



UNIVERSITY
OF TRENTO

DIPARTIMENTO DI INGEGNERIA E SCIENZA DELL'INFORMAZIONE

38123 Povo – Trento (Italy), Via Sommarive 14
<http://www.disi.unitn.it>

AN INTEGRATED STOCHASTIC MULTI-SCALING STRATEGY
FOR MICROWAVE IMAGING APPLICATIONS

P. Rocca and A. Massa

January 2011

Technical Report # DISI-11-275

An Integrated Stochastic Multi-Scaling Strategy for Microwave Imaging Applications

P. Rocca and A. Massa

Department of Information and Communication Technologies, University of Trento,
Via Sommarive 14, I-38050 Trento, Italy.

E-mail: andrea.massa@ing.unitn.it , Web-page: <http://www.eledia.ing.unitn.it>

This work presents an improved Multi-Scale algorithm for microwave imaging of two-dimensional scatterers. The proposed methodology includes a feedback between high- and low-resolution reconstructions in order to correlate the iterative reconstruction steps. Towards this end, the appealing features of a Particle Swarm-based algorithm are fully exploited. Such an integration is aimed at better matching a suitable representation of the unknowns with the global optimization properties of the stochastic optimizer to allow faithful reconstructions.

Introduction

The reconstruction of the electromagnetic properties of an unknown area starting from the measurement of the diffused electromagnetic field is still an open problem affected by some severe obstacles. In fact, the non-linear nature of the mathematical model that describes the scattering phenomena complicates the inversion procedure intrinsically characterized by the ill-posedness [1]. Moreover, the information content of the scattered field [2] even in multi-view systems [3] is limited. Therefore a key issue in every microwave imaging problem is the optimal representation of the unknowns in order to fully exploit all the available information. Toward this end, several multi-resolution approaches have been recently proposed in order to guarantee a high resolution only in the regions of interest (RoIs). Some approaches [4] define a multi-resolution discretization of the investigation domain starting from *a-priori* assumptions. Similarly, E. Miller proposed a wavelet expansion [5], [6] to deal with different statistical configurations of the scenario under test. On the contrary, the approach presented in [7] avoids *a-priori* hypotheses and it considers an adaptive distribution of the spatial unknowns according to the information gained during an iterative multi-steps process.

In the same framework, this paper proposes an improved methodology with respect to that presented in [7] aiming at overcoming some limitations. Firstly, a sort of feedback mechanism has been included in order to contemporarily take into account all the different resolution grids adopted for the unknowns expansions. Consequently, because of the nonlinear nature of the cost function at hand, an innovative optimization approach has been used for the cost function minimization. As a matter of fact, although a multi-resolution strategy intrinsically reduces the search space (with respect to a standard “bare” approach) thus limiting the occurrence of local-minima, it does not completely eliminate the false-solutions problems. Therefore, the optimization block of the multi-scaling methodology has been redesigned by inserting a global optimization technique based on the particle swarm optimizer (PSO) [8].

Mathematical Formulation

Let us consider a two-dimensional geometry. An unknown distribution of the object function $\tau(x, y)$ has to be reconstructed in an unknown region D_I starting from a set measures of the scattered electric field, $E_{scatt}^v(x_m, y_m), m=1, \dots, M^{(v)}$, collected in a measurement domain D_M . By assuming a multi-view/multi-illumination acquisition setup, the scenario under test is probed by a set of different incident fields, $E_{inc}^v(x, y), v=1, \dots, V$. The arising scattering phenomena can be described by the following system of nonlinear integral equations [1]:

$$\begin{aligned} E_{scatt}^v(x_m, y_m) &= \Omega_{ext} \left\{ \tau(x_n, y_n), E_{tot}^v(x_n, y_n) \right\} & x_m, y_m \in D_M \\ E_{inc}^v(x_n, y_n) &= E_{tot}^v(x_n, y_n) - \Omega_{int} \left\{ \tau(x_n, y_n), E_{tot}^v(x_n, y_n) \right\} & x_n, y_n \in D_I \\ & & n = 1, \dots, N \end{aligned} \quad (1)$$

which turns out to be ill-posed and ill-conditioned without a unique solution. To overcome these drawbacks, the problem is commonly addressed by looking for the configuration of the unknowns that minimizes a suitable cost function (3) forcing the solution to fit the available scattering data

$$\begin{aligned} \Phi(\underline{u}) &= \frac{\sum_{v=1}^V \sum_{m=1}^M \left| E_{scatt}^v(x_m, y_m) - \Omega_{ext} \left\{ \tau(x_n, y_n), E_{tot}^v(x_n, y_n) \right\} \right|^2}{\sum_{v=1}^V \sum_{m=1}^M \left| E_{scatt}^v(x_m, y_m) \right|^2} + \\ & \frac{\sum_{v=1}^V \sum_{n=1}^N \left| E_{inc}^v(x_n, y_n) - E_{tot}^v(x_n, y_n) + \Omega_{int} \left\{ \tau(x_n, y_n), E_{tot}^v(x_n, y_n) \right\} \right|^2}{\sum_{v=1}^V \sum_{n=1}^N \left| E_{inc}^v(x_n, y_n) \right|^2} \end{aligned} \quad (3)$$

