A PSO-based Method for Radiation Pattern Synthesis through Interval Analysis Tools

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Abstract

In an antenna array design scenario the reliability of the excitations' amplitudes and phases values is very important in order to satisfy the design constraints on the array's radiation pattern. The literature proposes some probability curves and statistical descriptions methods that can help the designer to predict the effect of the errors, at the cost of an high computational burden. In this report, Interval Arithmetic is exploited to overcome this problem and Interval Analysis is applied to a mask constrained power synthesis procedure in order to synthesize a beam shaped linear array robust with respect to the excitations' errors using a PSO optimizer. A preliminary numerical example is reported to show the behaviour of the proposed method.

1 Interval Power Synthesis Optimization

The synthesis of an antenna consist in determining a set of design parameters, such that the field radiated by the antenna satisfy some specifications in the far field. In the case of an antenna array the design parameters are the element's excitations and/or the geometry of the array.

A useful approach is to obtain a shaped beam starting from a desired array pattern. Thus to find the excitations which produce an array factor that best approximates the desired one.

The error between the desired pattern and the field or the power of the synthesized pattern is mathematically a distance in a suitable functional space S. This distance, in order to fit the requirements, must be minimized. For such a problem, $L^2(S)$, the space of the functions quadratically integrable in a set S, it's a good option. The distance between the functions $f(\xi)$ and $g(\xi)$ in $L^2(S)$ is, for $\xi \in S$:

$$||f - g|| = \sqrt{\int_S |f(\xi) - g(\xi)|^2 d\xi} \tag{1}$$

and is called Mean Square Error (MSE). Functions $f(\xi)$ and $g(\xi)$ are the required and synthesized patterns respectively.

1.1 Mask Constrained Power Synthesis

The method chosen in this work for the synthesis of an antenna array is the Mask Constrained Power Synthesis which consist in determining an array pattern belonging to a given mask. Usually the mask is composed by two bounds: an Upper Mask $UM(u)$ and a Lower Mask $LM(u)$ that define the constraints on the side lobe level, on the beam width, and on the beam shape of the antenna to be designed. Every mask is defined by the side lobe level (SLL); the lower mask height (LM-Height), the upper mask beamwidth (UMBW) and by the lower mask beamwidth (LMBW).

The distance used to measure the goodness of a set of excitations is the MSE given in (1). Thus given a set of excitations $I = \{I_1, I_2, ..., I_n\}$we can obtain the corresponding array factor $AF(u)$ and then we can calculate the upper $UD(u)$ and lower $LD(u)$ distances between the power pattern and the Upper and Lower Masks respectively:

$$UD(u) = \begin{cases} |AF(u)|^2 - UM(u) \; if \; |AF(u)|^2 > UM(u) \\ 0 \qquad\qquad otherwise \end{cases} \tag{2}$$

$$LD(u) = \begin{cases} LM(u) - |AF(u)|^2 \; if \; LM(u) > |AF(u)|^2 \\ 0 \qquad\qquad otherwise \end{cases} \tag{3}$$

The overall distance is calculated as:

$$\Phi = \Phi_1 + \Phi_2 \tag{4}$$

with:

