Detecting Failures in Planar Phased Arrays: a Bayesian Compressive Sensing Approach

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Abstract

In this work, the detection of failures in planar phased antenna arrays is dealt with. Towards this goal, the inverse problem at hand is formulated within a probabilistic framework and it is efficiently solved through a Bayesian compressive sensing (*BCS*) method. More in detail, starting from the knowledge of the failure-free (i.e., "gold") pattern and of that radiated by the antenna under test (*AUT*), the reconstruction of the faulty radiators is seen as a sparse retrieval problem whose solution does not require the compliance of the restricted isometry property (*RIP*) by the measurement operator. Some preliminary numerical results are shown to assess the effectiveness of the proposed array diagnosis tool.

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1 Mathematical Formulation

Let us consider a planar array of N elements located in $\mathbf{r}_n = (x_n, y_n)$, $n = 1, ..., N$ (Fig. 1).

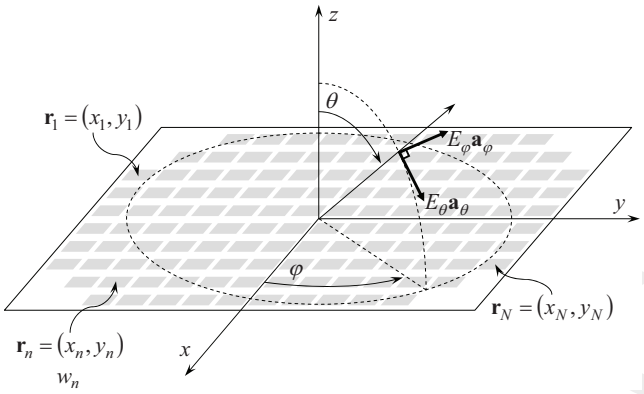

Figure 1: Geometry of the ideal planar array (*gold array*).

The far-field pattern radiated by the *failure-free* antenna (denoted in the following as the *gold array*) is given by

$$\mathbf{E}(u, v) = E_\theta(u, v)\,\mathbf{a}_\theta + E_\varphi(u, v)\,\mathbf{a}_\varphi \tag{1}$$

where $\mathbf{a}_\theta$ and $\mathbf{a}_\varphi$ are the spherical unit vectors, $E_{\theta/\varphi}$ is the electric field component along $\mathbf{a}_{\theta/\varphi}$, while $u = \sin\theta\cos\varphi$ and $v = \sin\theta\sin\varphi$ are direction cosines (Fig. 1). Assuming that $\mathbf{E}(u, v)$ is known at K sampling directions (u_k, v_k), $k = 1, ..., K$, the samples of each q-th pattern component in (1) (i.e., $\underline{E}_q = \{E_q(u_k, v_k)\,;\, k = 1, ..., K\}$, $q = \{\theta; \varphi\}$) are related to the *gold array* excitations $\underline{w} = \{w_n \in \mathbb{C};\, n = 1, ..., N\}$ by means of the following expression

$$\underline{E}_q = \underline{\underline{G}}_q\,\underline{w}; \quad q = \{\theta; \varphi\}\,. \tag{2}$$

In (2) $\underline{\underline{G}}_q$ is the $(K \times N)$ "measurement matrix" for the q-th field component, whose (k, n)-th entry is defined as

$$\begin{aligned} G_{q,kn} &= F_q^{(n)}(u_k, v_k)\, e^{j\frac{2\pi}{\lambda}(x_n u_k + y_n v_k)} \\ k &= 1, ..., K;\; n = 1, ..., N;\; q = \{\theta; \varphi\} \end{aligned} \tag{3}$$

λ being the free-space wavelength at the working frequency, while $F_q^{(n)}(u_k, v_k)$, $k = 1, ..., K$, are samples of the q-th component ($q = \{\theta; \varphi\}$) of the n-th *embedded* element pattern

$$\begin{aligned} \mathbf{F}^{(n)}(u, v) = F_\theta^{(n)}(u, v)\,\mathbf{a}_\theta + F_\varphi^{(n)}(u, v)\,\mathbf{a}_\varphi \\ n = 1, ..., N. \end{aligned} \tag{4}$$

More precisely, each $\mathbf{F}^{(n)}(u, v)$ $(n = 1, ..., N)$ corresponds to the far-field pattern radiated by the planar arrangement when setting its excitations to

$$w_p = \begin{cases} 1 & \text{if } p = n \\ 0 & \text{otherwise} \end{cases}; \quad p = 1, ..., N \tag{5}$$

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and loading all $(N-1)$ zero-excitation elements by the generator impedance [1] . It is worth pointing out that in case of *real* elementary radiators Eq. (2) provides an *exact* representation of the radiated far-field. As a matter of fact, the patterns $\mathbf{F}^{(n)}(u,\, v)$, $n = 1, ..., N$, do not only model the radiation behavior of the considered antennas (e.g., dipoles, patches, etc.), but they also describe all the effects of radiative mutual coupling (MC).

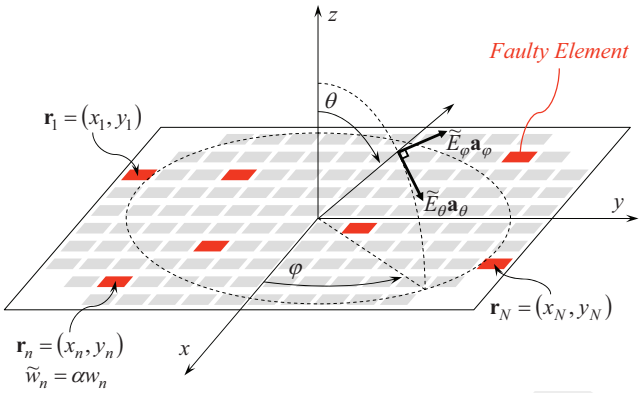

Figure 2: Geometry of the antenna under test (AUT).

Let us now consider that a small subset of $N_f = \Phi N << N$ elements ($0\% < \Phi << 100\%$ being the array *failure rate*) of the antenna under test (AUT) is failed (Fig. 2). Accordingly, indicating with $\underline{\widetilde{w}} = \{\widetilde{w}_n \in \mathbb{C};\, n = 1, ..., N\}$ the AUT excitations, it turns out that

$$\underline{\widetilde{w}} = \mathrm{diag}\left(\underline{\chi}\right) \underline{w} \tag{6}$$

where the entries of $\underline{\chi} = \{\chi_n;\, n = 1, ..., N\}$ are equal to $\chi_n = \kappa$ whether the n-th element is damaged [κ being the antenna *failure factor* describing a total ($\kappa = 0 \rightarrow \widetilde{w}_n = 0$) or a partial ($0 < \kappa < 1 \rightarrow \widetilde{w}_n < w_n$) failure], $\chi_n = 1$ otherwise (i.e., for the $(N - N_f)$ healthy radiators). Accordingly, since $\underline{\widetilde{w}} \neq \underline{w}$ for N_f entries, a deviation of the AUT far-field behavior from that of the *gold array* is observed, the (noisy) measured AUT pattern samples being equal to

$$\underline{\widetilde{E}}_q = \underline{\underline{G}}_q \underline{\widetilde{w}} + \underline{H}_q; \quad q = \{\theta;\, \varphi\} \tag{7}$$

where $\underline{\widetilde{E}}_q = \left\{\widetilde{E}_q(u_k,\, v_k);\, k = 1, ..., K\right\}$, while $\underline{H}_q = \{H_q(u_k,\, v_k);\, k = 1, ..., K\}$ contains the samples of an additive zero-mean Gaussian noise. Under these hypotheses, it is possible to define the *differential pattern* samples

$$\begin{aligned} \underline{\Delta E}_q &= \{\Delta E_q(u_k,\, v_k);\, k = 1, ..., K\} = \\ &= \left(\underline{E}_q - \underline{\widetilde{E}}_q\right) = \underline{\underline{G}}_q \underline{d} - \underline{H}_q; \quad q = \{\theta;\, \varphi\} \end{aligned} \tag{8}$$

corresponding to the far-field distribution generated by a fictitious *differential antenna* (Fig. 3) excited by the set of complex coefficients

$$\underline{d} = \{d_n = (w_n - \widetilde{w}_n);\, n = 1, ..., N\}. \tag{9}$$

[1] A local coordinate system centered on $\mathbf{r}_n$ is considered in the definition of each n-th embedded element pattern $\mathbf{F}^{(n)}(u,\, v)$ in (4), for $n = 1, ..., N$.

