

Multi-Frequency Deterministic Inversion of Wideband GPR Data: Achievable Performance in the Presence of Unknown Background Media

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Abstract

This work deals with the retrieval of the electromagnetic properties of buried investigation domains starting from the processing of wideband ground penetrating radar (*GPR*) data. More precisely, the achievable performance by two deterministic multi-zooming (*IMSA*) conjugate gradient (*CG*)-based techniques are assessed when no exact knowledge of the electric permittivity of the background is available to perform the inversion. More precisely, the first analyzed technique is based on a multi-frequency (*MF*) approach (i.e., the *MF-IMSA-CG*), while the second one is a state-of-the-art frequency-hopping (*FH*) based methodology (i.e., the *FH-IMSA-CG*). Some significant numerical results are reported in order to understand what is the impact of a wrong guess of the hosting medium under several noise conditions for both *MF* and *FH*-based inversion methods.

1 Definitions

1.1 Glossary

- D_{inv} : investigation domain;
- D_{obs} : observation domain;
- N : number of discretization cells in D_{ind} ;
- V : number of views;
- M : number of measurement points;
- F : number of frequencies considered for the inversion;
- (x_v, y_v) : coordinates of the v -th source ($v = 1, \dots, V$).
- (x_m^v, y_m^v) : coordinates of the m -th measurement point for the v -th view v , ($m = 1, \dots, M$);
- $\varepsilon_{ra} = \frac{\varepsilon_a}{\varepsilon_0}$: relative electric permittivity for the upper half-space ($y > 0$);
- σ_a : conductivity for the upper half-space ($y > 0$);
- $\varepsilon_{rb} = \frac{\varepsilon_b}{\varepsilon_0}$: background relative electric permittivity;
- σ_b : background conductivity;

2 Wrong guess of the background permittivity

2.1 Goal of this section

This analysis is devoted at verifying what are the achievable performances by the $MF - IMSA - CG$ [13] and the $FH - IMSA - CG$ [3] approaches when a wrong guess of the background permittivity is used for the inversion

$$\varepsilon_{rB}^{guess} \neq \varepsilon_{rB}^{actual}$$

The following values of ε_{rB}^{guess} will be considered in the following:

1. $\varepsilon_{rB}^{guess} = \varepsilon_{rB}^{actual} \pm 5\%$
2. $\varepsilon_{rB}^{guess} = \varepsilon_{rB}^{actual} \pm 10\%$
3. $\varepsilon_{rB}^{guess} = \varepsilon_{rB}^{actual} \pm 20\%$

2.2 ”I-Shaped” object ($\varepsilon_{r,obj} = 5.5$, $\sigma_{obj} = 10^{-3}$ [S/m])

2.2.1 Parameters

Background

Inhomogeneous and nonmagnetic background composed by two half spaces

- Upper half space ($y > 0$ - air): $\varepsilon_{ra} = 1.0$, $\sigma_a = 0.0$;
- Lower half space ($y < 0$ - soil): $\varepsilon_{rb} = 4.0$, $\sigma_b = 10^{-3}$ [S/m];

Investigation domain (D_{inv})

- Side: $L_{D_{inv}} = 0.8$ [m];
- Barycenter: $(x_{bar}^{D_{inv}}, y_{bar}^{D_{inv}}) = (0.00, -0.4)$ [m];

Time-Domain forward solver ($FDTD - GPRMax2D$)

- Side of the simulated domain: $L = 6$ [m];
- Number of cells: $N^{FDTD} = 750 \times 750 = 5.625 \times 10^5$;
- Side of the $FDTD$ cells $l^{FDTD} = 0.008$ [m];
- Simulation time window: $T^{FDTD} = 20 \times 10^{-9}$ [sec];
- Time step: $\Delta t^{FDTD} = 1.89 \times 10^{-11}$ [sec];
- Number of time samples: $N_t^{FDTD} = 1060$;
- Boundary conditions: perfectly matched layer (PML);
- Source type: Gaussian mono-cycle (first Gaussian pulse derivative, called ”Ricker” in $GPRMax2D$)
 - Central frequency: $f_0 = 300$ [MHz];
 - Source amplitude: $A = 1.0$ [A];