$$\Phi_1 = \int_{-1}^{1} |UD(u)|^2 du \tag{5}$$

$$\Phi_2 = \int_{-1}^{1} |LD(u)|^2 du \tag{6}$$

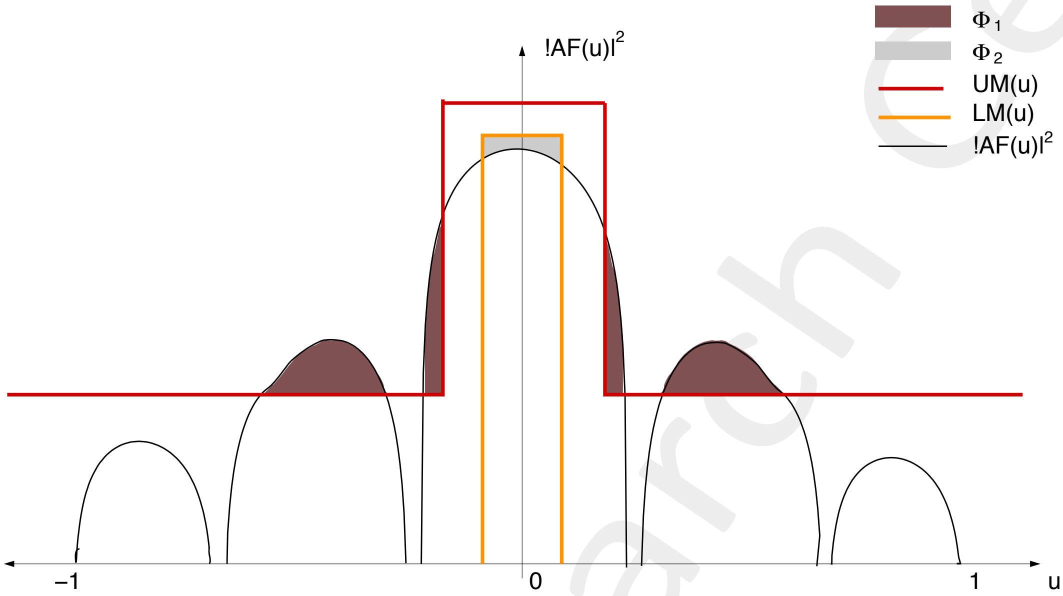

Figure 5.1.1. Mask constrained Power Synthesis

1.2 Mask Constrained Power Synthesis exploiting Interval analysis

In this work the final goal is the synthesis of a linear array using a mask constrained power synthesis method, exploiting Interval Analysis. The way Interval Analysis is applied in the synthesis procedure is visible in the distance calculation. Using Interval Analysis we derive from the given set of excitations I, in addition to the $|AF(u)|^2$, the infimum $|AF(u)|^2_{inf}$ and the supremum $|AF(u)|^2_{sup}$power patterns. Now the two distances are computed between the upper mask $UM(u)$ and $|AF(u)|^2_{sup}$ for the first contribution $D_{sup}(u)$; between the lower mask $LM(u)$ and $|AF(u)|^2_{inf}$ for the second contribution $D_{inf}(u)$. Therefore the two distances are:

$$D_{sup}(u) = \begin{cases} |AF(u)|^2_{sup} - UM(u) \; if \; |AF(u)|^2_{sup} > UM(u) \\ 0 \qquad\qquad otherwise \end{cases} \tag{7}$$

$$D_{inf}(u) = \begin{cases} LM(u) - |AF(u)|^2_{inf} \; if \; LM(u) > |AF(u)|^2_{inf} \\ 0 \qquad\qquad otherwise \end{cases} \tag{8}$$

Similarly as in (4) the final distance is:

$$\Phi = \Phi_{\text{sup}} + \Phi_{\text{inf}} \tag{9}$$

with:

$$\Phi_{\text{sup}} = \int_{-1}^{1} |D_{\text{sup}}(u)|^2 du \tag{10}$$

$$\Phi_{\text{inf}} = \int_{-1}^{1} |D_{\text{inf}}(u)|^2 du \tag{11}$$

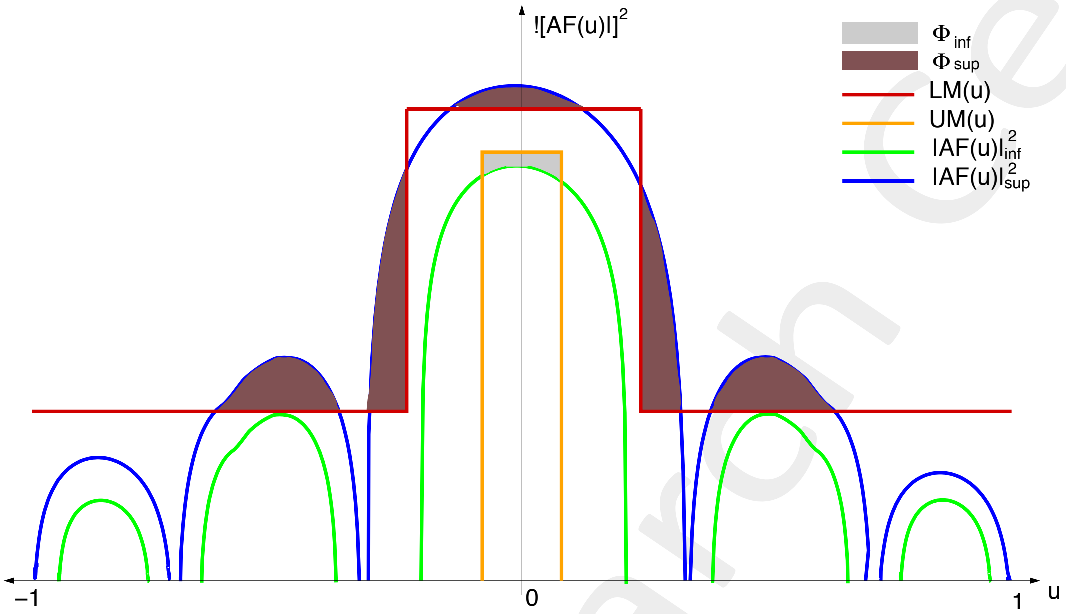

Figure 5.1.1. Mask constrained Power Synthesis using an Interval Pattern

Now we can formulate the synthesis problem using interval analysis in this way:

