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January 2011

Technical Report # DISI-11-251

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Abstract

Inverse scattering data, even though collected in a controlled environment, are usually corrupted by electromagnetic noise, which strongly affects the effectiveness of the reconstruction techniques because of the intrinsic ill-positioning of the problem. In order to limit the effects of the noise on the retrieval procedure and to fully exploit the limited information content available from the measurements, an innovative inversion scheme based on the integration of an adaptive multi-scale procedure and a fuzzy-logic-based decision strategy is proposed. The approach is based on an adaptive, coarse-to-fine successive representation of the unknown object obtained through a sequence of nonlinear reconstructions where suitable weighting coefficients are defined using fuzzy logic. Numerical examples from synthetic and experimental test cases are given to illustrate the advantages brought by the proposed approach in terms of reconstruction quality.

Key words: inverse scattering, fuzzy-logic, iterative multi-scaling approach.

1. INTRODUCTION

Electromagnetic noise unavoidably corrupts inverse scattering data even when collected in a controlled environment. As a consequence, effectiveness of inversion strategies is strongly affected because of the intrinsic ill-posedness and ill-conditioning of the problem in hand even because the amount of information collectable from the scattering experiments is limited [1]. In general, microwave imaging algorithms do not consider the actual reliability of the data. As a matter of fact, a direct evaluation of the data reliability is usually not available or it is difficult, complex and expensive to quantify using traditional mathematics or in terms of experimental rules. As a consequence, it is an usual and accepted rule to simply neglect it. However, and contrary to standard mathematics, fuzzy theory (see, e.g., [2] and [3]) seems to be a useful tool to exploit such information and to extract the information-content coming from noisy data. Accordingly, an innovative approach to effectively take into account the presence of noise has been presented in [4] where crisp numerical inputs are suitably mapped into crisp numerical outputs employing the Fuzzy-Logic operations based on fuzzy concepts and rules. The fuzzy-logic system is able to define suitable regularization parameters that weight the scattering data according to their degree of truth. Moreover, a multi-step iterative algorithm [5] has been used in order to adaptively optimize the resolution level and properly exploit the information content of the fuzzy-processed data. In order to assess the benefits of the integration of the fuzzy logic operations into reference inverse scattering techniques, selected numerical experiments will be presented and discussed in the following.

2. INVERSE PROBLEM FORMULATION

Let us consider a cylindrical two-dimensional geometry where a set of V transverse-magnetic (TM) plane waves with $\mathbf{E}^{v}_{inc}(x, y)$ $v = 1, \ldots V$ successively illuminates an investigation domain D where unknown inhomogeneous dielectric objects are present. The background medium is considered to be homogeneous and lossless with dielectric properties equal to that of the vacuum ε_0 and μ_0. The unknown scatterer is described by the object function τ given by

$$\tau(x,y) = [\varepsilon_R(x,y) - 1] - j\frac{\sigma(x,y)}{2\pi f \varepsilon_0}$$

where ε_R and σ are the relative dielectric permittivity and the electric conductivity of the scatterer, respectively, and f is the frequency of the electromagnetic source.

The field scattered from the scenario under test is collected in an external observation domain S at a set of M uniformly distributed measurement points . The problem is described by the well-known integral equations (see, e.g., [6]):

$$E^v_{scatt}(x,y) = k_0^2 \int_D G_{2D}(x,y|x',y')$$
$$\tau(x',y') E^v_{tot}(x',y')\, dx'dy'$$

$$E^v_{inc}(x,y) = E^v_{tot}(x,y) -$$
$$k_0^2 \int_D G_{2D}(x,y|x',y')\, \tau(x',y')\, E^v_{tot}(x',y')\, dx'dy'$$

where G_{2D} denotes the Green function of the background and E^v_{tot} is the total field within the investigation domain. In order to achieve a suitable resolution, a multi-scaling approach [5] can be used. The $IMSA$ (Iterative Multi Scaling Approach) consists of a sequence of S successive reconstructions ($s = 1, \ldots, S$) of the unknowns (field and contrast) through the minimization of a multi-resolution cost functional (see [5]) proportional to the mismatching between the measured and reconstructed data.