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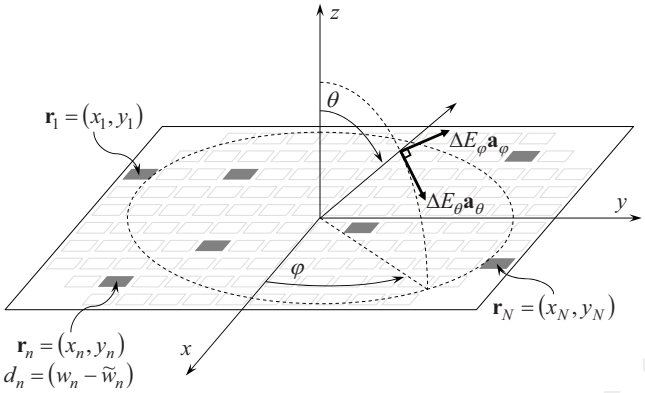

Figure 3: Geometry of the *differential* antenna.

It is worth observing that the *failure vector* $\underline{d} \in \mathbb{C}^N$ is intrinsically sparse since it is characterized by few non-null entries (i.e., those corresponding to the N_f failed radiators, for which $\tilde{w}_n \neq w_n \Rightarrow d_n \neq 0$), its ℓ_0-norm being equal to $\|\underline{d}\|_0 = N_f << N$ [3]. Accordingly, it is possible to exploit such an *a-priori* information by formulating the planar array diagnosis problem as follows:

> ***Planar Array Diagnosis Problem*** - Given the samples of the *differential pattern*, $\underline{\Delta E}_q$, $q = \{\theta; \varphi\}$, and of the *embedded* element patterns, $\underline{F}_q^{(n)} = \left\{ F_q^{(n)}(u_k, v_k); k = 1, ..., K \right\}$, $n = 1, ..., N$, $q = \{\theta; \varphi\}$, retrieve the unknown set $\underline{d}$ complying with
>
> $$\underline{\underline{G}}_q \underline{d} - \underline{\Delta E}_q = \underline{H}_q; \quad q = \{\theta; \varphi\} \tag{10}$$
>
> subject to $\underline{d}$ is *sparse*.

Given the linear nature of (10) as well as the sparseness of the unknown, a BCS solution approach is exploited to effectively solve the problem at hand without requiring - unlike standard CS-based approaches [5][6] - any compliance of the RIP condition by the two matrix operators $\underline{\underline{G}}_{\theta/\varphi}$, as detailed in the following.

Since the applicability of available BCS solvers is limited to real-valued linear formulations, in order to solve the planar array diagnosis problem Eq. (10) is rearranged as follows

$$\underline{\underline{\mathcal{G}}}_q \underline{\delta} - \underline{\Psi}_q = \underline{\mathcal{H}}_q; \quad q = \{\theta; \varphi\} \tag{11}$$

where $\underline{\delta} = [\Re\{\underline{d}\}, \Im\{\underline{d}\}]^T \in \mathbb{R}^{2N}$ comprises the real (i.e., $\Re\{\underline{d}\} = [\Re\{d_n\}; n = 1, ..., N]$) and imaginary ($\Im\{\underline{d}\} = [\Im\{d_n\}; n = 1, ..., N]$) parts of the unknown vector $\underline{d}$, while $\underline{\Psi}_q = \left[\Re\left\{\underline{\Delta E}_q\right\}, \Im\left\{\underline{\Delta E}_q\right\}\right]^T \in \mathbb{R}^{2K}$, $\underline{\mathcal{H}}_q = \left[\Re\left\{\underline{H}_q\right\}, \Im\left\{\underline{H}_q\right\}\right]^T \in \mathbb{R}^{2K}$, and

$$\underline{\underline{\mathcal{G}}}_q = \begin{bmatrix} \Re\left\{\underline{\underline{G}}_q\right\} & -\Im\left\{\underline{\underline{G}}_q\right\} \\ \Im\left\{\underline{\underline{G}}_q\right\} & \Re\left\{\underline{\underline{G}}_q\right\} \end{bmatrix} \in \mathbb{R}^{2K \times 2N}; \quad q = \{\theta; \varphi\} \tag{12}$$

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$.^T$ being the transpose operator. Accordingly, the problem at hand is formulated within the Bayesian framework, retrieving an estimation of $\underline{\delta}$ as follows

$$\widehat{\underline{\delta}} = \frac{1}{2} \sum_{q=\{\theta;\,\varphi\}} \left\{ \frac{1}{\hat{\sigma}_q^2} \left[\frac{\underline{\underline{\mathcal{G}}}_q^T \underline{\underline{\mathcal{G}}}_q}{\hat{\sigma}_q^2} + \mathrm{diag}\left(\hat{\underline{\zeta}}_q\right) \right]^{-1} \underline{\underline{\mathcal{G}}}_q^T \underline{\Psi}_q \right\} \tag{13}$$

where $\hat{\sigma}_q^2$ and $\hat{\underline{\zeta}}_q = \left\{ \hat{\zeta}_{q,n};\, n = 1, ..., 2N \right\}$ are respectively the estimated noise variance and the set of BCS hyper-parameters, determined by maximizing through a fast relevance vector machine (RVM) solver the BCS *marginal likelihood* function

$$\mathcal{L}^{ST-BCS}\left(\sigma_q^2,\, \underline{\zeta}_q\right) = -\frac{1}{2}\left[2K \log 2\pi + \log\left|\underline{\underline{\mathcal{W}}}_q\right| + \underline{\Psi}_q^T \underline{\underline{\mathcal{W}}}_q^{-1} \underline{\Psi}_q \right] \tag{14}$$

where

$$\underline{\underline{\mathcal{W}}}_q = \sigma_q^2 + \underline{\underline{\mathcal{G}}}_q \left[\mathrm{diag}\left(\underline{\zeta}_q\right) \right]^{-1} \underline{\underline{\mathcal{G}}}_q^T. \tag{15}$$

Finally, the set of complex retrieved failures $\widehat{\underline{d}} = \left\{ \widehat{d}_n;\, n = 1, ..., N \right\}$ is derived from (13) by letting

$$\widehat{d}_n = \left(\widehat{\delta}_n + j\widehat{\delta}_{n+N} \right);\quad n = 1, ..., N. \tag{16}$$

It is worth pointing out that the posterior probability $\mathcal{P}\left(\underline{\delta} | \underline{\Psi}_q\right)$ is modelled as a multi-variate normal distribution $\mathcal{N}\left(\underline{\mu}_q,\, \underline{\underline{\mathcal{S}}}_q\right)$ with mean vector $\underline{\mu}_q = \sigma_q^{-2} \underline{\underline{\mathcal{S}}}_q \underline{\underline{\mathcal{G}}}_q^T \underline{\Psi}_q$ and co-variance matrix

$$\underline{\underline{\mathcal{S}}}_q = \left[\mathrm{diag}\left(\underline{\zeta}_q\right) + \sigma_q^{-2} \underline{\underline{\mathcal{G}}}_q^T \underline{\underline{\mathcal{G}}}_q \right]^{-1}. \tag{17}$$

Accordingly, $\underline{\underline{\mathcal{S}}}_q$ provides useful information about the *confidence of* the BCS diagnosis, since its diagonal entries, $\mathcal{S}_{q,nn}$, are inversely proportional to the degree of *reliability* of $\widehat{\delta}_n$, for $n = 1, ..., 2N$. Consequently, it is possible to compute the *total confidence* of the BCS solution as

$$\Gamma = \frac{1}{4N} \sum_{q=\{\theta;\,\varphi\}} \sum_{n=1}^{2N} \left(\mathcal{S}_{q,nn}\right)^2 \tag{18}$$

lower values of Γ indicating a higher *reliability* of the BCS diagnosis.

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2 Calibration of the *BCS* Diagnosis Method

The goal of this Section is the calibration of the main parameters of the developed diagnosis method, i.e.,

1. The noise variance initialization value, η;
2. The ratio between measurements and number of elements in the array: $\nu = \frac{K}{N}$.

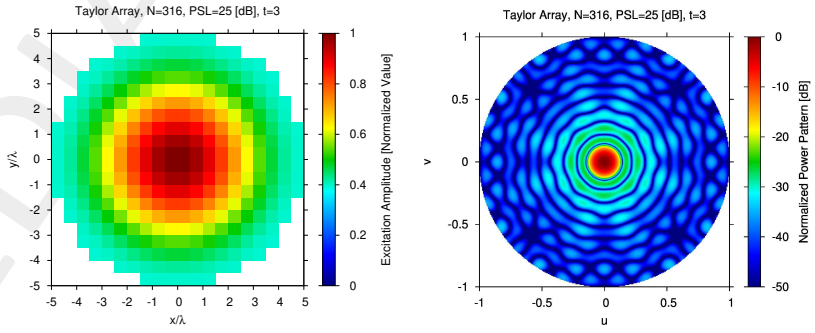
Throughout the whole numerical analysis, the "quality" of the array diagnosis will be quantitatively measured in terms of the normalized diagnosis error, defined as follows [3]

$$\xi = 100 \times \frac{1}{I} \sum_{i=1}^{I} \left[\frac{\sum_{n=1}^{N} \left| d_n^{(i)} - \widehat{d}_n^{(i)} \right|^2}{\sum_{n=1}^{N} \left| d_n^{(i)} \right|^2} \right] \tag{19}$$

where the apex i denotes the i-th ($i = 1, ..., I$; $I = 100$) realization of the process of randomly locating a fixed set of N_f failures (i.e., a value of the failure rate Φ) within the AUT.