- consider for simplicity just real excitations, so the unknowns of our problem is the vector of the excitations amplitudes $\overrightarrow{a} = \{a_n;\, n = 1,\, ...,\, N\}$
- I know that exist an upper and a lower bound for each nominal excitation's amplitude value, and for each a_n; $n = 1, ..., N$, i can calculate the upper bound $a_n + \delta a_n$ and the lower bound $a_n - \delta a_n$ using (**??**).
- I want to find the working points $\overrightarrow{a}^* = \{a_n^*;\, n = 1,\, ...,\, N\}$ so that

$$LM(u) < [|AF(u)|^2] < UM(u) \tag{12}$$

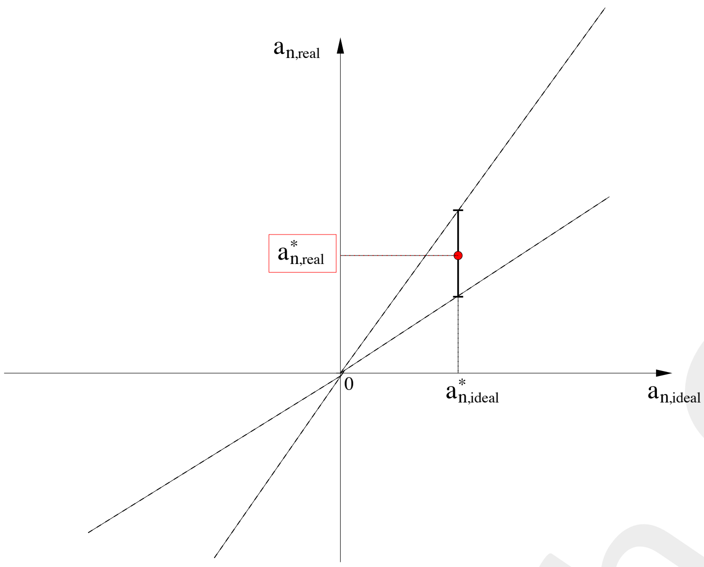

Figure 5.1.2. Ideal and real amplitude values relation.

2 Numerical Example

2.1 Test Case #1 - Pencil Beam - $N = 10$ - $\delta a_n = 1\%$

Array Geometry:

- Number of Elements: $N = 10$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 1%
- Phase Error: $\delta \varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.46 u
- LMBW: 0.23 u
- SLL: -20 dB
- LM-Height: -5 dB