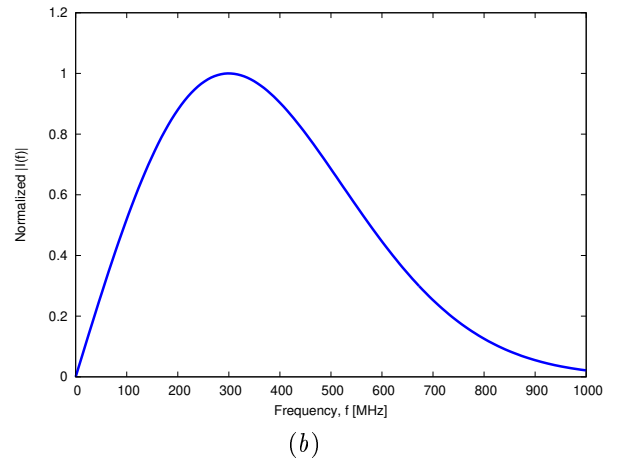
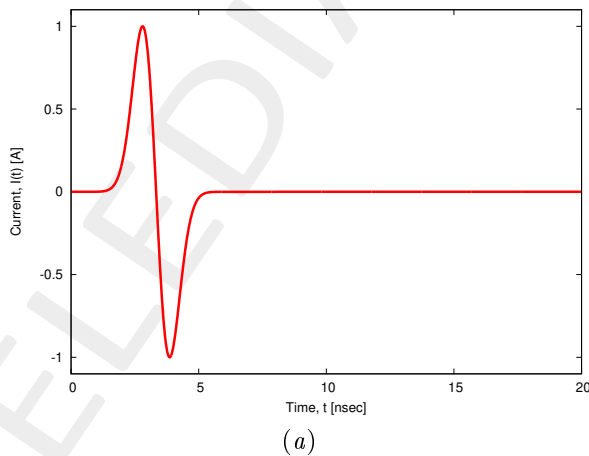


Figure 1: $GPRMax2D$ excitation signal. (a) Time pulse, (b) normalized frequency spectrum.

Frequency parameters

- Frequency range: $f \in [f_{min}, f_{max}] = [200.0, 600.0]$ [MHz] [?] (-3 [dB] bandwidth of the Gaussian Monocycle excitation centered at $f_0 = 300$ [MHz]);
- Frequency step: $\Delta f = 100$ [MHz] ($F = 5$ frequency steps in $[f_{min}, f_{max}]$);

f [MHz]	λ_a [m]	λ_b [m]	f^* [MHz]
200.0	1.50	0.75	200.5
300.0	1.00	0.50	297.6
400.0	0.75	0.37	401.1
500.0	0.60	0.30	498.1
600.0	0.50	0.25	601.6

Table 1: Considered frequencies and corresponding wavelength in the upper medium (λ_a , free space) and in the lower medium (λ_b , soil). f^* is the nearest frequency sample available from transformed time-domain data, and represents the real frequency considered by the inversion algorithm.

Scatterer

- Electromagnetic properties: $\varepsilon_{r,obj} = 5.5$, $\sigma_{obj} = 10^{-3}$ [S/m] ($\sigma_{obj} = \sigma_b$);
- Contrast function: $\tau = 1.5 + j0.0$

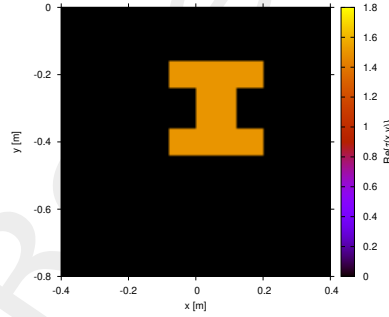


Figure 2: Actual object.

Measurement setup

- Considered frequency: $f_{min} = 200$ [MHz], $\lambda_b = 0.75$ [m].
- $\#DoFs = 2ka = \frac{2\pi}{\lambda_b} L \sqrt{2} = \frac{2\pi}{0.75} 0.8 \sqrt{2} \simeq 9.5$;
- Number of views (sources): $V = 10$;
 - $\min \{x_v\} = -0.5$ [m], $\max \{x_v\} = 0.5$ [m];
 - height: $y_v = 0.1$ [m], $\forall v = 1, \dots, V$;
- Number of measurement points: $M = 9$;
 - $\min \{x_m\} = -0.5$ [m], $\max \{x_m\} = 0.5$ [m];

- height: $y_m = 0.1$ [m], $\forall m = 1, \dots, M$;

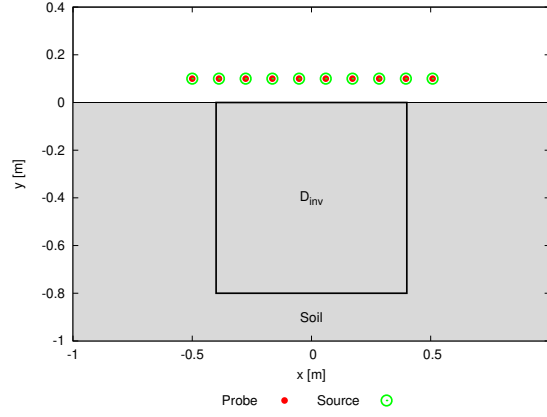


Figure 3: Location of the measurement points ($M = 9$) and of the sources ($V = 10$). Only one source is active for each view.