2.1 Parameters

- Gold array
  - Total number of elements: $N = 316$;
  - Type of elements: isotropic/ideal[2]
  - Spacing along x and y: $d_x = d_y = 0.5$ [λ];
  - Excitation tapering: Taylor;
    - Radius: $R = 5$ [λ];
    - Transition index: $t = 3$;
    - Peak sidelobe level: $PSL = 25$ [dB]



(*a*) (*b*)

Figure 4: Sensitivity Analysis (Taylor Array, $N = 316$, $PSL = 25$ [dB], $t = 3$) - (*a*) Array excitations and (*b*) normalized power pattern of the expected array (gold antenna).

[2] In order to model *isotropic* radiators, let us assume that in (4) the embedded elements patterns are equal to $F_\theta^{(n)}(u, v) = 1$ and $F_\varphi^{(n)}(u, v) = 0$, for $n = 1, ..., N$.

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- Failed Array
  - Number of failures: $N_f = 13$;
  - Failure rate: $\Phi = \frac{N_f}{N} \simeq 4.\%$;
  - Failure factor: $\kappa = 0$ (total failures);
- Measurement set-up
  - Type of sampling: uniform sampling in the (u, v) plane;
  - Number of points along u and v: see table below (calibrated parameter);

K_u	K_v	K	$\nu = \frac{K}{N}$
18	18	216	0.68
19	19	253	0.80
20	20	276	0.87
21	21	317	1.00
22	22	332	1.05

Table 1: Sensitivity Analysis (Taylor Array, $N = 316$, $PSL = 25$ [dB], $t = 3$) - Number of sampling points in $-1.0 \leq u \leq 1.0$ (K_u) and in $-1.0 \leq v \leq 1.0$ (K_v), total number of sampling points falling in the visible range (K), and ratio between measurements and number of elements ($\nu = \frac{K}{N}$).

- *BCS* solver
  - Noise variance: $\eta = \left\{10^{-9}; 5 \times 10^{-9}; 10^{-8}; ...; 10^{1}\right\}$ (calibrated parameter);
  - Tolerance factor: $\iota = 10^{-8}$;
- Signal-to-Noise-Ratio: $SNR = \{20; 30; 40; 50; 60\}$.

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2.2 Results

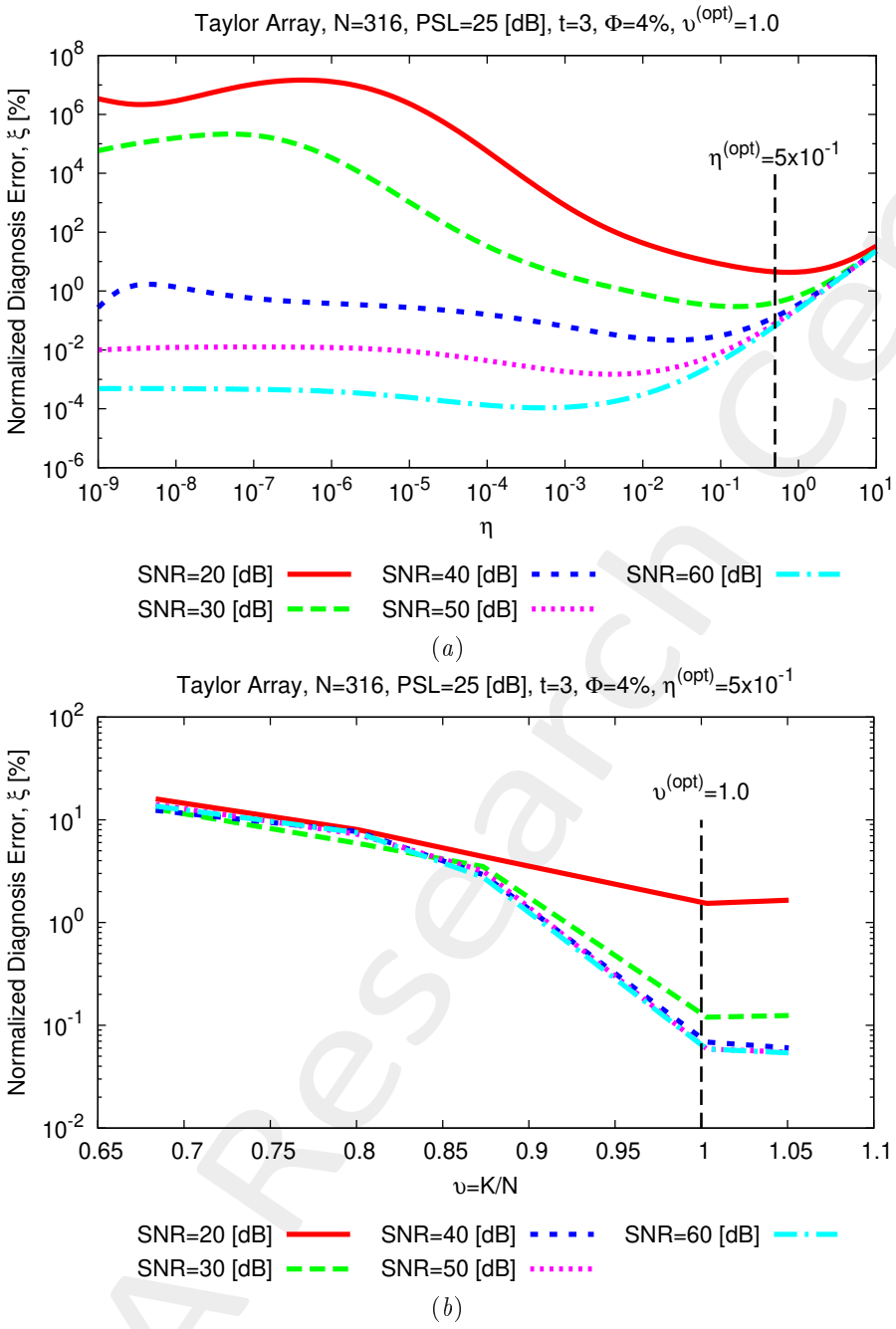

Figure 5: Sensitivity Analysis (Taylor Array, $N = 316$, $PSL = 25$ [dB], $t = 3$) - Behavior of the average diagnosis error versus (a) the BCS noise variance, η, and (b) the ratio $\nu = \frac{K}{N}$.

According to the obtained results, the optimal (η, ν) pair is

$$\left(\eta^{(opt)}, \nu^{(opt)}\right) = \left(5 \times 10^{-1}, 1.0\right) \tag{20}$$

Such a configuration will be considered for the successive numerical validations.

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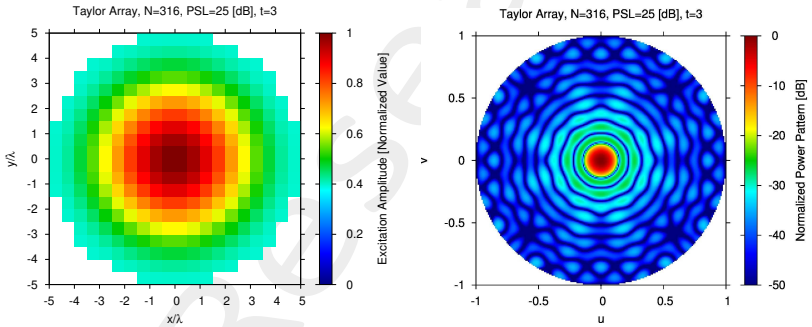
3 Preliminary Assessment: Analysis vs. Array Failure Rate

The purpose of this Section is to show a preliminary numerical assessment of the proposed BCS planar array diagnosis method. Towards this end, a variation of the number of failed elements (i.e., of the array failure factor, Φ) and of the SNR on measured far-field samples will be considered.

3.1 Parameters

- Gold array
  - Total number of elements: $N = 316$;
  - Type of elements: isotropic/ideal[3]
  - Spacing along x and y: $d_x = d_y = 0.5$ [λ];
  - Excitation tapering: Taylor;
    - Radius: $R = 5$ [λ];
    - Transition index: $t = 3$;
    - Peak sidelobe level: $PSL = 25$ [dB]



(a) (b)

Figure 6: (a) Array excitations and (b) normalized power pattern of the expected array (gold antenna).

