Multi-Resolution Processing of Multi-Frequency GPR Data for Robust Buried Object Imaging

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Abstract

This work presents an innovative *GPR* microwave imaging technique aimed at retrieving the electromagnetic properties of inaccessible domains buried below a planar interface. The arising two-dimensional (2-*D*) inverse scattering problem is solved taking into account for the wide-band nature of *GPR* data by exploiting a multi frequency (*MF*) solution approach. Moreover, a customized multi-resolution particle swarm optimizer (*IMSA-PSO*) is exploited in order to minimize the *MF* cost function by adaptively refining the image resolution only in the identified regions of interest (*RoI*s). A set of numerical experiments is shown in order to verify the effectiveness of the developed *MF-IMSA-PSO* technique when the background permittivity is not exactly known. A comparative assessment with respect to a deterministic local search-based microwave imaging technique is given, as well, to highlight the superior performances yielded by the exploitation of the *PSO* solver.

1 Definitions

1.1 Glossary

- SF: Single-Frequency;
- FH: Frequency-Hopping;
- MF: Multi-Frequency;
- P: Swarm dimension;
- U: Total number of unknowns;
- S: Maximum number of $IMSA$ zooming steps;
- s^{best}: Last performed $IMSA$ zooming step ($s^{best} \leq S$);
- η_{th}: $IMSA$ zooming threshold;
- D_{inv}: Investigation domain;
- D_{obs}: Observation domain;
- L: Side of the investigation domain;
- N: Number of discretization cells in D_{ind};
- V: Number of views;
- M: Number of measurement points;
- F: Number of frequencies considered for the inversion;
- $\mathbf{r}^{(v)} = \left(x^{(v)}, y^{(v)}\right)$: Coordinates of the v-th source ($v = 1, \dots, V$).
- $\mathbf{r}_m^{(v)} = \left(x_m^{(v)}, y_m^{(v)}\right)$: Coordinates of the m-th measurement point for the v-th view v, ($m = 1, \dots, M$);
- $\varepsilon_{ra} = \frac{\varepsilon_a}{\varepsilon_0}$: Relative electric permittivity for the upper half-space ($y > 0$);
- σ_a: Conductivity for the upper half-space ($y > 0$);
- $\varepsilon_{rb} = \frac{\varepsilon_b}{\varepsilon_0}$: Background relative electric permittivity;
- σ_b: Background conductivity;
- $E_{inc}^{(v)}(\mathbf{r}_n; f)$: Measured internal incident field inside the n-th cell, for the v-th view at frequency f;
- $\widetilde{E}_{inc}^{(v)}(\mathbf{r}_n; f)$: Computed internal incident field inside the n-th cell, for the v-th view at frequency f;
- $E_{scatt}^{(v)}\left(\mathbf{r}_m^{(v)}; f\right)$: Measured external scattered by the m-th measurement point, for the v-th view at frequency f;
- $\widetilde{E}_{scatt}^{(v)}\left(\mathbf{r}_m^{(v)}; f\right)$: Measured external scattered by the m-th measurement point, for the v-th view at frequency f.

1.2 Contrast function

The contrast function at frequency f is defined as

$$\tau(\mathbf{r}; f) = \frac{\varepsilon_{eq}(\mathbf{r}) - \varepsilon_{eqb}}{\varepsilon_0} = [\varepsilon_r(\mathbf{r}) - \varepsilon_{rb}] + j\left[\frac{\sigma_b - \sigma(\mathbf{r})}{2\pi f \varepsilon_0}\right]$$

where

- $\mathbf{r} = (x, y)$: position vector;
- $\Re\{\tau(\mathbf{r}; f)\} = [\varepsilon_r(\mathbf{r}) - \varepsilon_{rb}]$;
- $\Im\{\tau(\mathbf{r}; f)\} = \left[\frac{\sigma_b - \sigma(\mathbf{r})}{2\pi f \varepsilon_0}\right]$;
- $\varepsilon_{eq}(\mathbf{r}) = \varepsilon_0 \varepsilon_r(\mathbf{r}) - j\frac{\sigma(\mathbf{r})}{2\pi f}$;
- $\varepsilon_{eqb} = \varepsilon_0 \varepsilon_{rb} - j\frac{\sigma_b}{2\pi f}$;
- $\varepsilon_r(\mathbf{r})$: relative electric permittivity at position $\mathbf{r}$;
- $\sigma(\mathbf{r})$: conductivity at position $\mathbf{r}$;

NOTE: we assume that $\varepsilon_r(\mathbf{r})$ and $\sigma(\mathbf{r})$ are **not frequency dependent** (non-dispersive mediums).

1.2.1 Contrast function and reference frequency f_{ref} (MF approaches)

The contrast function at a generic frequency f can be expressed by means of the contrast function computed for a selected reference frequency

$$f = f_{ref} \tag{1}$$

as follows

$$\tau(\mathbf{r}; f) = \Re\{\tau(\mathbf{r}; f_{ref})\} + j\frac{f_{ref}}{f}\Im\{\tau(\mathbf{r}; f_{ref})\}. \tag{2}$$

This allows to reduce the number of unknowns when dealing with multi-frequency techniques, since we can just consider the contrast function at the reference frequency.

1.3 Cost function & unknowns

1.3.1 Multi-Frequency (MF) approaches

These approaches jointly consider data at F frequencies. The functional minimized by the inversion algorithm is defined as

$$\Phi(\mathbf{x}) = \Phi_{state}(\mathbf{x}) + \Phi_{data}(\mathbf{x}) \tag{3}$$

where $\Phi_{state}(\mathbf{x})$ and $\Phi_{data}(\mathbf{x})$ are respectively the data and state terms of the cost function, defined as

$$\Phi_{state}(\mathbf{x}) = \frac{\sum_{j=1}^{F}\sum_{v=1}^{V}\sum_{n=1}^{N}\left|E_{inc}^{(v)}(\mathbf{r}_n; f_j) - \widetilde{E}_{inc}^{(v)}(\mathbf{r}_n; f_j)\right|^2}{\sum_{j=1}^{F}\sum_{v=1}^{V}\sum_{n=1}^{N}\left|E_{inc}^{(v)}(\mathbf{r}_n; f_j)\right|^2} \tag{4}$$

$$\Phi_{data} = \frac{\sum_{j=1}^{F}\sum_{v=1}^{V}\sum_{m=1}^{M}\left|E_{scatt}^{(v)}\left(\mathbf{r}_m^{(v)}; f_j\right) - \widetilde{E}_{scatt}^{(v)}\left(\mathbf{r}_m^{(v)}; f_j\right)\right|^2}{\sum_{j=1}^{F}\sum_{v=1}^{V}\sum_{m=1}^{M}\left|E_{scatt}^{(v)}\left(\mathbf{r}_m^{(v)}; f_j\right)\right|^2} \tag{5}$$