Inverse solver parameters

- **Shared parameters**

- Weight of the state term of the functional: 1.0;
- Weight of the data term of the functional: 1.0;
- Convergence threshold: 10^{-10} ;
- Variable ranges:
 - * $\sigma \in [8.0 \times 10^{-4}, 1.2 \times 10^{-3}]$ [S/m];
 - * $\Re \{E_{tot}^{int}\} \in [-8, 8]$, $\Im \{E_{tot}^{int}\} \in [-8, 8]$;
- Degrees of freedom:
 - * Considered frequency: $f_{min} = 200$ [MHz], $\lambda_b = 0.75$ [m];
 - * $\frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda_b} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda_b}\right)^2 = 4\pi^2 \left(\frac{0.8}{0.75}\right)^2 \simeq 44.87$;
- Number of cells: $N = 49 = 7 \times 7$;
- Maximum number of *IMSA* steps: $S = 4$;
- Side ratio threshold: $\eta_{th} = 0.2$;

- **MF – IMSA – CG parameters**

- Maximum number of iterations: $I = 200$;

- **FH – IMSA – CG parameters**

- Maximum number of iterations: $I = 400$;

Signal to noise ratio (on $E_{tot}(t)$)

- $SNR = \{50, 40, 30, 20\}$ [dB] + Noiseless data.

2.2.2 $\varepsilon_{rB}^{guess} = 3.2$: Final reconstructions ($@f_{max} = 600$ [MHz])