- Failed Array
  - Failure factor: $\kappa = 0$ (total failures);
  - Failure rate: see table below;

[3] In order to model *isotropic* radiators, let us assume that in (4) the embedded elements patterns are equal to $F_\theta^{(n)}(u, v) = 1$ and $F_\varphi^{(n)}(u, v) = 0$, for $n = 1, ..., N$.

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N_f	$\Phi = \frac{N_f}{N}$
3	1%
6	2%
13	4%
25	8%
32	10%
38	12%
51	16%
63	20%

Table 2: Number of failures (N_f) and corresponding failure rate ($\Phi = \frac{N_f}{N}$).

- Measurement set-up
  - Type of sampling: uniform sampling in the (u, v) plane;
  - Number of points along u and v: $K_u = K_v = 21$;
  - Number of points in the visible range: $K = 317$;
  - Ratio between measurements and number of elements: $\nu = \frac{K}{N} \simeq 1.0$ ($\nu^{(opt)}$);
- *BCS* solver
  - Noise variance: $\eta = 5 \times 10^{-1}$ ($\eta^{(opt)}$);
  - Tolerance factor: $\iota = 10^{-8}$;
- Signal-to-Noise-Ratio: $SNR = \{10; 20; ...; 100\}$.

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3.2 Results

$\Phi = \frac{N_f}{N} = 1\%$ $(N_f = 3)$ - **Best and Worst** BCS **Reconstructions**

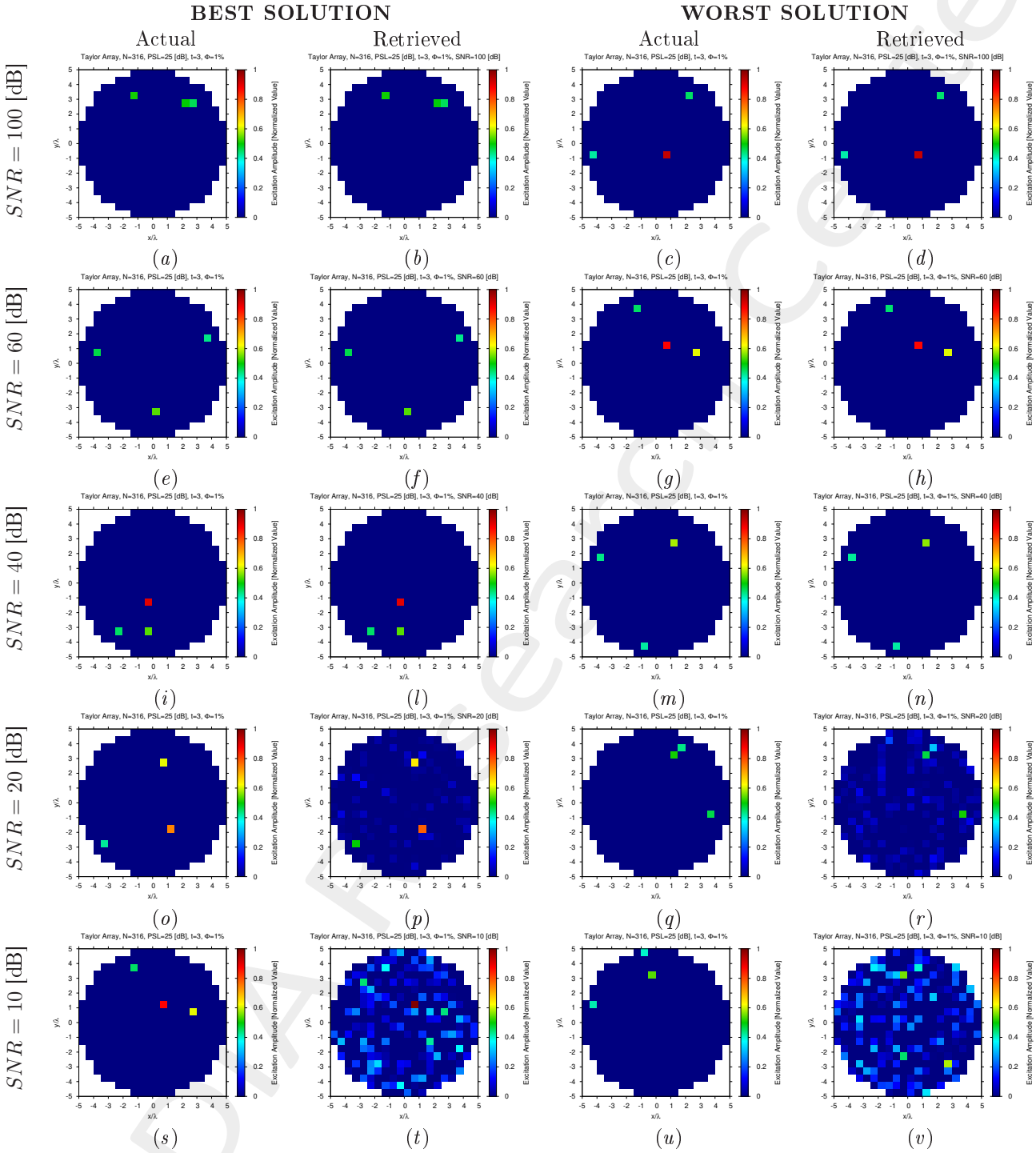

Figure 7: Taylor Array ($N = 316$, $PSL = 25$ [dB], $t = 3$, $\Phi = 1\%$) - Best and worst reconstructions under several SNR values.

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$\Phi = \frac{N_f}{N} = 2\%$ $(N_f = 6)$ - **Best and Worst** BCS **Reconstructions**

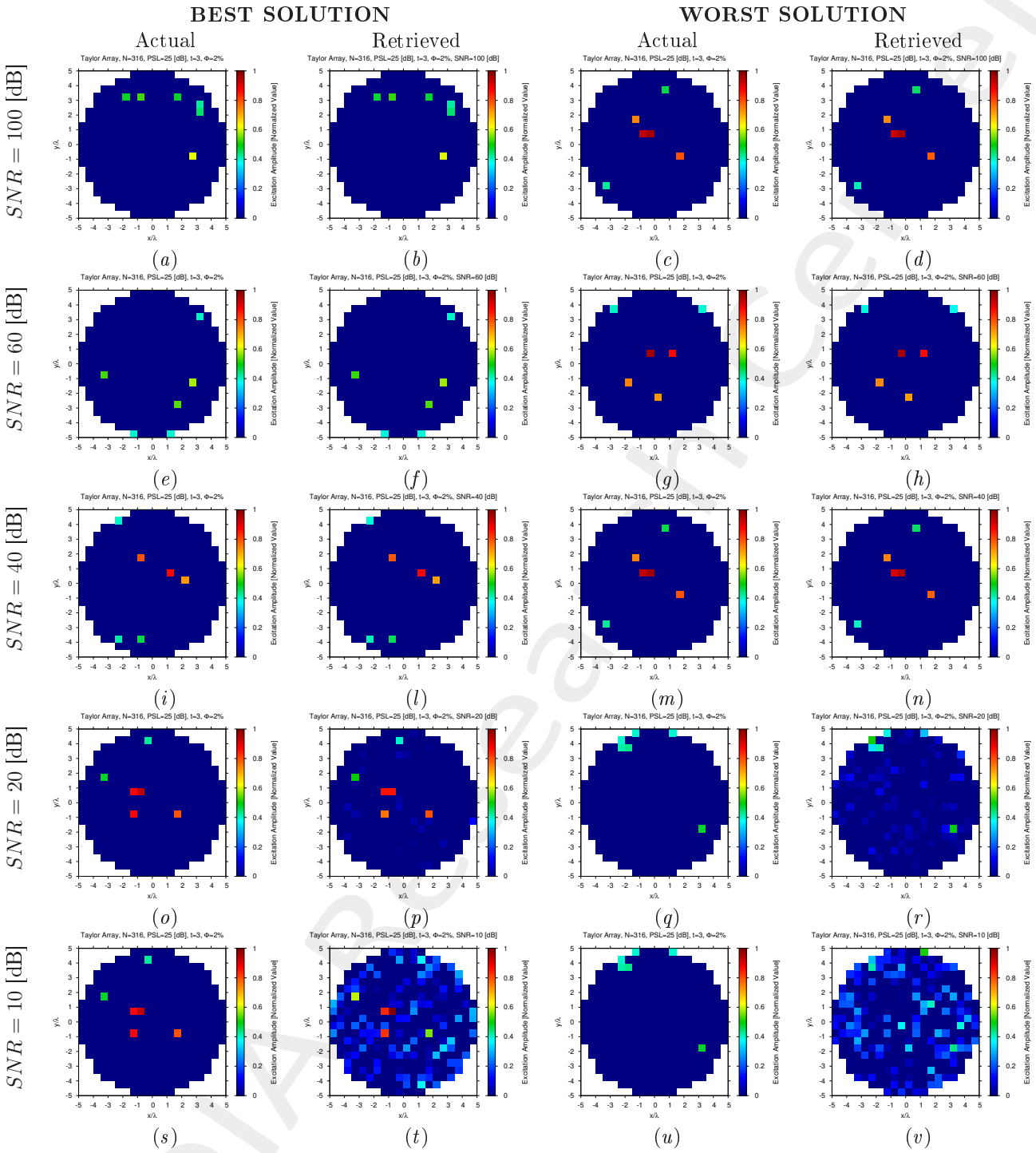

Figure 8: Taylor Array ($N = 316$, $PSL = 25$ [dB], $t = 3$, $\Phi = 2\%$) - Best and worst reconstructions under several SNR values.

