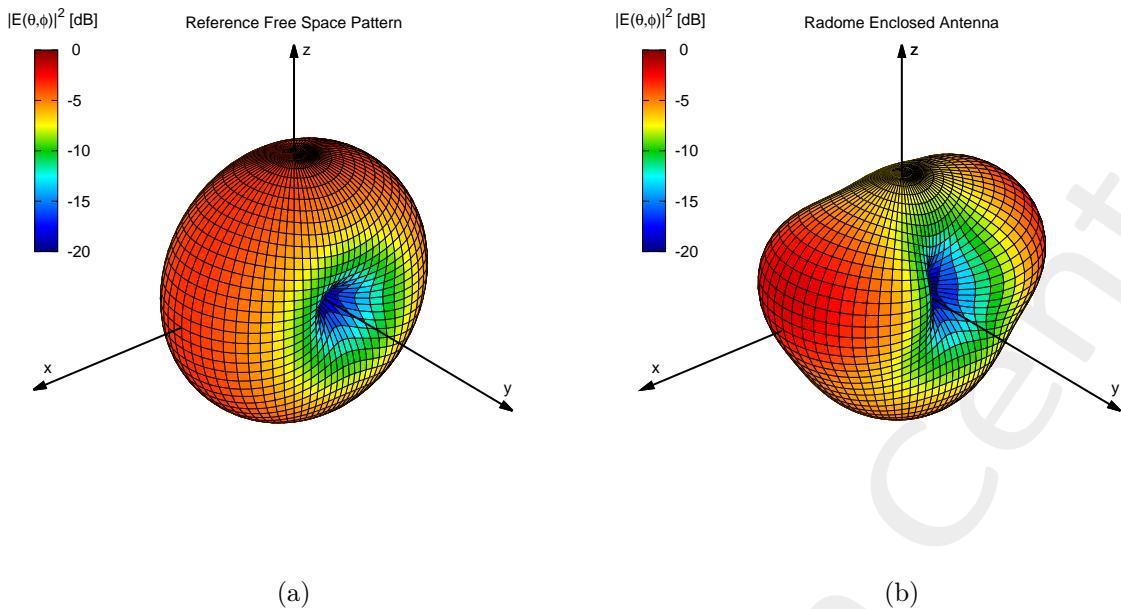


Figure 9: (*2-layer ogive radome*) – 3D plot of the power pattern of the antenna (a) in free space and (b) in presence of the radome.

