
An Inverse Source Approach for the Design of Reflectarrays: Proof-of-Concept

M. Salucci, G. Oliveri, and A. Massa

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Part I

Numerical Analysis

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1 Phase Range [-90:90] - Test Case 1 - 55x55 - Linear Polarization

1.1 K=800, P=100, I=100000

In the Fig. 1 is depicted the behaviour of the Cost Function varying the random seed. The best value of cost function is achieved by Seed=2 and is $\Phi = 5.089 \times 10^{-1}$.

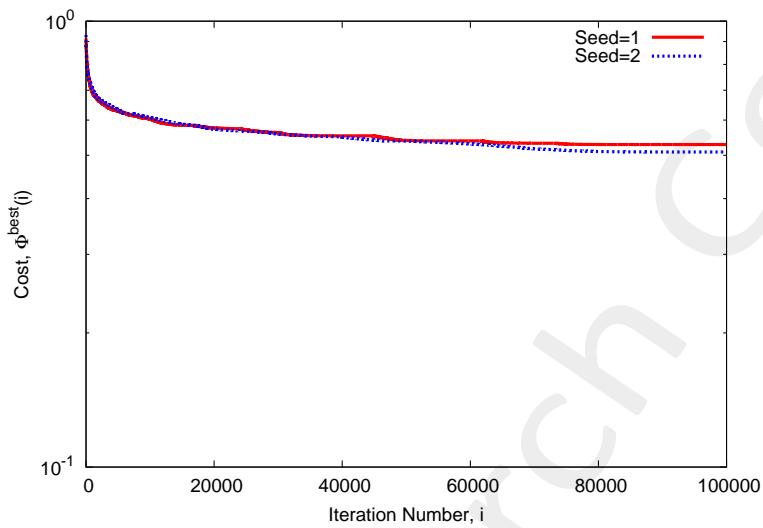


Figure 1: Cost Function behaviour at different random seed.

At this value of cost function the achieved performance on the Phase are showed in Fig. 2 and are numerically showed in table I.

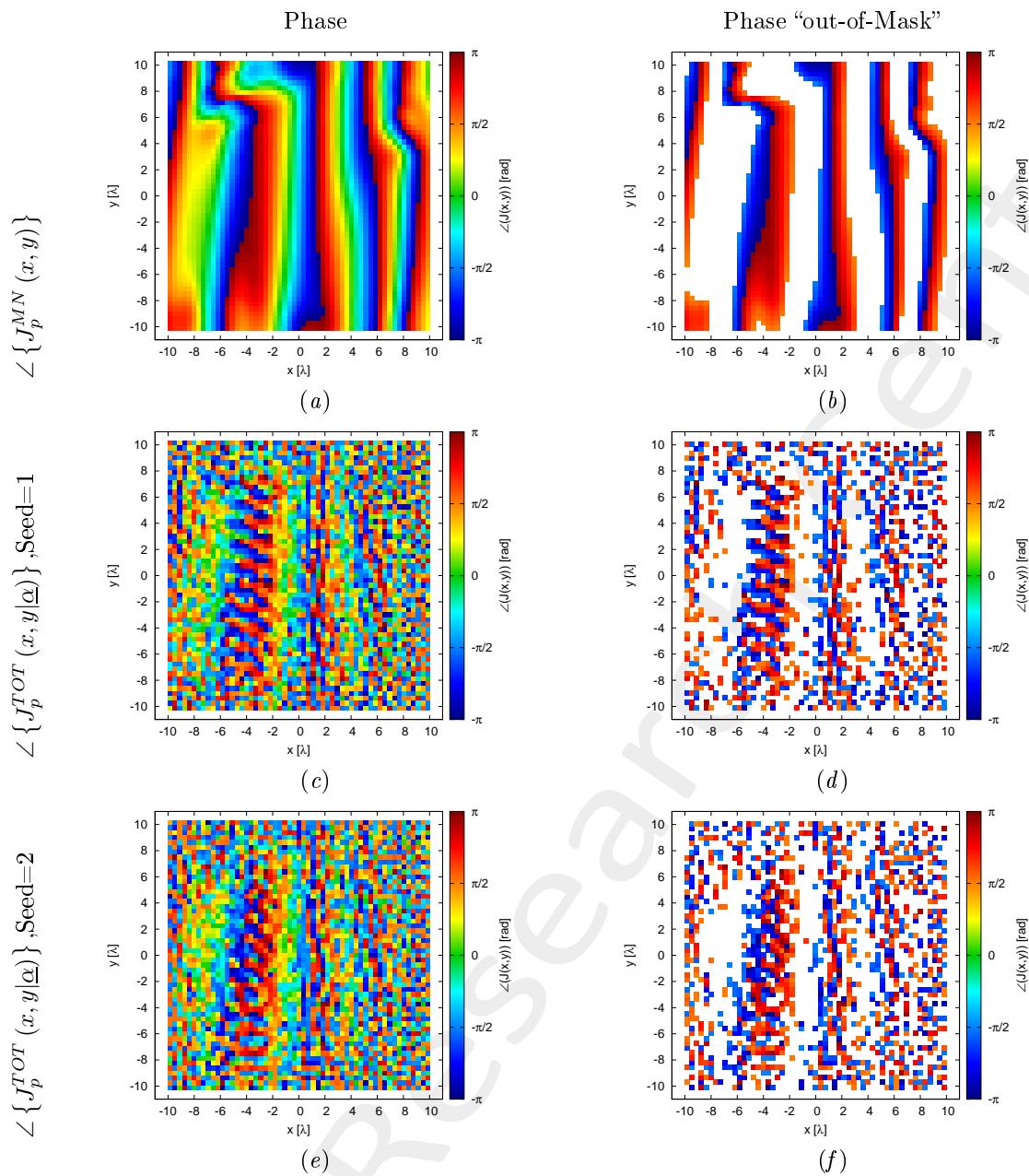


Figure 2: Phase (a)(c)(e) and value of the phase out of the minimization range (b)(d)(f) of the Minimum-Norm current ($\angle \{J_p^{MN}(x, y)\}$)(a)(b), of the total current for the random seed = 1(c)(d) and for the random seed = 2(e)(f).

Case	Φ	Number of value $> \phi_p^{MAX}(x, y)$	Number of value $< \phi_p^{MIN}(x, y)$	Phase Range		Time [s]
				Min [deg]	Max [deg]	
MN	1.0	899	663	-179.87	179.63	
Seed=1	5.288×10^{-1}	654	631	-178.70	179.57	2.13×10^5
Seed=2	5.089×10^{-1}	664	593	-179.05	178.27	2.20×10^5

Table I: Cost Function value and statistics about the result.

The verification of the radiated field is showed in Fig. 3 and numerically in table II.

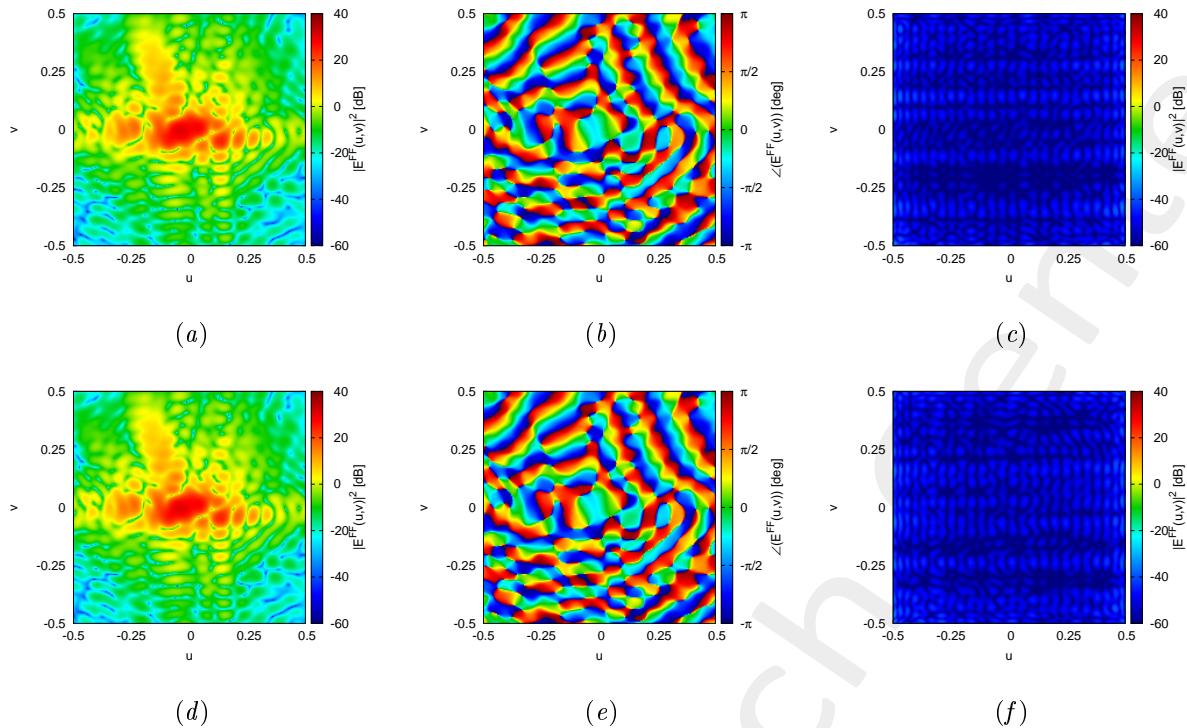


Figure 3: Magnitude (a)(d), Phase (b)(e) and Magnitude of the difference with respect to the original field (c)(f) of the seed=1 (a)(b)(c) and seed=2 (d)(e)(f).