The unknowns of the inversion problem are

$$\mathbf{x} = \left\{\tau(\mathbf{r}; f_{ref}); E_{tot}^{(v)}(\mathbf{r}_n; f_j)\right\} \qquad n = 1, ..., N; v = 1, ..., V; j = 1, ..., F. \tag{6}$$

The total number of unknowns for MF-based approaches is then given by

$$U_{MF} = 2N(1 + VF). \tag{7}$$

1.4 Reconstruction errors

The following integral error is defined

$$\Xi_{reg} = \frac{1}{N_{reg}}\sum_{n=1}^{N_{reg}}\frac{|\tau_n^{act} - \tau_n^{rec}|}{|\tau_n^{act} + 1|} \tag{8}$$

where reg indicates if the error computation covers

- the overall investigation domain ($reg \Rightarrow tot$),
- the actual scatterer support ($reg \Rightarrow int$),
- or the background region ($reg \Rightarrow ext$).

2 Wrong guess of the background permittivity

2.1 Goal of this section

This analysis is devoted at verifying what are the achievable performances by the $MF - IMSA - PSO$ when a wrong guess of the background permittivity is used for the inversion

$$\varepsilon_{rB}^{guess} \neq \varepsilon_{rB}^{actual}$$

The following values of ε_{rB}^{guess} will be considered in the following:

1. $\varepsilon_{rB}^{guess} = \varepsilon_{rB}^{actual} \pm 5\%$
2. $\varepsilon_{rB}^{guess} = \varepsilon_{rB}^{actual} \pm 10\%$
3. $\varepsilon_{rB}^{guess} = \varepsilon_{rB}^{actual} \pm 20\%$

2.2 "$\mathcal{I}$-Shaped" object ($\varepsilon_{r,obj} = 5.5$, $\sigma_{obj} = 10^{-3}$ [S/m])

2.2.1 Parameters

Background

Inhomogeneous and nonmagnetic background composed by two half spaces

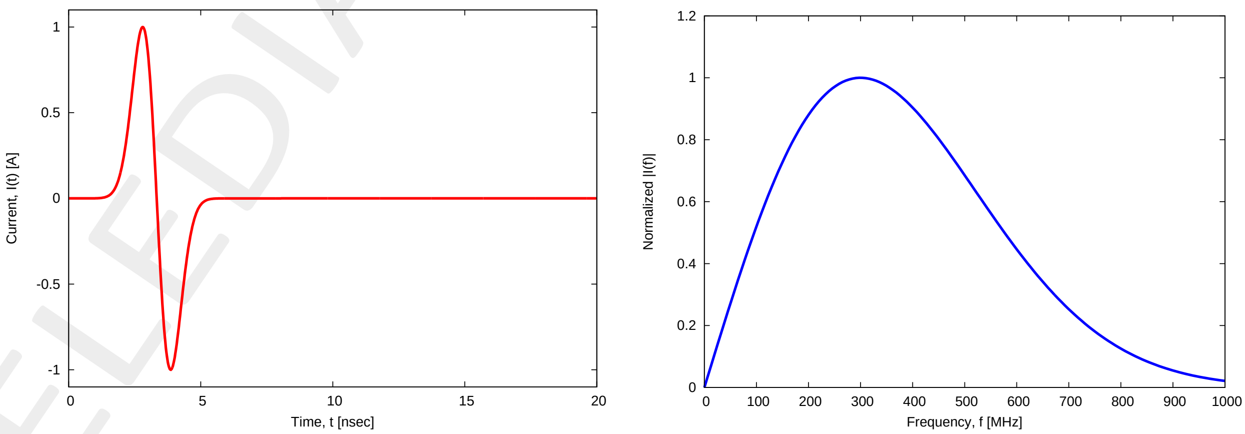
- Upper half space ($y > 0$ - air): $\varepsilon_{ra} = 1.0$, $\sigma_a = 0.0$;
- Lower half space ($y < 0$ - soil): $\varepsilon_{rb} = 4.0$, $\sigma_b = 10^{-3}$[S/m];

Investigation domain (D_{inv})

- Side: $L_{D_{inv}} = 0.8$ [m];
- Barycenter: $\left(x_{bar}^{D_{inv}}, y_{bar}^{D_{inv}}\right) = (0.00, -0.4)$ [m];

Time-Domain forward solver (*FDTD - GPRMax2D*)

- Side of the simulated domain: $L = 6$ [m];
- Number of cells: $N^{FDTD} = 750 \times 750 = 5.625 \times 10^5$;
- Side of the $FDTD$ cells $l^{FDTD} = 0.008$ [m];
- Simulation time window: $T^{FDTD} = 20 \times 10^{-9}$ [sec];
- Time step: $\Delta t^{FDTD} = 1.89 \times 10^{-11}$ [sec];
- Number of time samples: $N_t^{FDTD} = 1060$;
- Boundary conditions: perfectly matched layer (PML);
- Source type: Gaussian mono-cycle (first Gaussian pulse derivative, called "Ricker" in $GPRMax2D$)
  - Central frequency: $f_0 = 300$ [MHz];
  - Source amplitude: $A = 1.0$ [A];



(a) (b)

Figure 1: $GPRMax2D$ excitation signal. (a) Time pulse, (b) normalized frequency spectrum.

Frequency parameters

- Frequency range: $f \in [f_{min}, f_{max}] = [200.0, 600.0]$ [MHz] [?] (-3 [dB] bandwidth of the Gaussian Monocycle excitation centered at $f_0 = 300$ [MHz]);
- Frequency step: $\Delta f = 100$ [MHz] ($F = 5$ frequency steps in $[f_{min}, f_{max}]$);

f [MHz]	λ_a [m]	λ_b [m]	f^* [MHz]
200.0	1.50	0.75	200.5
300.0	1.00	0.50	297.6
400.0	0.75	0.37	401.1
500.0	0.60	0.30	498.1
600.0	0.50	0.25	601.6

Table 1: Considered frequencies and corresponding wavelength in the upper medium (λ_a, free space) and in the lower medium (λ_b, soil). f^* is the nearest frequency sample available from transformed time-domain data, and represents the real frequency considered by the inversion algorithm.