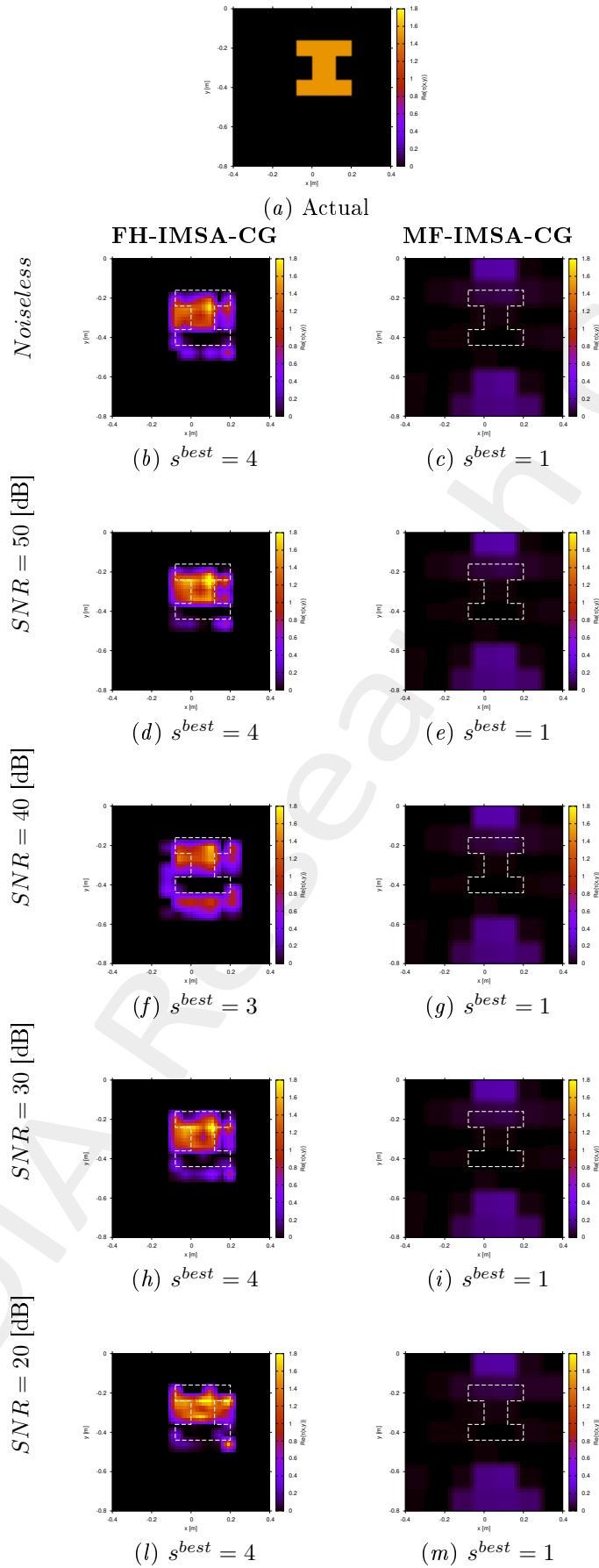


Figure 4: *FH-IMSA-CG* vs. *MF-IMSA-CG*: Retrieved dielectric profiles at the *IMSA* convergence step (s^{best}).

2.2.3 $\varepsilon_{rB}^{guess} = 3.6$: Final reconstructions ($@f_{max} = 600$ [MHz])

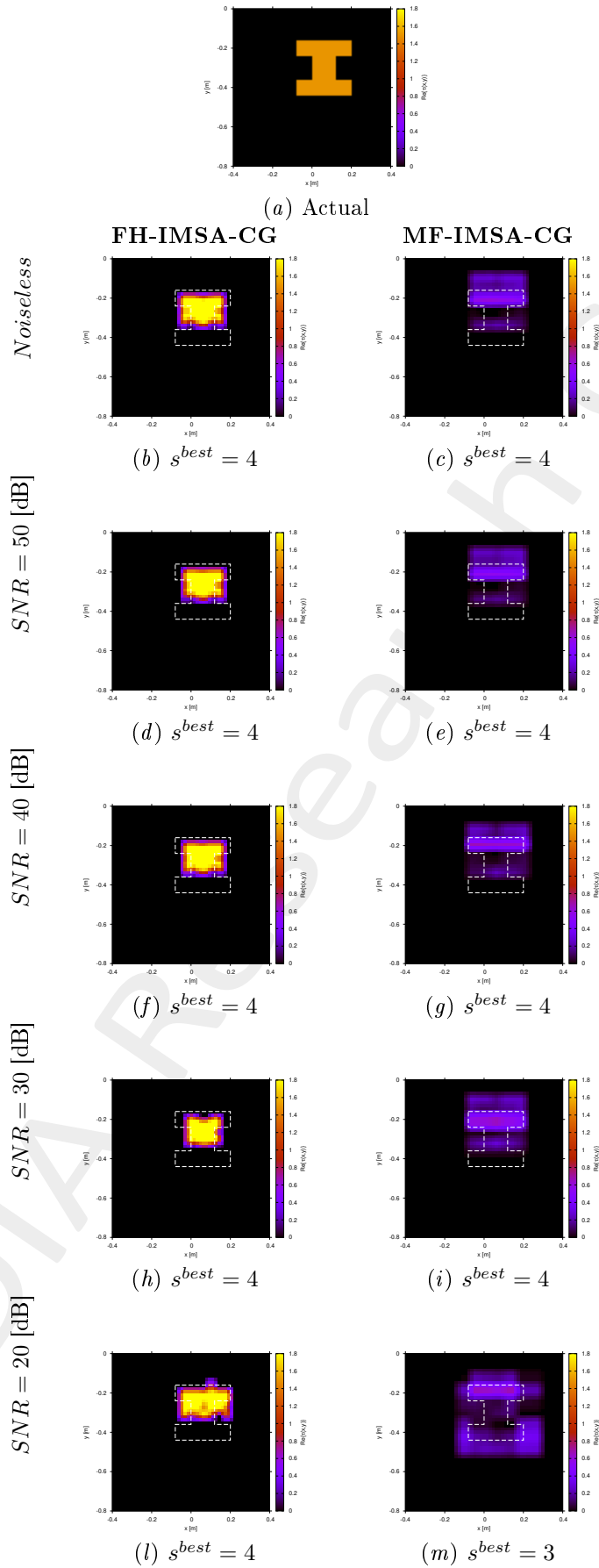


Figure 5: *FH-IMSA-CG* vs. *MF-IMSA-CG*: Retrieved dielectric profiles at the *IMSA* convergence step (s^{best}).