Unfortunately, in real applications data are usually corrupted by equivalent sources of noise and, in order to take into account this inaccuracies and the uncertainty of the measurement process itself, we decided to use a fuzzy representation [3] of such uncertainty. This is done by means of a fuzzy-logic-based strategy, as described in [4]. The proposed system is composed by two separated blocks. First, the scattering data are normalized (normalization block) with respect to the maximum values of fields recorded in the observation and investigation domain and then a fuzzy logic inference process is carried out, according to a specific set of heuristically defined rules (see [7] for details) and according to the uncertainty that one associates to every measurement. The outputs of the process are the fuzzy coefficients that allow one to take into consideration the presence of the noise on the data and consequently rely, with a certain degree, to the data and state equations. In order to include in the scheme the FL system, the $IMSA$ cost function is redefine and the normalized misfits in the data and state equation are weighted using the fuzzy coefficients as follows

$$\Phi^{(s)} = \frac{\sum_{v=1}^{V}\sum_{m=1}^{M} \alpha_{m,v} \left|E^v_{scatt}(x,y) - \Phi_{Data}\right|^2}{\sum_{v=1}^{V}\sum_{m=1}^{M} \left|E^v_{scatt}(x,y)\right|^2} +$$
$$\frac{\sum_{v=1}^{V}\sum_{r=1}^{R(s)}\sum_{n(r)=1}^{N(r)} w(x',y')\beta_{n,v}\left|E^v_{inc}(x',y') - \Phi_{State}\right|^2}{\sum_{v=1}^{V}\sum_{r=1}^{R(s)}\sum_{n(r)=1}^{N(r)}\left|w(x',y')E^v_{inc}(x',y')\right|^2}$$

where (x,y) and (x',y') refer to points of the observation and investigation domain respectively, while

$$\Phi_{Data} = \sum_{r=1}^{R(s)}\sum_{n(r)=1}^{N(r)} w\left(x_{n(r)}, y_{n(r)}\right)$$
$$\tau\left(x_{n(r)}, y_{n(r)}\right) E^v_{tot}\left(x_{n(r)}, y_{n(r)}\right) G_{2D}\left(k_0 \rho_{m(v)n(r)}\right)$$

and

$$\Phi_{State} = E^v_{tot}(x',y') - \sum_{p(r)=1}^{N(r)} \tau\left(x_{p(r)}, y_{p(r)}\right)$$
$$E^v_{tot}\left(x_{p(r)}, y_{p(r)}\right) G_{2D}\left(k_0 \rho_{n(r)p(r)}\right)$$

w being a weighting function

$$w\left(x_{n(r)}, y_{n(r)}\right) = \begin{cases} 0 & if \quad \left(x_{n(r)}, y_{n(r)}\right) \notin D^{(s-1)} \\ 1 & if \quad \left(x_{n(r)}, y_{n(r)}\right) \in D^{(s-1)} \end{cases}$$

and $D^{(s-1)}$ the support of the Region-of-Interest (RoI) where the unknown scatterer has been detected at the $(s-1)$-th step (see [5] for details). The iterative process is repeated until a "*stationary*" condition [10] is verified. Although global minimization techniques [9] have been shown to be very effective in solving this kinds of problems, in this work a conjugate gradient technique has been exploit in order to better focus on the advantages brought by the fuzzy-logic system avoiding the misleading random effects of stochastic processes.

3. NUMERICAL RESULTS

The effectiveness and robustness of the $IMSA-Fuzzy$ approach have been assessed through an extensive set of numerical simulations and in the following a representative selection of such experiments concerned with synthetically-generated as well as experimental scenarios will be presented.

In the first example, the scatterer is a square homogeneous dielectric ($\tau = 1.5$) cylinder of side $l = 1.2\,\lambda_0$ located at $x_c = 0.4\,\lambda_0$ and $y_c = 1.0\,\lambda_0$ in an investigation domain $D = 4\,\lambda_0$ in size. D has been illuminated by $V = 8$ plane waves and the scattering data have been collected in $M_v = 50$ $v = 1, \ldots V$ measurement points on the circular observation domain S, $r_{obs} = 2.93\,\lambda_0$ in radius.