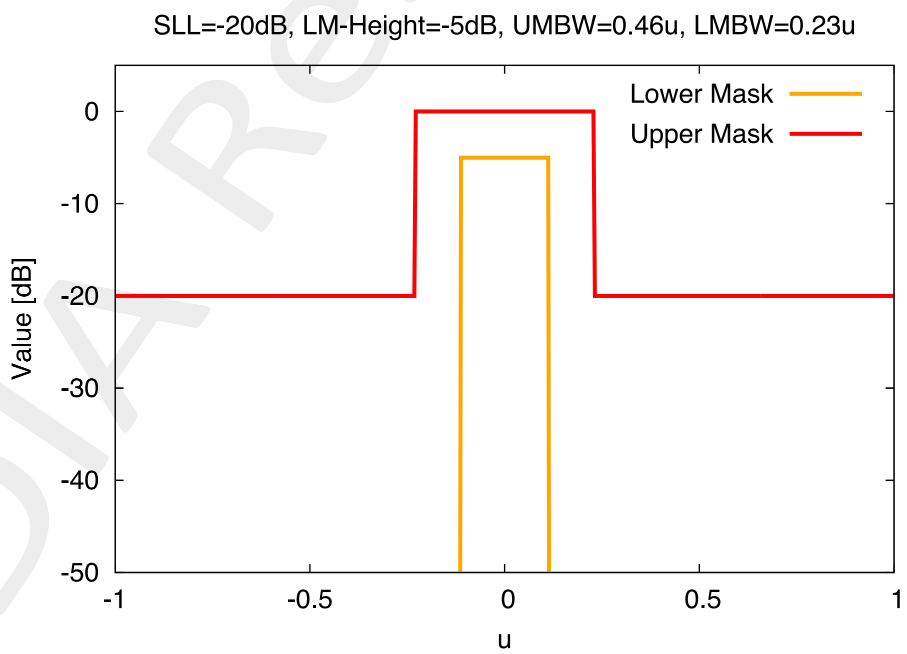

Figure 6.1.1. Power Synthesis Mask

2.1.1 PSO Parameters:

- Unknown Number: 10
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 200
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.1.1. Max and Min excitations amplitudes values, for the PSO

2.1.2 Excitations:

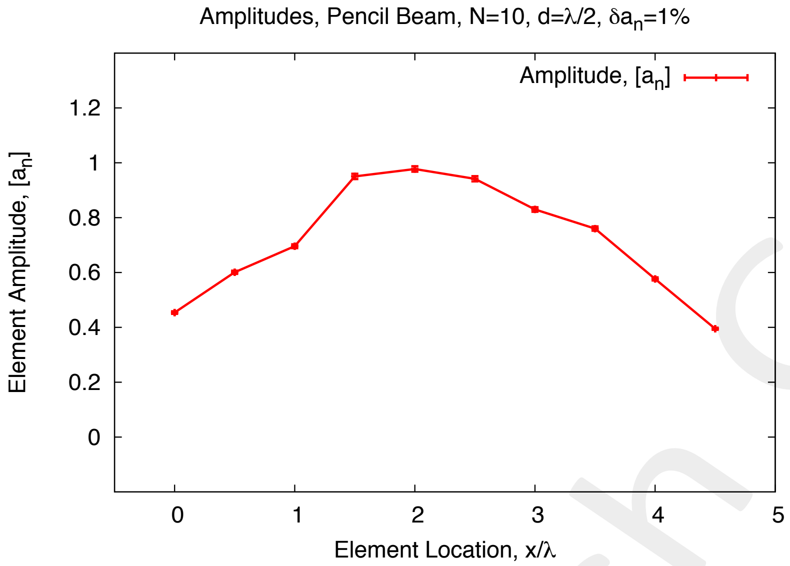

Figure 6.1.2.1. Optimal particle's excitations amplitudes

2.1.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (4). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