2.2.4 $\varepsilon_{rB}^{guess} = 3.8$: Final reconstructions ($@f_{max} = 600$ [MHz])

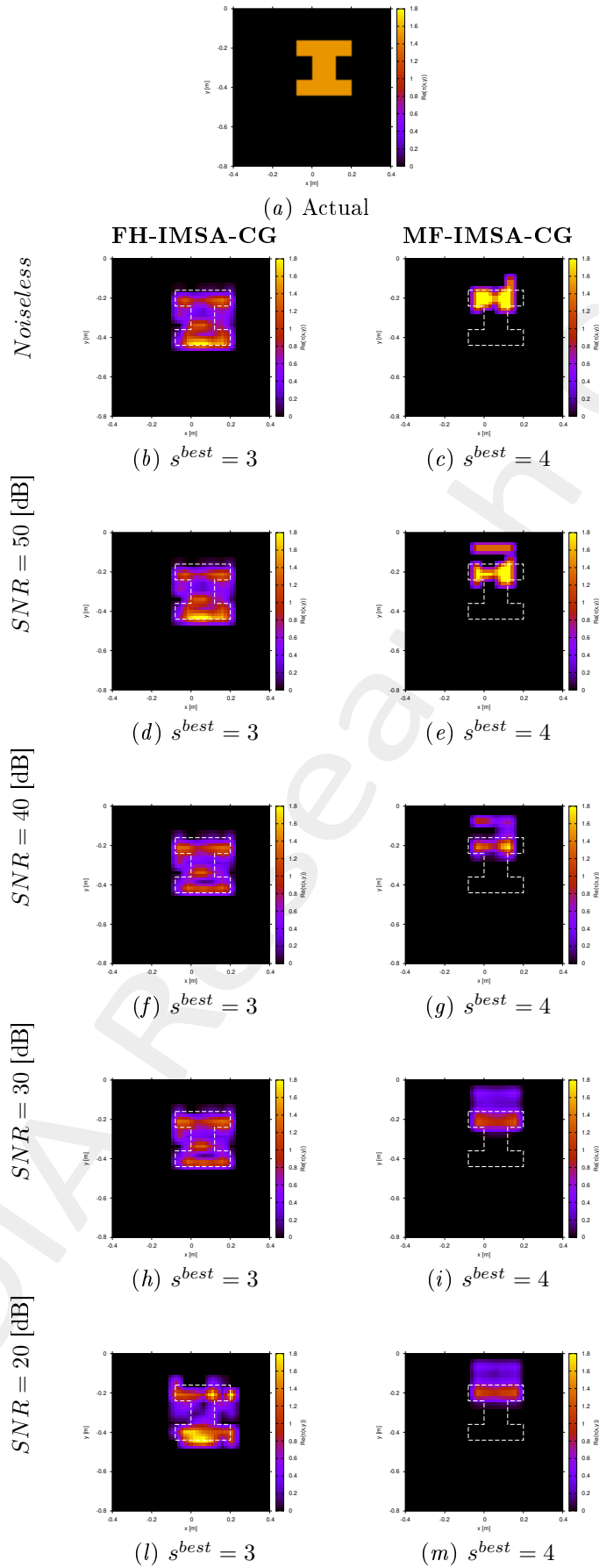


Figure 6: *FH-IMSA-CG* vs. *MF-IMSA-CG*: Retrieved dielectric profiles at the *IMSA* convergence step (s^{best}).

2.2.5 $\varepsilon_{rB}^{guess} = 4.2$: Final reconstructions ($@f_{max} = 600$ [MHz])