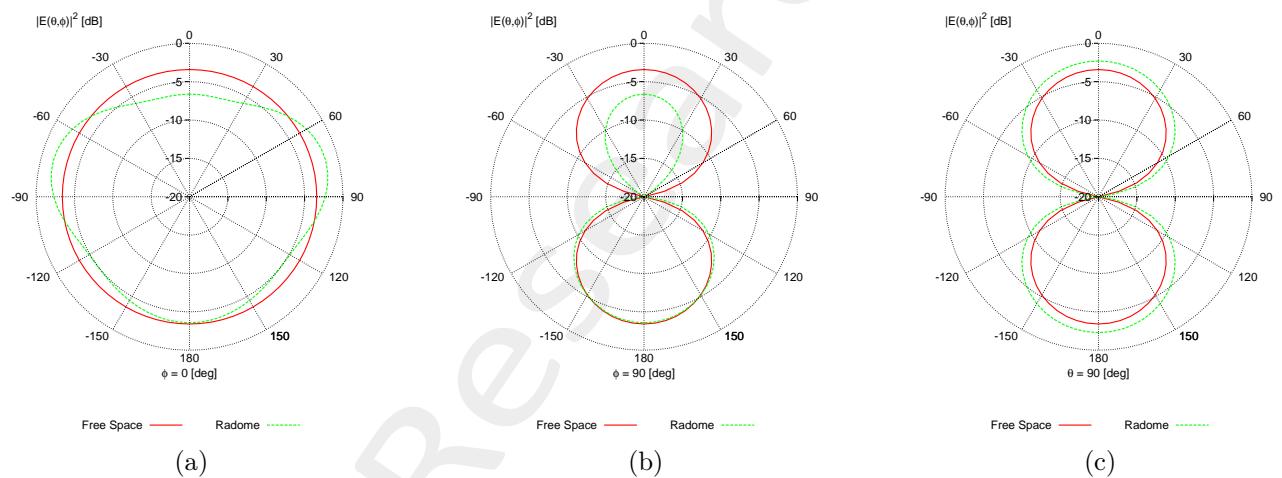


Figure 10: (*2-layer ogive radome*) – Polar plot of the power pattern of the antenna in free space and in presence of the radome: (a) $\phi = 0$ [deg] plane, (b) $\phi = 90$ [deg] plane and (c) $\theta = 90$ [deg] plane.

2.2.3 3-layers structure

Simulation Parameters

- Frequency: $f = 2.00 \times 10^8$ [Hz]
- Free space wavelength: $\lambda = 1.5$ [m]
- EM Solver: *FEM* (Finite Element Method)

Radome Parameters

- Number of layer: $N = 3$
- Radome length: $L = 1.75$ [m] $\simeq 1.17\lambda$
- Radome base diameter: $D = 1.6$ [m] $\simeq 1.07\lambda$
- Curvature type: $\nu = 1.449$ (tangent ogive)
- Radome thickness: $t = 1.19 \times 10^{-1}$ [m] $\simeq \frac{\lambda}{13}$
 - $t_1 = 4.64 \times 10^{-2}$ [m]
 - $t_2 = 3.79 \times 10^{-2}$ [m]
 - $t_3 = 2.76 \times 10^{-2}$ [m]
- Radome relative permittivity:
 - $\epsilon_1 = 4.80$
 - $\epsilon_2 = 2.84$
 - $\epsilon_3 = 4.89$

Antenna Parameters

- Dipole centered in $(x, y, z) = (0, 0, 0)$ and directed along \hat{y}
- Dipole length: $l_d = 0.75$ [m] $= \frac{\lambda}{2}$

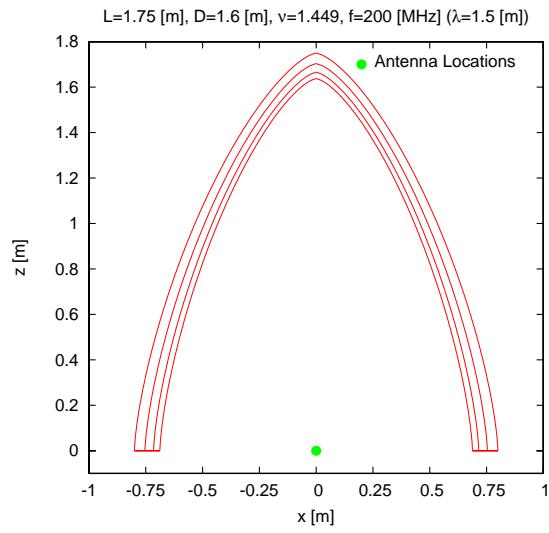


Figure 11: (3-layer ogive radome) – Geometry of the radome.