Seed	ξ
1	1.91×10^{-3}
2	2.03×10^{-3}

Table II: Integral error of the difference between the original field and the one radiated by the total current.

1.2 K=800, P=200, I=100000

In the Fig. 4 is depicted the behaviour of the Cost Function varying the random seed. The best value of cost function is achieved by Seed=2 and is $\Phi = 5.004 \times 10^{-1}$.

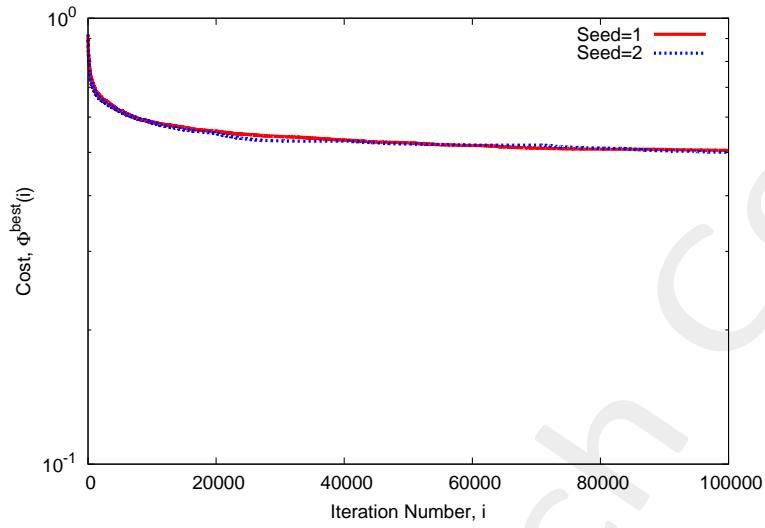


Figure 4: Cost Function behaviour at different random seed.

At this value of cost function the achieved performance on the Phase are showed in Fig. 5 and are numerically showed in table III.

$$\angle \{ J_p^{MN} (x, y) \} \quad \angle \{ J_p^{TOT} (x, y|_{\underline{\alpha}}) \}, \text{Seed=1}$$

$$\angle \{ J_p^{TOT} (x, y|_{\underline{\alpha}}) \}, \text{Seed=2}$$

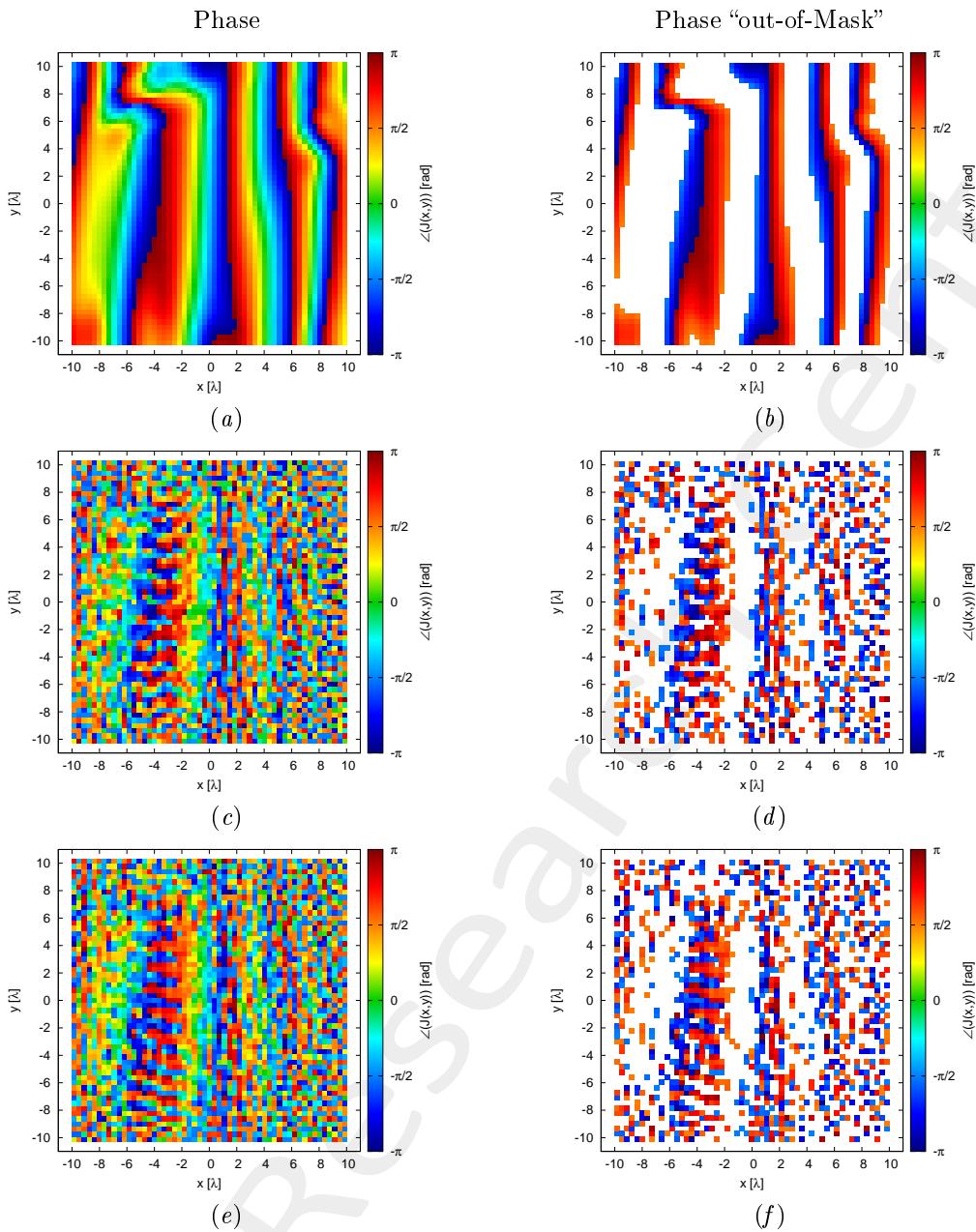


Figure 5: Phase (a)(c)(e) and value of the phase out of the minimization range (b)(d)(f) of the Minimum-Norm current ($\angle \{ J_p^{MN} (x, y) \}$)(a)(b), of the total current for the random seed = 1(c)(d) and for the random seed = 2(e)(f).

Case	Φ	Number of value $> \phi_p^{MAX} (x, y)$	Number of value $< \phi_p^{MIN} (x, y)$	Phase Range		Time [s]
				Min [deg]	Max [deg]	
MN	1.0	899	663	-179.87	179.63	
Seed=1	5.052×10^{-1}	645	588	-179.55	177.71	4.36×10^5
Seed=2	5.004×10^{-1}	664	581	-179.92	178.12	4.22×10^5

Table III: Cost Function value and statistics about the result.

The verification of the radiated field is showed in Fig. 6 and numerically in table IV.

