UNIVERSITY
OF TRENTO

DIPARTIMENTO DI INGEGNERIA E SCIENZA DELL'INFORMAZIONE

38123 Povo – Trento (Italy), Via Sommarive 14
http://www.disi.unitn.it

DETECTION, LOCATION AND RECONSTRUCTION OF MULTICRACKS BY MEANS OF A GA-BASED ELECTROMAGNETIC TECHNIQUE

M. Benedetti, M. Pastorino, A. Rosani, A. Massa

January 2011

Technical Report # DISI-11-231

DETECTION, LOCATION AND RECONSTRUCTION OF MULTICRACKS BY MEANS OF A GA-BASED ELECTROMAGNETIC TECHNIQUE

Manuel Benedetti[1], Matteo Pastorino[2], Andrea Rosani[1], and Andrea Massa[1]

[1]Department of Information and Communication Technology,
University of Trento, Via Sommarive 14, 38050 Trento, Italy
Phone: +39 0461 882067, Fax: +39 0461 882093
Email: andrea.massa@ing.unitn.it, {manuel.benedetti, andrea.rosani}@dit.unitn.it

[2]Department of Biophysical and Electronic Engineering,
University of Genoa, Via Opera Pia 11a, 16145 Genoa, Italy
Phone: +39 010 3532242, Fax: +39 010 3532245
E-mail: pastorino@dibe.unige.it

Abstract. In this work, an approach for the reconstruction of both geometrical and dielectric properties of multiple defects inside a known host medium is analyzed. In particular, the GA-based integrated strategy is applied starting from the knowledge of the scattered field in an external measurement domain in order to determine positions, sizes and dielectric permittivities of the defects, as well as the related field distribution. In order to evaluate the effectiveness in both locating the defects and reconstructing the corresponding permittivity profiles, a selected set of numerical results is presented.

Keywords: Non-destructive Testing and Evaluation, Microwave Imaging, Genetic Algorithm, Multicrack Detection

1. INTRODUCTION

In order to inspect industrial products, effective non-destructive evaluation and testing (NDE/NDT) techniques are needed. Different approaches have been proposed in the framework of microwave-based strategies [1-2], because of the non-invasive measurements and the possibility of examining large “volumes” of products with an unsupervised process. Moreover, suitable inverse scattering approaches have been developed in order to look for a complete image of the region under test [3-4]. However, these techniques have to be further developed in order to address more effectively several methodological issues as well as some bottlenecks concerned with the computational costs. Towards this end, recent advances are focused on the exploitation of the *a-priori* information in order to reduce the number of unknowns [5-6]. In this framework, two GA-based optimization techniques for the retrieval of positions and sizes of multiple defects inside known host medium have been proposed in [7]. The first technique, called Hierarchical Strategy (HS), is characterized by parallel optimization sub-processes, while the other approach, called Integrated Strategy (IS), consists of a single optimization process. In order to deal with more realistic situations, this work is aimed at discussing the application of the IS to more complex problems than those proposed in [7], where the dielectric permittivity of the defects is an unknown, as well. Therefore, more complicated numerical experiments have been considered in order to evaluate the capabilities of properly retrieving both the geometric (i.e., positions and sizes) and dielectric parameters of multiple defective shapes.

2. MATHEMATICAL FORMULATION

Let us consider a region Ω characterized by an object function $\tau_\Omega = \varepsilon_\Omega - 1 - j\frac{\sigma_\Omega}{2\pi f \varepsilon_0}$, ε_Ω and σ_Ω being the relative permittivity and conductivity, respectively. A set of C defects D_i, $(i=1,\dots,C)$ characterized by unknown positions, shapes and dielectric parameters belongs to Ω. Such a scenario is illuminated by V electromagnetic TM plane waves $\underline{E}^v_{inc}(x,y) = E^v_{inc}(x,y)\hat{\underline{z}}$, inducing an electromagnetic field, $\underline{E}^v_{tot}(x,y)$, given by

$$E^v_{tot}(x,y) = E^v_{inc}(x,y) + \int\int_\Omega \tau(x',y') E^v_{tot(c)}(x',y') G_0(x',y'/x,y) dx'dy' \tag{1}$$