Scatterer

- Electromagnetic properties: $\varepsilon_{r,obj} = 5.5$, $\sigma_{obj} = 10^{-3}$ [S/m] ($\sigma_{obj} = \sigma_b$);
- Contrast function: $\tau = 1.5 + j0.0$

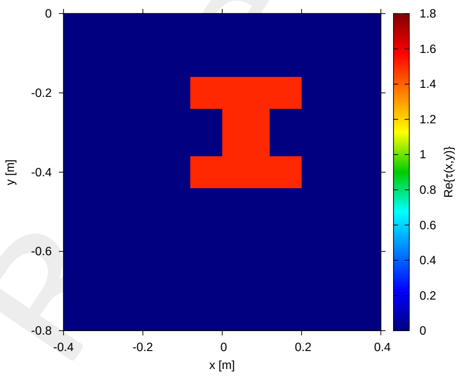


Figure 2: Actual object.

Measurement setup

- Considered frequency: $f_{min} = 200$ [MHz], $\lambda_b = 0.75$ [m]. [1]
- $\#DoFs = 2ka = \frac{2\pi}{\lambda_b} L\sqrt{2} = \frac{2\pi}{0.75} 0.8\sqrt{2} \simeq 9.5$;
- Number of views (sources): $V = 10$;
  - $\min\{x_v\} = -0.5$ [m], $\max\{x_v\} = 0.5$ [m];
  - height: $y_v = 0.1$ [m], $\forall v = 1, \dots, V$;
- Number of measurement points: $M = 9$;

[1]**NOTE:** This choice is done in order to keep the number of unknowns lower than 5000.

- $\min\{x_m\} = -0.5$ [m], $\max\{x_m\} = 0.5$ [m];
- height: $y_m = 0.1$ [m], $\forall m = 1, \ldots, M$;

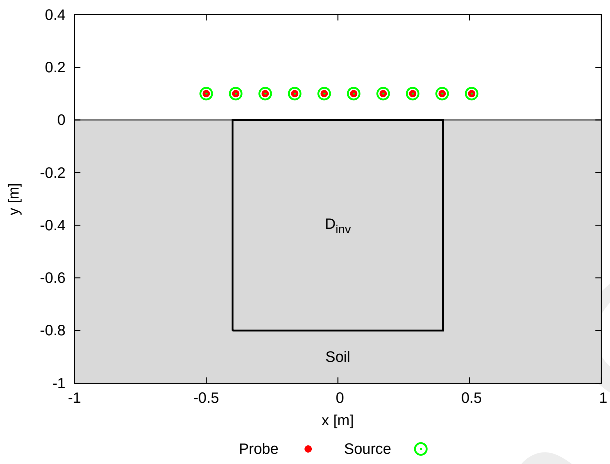


Figure 3: Location of the measurement points ($M = 9$) and of the sources ($V = 10$). Only one source is active for each view.

Inverse solver parameters

- **Shared parameters**
  - Number of unknowns: $U = 2N(1 + VF) = 4998$;
  - Weight of the state term of the functional: 1.0;
  - Weight of the data term of the functional: 1.0;
  - Weight of the penalty term of the functional: 0.0;
  - Convergence threshold: 10^{-10};
  - Variable ranges:
    - $\sigma \in \left[8.0 \times 10^{-4}, 1.2 \times 10^{-3}\right]$ [S/m];
    - $\Re\left\{E_{tot}^{int}\right\} \in [-8, 8]$, $\Im\left\{E_{tot}^{int}\right\} \in [-8, 8]$;
  - Degrees of freedom:
    - Considered frequency: $f_{min} = 200$ [MHz], $\lambda_b = 0.75$ [m];
    - $\frac{(2ka)^2}{2} = \frac{\left(2 \times \frac{2\pi}{\lambda_b} \times \frac{L\sqrt{2}}{2}\right)^2}{2} = 4\pi^2 \left(\frac{L}{\lambda_b}\right)^2 = 4\pi^2 \left(\frac{0.8}{0.75}\right)^2 \simeq 44.87$;
  - Number of cells: $N = 49 = 7 \times 7$;
  - Maximum number of $IMSA$ steps: $S = 4$;
  - Side ratio threshold: $\eta_{th} = 0.2$;
- $MF-IMSA-PSO$ **parameters**
  - Maximum number of iterations: $I = 20000$;

- Swarm dimension: $P = \frac{5}{100} \times U = 250$ ($5\%U$ - as in [?]);
- $C_1 = C_2 = 2.0$ (as in [?]);
- Inertial weight: $w = 0.4$ (constant, as in [?]);
- Velocity clamping: enabled;

- $MF - IMSA - CG$ **parameters**
  - Maximum number of iterations: $I = 200$;

Signal to noise ratio (on $E_{tot}(t)$)

- $SNR = \{50, 40, 30, 20\}$ [dB] + Noiseless data.

2.2.2 $\varepsilon_{rB}^{guess} = 3.2$ - $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Final reconstructions