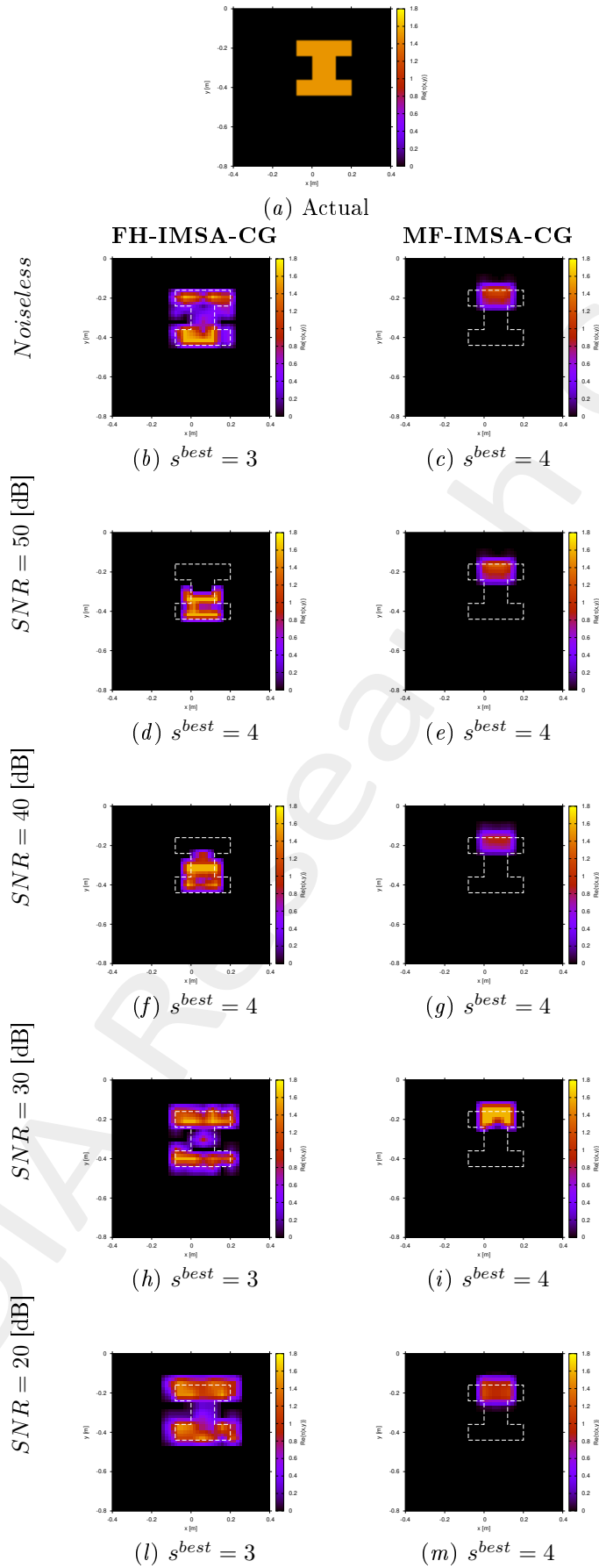


Figure 7: $FH-IMSA-CG$ vs. $MF-IMSA-CG$: Retrieved dielectric profiles at the $IMSA$ convergence step (s^{best}).

2.2.6 $\varepsilon_{rB}^{guess} = 4.4$: Final reconstructions ($@f_{max} = 600$ [MHz])