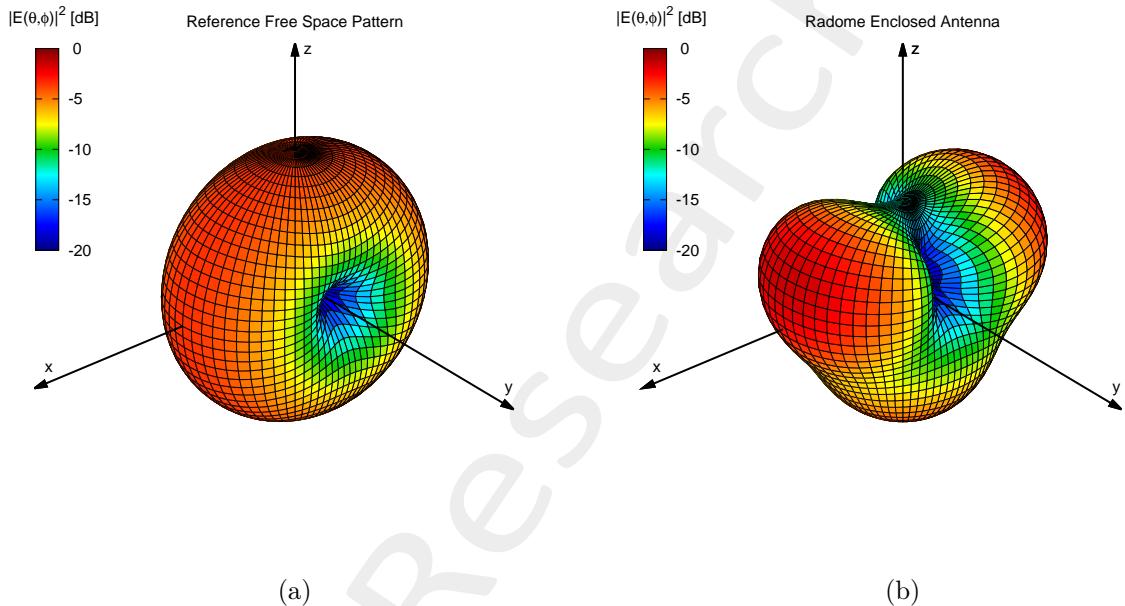


Figure 12: (3-layer ogive radome) – 3D plot of the power pattern of the antenna (a) in free space and (b) in presence of the radome.

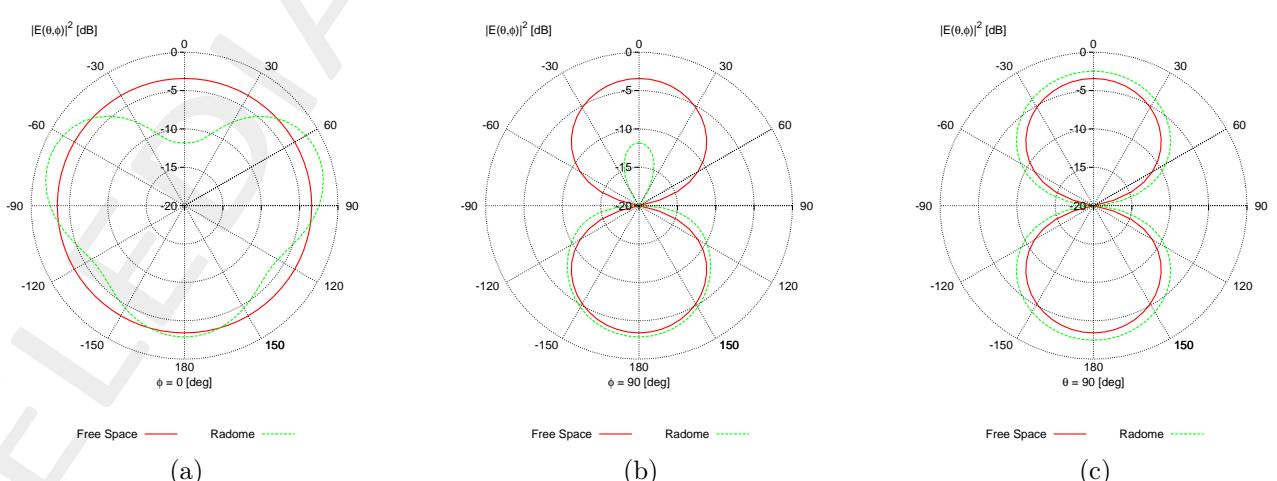


Figure 13: (3-layer ogive radome) – Polar plot of the power pattern of the antenna in free space and in presence of the radome: (a) $\phi = 0$ [deg] plane, (b) $\phi = 90$ [deg] plane and (c) $\theta = 0$ [deg] plane.

More information on the topics of this document can be found in the following list of references.

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