In order to show the effect of the reliability indexes $\alpha_{m,v}$ and $\beta_{n,v}$ on the reconstruction capabilities of the imaging procedure, the grey-scale representations of the reconstructions obtained with the reference $IMSA$ with and without the FLS in different noisy conditions are reported in Fig. 1. When the noise level is low ($SNR \geq 20\,dB$), the FL block behaves, as requested, in a "transparent" way and its effect in terms of reconstruction accuracy appears almost negligible as pictorially shown in Figs. 1(*a*) and 1(*b*). On the other hand, and as expected, the fuzzy system, which acts before the cost function minimization, significantly impacts when $SNR < 20\,dB$. When the level of noise increases, the dielectric profile reconstructed by means of the standard $IMSA$ presents some inhomogeneities and artifacts while the images retrieved using the FL system turn out to be more homogeneous.

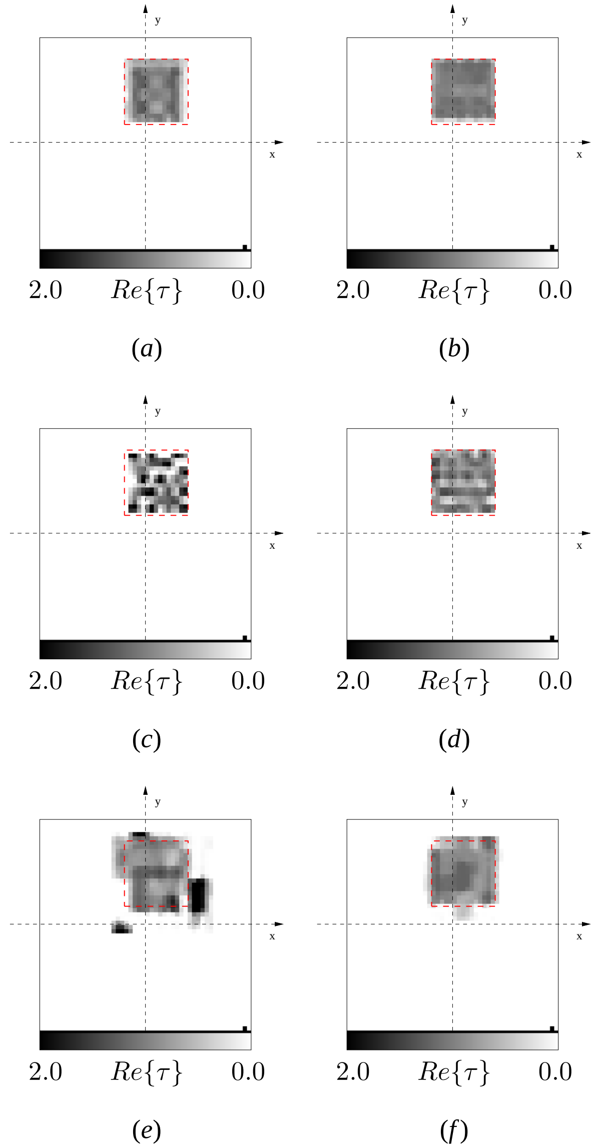

*Figure 1. Reconstruction of an off-centered square homogeneous dielectric ($\tau = 1.5$) cylinder - Samples of the dielectric profiles reconstructed by using the $IMSA$ (*left column*) and the $IMSA-Fuzzy$ method (*right column*): (*a)(b) $SNR = 20\,dB$, (c)(d) $SNR = 10\,dB$, and (e)(f) $SNR = 5\,dB$.*

The performances of the two strategies are pictorially summarized in Fig. 2 where the plot of the quantitative error figures $\overline{\zeta}_{tot}$ (defined as in [10]) versus SNR are shown. The analysis of the cost functional clearly indicates that also from a computational point of view, the $IMSA$ technique positively benefits from the introduction of the reliability coefficients $\alpha_{m,v}$ and $\beta_{n,v}$. Such a positive effect is further pointed out from Figure 2 where the averaged total number of iterations needed for achieving the stationary condition is reported.