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$\Phi = \frac{N_f}{N} = 4\%$ ($N_f = 13$) - **Best and Worst** BCS **Reconstructions**

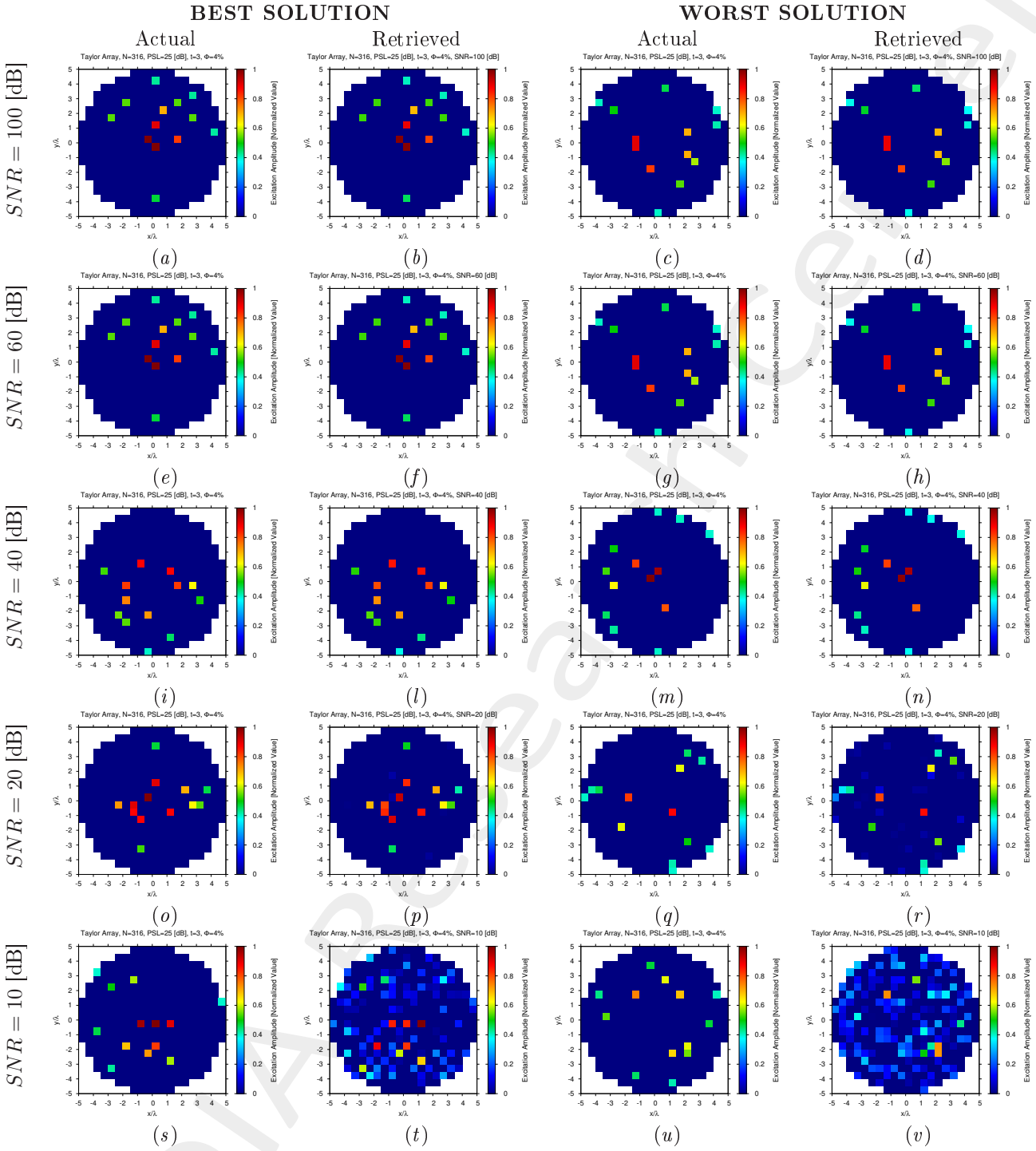

Figure 9: Taylor Array ($N = 316$, $PSL = 25$ [dB], $t = 3$, $\Phi = 4\%$) - Best and worst reconstructions under several SNR values.

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$\Phi = \frac{N_f}{N} = 8\%$ $(N_f = 25)$ - **Best and Worst** BCS **Reconstructions**

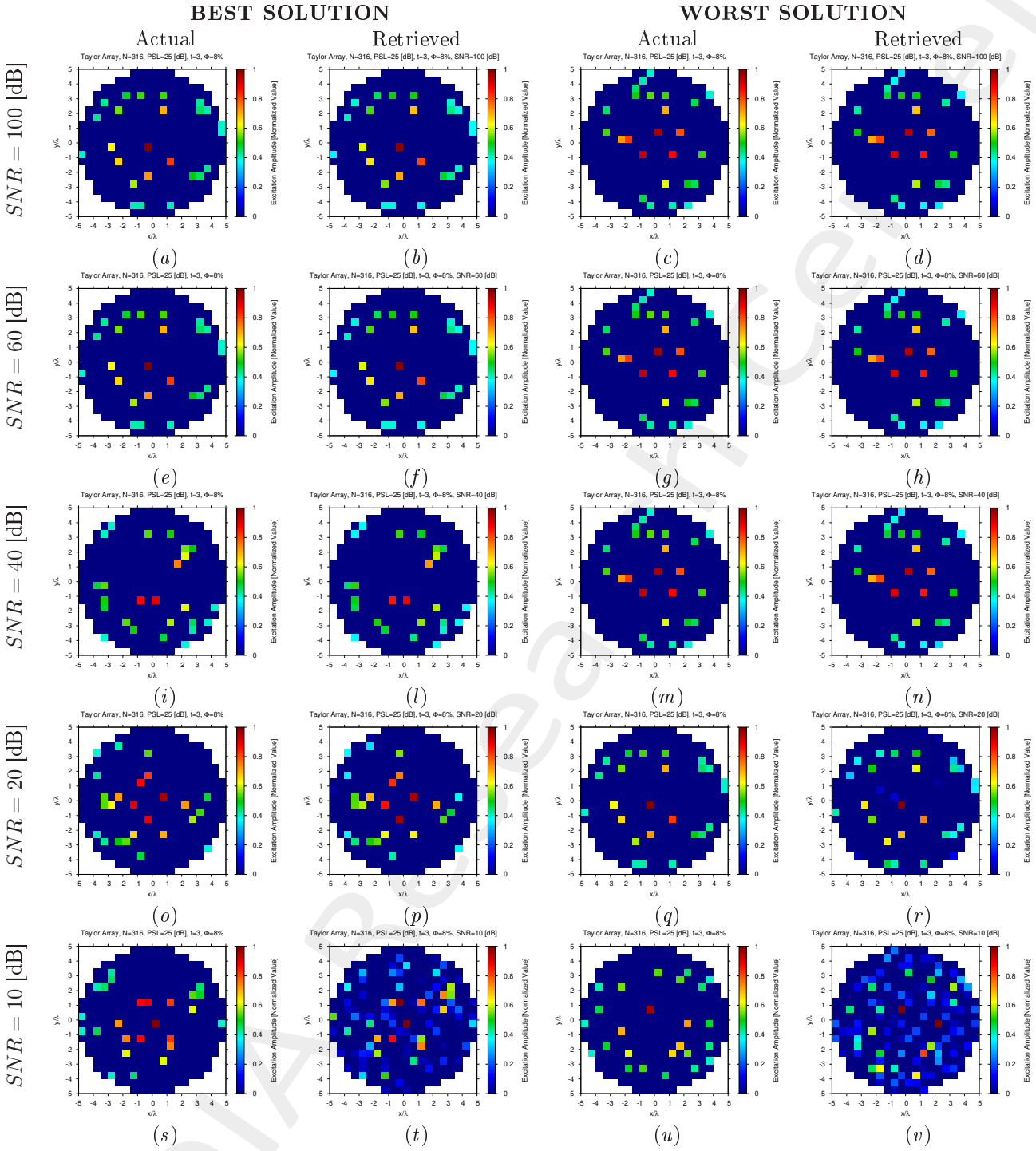

Figure 10: Taylor Array ($N = 316$, $PSL = 25$ [dB], $t = 3$, $\Phi = 8\%$) - Best and worst reconstructions under several SNR values.