where G_0 is the free-space Green's function and $\tau(x,y) = \varepsilon(x,y) - 1 - j\frac{\sigma(x,y)}{2\pi f \varepsilon_0}$, f being the working frequency. By assuming that, in each region D_i, a differential equivalent current density radiates in an inhomogeneous medium, equation (1) becomes [6]

$$E^v_{tot}(x,y) = E^v_{inc(cf)}(x,y) + \sum_{i=1}^{C}\int\int_{D_i} \tau_{D_i}(x',y') E^v_{tot(c),i}(x',y') G_1(x',y'/x,y) dx'dy' \tag{2}$$

where $E^v_{inc(cf)}(x,y)$ is the total electric field in the unperturbed scenario and G_1 is the inhomogeneous Green's function. The second term in the right side of (2) describes the electromagnetic field induced by the differential object function $\tau_{D_i}(x,y) = \tau(x,y) - \tau_\Omega(x,y)$, $(x,y) \in D_i$, with $i=1,\dots,C$. In order to reduce the number of unknowns, $\tau_{D_i}(x,y)$ is expressed through a set of representative parameters: the centers of the defective shapes $(\tilde{x}_i, \tilde{y}_i)$, their lengths, sides and orientations $(\tilde{L}_i, \tilde{W}_i, \tilde{\theta}_i)$, and their dielectric permittivities $\tilde{\varepsilon}_i$.
Consequently

$$\tau_{D_i}(x,y) = \begin{cases} [\varepsilon_\Omega - \tilde{\varepsilon}_i] - j\frac{[\sigma_\Omega - \tilde{\sigma}_i]}{2\pi f \varepsilon_0} & if\ \ X \in \left[-\frac{\tilde{L}_i}{2}, \frac{\tilde{L}_i}{2}\right] and\ Y \in \left[-\frac{\tilde{W}_i}{2}, \frac{\tilde{W}_i}{2}\right] \\ 0 & otherwise \end{cases} \tag{3}$$

being

$$\begin{aligned} X &= (x - \tilde{x}_i)\cos\tilde{\theta}_i + (y - \tilde{y}_i)\sin\tilde{\theta}_i \\ Y &= (x - \tilde{x}_i)\sin\tilde{\theta}_i + (y - \tilde{y}_i)\cos\tilde{\theta}_i \end{aligned} \tag{4}$$

Therefore, the array of unknowns is defined as follows

$$\chi = \left\{C; \Psi_i, i=1,\dots,C; \left[E^v_{tot(c),i}\right], i=1,\dots,C\right\} \tag{5}$$

where $\Psi_i = \left[\left(\tilde{x}_i, \tilde{y}_i\right); \left(\tilde{L}_i; \tilde{W}_i; \tilde{\theta}_i\right); \tilde{\varepsilon}_i\right]$.

In order to numerically deal with such a problem, the region H is partitioned in N sub-domains. The optimal solution χ_{opt} is determined as the result of an optimization procedure by minimizing the following cost function:

$$\Phi(\chi) = \left\{ \frac{\left\| \left[E_{tot}^{v}\right] - \left[E_{tot(cf)}^{v}\right] - \sum_{i=1}^{C} \left[G_{1,i}\right] \left[\tau_{D_i}\right] \left[E_{tot,i}^{v}\right] \right\|_O^2}{\left\| \left[E_{tot}^{v}\right] - \left[E_{inc}^{v}\right] \right\|_O^2} \right\} + \\ + \left\{ \frac{\left\| \left[E_{tot(cf)}^{v}\right] + \left[E_{tot}^{v}\right] - \sum_{i=1}^{C} \left[G_{1,i}\right] \left[\tau_{D_i}\right] \left[E_{tot,i}^{v}\right] \right\|_\Omega^2}{\left\| \left[E_{inc}^{v}\right] \right\|_\Omega^2} \right\} \tag{6}$$

providing a measure of the misfit between the estimated fields and the scattering data collected at M locations of the observation domain O [i.e., $E_{tot}^{v}(x_m, y_m)$ and $E_{tot(cf)}^{v}(x_m, y_m)$, m=1,…,M] as well as those collected at N positions inside the investigation domain Ω [i.e., $E_{inc}^{v}(x_n, y_n)$, n=1,…,N].