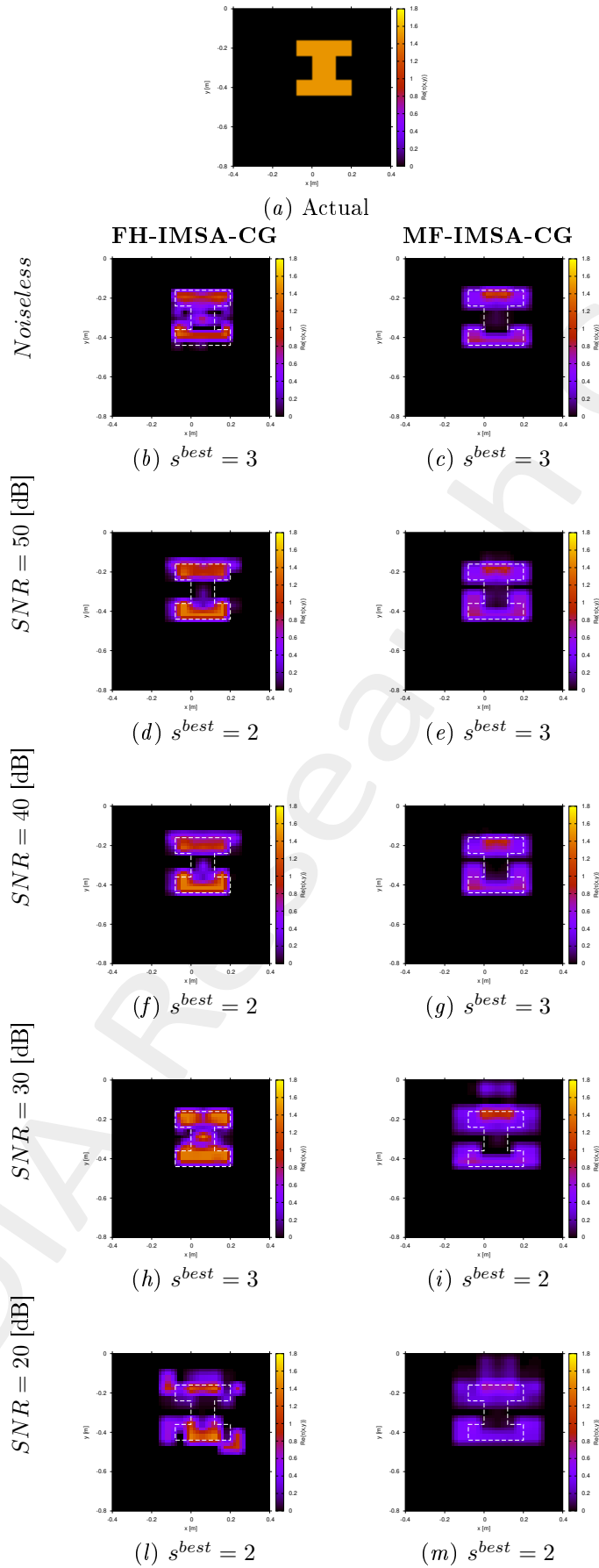


Figure 8: *FH-IMSA-CG* vs. *MF-IMSA-CG*: Retrieved dielectric profiles at the *IMSA* convergence step (s^{best}).

2.2.7 $\varepsilon_{rB}^{guess} = 4.8$: Final reconstructions ($@f_{max} = 600$ [MHz])

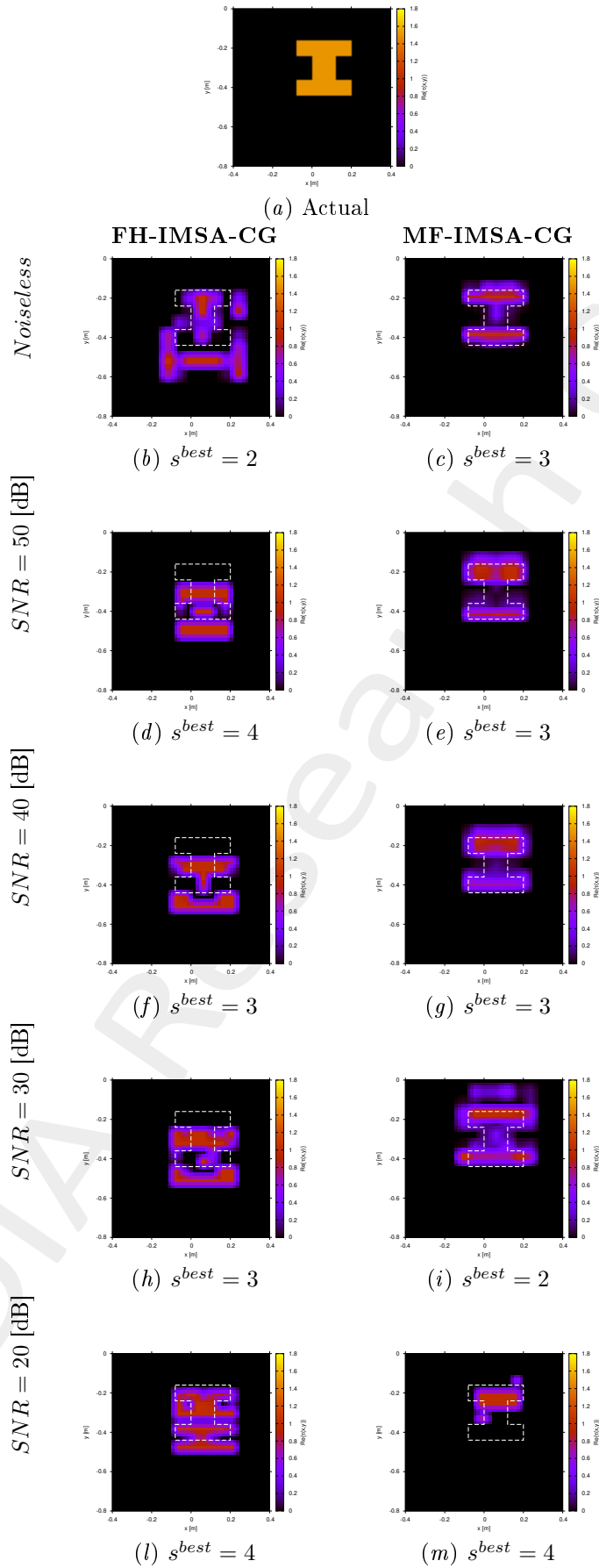


Figure 9: *FH-IMSA-CG* vs. *MF-IMSA-CG*: Retrieved dielectric profiles at the *IMSA* convergence step (s^{best}).