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$\Phi = \frac{N_f}{N} = 10\%$ $(N_f = 32)$ - **Best and Worst** BCS **Reconstructions**

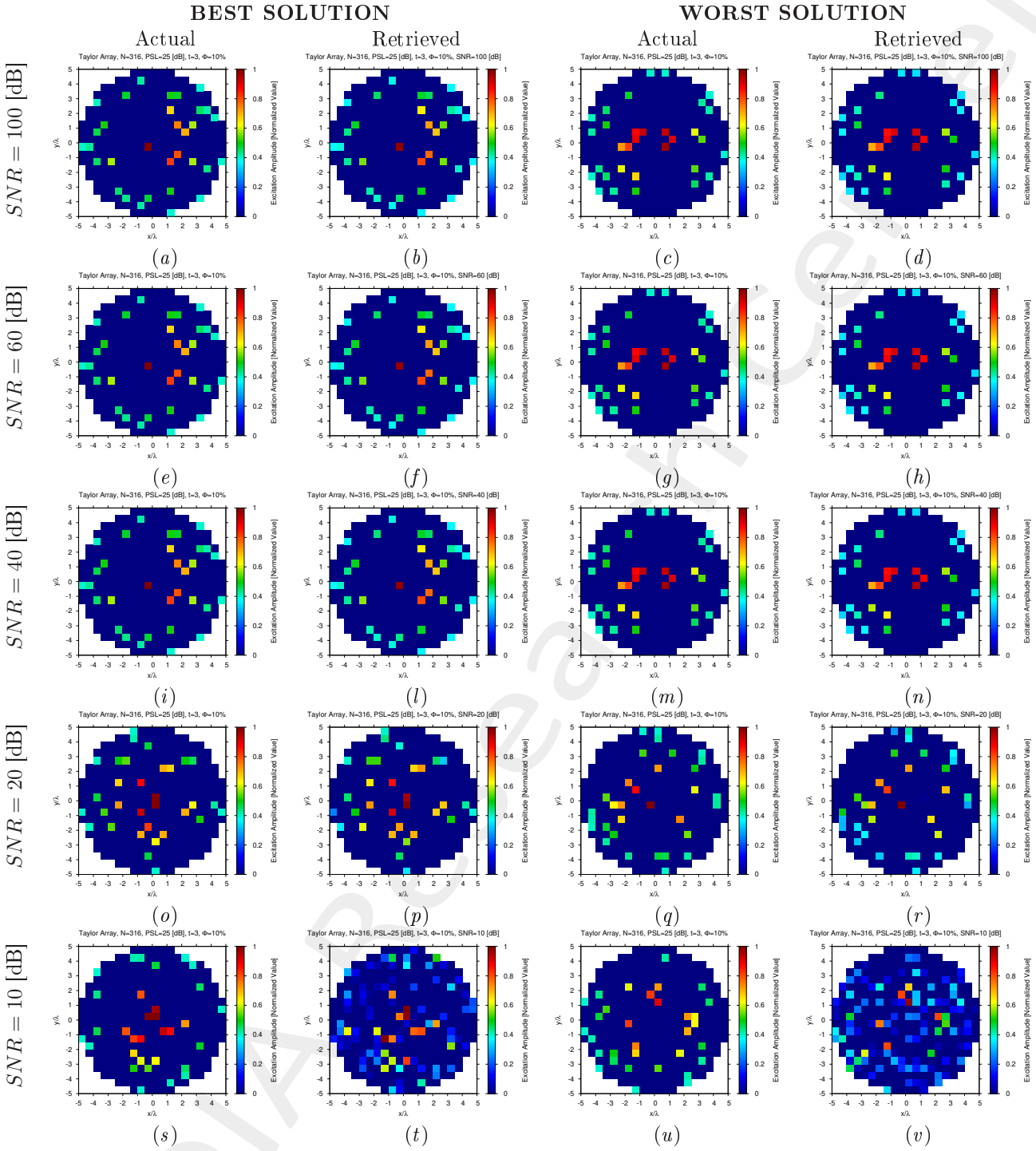

Figure 11: Taylor Array ($N = 316$, $PSL = 25$ [dB], $t = 3$, $\Phi = 10\%$) - Best and worst reconstructions under several SNR values.

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$\Phi = \frac{N_f}{N} = 12\%$ $(N_f = 38)$ - **Best and Worst** BCS **Reconstructions**

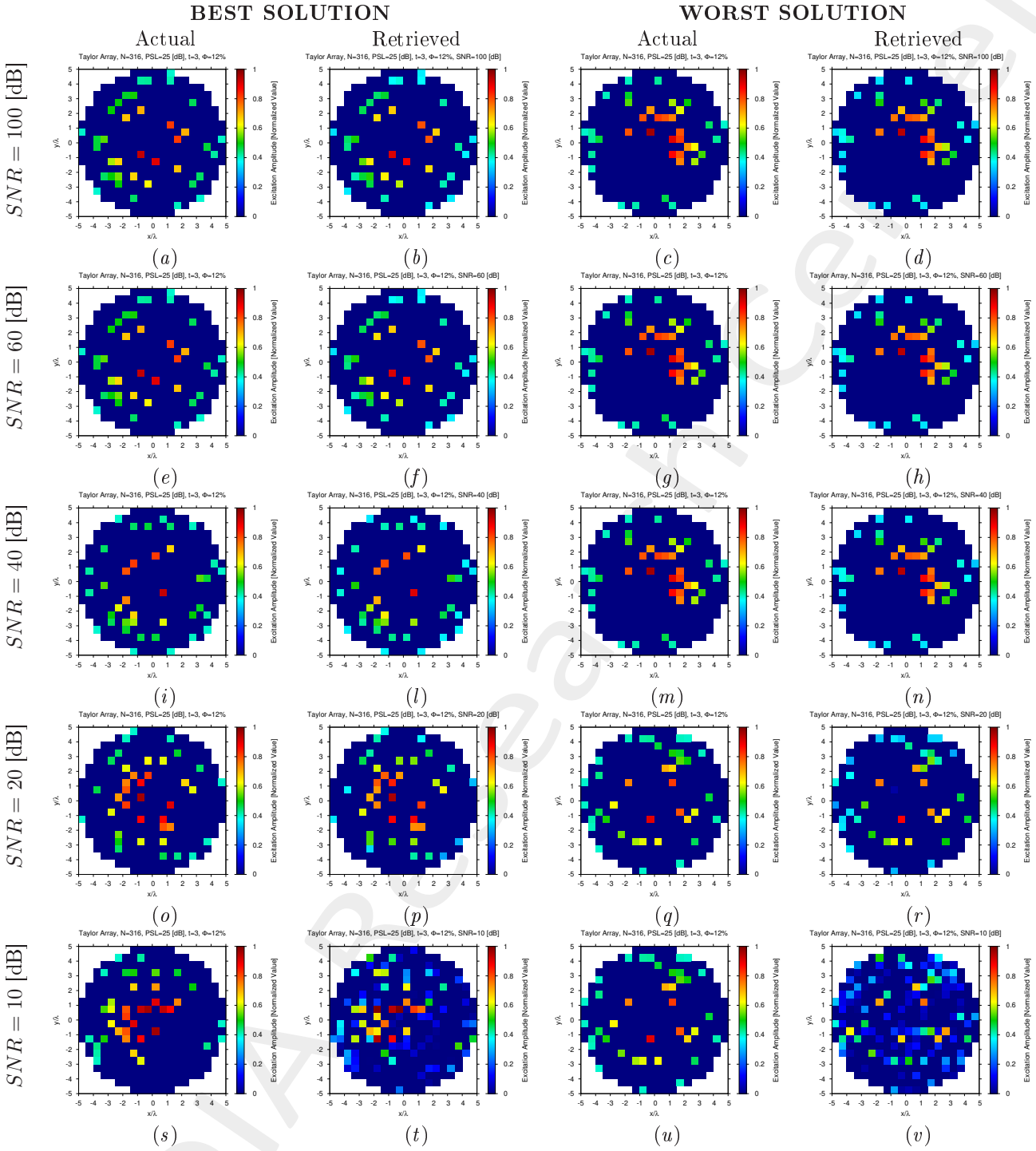

Figure 12: Taylor Array ($N = 316$, $PSL = 25$ [dB], $t = 3$, $\Phi = 12\%$) - Best and worst reconstructions under several SNR values.