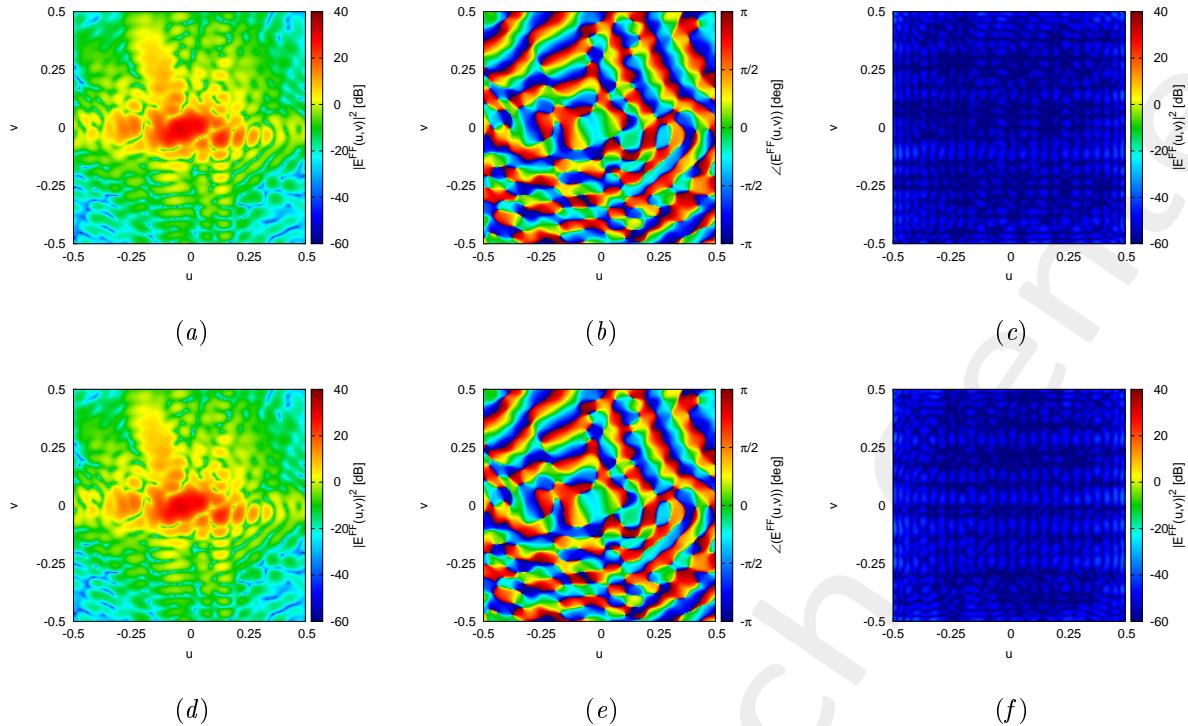


Figure 6: Magnitude (a)(d), Phase (b)(e) and Magnitude of the difference with respect to the original field (c)(f) of the seed=1 (a)(b)(c) and seed=2 (d)(e)(f).

Seed	ξ
1	1.73×10^{-3}
2	1.96×10^{-3}

Table IV: Integral error of the difference between the original field and the one radiated by the total current.

1.3 K=800, P=400, I=100000

In the Fig. 7 is depicted the behaviour of the Cost Function varying the random seed. The best value of cost function is achieved by Seed=2 and is $\Phi = 4.511 \times 10^{-1}$.

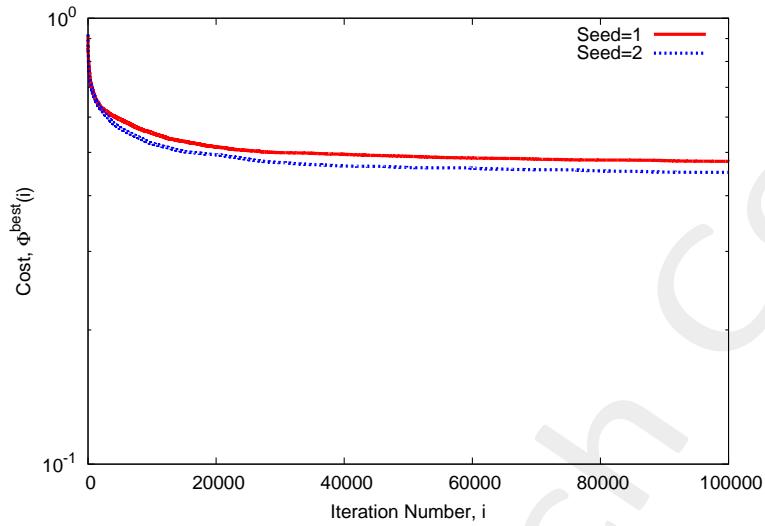


Figure 7: Cost Function behaviour at different random seed.

At this value of cost function the achieved performance on the Phase are showed in Fig. 8 and are numerically showed in table V.

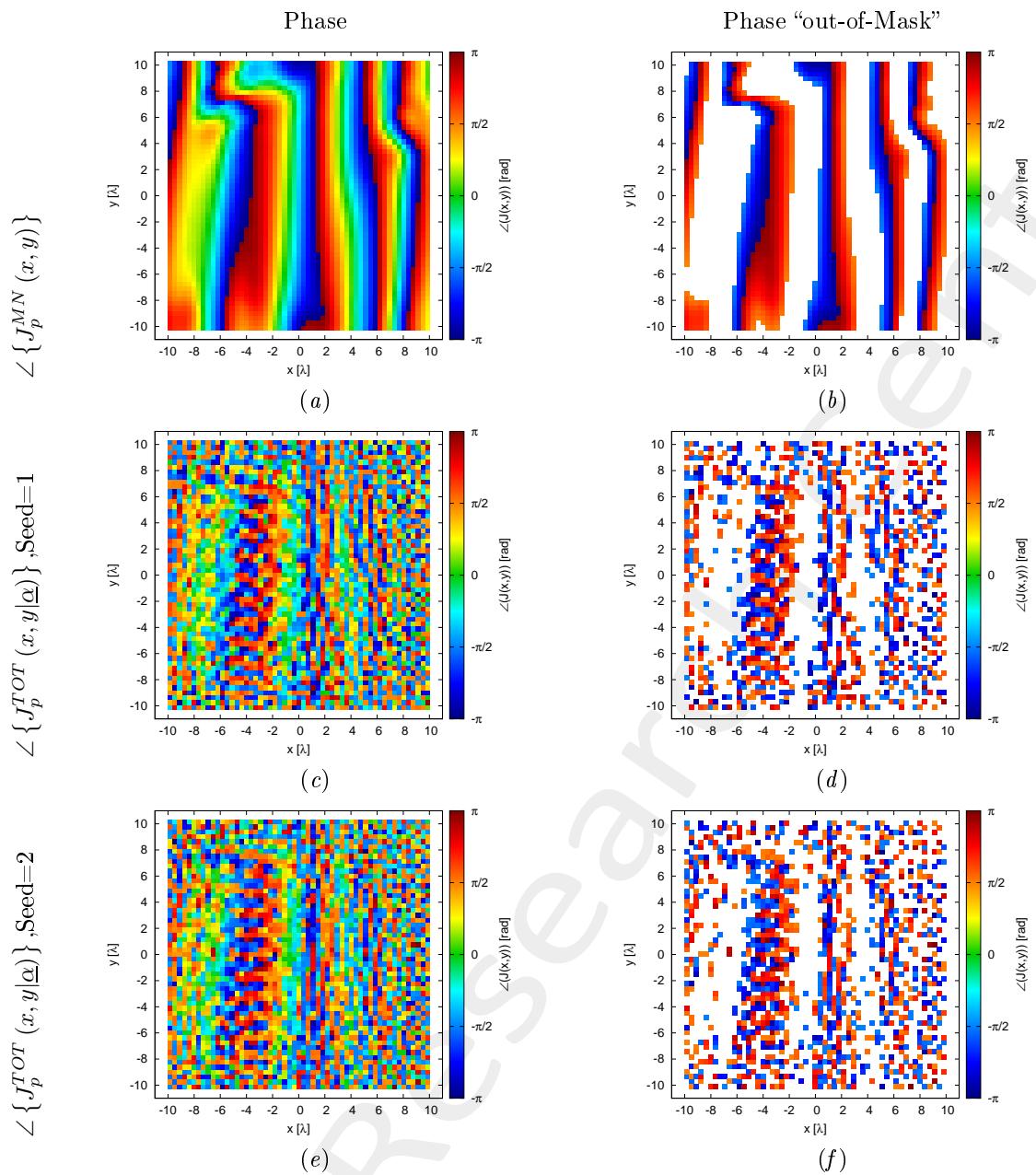


Figure 8: Phase (a)(c)(e) and value of the phase out of the minimization range (b)(d)(f) of the Minimum-Norm current ($\angle \{J_p^{MN}(x, y)\}$)(a)(b), of the total current for the random seed = 1(c)(d) and for the random seed = 2(e)(f).