As far as the minimization of (6) is concerned, the Integrated Strategy (IS) proposed in [7] has been considered. Such a technique consists in a GA-based iterative procedure, where a population $\underline{\chi}$ of Q trial solutions code a different number of defects, from 1 up to C_{max}

$$\underline{\chi} = \left\{\chi_q; q = 1, \ldots, Q\right\} = \\ = \left\{\left(C_q; \Psi_i, i = 1, \ldots, j; \left[E_{tot(c),i}^{v}\right], i = 1, \ldots, C_q\right); C_q \in \left[1, \ldots, C_{max}\right]\right\} \tag{7}$$

At the iteration k=0 (k being the iteration index), a random initialization generates the first set of trial solutions, $\underline{\chi}^0$. Then, the following steps are carried out until a stopping criterion holds true ($k<K_{max}$ or $\Phi(\chi_{opt}) < \Phi_{th}$, $\chi_{opt} = \arg\left\{\min_k \left\{\min_q \left[\Phi\left(\chi_q^k\right)\right]\right\}\right\}$):

- the iteration index is updated (k=k+1);
- a set of Q/2 individuals, $\underline{\chi}_b^k$, coding the same number C_{opt}^{k-1} of defects of χ_{opt}^{k-1} is randomly generated;
- a $Q/2$-sized population ($\underline{\chi}_o^k$) partitioned into C_{max}-1 equally-sized subsets with individuals coding the same number of cracks C_l (C_l=1,…,C_{max}, $C_l \neq C_{opt}^{k-1}$) is computed from χ_{opt}^{k-1} by applying random operators [7];
- standard *selection*, *mutation*, and *elitism* are applied to the set of trial solutions $\left\{\underline{\chi}_b^k \cup \underline{\chi}_o^k\right\}$ in order to get $\underline{\chi}^k$ and to identify the best solution $\chi_{\min}^k = \arg\left\{\min_{q=1,\ldots,Q} \Phi\left(\chi_q^k\right)\right\}$.

3. NUMERICAL RESULTS

Let us consider a square non-dissipative host medium of size $L_{\Omega} = 0.8\lambda$ characterized by a dielectric permittivity $\varepsilon_{\Omega} = 2.0$. The region Ω has been discretized with N=256 sub-domains, while M=50 measurement points and V=4 views have been used. The actual scenario is composed by three void defects located at $(x_1/\lambda = 0.2\,;\, y_1/\lambda = 0.16)$, $(x_2/\lambda = -0.2\,;\, y_2/\lambda = 0.16)$, and $(x_3/\lambda = 0\,;\, y_3/\lambda = -0.16)$. Their area A_c/λ^2 has been varied within the range [0.01;0.0625] in order to evaluate the dependence of the reconstruction accuracy of the IS on the cracks dimension. As far as the GA-based optimization is concerned, the parameters adopted in [7] have been considered and the permittivity $\tilde{\varepsilon}_i$ of the defects was looked for between 1.0 and 2.0, being the maximum number of defects set to C_{max}=3.

Figure 1 shows the samples of the reconstructions obtained for (*b*) A_c/λ^2=0.01 and (*d*) A_c/λ^2 = 0.0625. Quantitatively, the result depicted in fig. 1(*b*) is characterized by a *localization error* δ [7] equal to 3%, while the *area error* Δ is lower than 50%. The values of ε_i, with i=1,…,C, have been overestimated, since $1.46 < \hat{\varepsilon}_i < 1.57$. When A_c/λ^2 = 0.0625 [fig. 1(*d*)], the proposed reconstruction is characterized by δ= 5% and the permittivity of the cracks has been correctly estimated. On the other hand, the reconstructed shapes are characterized by a wrong orientation and the error Δ is equal to 43%.