where $\underline{u} = \left\{ \tau(x_n, y_n), E_{tot}^v(x_n, y_n); n=1, \dots, N; v=1, \dots, V \right\}$. In order to better exploit the information available from scattered data [2], an innovative approach aimed at fully exploiting the features of the adaptive multi-resolution strategy proposed in [7] will be adopted. Such a method considers a new processing of the data through an integration of the multi-steps procedure with an evolutionary optimization algorithm. The key-features of the approach can be summarized as follows.

Low-Resolution Reconstruction. A ‘‘rough’’ reconstruction is achieved by means of the minimization of $\Phi(\underline{u})$ by considering a uniform discretization of the investigation area with $N^{(1)}$ equal square basis functions.

Iterative Multi-Resolution Procedure. Starting from the initial estimate $\underline{u}^{(1)}$ obtained through the ‘‘Low-Resolution Reconstruction’’, the following operations are iterated at each step ‘‘s’’ of the multi-scaling procedure:

RoIs Detection. According to a clustering method based on the analysis of the image-histogram [9], $T^{(s)}$ RoIs are identified by defining their centers $(x_{c(t)}^{(s)}, y_{c(t)}^{(s)})$ and $L_{(t)}^{(s)}$ sides ($t=1, \dots, T^{(s)}$).

Combined Multi-Resolution Expansion. A finer expansion is used in the estimated RoIs so that a higher resolution level $R(s)$ is adopted in those areas at the s -th step. Moreover,

the unknowns related to previous $(R - 1)$ resolution levels are not neglected, but they are simultaneously optimized to obtain a sort of retrieval feedback. Even though the number of unknowns grows during the multi-step procedure, (e.g., $N^{(1)} + \dots + N^{(R(s)-1)} + N^{(R(s))}$ is the number of unknowns at the s -th step), it should be noticed that they are not “blind” parameters since they are set according to the information gained at previous steps.

PSO-based Optimization. Previous works [7], [9] consider a deterministic optimizer for the minimization of the multi-resolution cost function since the multi-scaling technique limits the dimension of the solution-space and the occurrence of local minima. However, the presence of false solution cannot be completely avoided and a global optimization procedure is needed. Towards this end, the easy implementation and integration with the multi-scaling technique, as well as the simple tuning of the control parameters and the low computational cost, seem to indicate the PSO as a convenient choice.

According to the swarm logic, a set of $I^{(s)}$ trial solutions (called *swarm of particles* $p_i^{(s)}; i = 1, \dots, I^{(s)}$) are generated by defining their positions $\underline{u}_i^{(s)}$ (on the basis of the information acquired during the multi-step process) and velocities $\underline{g}_i^{(s)}$ in the solution space. Successively, K iterations of the PSO are performed by applying the evolutionary operators defined in [10] to achieve an estimate of the unknown dielectric profile,

$$\underline{u}^{(s)} = \arg \left\{ \min_{k=1, \dots, K} \left[\min_{i=1, \dots, I^{(s)}} \left[\Phi(\underline{u}_i^{(s)}) \right] \right] \right\}.$$

Termination Condition. The iterative multi-step scheme is terminated when there is a stationariness on the number of estimated RoIs and their geometrical parameters $(x_{c(t)}^{(s)}, y_{c(t)}^{(s)})$ and $L_{(t)}^{(s)}$ [9].

Numerical Validation

In this section, a selected test case is shown to preliminarily assess the potentialities of the proposed reconstruction method. It refers to the experimental database [11] and it is concerned with the multiple-object configuration of two homogeneous circular cylinders characterized by an object function $\tau(x, y) = 2.0 \pm 0.3$. Their radius measures 15 mm and they are located about 30 mm from the center of the experimental setup, while 90 mm is the distance between the centers of the cylinders. As far as the retrieval process is concerned, all the available data ($M = 49$, $V = 36$) at $f = 4 \text{ GHz}$ have been considered to reconstruct an investigation domain of $30 \times 30 \text{ cm}^2$. The enhancement of the reconstruction accuracy allowed by the IMSA-PSO integrated strategy can be noticed in Figure 1, where a comparison with the same multi scaling technique using a deterministic optimizer is shown. Even though the scatterers are correctly located in both the numerical experiments, $(x_{c(1)}^{(PSO)} = 12.4 \text{ mm}, y_{c(1)}^{(PSO)} = -45.3 \text{ mm}$ versus $x_{c(1)}^{(CG)} = 11.1 \text{ mm}, y_{c(1)}^{(CG)} = -49.5 \text{ mm}; x_{c(2)}^{(PSO)} = -1.3 \text{ mm}, y_{c(2)}^{(PSO)} = -49.2 \text{ mm}$ versus $x_{c(2)}^{(CG)} = 0.5 \text{ mm}, y_{c(2)}^{(CG)} = -46.4 \text{ mm})$ as well as satisfactory dimensioned (the reconstructed radii are equal to $\rho_1^{(PSO)} = 14.5 \text{ mm}$ and $\rho_2^{(PSO)} = 13.4 \text{ mm}$ for the IMSA-PSO, while they turn out to be $\rho_1^{(CG)} = 15.1 \text{ mm}$ and $\rho_2^{(CG)} = 16.0 \text{ mm}$ with the IMSA-CG), the PSO provides a more faithful and homogeneous retrieval of the object function (maximum value of the object