Case	Φ	Number of value $> \phi_p^{MAX}(x, y)$	Number of value $< \phi_p^{MIN}(x, y)$	Phase Range		Time [s]
				Min [deg]	Max [deg]	
MN	1.0	899	663	-179.87	179.63	
Seed=1	4.774×10^{-1}	630	581	-179.33	177.82	8.44×10^5
Seed=2	4.511×10^{-1}	650	576	-179.58	178.25	8.85×10^3

Table V: Cost Function value and statistics about the result.

The verification of the radiated field is showed in Fig. 9 and numerically in table VI.

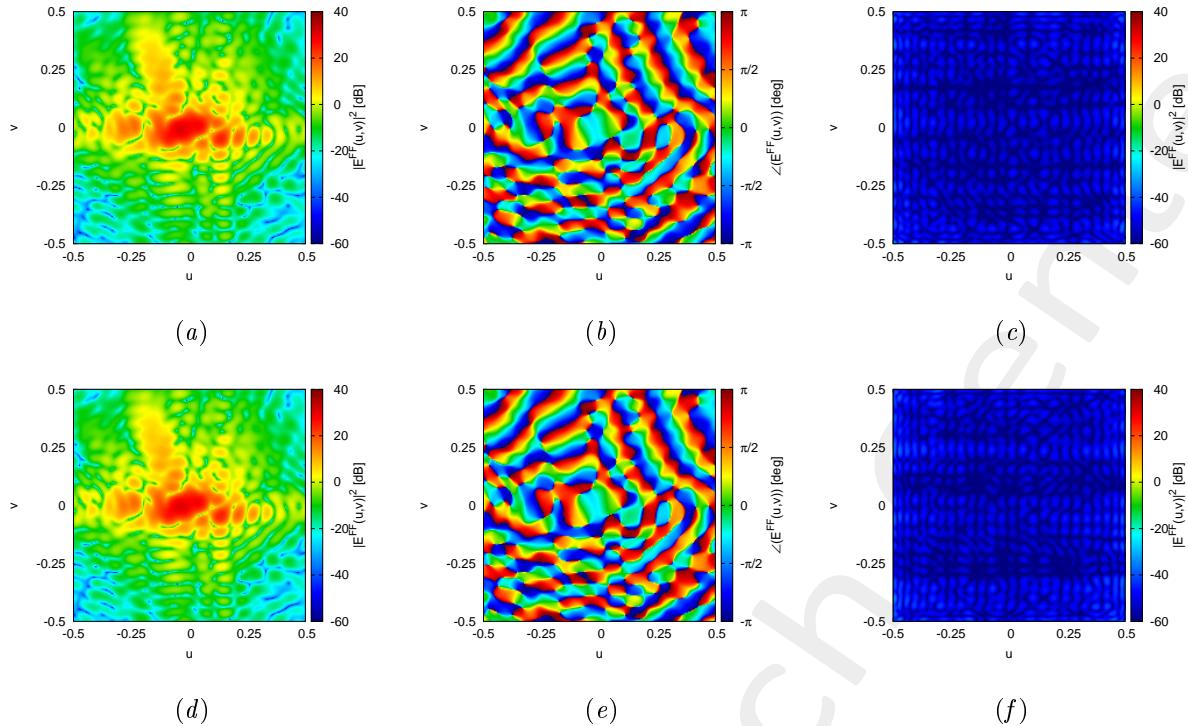


Figure 9: Magnitude (a)(d), Phase (b)(e) and Magnitude of the difference with respect to the original field (c)(f) of the seed=1 (a)(b)(c) and seed=2 (d)(e)(f).

Seed	ξ
1	1.79×10^{-3}
2	1.96×10^{-3}

Table VI: Integral error of the difference between the original field and the one radiated by the total current.

1.4 K=2000, P=250, I=100000

In the Fig. 10 is depicted the behaviour of the Cost Function varying the random seed. The best value of cost function is achieved by Seed=2 and is $\Phi = 4.460 \times 10^{-1}$.

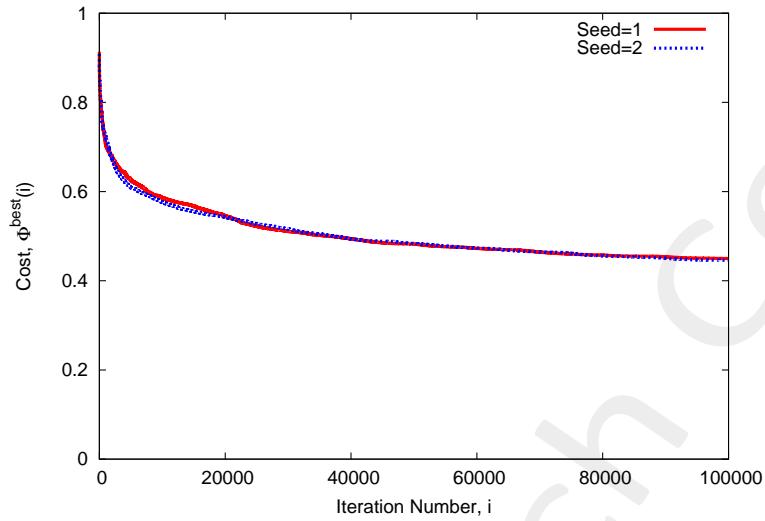


Figure 10: Cost Function behaviour at different random seed.

At this value of cost function the achieved performance on the Phase are showed in Fig. 11 and are numerically showed in table VII.

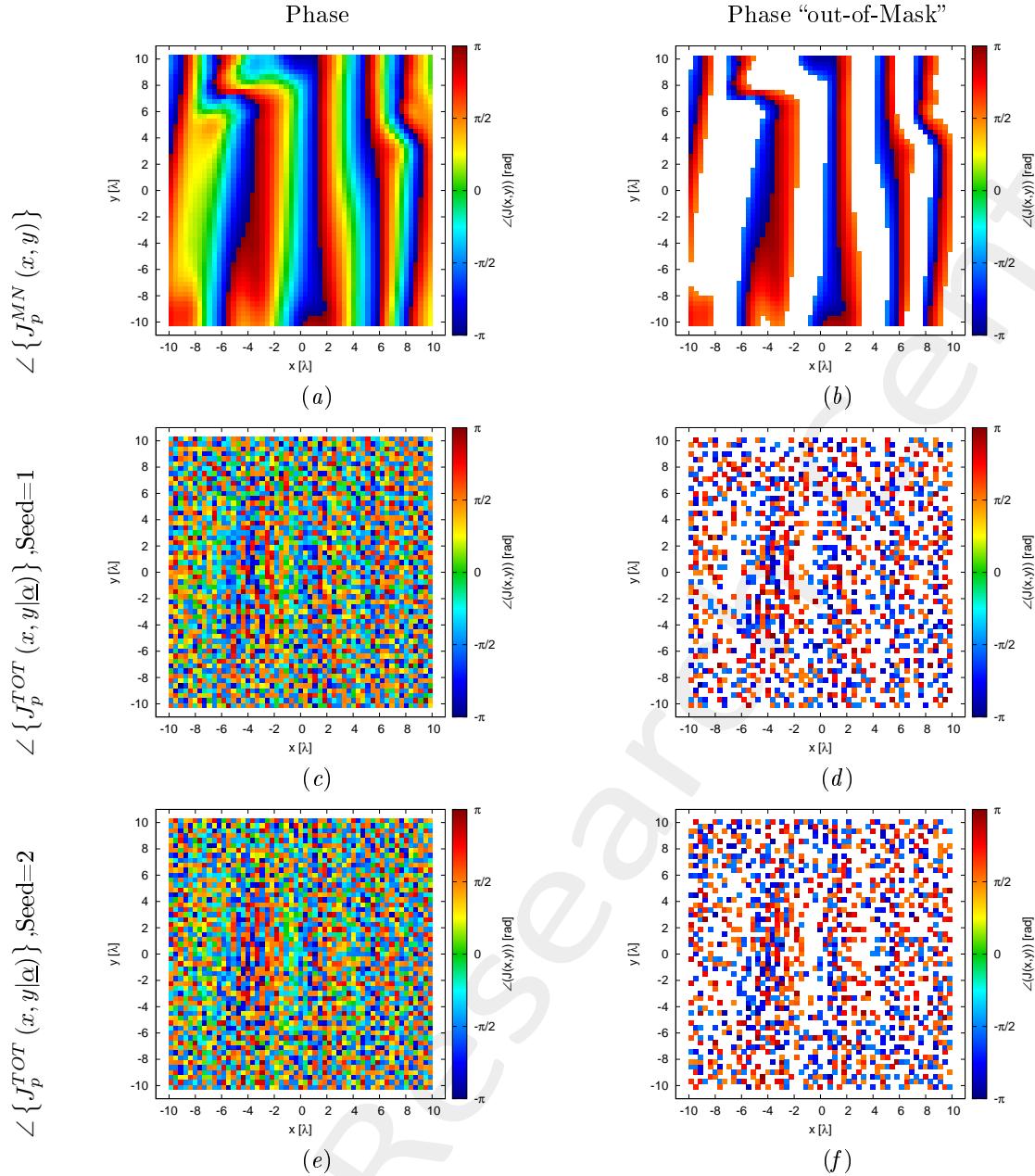


Figure 11: Phase (a)(c)(e) and value of the phase out of the minimization range (b)(d)(f) of the Minimum-Norm current ($\angle \{J_p^{MN}(x,y)\}$)(a)(b), of the total current for the random seed = 1(c)(d) and for the random seed = 2(e)(f).