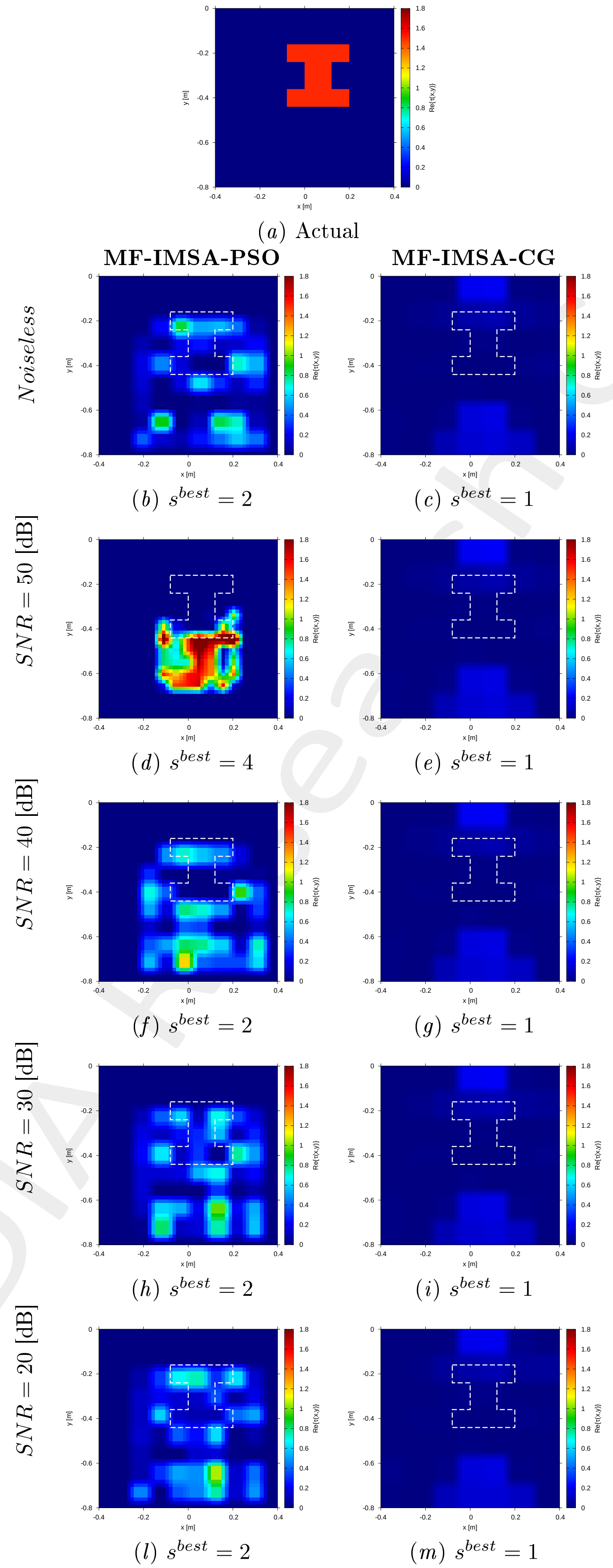


Figure 4: $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Retrieved dielectric profiles at the $IMSA$ convergence step (s^{best}).

2.2.3 $\varepsilon_{rB}^{guess} = 3.6$ - $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Final reconstructions

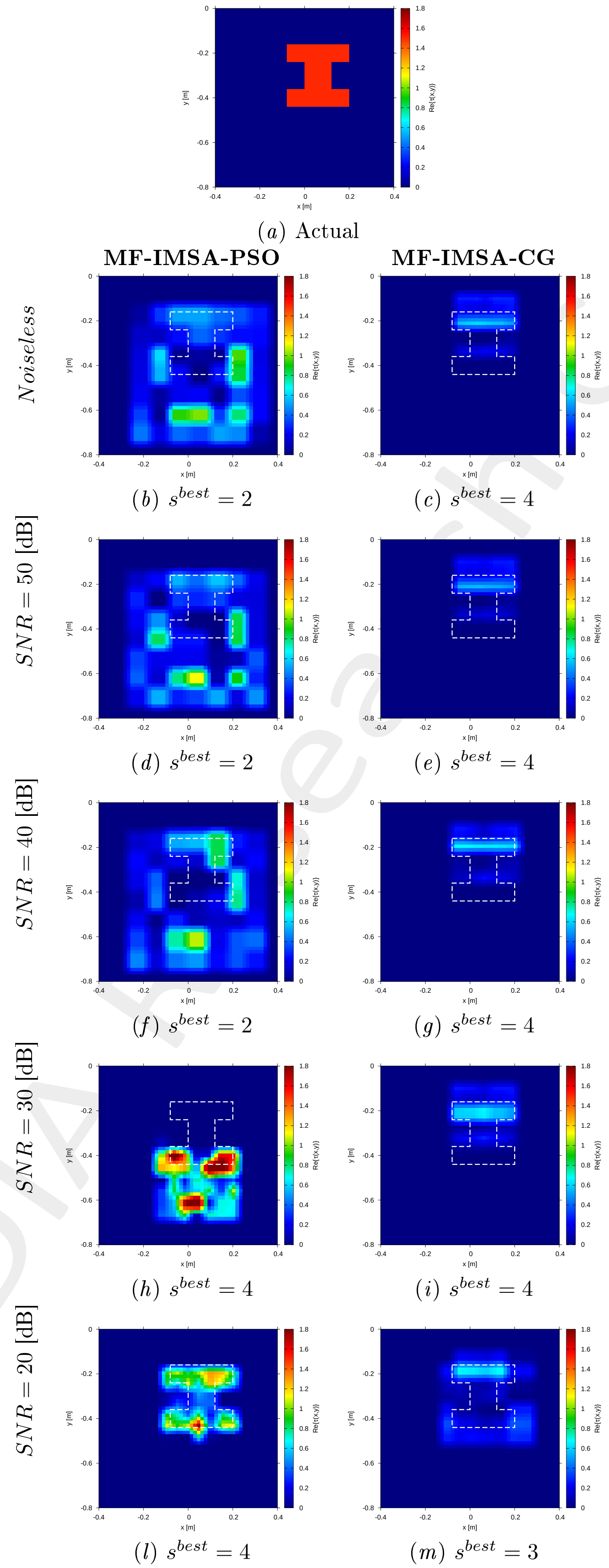


Figure 5: $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Retrieved dielectric profiles at the $IMSA$ convergence step (s^{best}).

2.2.4 $\varepsilon_{rB}^{guess} = 3.8$ - $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Final reconstructions