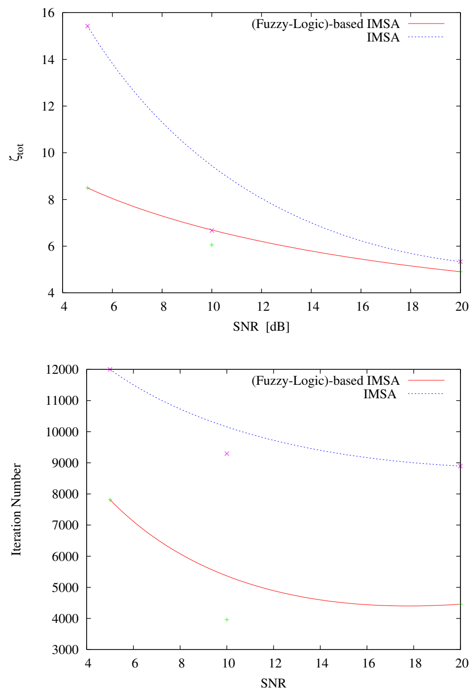

*Figure 2. Reconstruction of an off-centered square homogeneous dielectric ($\tau = 1.5$) cylinder - Behavior of the error figure (*top*) and average iteration number (*bottom*) versus SNR.*

The effectiveness of the $IMSA - Fuzzy$ technique when experimental scattering data are dealt with has also been addressed processing the scattering data provided by the Institut Fresnel in Marseille, France [11]. The imaging setup consists of a 2D bistatic measurement system with an emitter placed at a fixed position, while the receiver is rotated with an arm along the vertical axis of the cylindrical scatterer under test. Such a system allows to implement a multi-illumination/multi-view acquisition procedure characterized by $V = 36$ views and $M_v = 49$ sample measurement points. In particular, we considered the inversion of an off-centered homogeneous circular cylinder $d = 30\,mm$ in diameter ("*dielTM_dec8f.exp*"). Such an object is characterized by an object function equal to $\tau(x, y) = 2.0 \pm 0.3$ and it is located at $x_c = 0.0$, $y_c = -30\,mm$. As far as the investigation domain D is concerned, a square domain $30 \times 30\,cm^2$ has been assumed and the complete set of measures has been used. Although multiple-frequency schemes have been shown to be very effective in enhancing the quality of inversion procedures, in this work only mono-frequency data have been considered.

The first computational test has been performed by using the scattering data collected at $f = 3\,GHz$. Although the retrieved distributions turn out to be smoothed versions of the actual profile, it is possible to clearly detect the object under test as well as its location and shape. As far as the comparison between the $IMSA$ and the $IMSA - Fuzzy$ is concerned, the FL-based approach allows one to obtain a more homogeneous representation of the dielectric profile under test. However, small differences can be observed and the improvement in the reconstruction accuracy does not turn out as large as for the synthetic test cases. Such a behavior can be justified by a better SNR (greater than that of previous synthetic test cases) for the low-frequency scattering data when collected in a controlled-environment.

The reconstructions at an higher frequency ($f = 8\,GHz$) has also been carried out and the results are shown in Fig. 3. As pointed out by other authors, and further confirmed by the results reported, the reconstruction accuracy reduces. However, because of the decreasing of the SNR in correspondence with the increasing of the frequency f the improvement allowed by the FLS is non-negligible in terms of the qualitative imaging.