Case	Φ	Number of value $> \phi_p^{MAX}(x,y)$	Number of value $< \phi_p^{MIN}(x,y)$	Phase Range		Time [s]	
				Min [deg]	Max [deg]	parallel	no-parallel
MN	1.0	899	663	-179.87	179.63		
Seed=1	4.499×10^{-1}	580	540	-179.27	179.81	2.73×10^5	1.42×10^6
Seed=2	4.460×10^{-1}	623	555	-177.26	179.33	2.73×10^5	1.42×10^6

Table VII: Cost Function value and statistics about the result.

The verification of the radiated field is showed in Fig. 12 and numerically in table VIII.

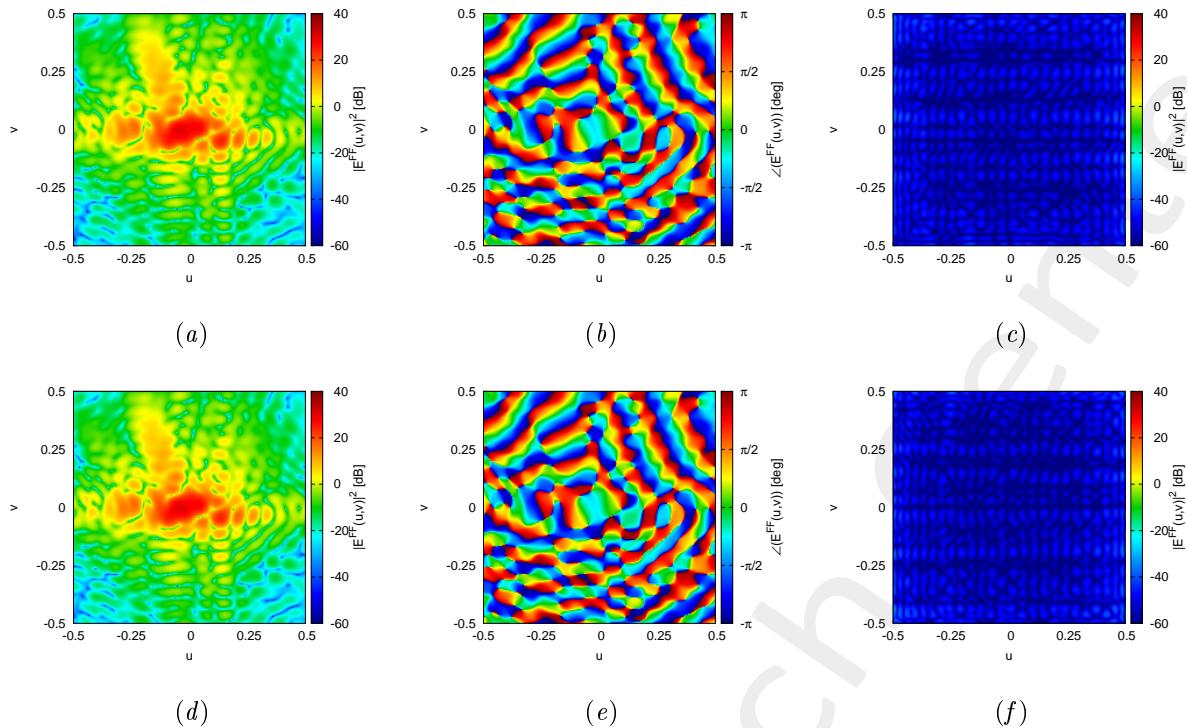


Figure 12: Magnitude (a)(d), Phase (b)(e) and Magnitude of the difference with respect to the original field (c)(f) of the seed=1 (a)(b)(c) and seed=2 (d)(e)(f).

Seed	ξ
1	1.96×10^{-3}
2	1.82×10^{-3}

Table VIII: Integral error of the difference between the original field and the one radiated by the total current.

1.5 K=2000, P=500, I=100000

In the Fig. 13 is depicted the behaviour of the Cost Function varying the random seed. The best value of cost function is achieved by Seed=2 and is $\Phi = 4.209 \times 10^{-1}$.

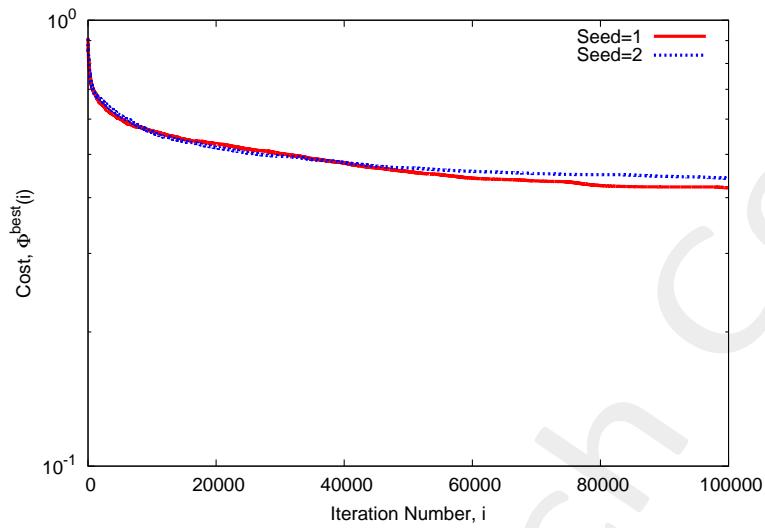


Figure 13: Cost Function behaviour at different random seed.

At this value of cost function the achieved performance on the Phase are showed in Fig. 14 and are numerically showed in table IX.