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$\Phi = \frac{N_f}{N} = 16\%$ $(N_f = 51)$ - **Best and Worst** BCS **Reconstructions**

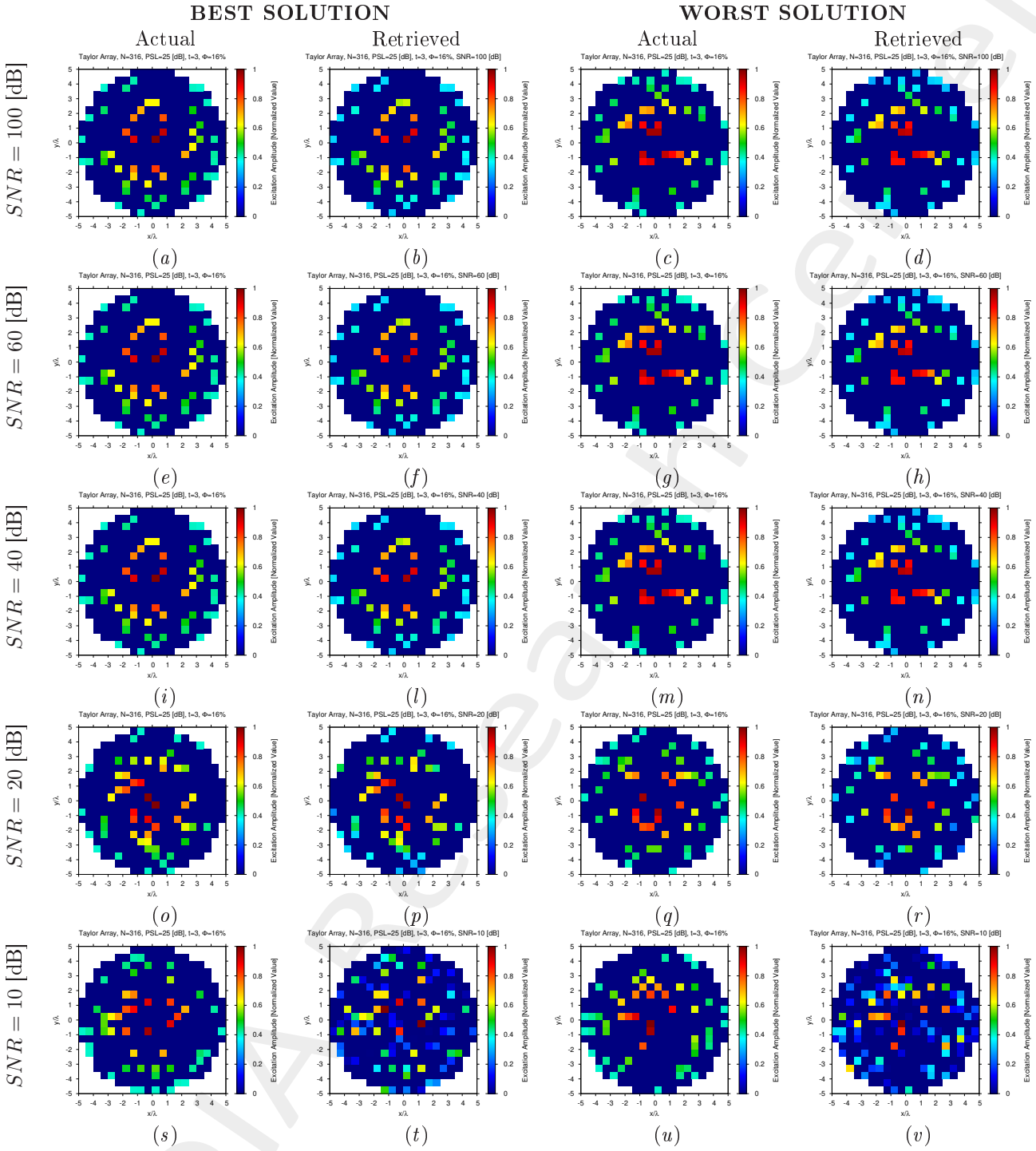

Figure 13: Taylor Array ($N = 316$, $PSL = 25$ [dB], $t = 3$, $\Phi = 16\%$) - Best and worst reconstructions under several SNR values.

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$\Phi = \frac{N_f}{N} = 20\%$ $(N_f = 63)$ - **Best and Worst** BCS **Reconstructions**

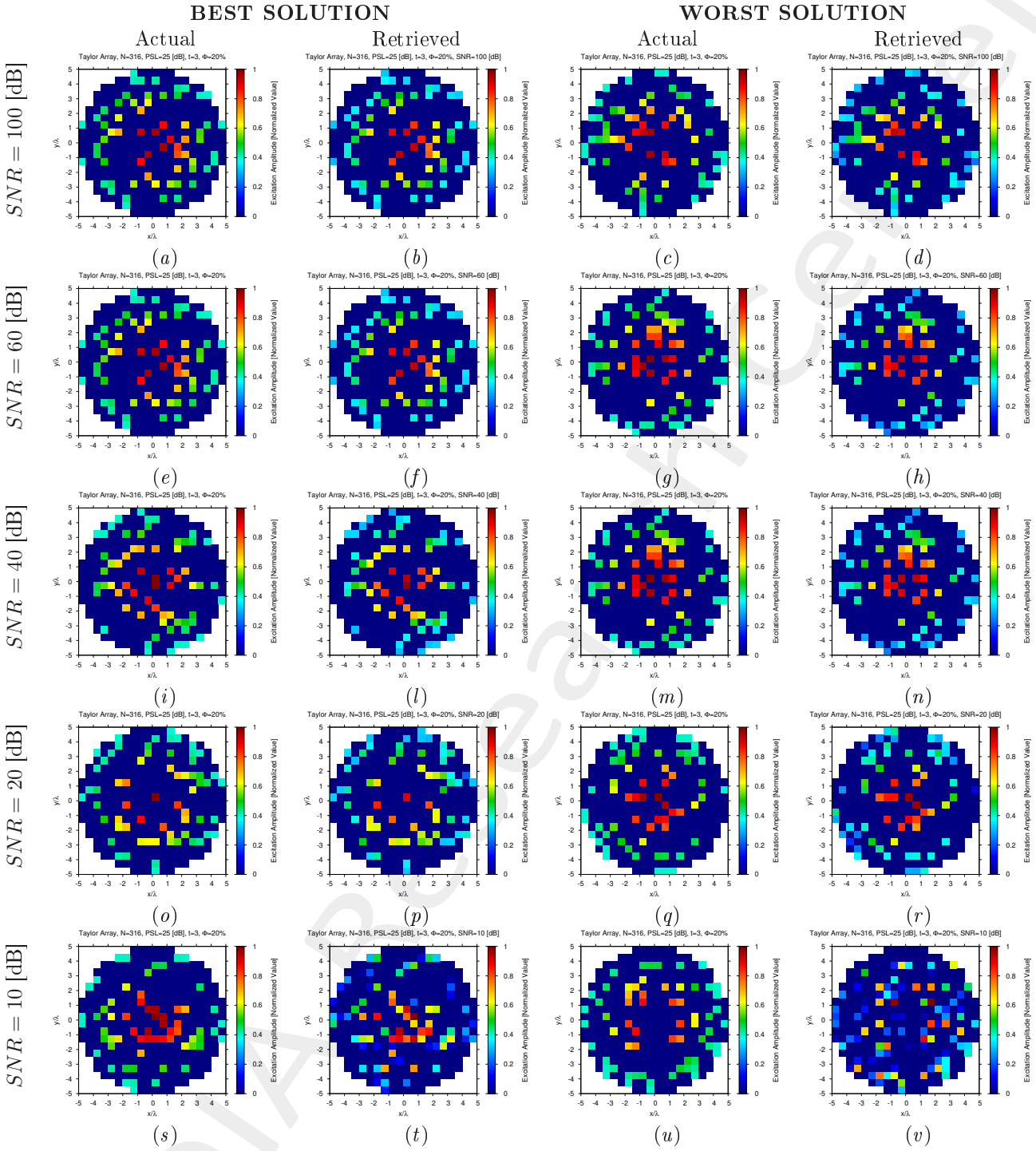

Figure 14: Taylor Array ($N = 316$, $PSL = 25$ [dB], $t = 3$, $\Phi = 20\%$) - Best and worst reconstructions under several SNR values.