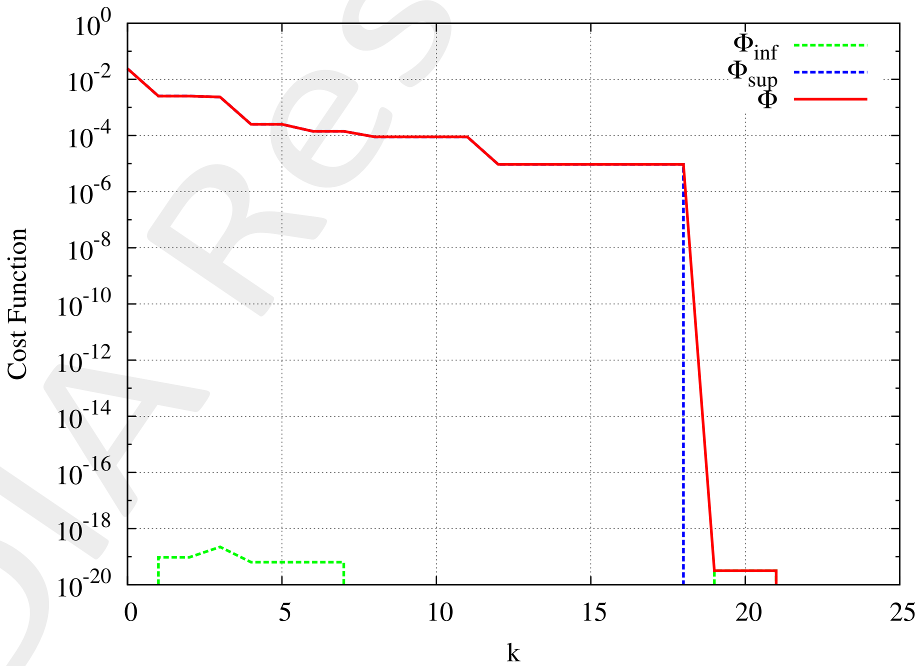

Figure 6.1.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
22	0	39 sec

2.1.4 Synthesized Interval Pattern:

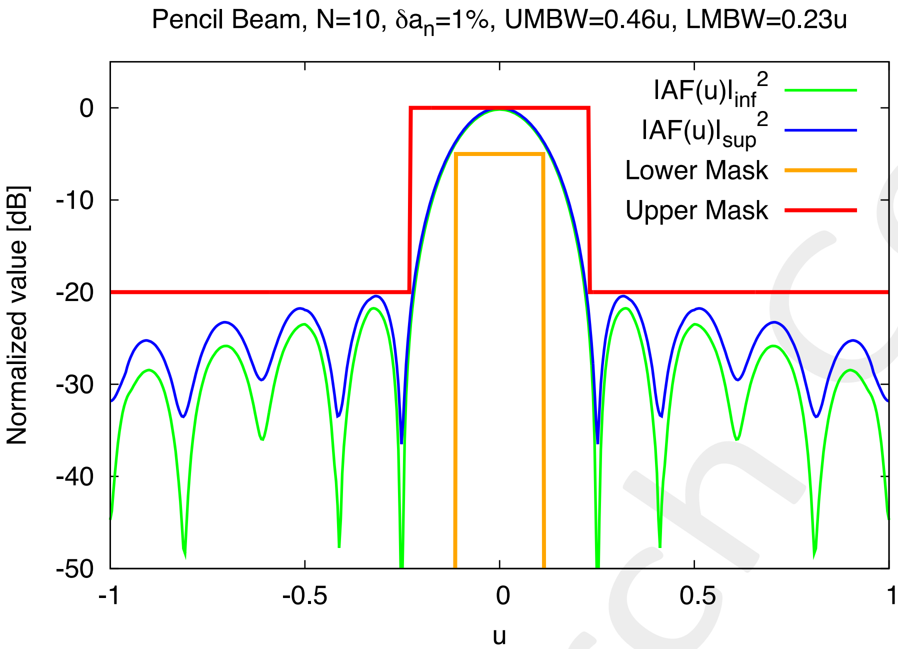

Figure 6.1.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	9.6	-21.1	0.208	-0.08	0.01186
Inf.	9.5	-21.7	0.208	-0.17	/
Sup.	9.8	-20.5	0.216	0.0	/

Table 6.1.4.1. Interval Pattern Parameters

2.1.5 Superior bound evolution

In the following figure the superior bound evolution during the optimization process of the previous test case, is plotted for some iterations.

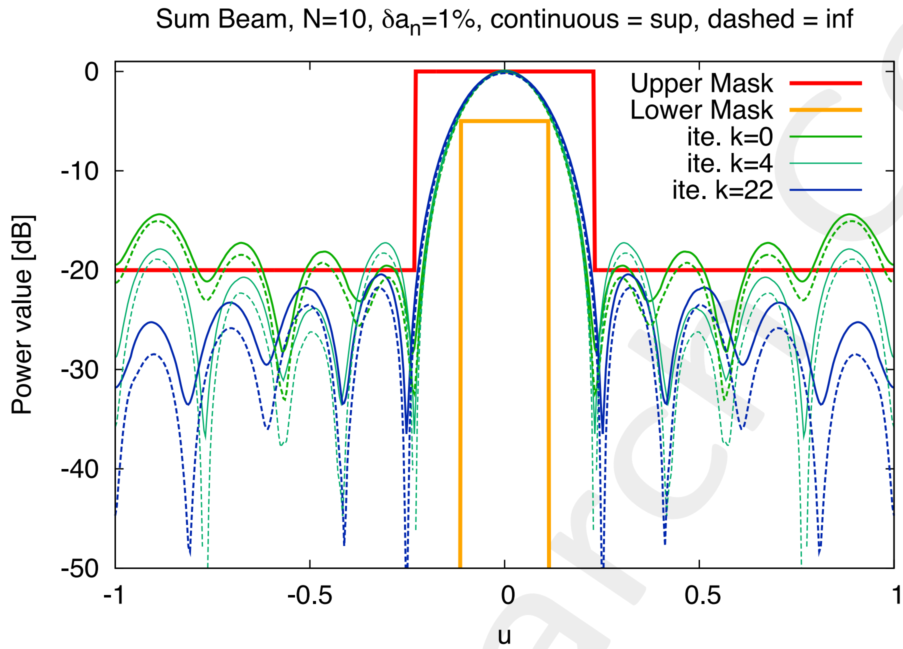

Figure 6.1.5.1. Evolution of the Superior Bound

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