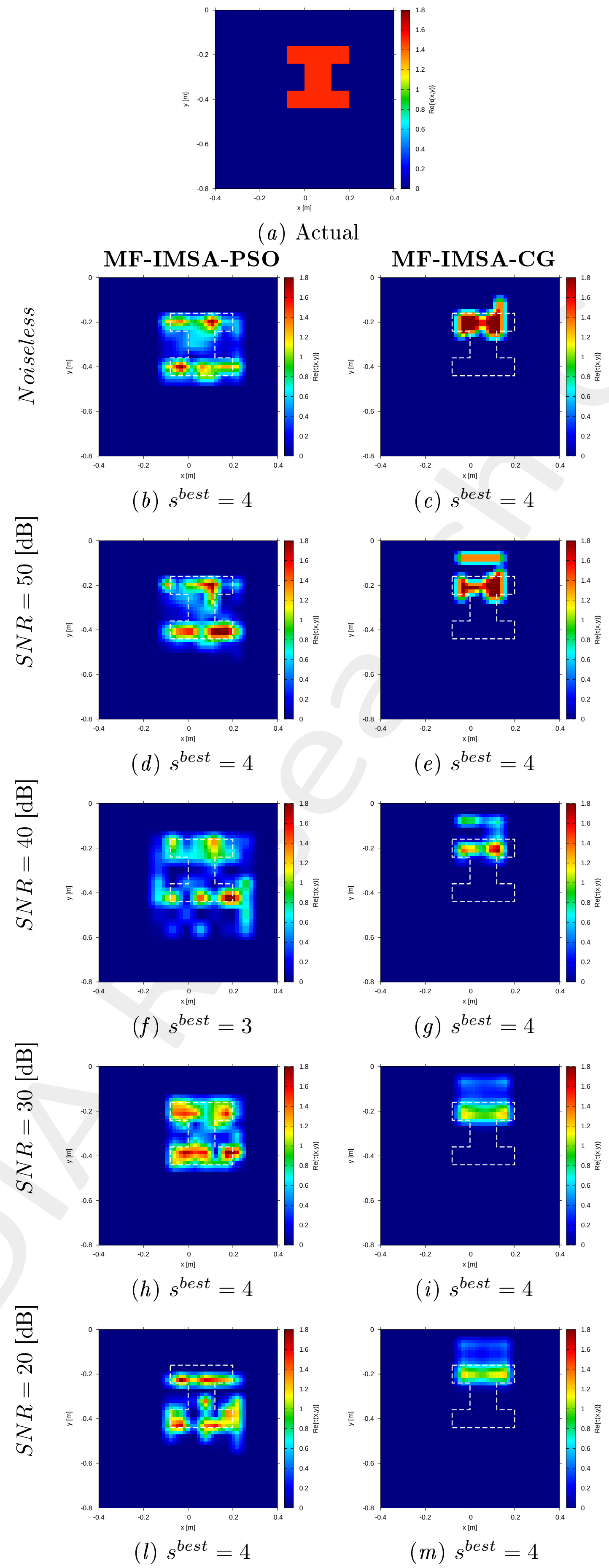
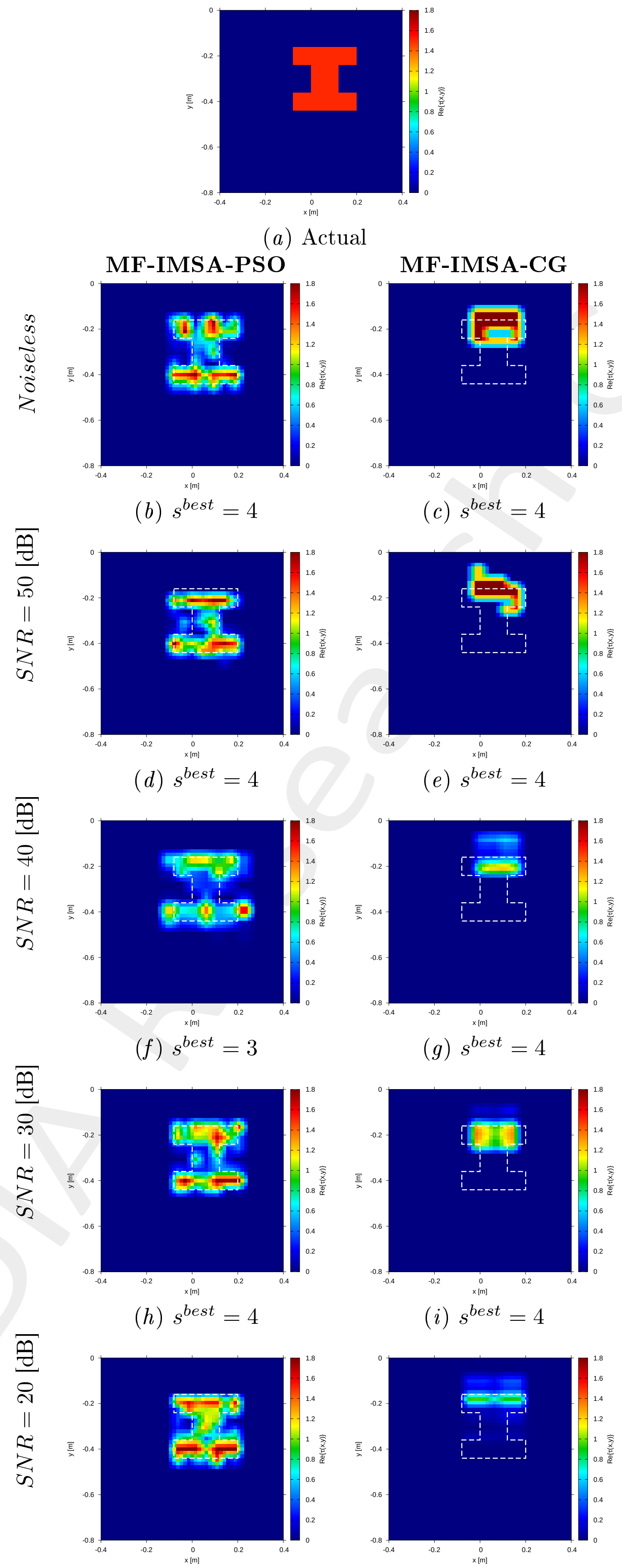


Figure 6: $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Retrieved dielectric profiles at the $IMSA$ convergence step (s^{best}).