function equal to $\tau_{\max}^{PSO} = 1.98$ versus $\tau_{\max}^{CG} = 2.10$), thus contributing to the improvement of the overall quantitative imaging of the structure under test.

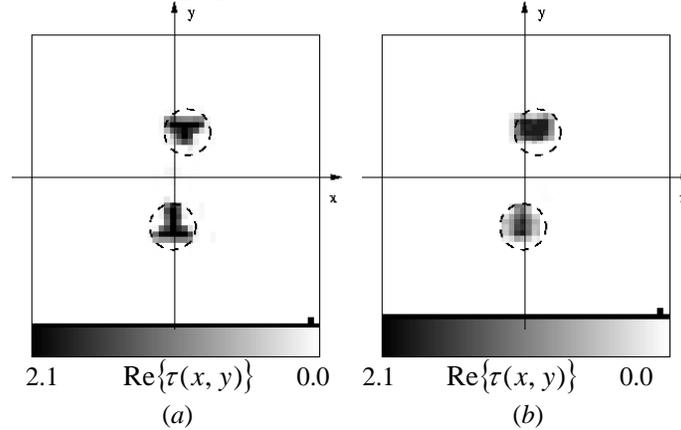


Figure 1. Real Database “Marseille”. Reconstruction of a multiple-object configuration by means of the iterative multi-scaling technique integrated with a conjugate gradient based algorithm, (a), and a PSO based algorithm (b).

References:

- [1] M. Bertero and P. Boccacci, *Introduction to Inverse Problem in Imaging*. IOP Publishing Ltd, Bristol, 1998.
- [2] O.M. Bucci and G. Franceschetti, “On the degrees of freedom of scattered fields,” *IEEE Trans. Antennas Propagat.*, vol. 37, pp. 918-926, Jul. 1989.
- [3] O.M. Bucci and T. Isernia, “Electromagnetic inverse scattering: Retrievable information and measurement strategies,” *Radio Sci.*, vol. 32, pp. 2123-2138, Nov.-Dec. 1997.
- [4] O. M. Bucci, L. Crocco, T. Isernia, and V. Pascazio, “Subsurface inverse scattering problems: Quantifying qualifying and achieving the available information,” *IEEE Tran. Geosci. Remote Sensing*, vol. 39, pp. 2527-2537, Nov. 2001.
- [5] E. L. Miller and A. Willsky, “A multiscale, statistically based inversion scheme for linearized inverse scattering problems,” *IEEE Trans. Geosci. Remote Sensing*, vol. 34, pp. 346-357, Mar. 1996.
- [6] E. L. Miller, “Statistically based methods for anomaly characterization in images observation of scattered fields,” *IEEE Trans. Image Processing*, vol. 8, pp. 92-101, Jan. 1999.
- [7] S. Caorsi, M. Donelli, D. Franceschini, and A. Massa, “A new methodology based on an iterative multiscaling for microwave imaging,” *IEEE Trans. Microwave theory Tech.*, vol. 51, pp. 1162-1173, Apr. 2003.
- [8] J. Kennedy, R. C. Eberhart, and Y. Shi, *Swarm Intelligence*. San Francisco: Morgan Kaufmann Publishers, 2001.
- [9] S. Caorsi, M. Donelli, and A. Massa, “Detection, location, and imaging of multiple scatterers by means of the iterative multiscaling method,” *IEEE Trans. Microwave Theory Tech.*, vol. 52, pp. 1217-1228, Apr. 2004.
- [10] J R. Robinson and Y. Rahmat-Sami, “Particle Swarm optimization in electromagnetics,” *IEEE Trans. Antennas Propagat.*, vol. 52, pp. 771-778, Mar. 2004.
- [11] K. Belkebir and M. Saillard, “Testing Inversion algorithms against experimental data,” *Inverse problems*, vol. 17, pp. 1565-1702, Dec. 2001.