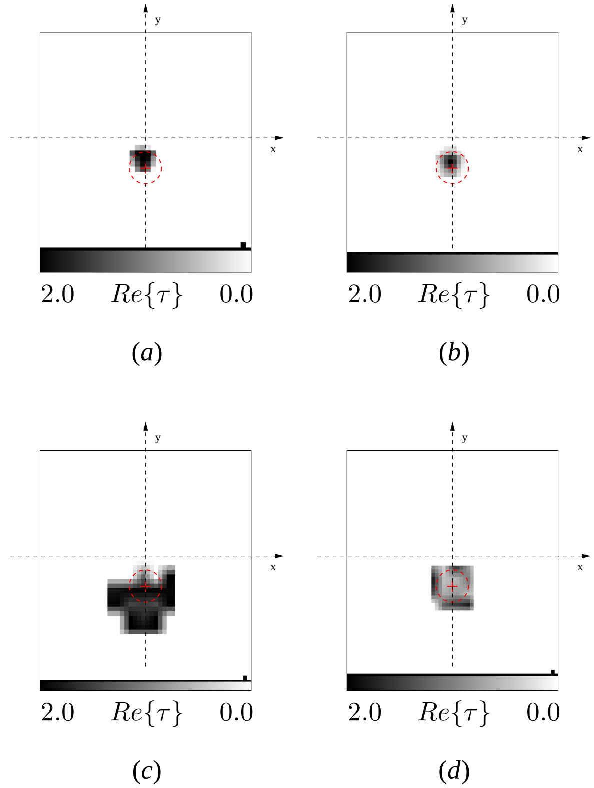

Figure 3. Reconstruction of an off-centered circular cylinder (Marseille dataset [11] - dielTM_dec8f.exp*)* $f = 3\,GHz$*, IMSA* (a) *and Fuzzy-Logic-enhanced IMSA (*b), $f = 8\,GHz$*, IMSA* (c) *and Fuzzy-Logic-enhanced IMSA* (d)*.*

4. CONCLUSIONS

In this paper, a further assessment of the effectiveness of a technique integrating the iterative multi-scaling approach with a fuzzy-logic strategy to estimate the uncertainty associated with scattering measurements has been presented. The exploitation of the information content available from noisy-corrupted scattering data through a fuzzy-logic-based system and its integration with a multi-resolution representation of the profile under test yields reliable reconstructions. The numerical assessment, carried out on different conditions and data sets showed the effectiveness of the proposed technique providing satisfying reconstruction accuracy, robustness to noise and significant computational savings.

REFERENCES

[1] O.M. Bucci and T. Isernia, "Electromagnetic inverse scattering: Retrievable information and measurement strategies," *Radio Science*, vol. 32, pp. 2123-2138, Nov.-Dec. 1997.

[2] L. A. Zadeh, "Fuzzy Sets", *Information and Control*, vol. 8, pp. 338-353, 1965.

[3] J. M. Mendel, "Fuzzy logic system for engineer - A tutorial", *Proc. IEEE*, vol. 83, pp. 345-377, March 1995.

[4] A. Casagranda, D. Franceschini, and A. Massa, "Assessment of the reliability and exploitation of the information content of inverse scattering data through a fuzzy-logic-based strategy - Preliminary results," *IEEE Trans. Geosci. Remote Sensing Lett.*, vol. 2, pp. 36-39, Jan. 2005.

[5] S. Caorsi, M. Donelli, D. Franceschini, and A. Massa, "A New Methodology Based on ad Iterative Multiscaling for Microwave Imaging," *IEEE Trans. Microwave Theory Tech.*, vol. 51, April 2003.

[6] D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*. Springer, Berlin, 1992.

[7] R. Azaro, A. Casagranda, D. Franceschini and A. Massa, "An innovative fuzzy-logic-based strategy for an effective exploitation of noisy inverse scattering data", *Progress in Electromagnetics Research*, PIERS 54, pp. 283–302. 2005.

[8] S. Caorsi, A. Massa, M. Pastorino, and A. Rosani, "Microwave medical imaging: potentialities and limitations of a stochastic optimization technique,"*IEEE Trans. Microwave Theory Tech.*, vol. 52, pp. 1908-1916, Aug. 2004.

[9] M. Pastorino, A. Massa, and S. Caorsi, “A microwave inverse scattering technique for image reconstruction based on a genetic algorithm,” *IEEE Trans. Instrum. Meas.*, vol. 49, pp. 573-578, 2000.

[10] S Caorsi, M. Donelli, and A. Massa, “Detection, location, and imaging of multiple scatterers by means of the iterative multiscaling method,” *IEEE Trans. Microwave Theory Tech.*, vol. 52, pp. 1217-1228, Apr. 2004.

[11] K. Belkebir, S. Bonnard, F. Sabouroux, and M. Saillard, “Validation of 2D inverse scattering algorithms from multi-frequency experimental data,” *J. Electromagn. Waves Appl.*, vol. 14, pp. 1637-1668, Dec. 2000.