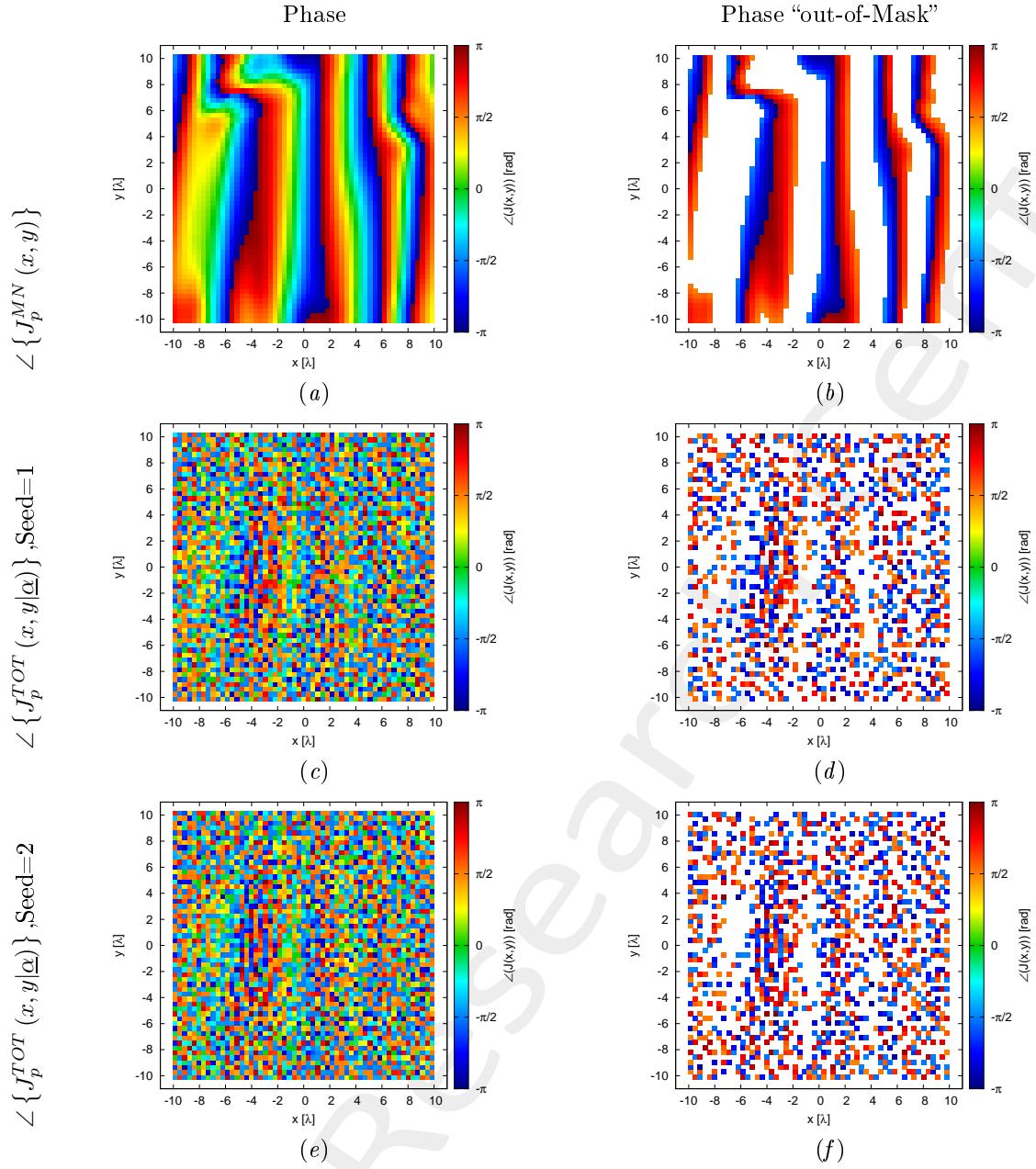


Figure 14: Phase (a)(c)(e) and value of the phase out of the minimization range (b)(d)(f) of the Minimum-Norm current ($\angle \{J_p^{MN}(x, y)\}\right)$ (a)(b), of the total current for the random seed = 1(c)(d) and for the random seed = 2(e)(f).

Case	Φ	Number of value $> \phi_p^{MAX}(x, y)$	Number of value $< \phi_p^{MIN}(x, y)$	Phase Range		Time [s]	
				Min [deg]	Max [deg]	parallel	no-parallel
MN	1.0	899	663	-179.87	179.63		
Seed=1	4.417×10^{-1}	587	546	-175.20	175.83	3.93×10^5	3.07×10^6
Seed=2	4.209×10^{-1}	568	540	-177.56	178.58	4.35×10^5	3.05×10^6

Table IX: Cost Function value and statistics about the result.

The verification of the radiated field is showed in Fig. 15 and numerically in table X.

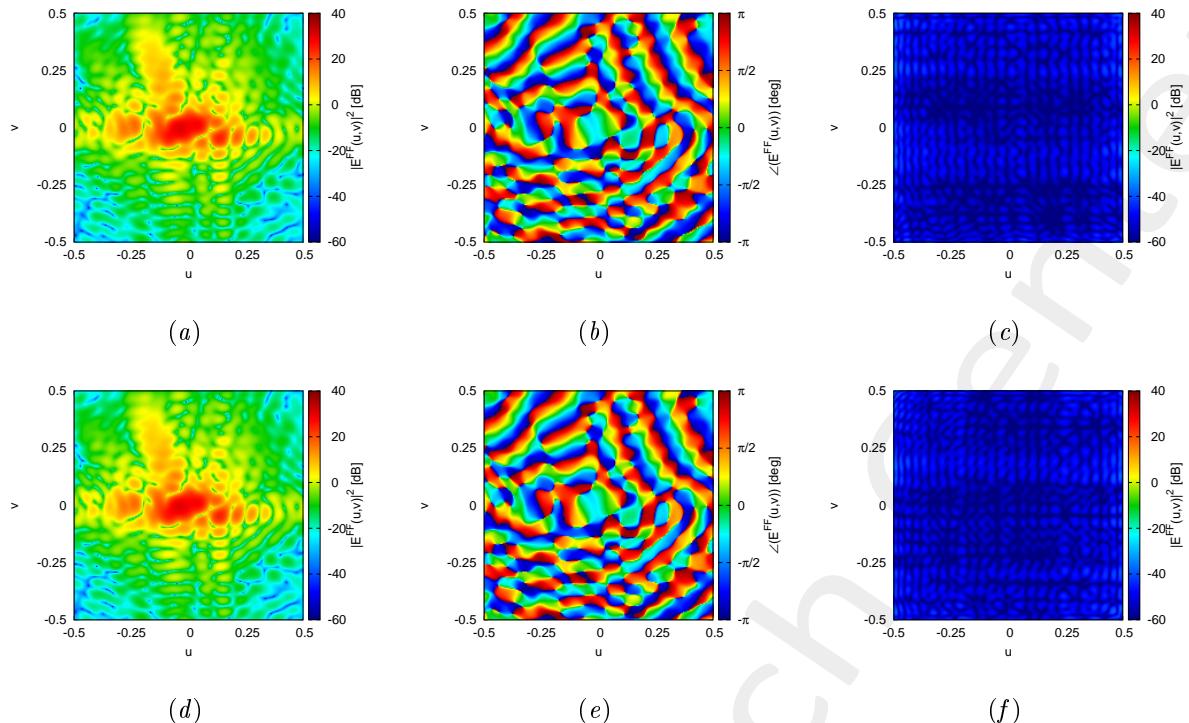


Figure 15: Magnitude (a)(d), Phase (b)(e) and Magnitude of the difference with respect to the original field (c)(f) of the seed=1 (a)(b)(c) and seed=2 (d)(e)(f).

Seed	ξ
1	1.88×10^{-3}
2	1.90×10^{-3}

Table X: Integral error of the difference between the original field and the one radiated by the total current.

1.6 K=2000, P=1000, I=100000

In the Fig. 16 is depicted the behaviour of the Cost Function varying the random seed. The best value of cost function is achieved by Seed=1 and is $\Phi = 3.953 \times 10^{-1}$.

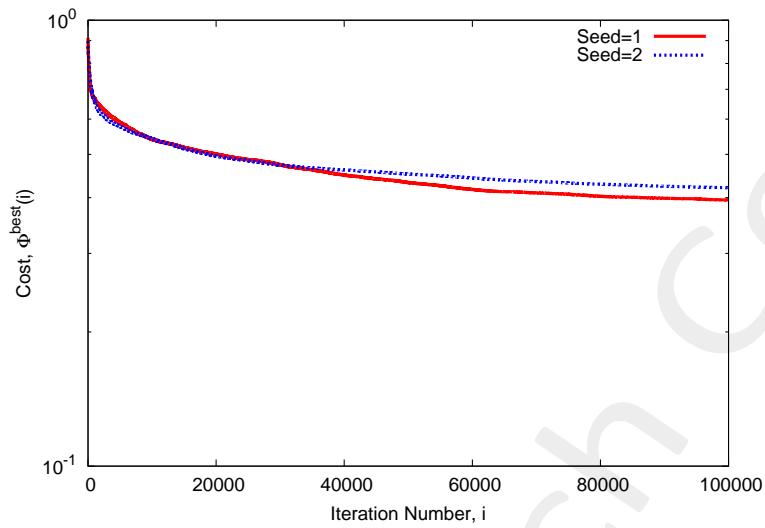


Figure 16: Cost Function behaviour at different random seed.

At this value of cost function the achieved performance on the Phase are showed in Fig. 17 and are numerically showed in table XI.