2.2.5 $\varepsilon_{rB}^{guess} = 4.0$ - $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Final reconstructions



(a) Actual

(b) $s^{best} = 4$ (c) $s^{best} = 4$

(d) $s^{best} = 4$ (e) $s^{best} = 4$

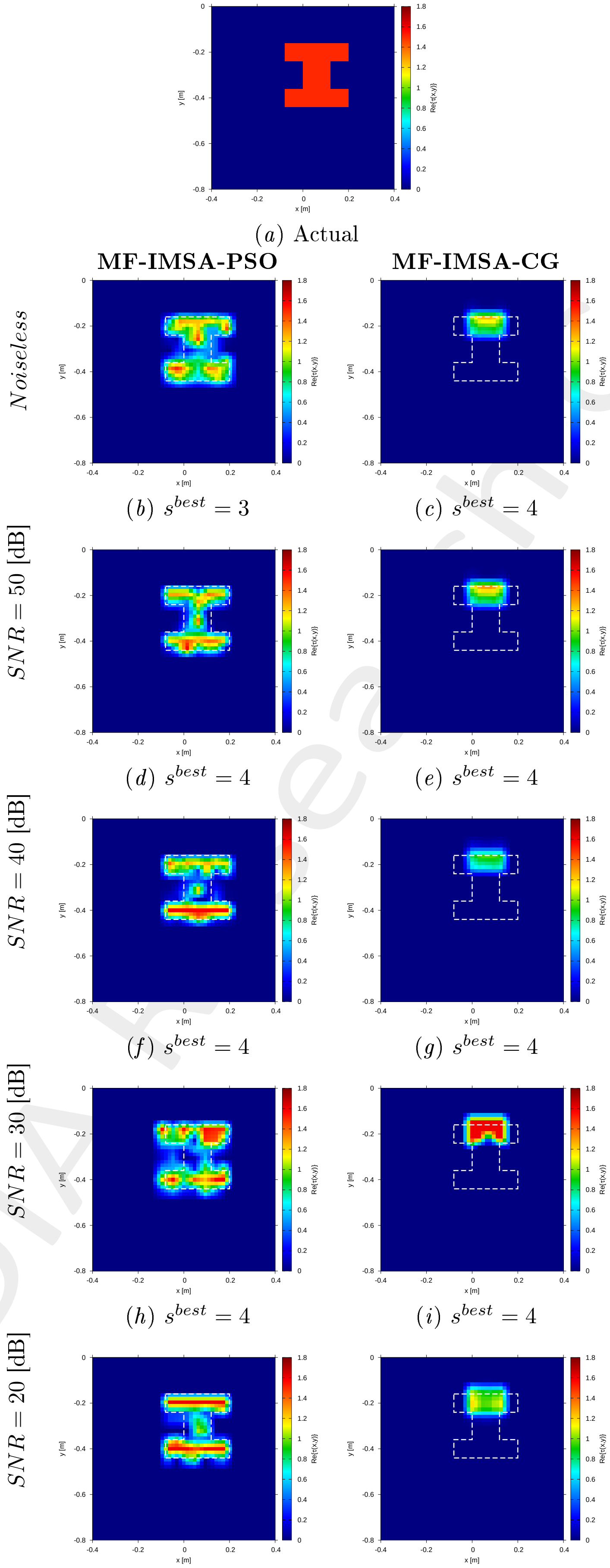
(f) $s^{best} = 3$ (g) $s^{best} = 4$

(h) $s^{best} = 4$ (i) $s^{best} = 4$

(l) $s^{best} = 4$ (m) $s^{best} = 4$

Figure 7: $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Retrieved dielectric profiles at the $IMSA$ convergence step (s^{best}).

2.2.6 $\varepsilon_{rB}^{guess} = 4.2$ - $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Final reconstructions



(a) Actual

(b) $s^{best} = 3$ (c) $s^{best} = 4$

(d) $s^{best} = 4$ (e) $s^{best} = 4$

(f) $s^{best} = 4$ (g) $s^{best} = 4$

(h) $s^{best} = 4$ (i) $s^{best} = 4$

(l) $s^{best} = 4$ (m) $s^{best} = 4$

Figure 8: $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Retrieved dielectric profiles at the $IMSA$ convergence step (s^{best}).

2.2.7 $\varepsilon_{rB}^{guess} = 4.4$ - $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Final reconstructions