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Diagnosis Error and Confidence Level

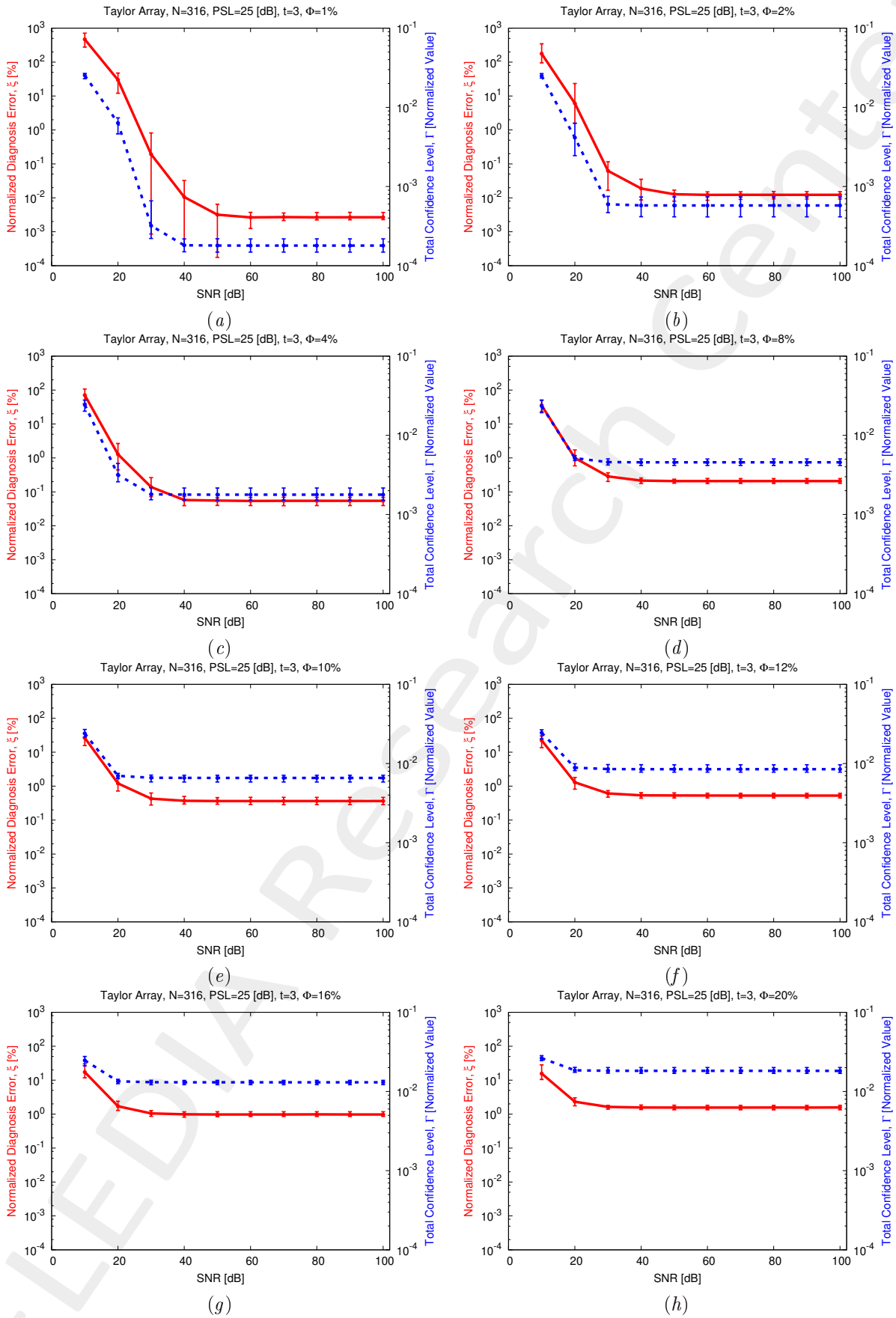

Figure 15: Taylor Array ($N = 316$, $PSL = 25$ [dB], $t = 3$) - Behavior of the average, minimum and maximum diagnosis error (ξ) and total confidence level (Γ) versus the SNR, for (a) $\Phi = 1\%$, (b) $\Phi = 2\%$, (c) $\Phi = 4\%$, (d) $\Phi = 8\%$, (e) $\Phi = 10\%$, (f) $\Phi = 12\%$, (g) $\Phi = 16\%$, and (h) $\Phi = 20\%$.

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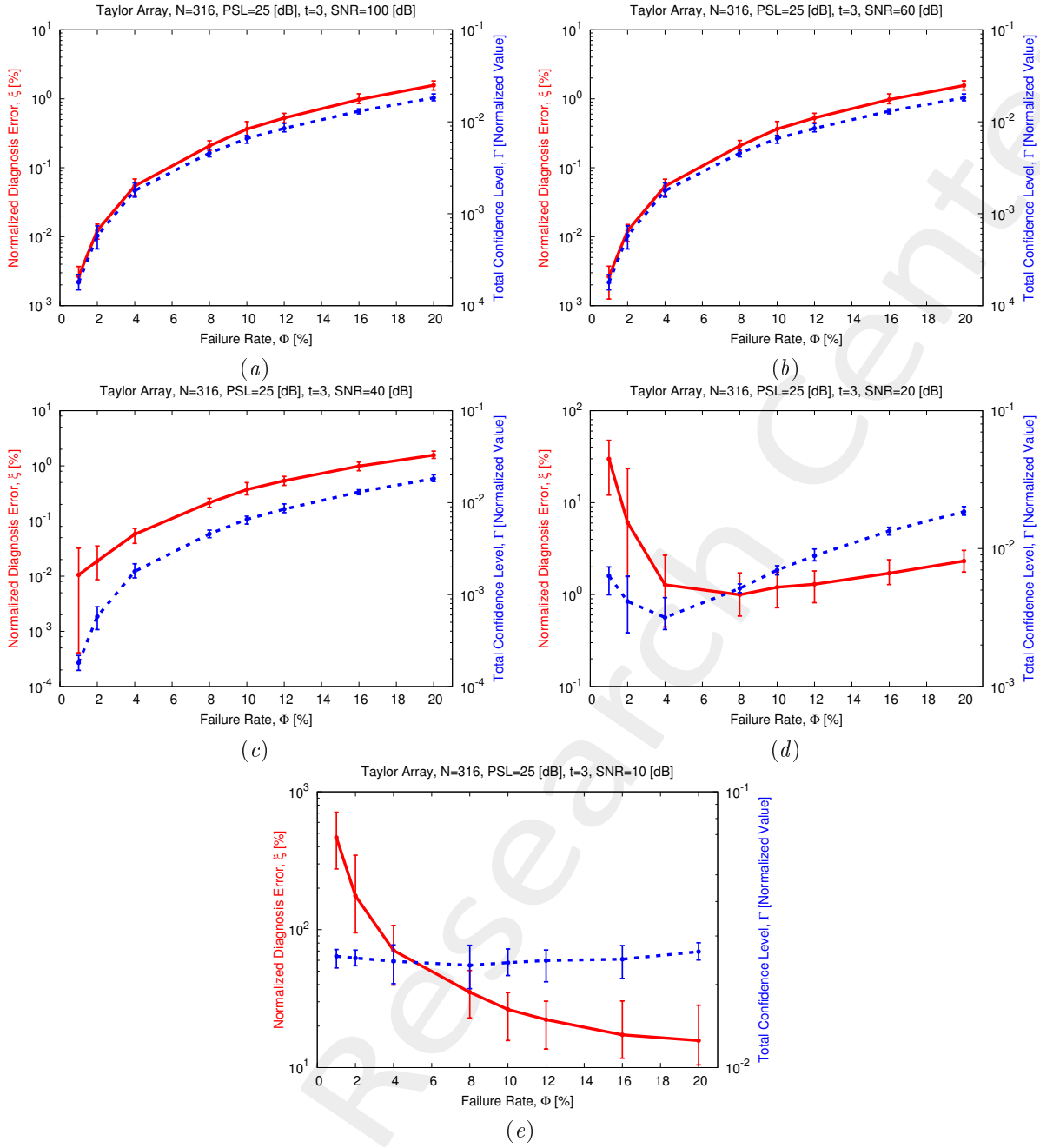

Figure 16: Taylor Array ($N = 316$, $PSL = 25$ [dB], $t = 3$) - Behavior of the average, minimum and maximum diagnosis error (ξ) and total confidence level (Γ) versus the failure rate (Φ), for (a) $SNR = 100$ [dB], (b) $SNR = 60$ [dB], (c) $SNR = 40$ [dB], (d) $SNR = 20$ [dB], and (e) $SNR = 10$ [dB].

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More information on the topics of this document can be found in the following list of references.

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