2.2.8 Reconstruction Errors vs. ε_{rB}^{guess} ($@f_{max} = 600$ [MHz])

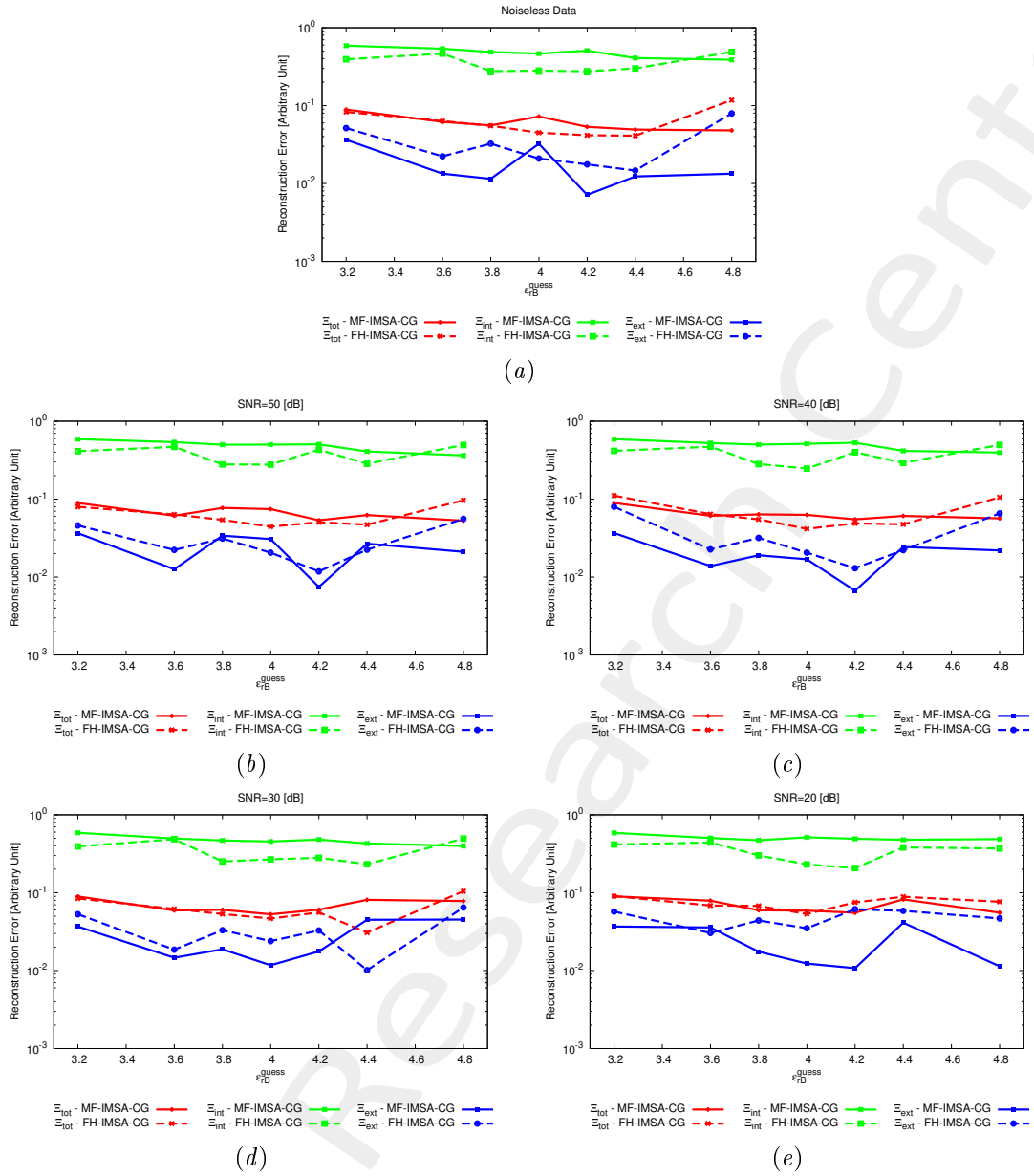


Figure 10: *FH-IMSA-CG* vs. *MF-IMSA-CG*: Reconstruction errors vs. ε_{rB}^{guess} .

More information on the topics of this document can be found in the following list of references.

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