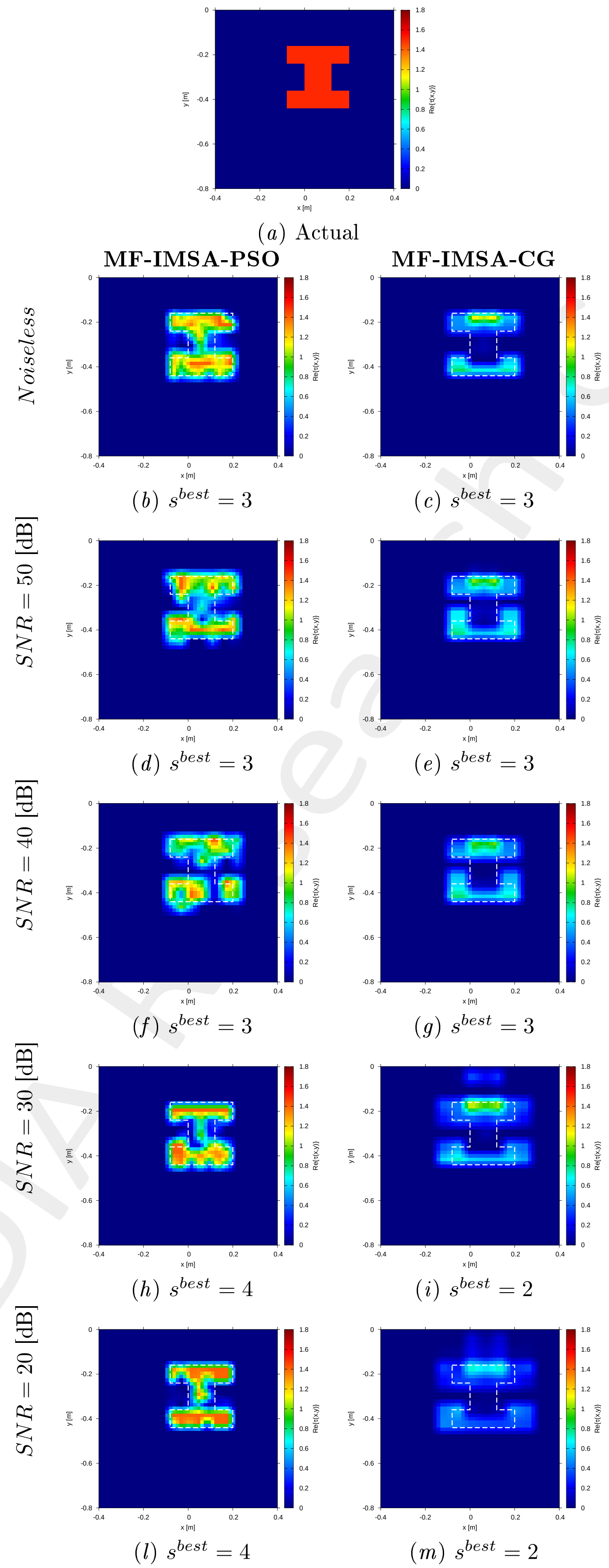
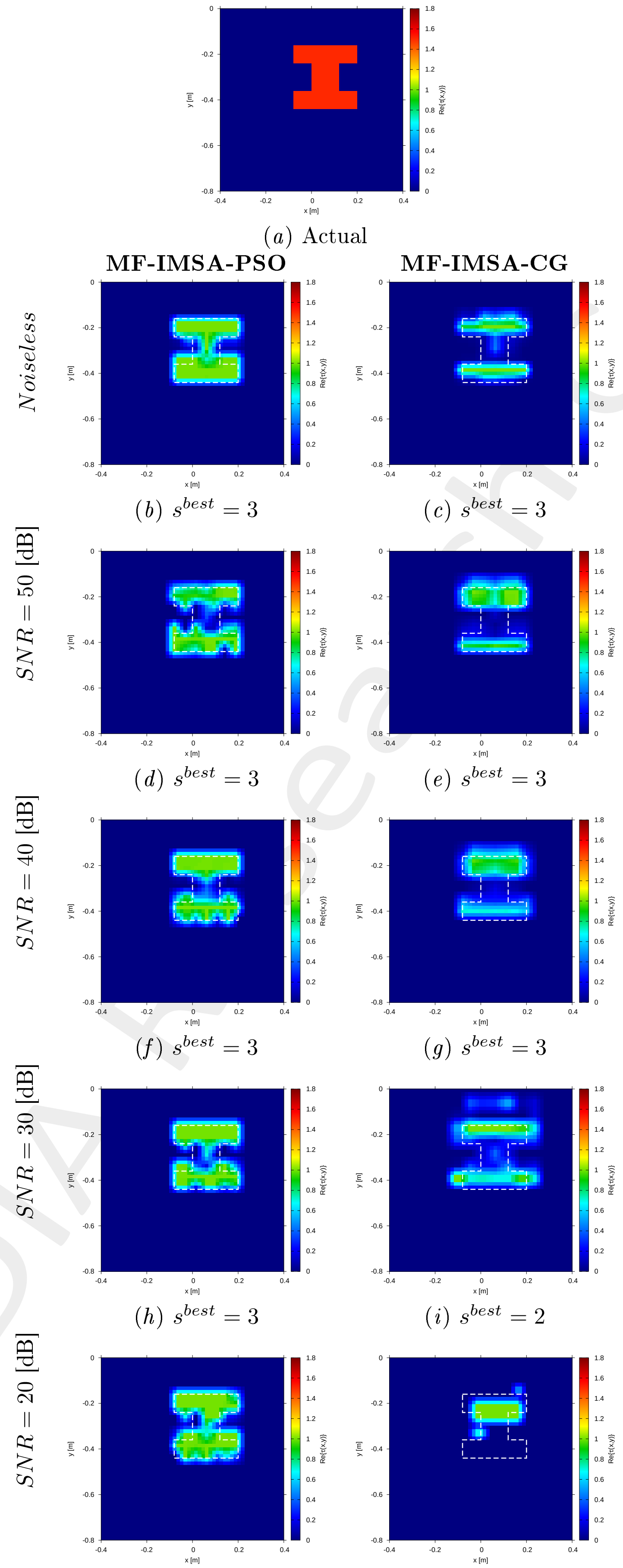


Figure 9: $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Retrieved dielectric profiles at the $IMSA$ convergence step (s^{best}).

2.2.8 $\varepsilon_{rB}^{guess} = 4.8$ - $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Final reconstructions



(a) Actual

(b) $s^{best} = 3$ (c) $s^{best} = 3$

(d) $s^{best} = 3$ (e) $s^{best} = 3$

(f) $s^{best} = 3$ (g) $s^{best} = 3$

(h) $s^{best} = 3$ (i) $s^{best} = 2$

(l) $s^{best} = 3$ (m) $s^{best} = 4$

Figure 10: $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Retrieved dielectric profiles at the $IMSA$ convergence step (s^{best}).

2.2.9 $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Errors vs. ε_{rB}^{guess}