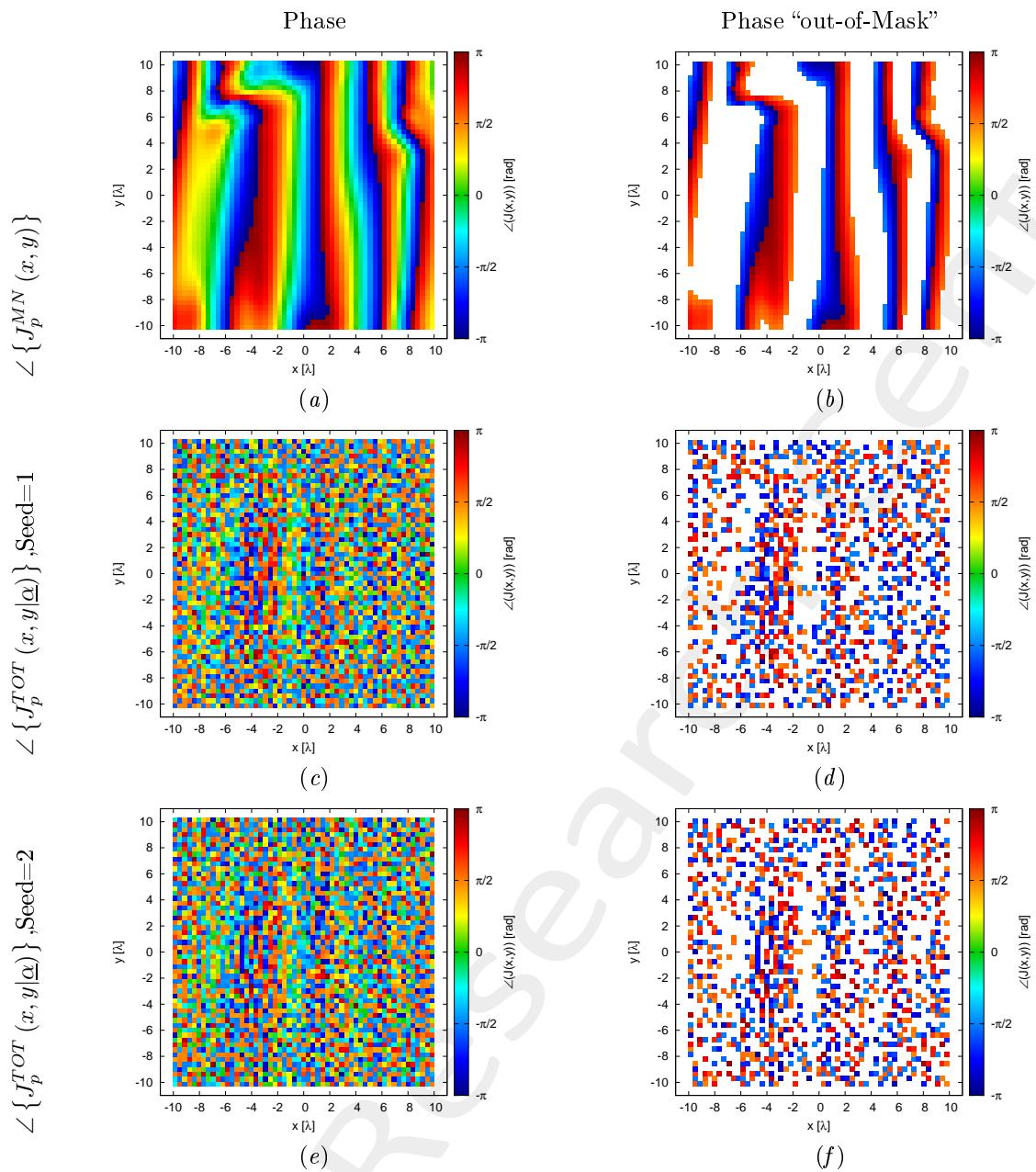


Figure 17: Phase (a)(c)(e) and value of the phase out of the minimization range (b)(d)(f) of the Minimum-Norm current ($\angle \{J_p^{MN}(x, y)\}\right)$ (a)(b), of the total current for the random seed = 1(c)(d) and for the random seed = 2(e)(f).

Case	Φ	Number of value $> \phi_p^{MAX}(x, y)$	Number of value $< \phi_p^{MIN}(x, y)$	Phase Range		Time [s]	
				Min [deg]	Max [deg]	parallel	no-parallel
MN	1.0	899	663	-179.87	179.63		
Seed=1	3.953×10^{-1}	557	544	-174.71	176.08	7.86×10^5	5.81×10^6
Seed=2	4.212×10^{-1}	561	509	-177.54	179.63	7.83×10^5	5.79×10^6

Table XI: Cost Function value and statistics about the result.

The verification of the radiated field is showed in Fig. 18 and numerically in table XII.

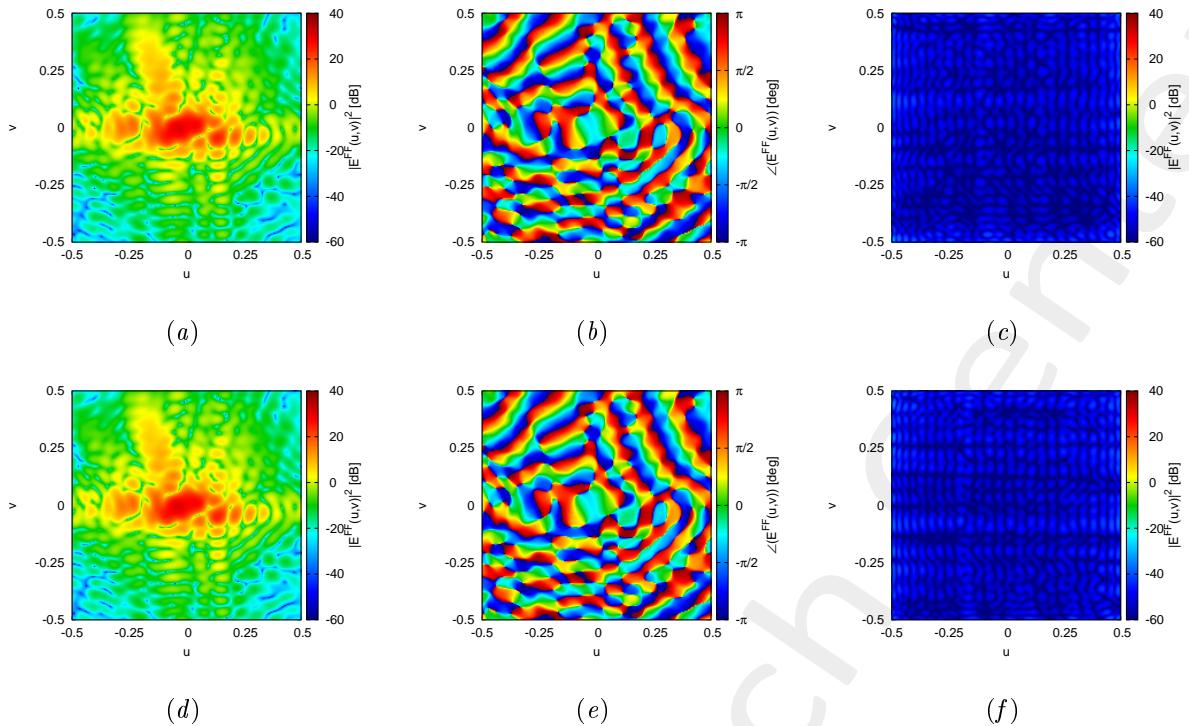


Figure 18: Magnitude (a)(d), Phase (b)(e) and Magnitude of the difference with respect to the original field (c)(f) of the seed=1 (a)(b)(c) and seed=2 (d)(e)(f).

Seed	ξ
1	1.88×10^{-3}
2	2.06×10^{-3}

Table XII: Integral error of the difference between the original field and the one radiated by the total current.

1.7 Comparison

In the Fig. 19 is depicted a summary of the behaviour of the cost function at different population and number of coefficient and seed.

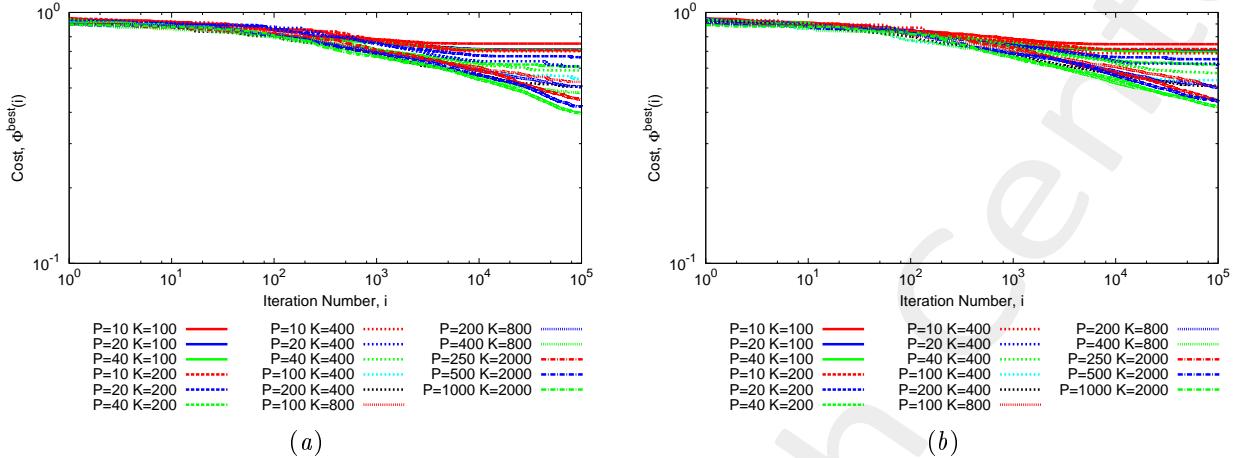


Figure 19: Cost Function behaviour at different population and number of coefficient for Seed=1(a) and Seed=2(b).