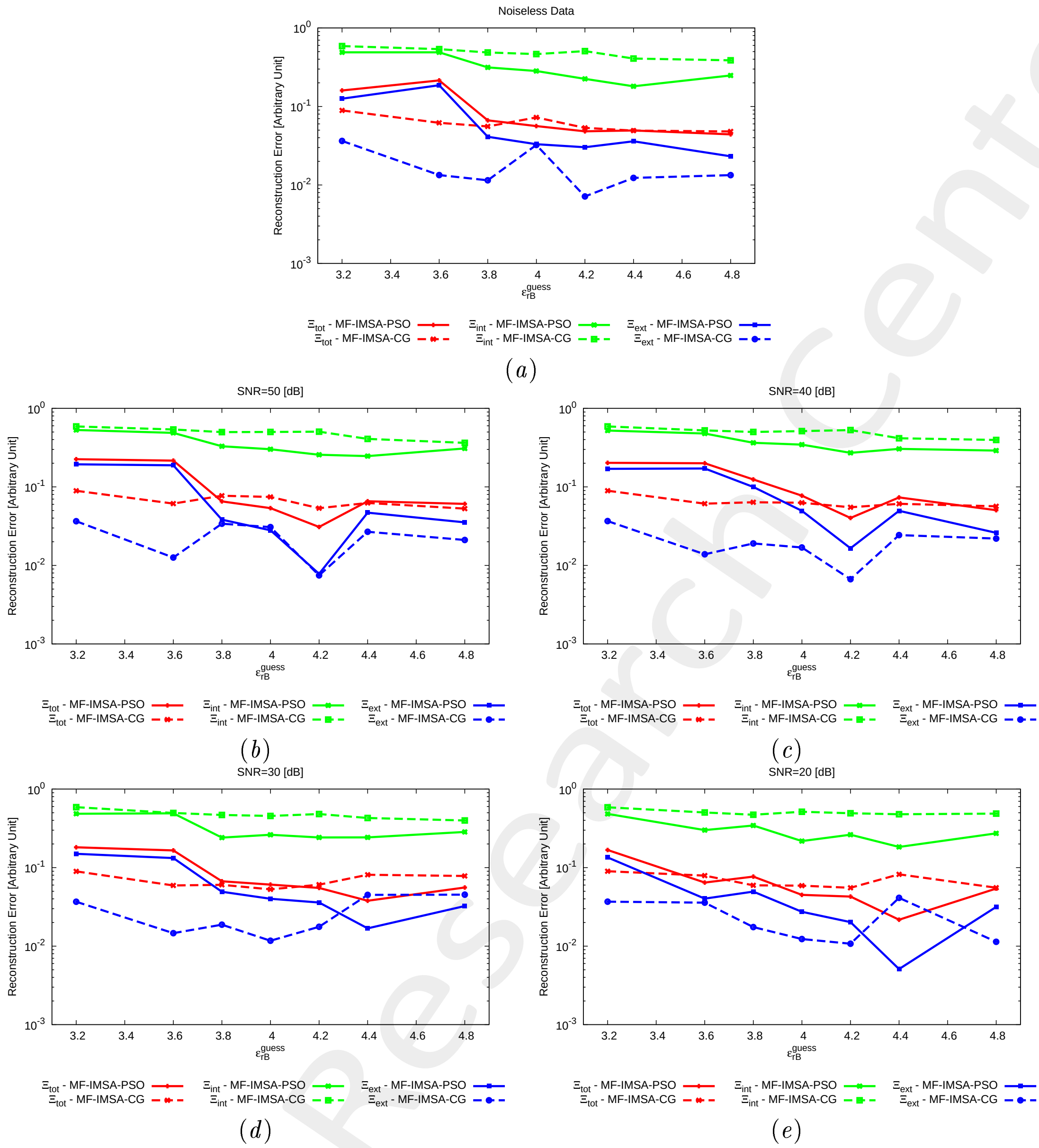


Figure 11: $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Reconstruction errors vs. the guess of the background relative permittivity (ε_{rB}^{guess}).

2.2.10 $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Errors vs. SNR

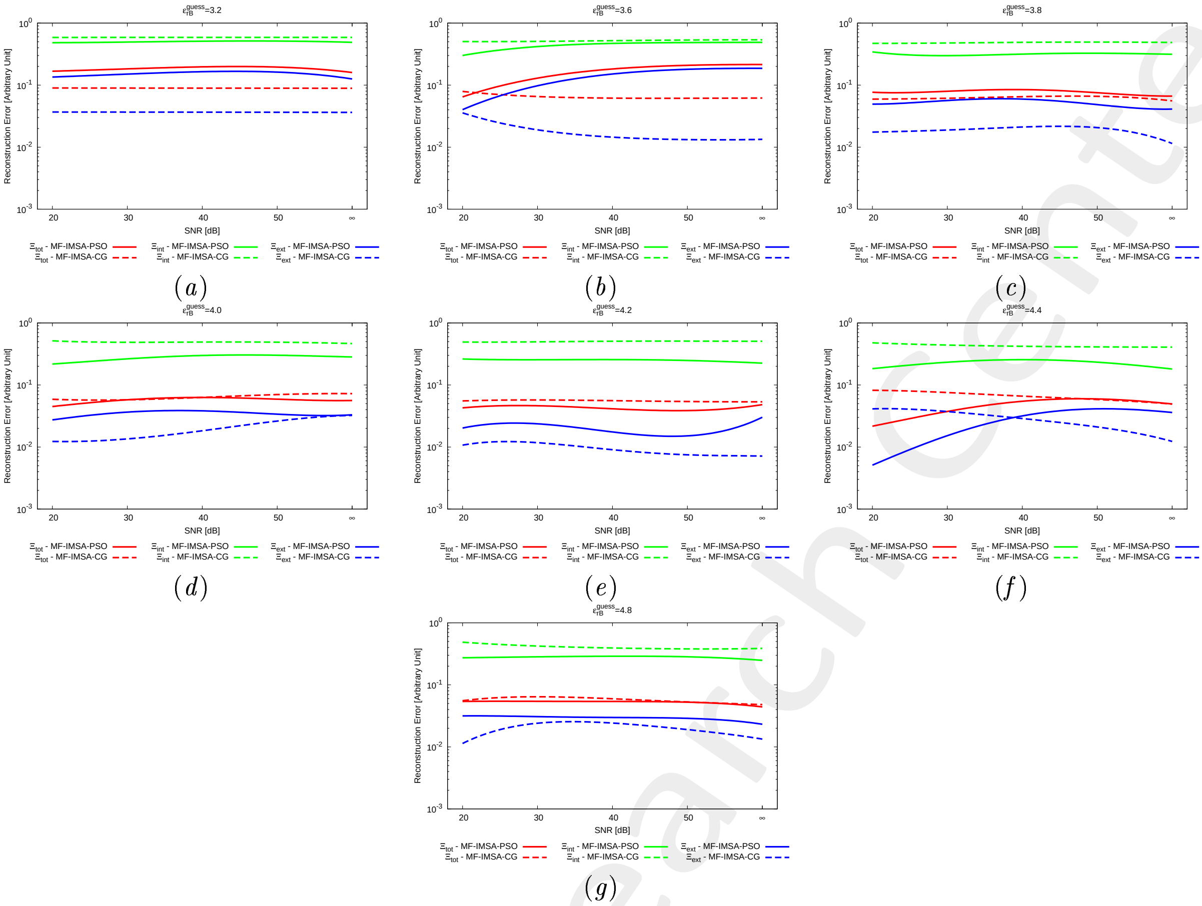


Figure 12: $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Reconstruction errors vs. SNR.

3 Conclusions

The reported numerical validation has shown that the proposed $MF - IMSA - PSO$ is able to retrieve an acceptable image of the investigation domain even when considering a wrong guess of the soil relative permittivity (i.e., by letting ε_{rB}^{guess} different from the actual/nominal relative permittivity of the background medium, $\varepsilon_{rB}^{guess} \neq \varepsilon_{rB}^{actual}$). However, better reconstructions are obtained when the background permittivity is over-estimated ($\varepsilon_{rB}^{guess} > \varepsilon_{rB}^{actual}$) with respect to an under-estimation ($\varepsilon_{rB}^{guess} < \varepsilon_{rB}^{actual}$). Finally, on average a significant improvement is obtained by the $MF - IMSA - PSO$ with respect to the $MF - IMSA - CG$ [5], thanks to the superior capabilities of the PSO in minimizing the highly non-linear MF cost function.

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