Observations:

- The cost function is lower for the test cases that involve the higher number of coefficient and the higher population;
- In Fig. 19 it is shown that the cost function is decreased increasing the number of iteration;
- the overall results are not yet satisfactory and we decide to increase the range of wanted phase to $[-135, 135]$.

1.8 K=2000, P=250, I=100000, 20 seed

In this Section is increased the number of seed in order to have a statistical behaviour of the test case with the population almost at 6% of the number of coefficients.

In the Fig. 20 is depicted the behaviour of the Cost Function varying the random seed. The best value of cost function is achieved by Seed=15 and is $\Phi = 4.187 \times 10^{-1}$.

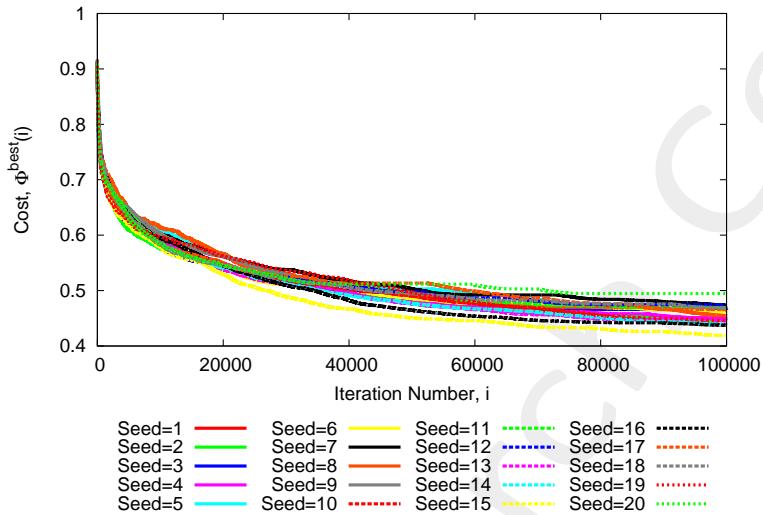


Figure 20: Cost Function behaviour at different random seed.

At this value of cost function the achieved performance on the Phase are showed in Fig. 21 and are numerically showed in table XIII.

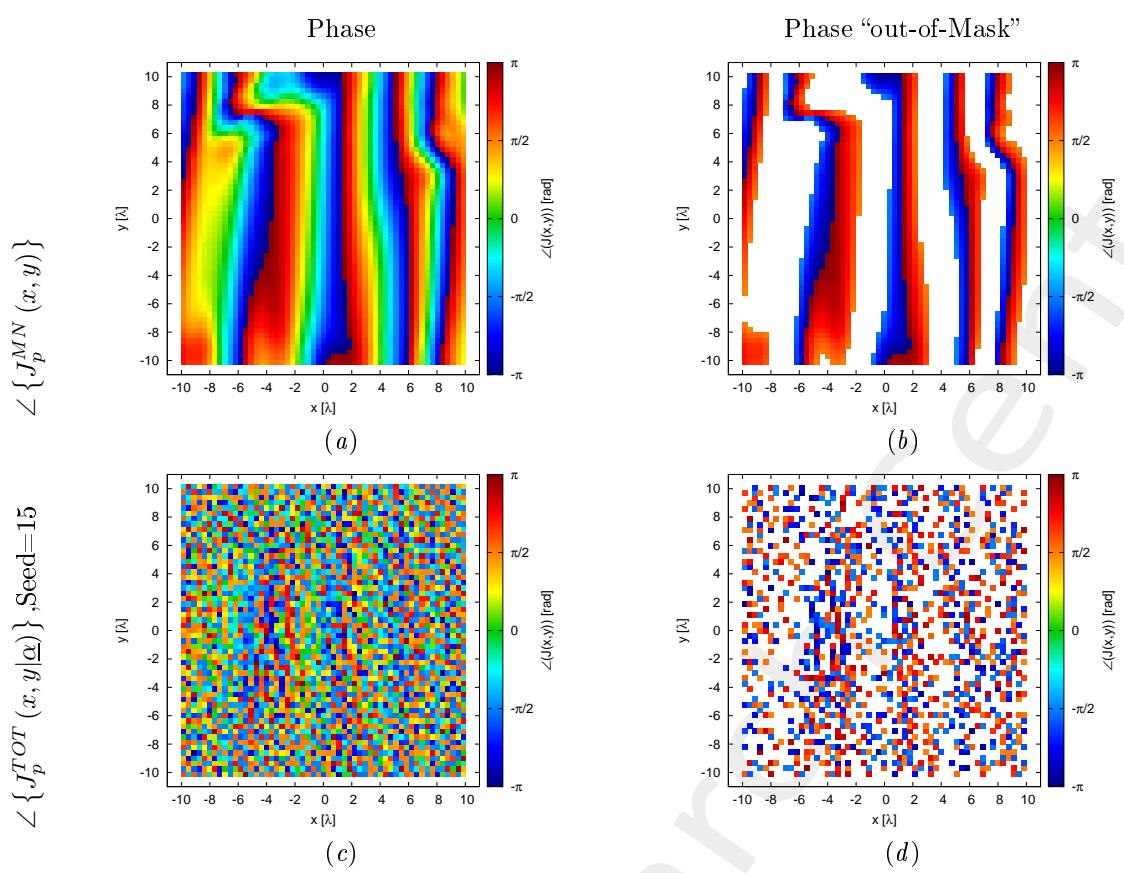


Figure 21: Phase (a)(c)(e) and value of the phase out of the minimization range (b)(d)(f) of the Minimum-Norm current ($\langle J_p^{MN}(x,y) \rangle$)(a)(b), of the total current for the best random seed = 15(c)(d).

Case	Φ	Number of value $> \phi_p^{MAX}(x,y)$	Number of value $< \phi_p^{MIN}(x,y)$	Phase Range		Time [s]	
				Min [deg]	Max [deg]	parallel	no-parallel
MN	1.0	899	663	-179.87	179.63		
Seed=15	4.187×10^{-1}	586	534	-177.74	179.57	1.98×10^5	1.42×10^6

Table XIII: Cost Function value and statistics about the result.

The verification of the radiated field is showed in Fig. 22 and numerically in table XIV.

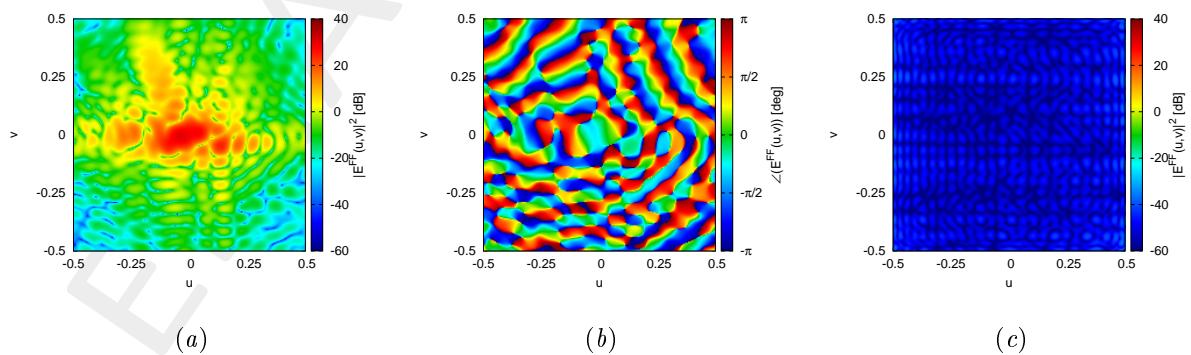


Figure 22: Magnitude (a), Phase (b) and Magnitude of the difference with respect to the original field (c) of the seed=15 (a)(b)(c).

Seed	ξ
15	2.25×10^{-3}

Table XIV: Integral error of the difference between the original field and the one radiated by the total current.

Moreover, the statistics on the cost function with different seed is described in Tab. XV

Average Φ	Minimum Φ	Maximum Φ	Median Φ	Variance Φ
4.551×10^{-1}	4.187×10^{-1}	4.946×10^{-1}	4.499×10^{-1}	2.988×10^{-4}

Table XV: Cost function statistics.

More information on the topics of this document can be found in the following list of references.

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