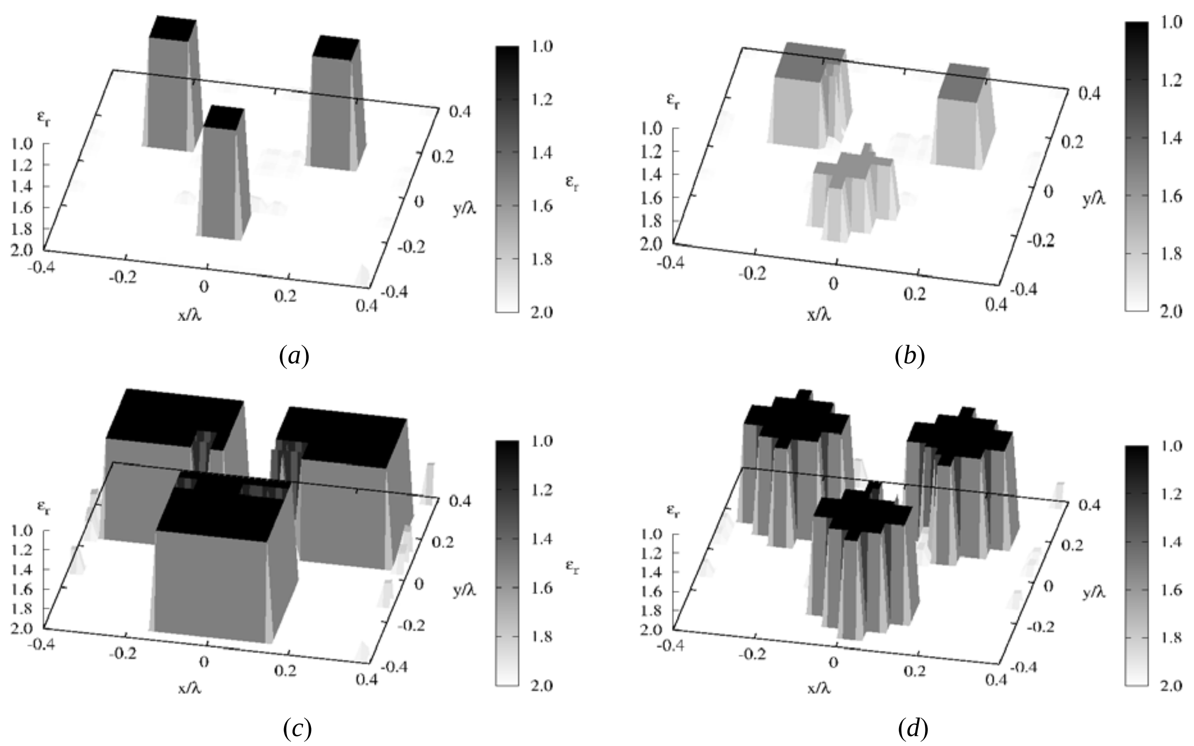

Figure 1 - Reconstructions obtained when varying the area of the defects. (*a*)(*b*) A_c/λ^2=0.01 and (*c*)(*d*) A_c/λ^2=0.0625. (*a*)(*c*) original profile of the permittivity. (*b*)(*d*) reconstructed defects.

4. CONCLUSIONS

In this work, a numerical assessment of a GA-based approach for the reconstruction of defective shapes located inside industrial products has been presented. The achieved results confirm the effectiveness of the approach both in locating and reconstructing the dielectric distributions of the cracks. Future works will be aimed at defining suitable genetic operators to control the aggregation/disjointing of adjacent defects.

REFERENCES

1. R. Zoughi, *Microwave Nondestructive Testing and Evaluation*, Kluwer Academic Publishers, The Netherlands, 2000.
2. S. J. Lockwood and H. Lee, "Pulse-echo microwave imaging for NDE of civil structures: Image reconstruction, enhancement, and object recognition," *Int. J. Imaging System Technol.*, vol. 8, pp. 407-412, 1997.
3. W. C. Chew and Y. M. Wang, "Reconstruction of two-dimensional permittivity distribution using the distorted Born iterative method," *IEEE Trans. on Medical Imaging*, vol. 9, pp. 218-225, 1990.
4. C. C. Chiu and P. T. Liu, "Image reconstruction of perfectly conducting cylinder by the genetic algorithm," *IEE Proc. Microw. Antennas Propag.*, vol. 3, p. 143, 1996.
5. S. Caorsi, A. Massa, and M. Pastorino, "A crack identification microwave procedure based on a genetic algorithm for nondestructive testing," *IEEE Trans. Antennas Propagat.*, vol. 49, pp. 1812-1820, 2001.
6. S. Caorsi, A. Massa, M. Pastorino, and M. Donelli, "Improved microwave imaging procedure for nondestructive evaluations of two-dimensional structures," *IEEE Trans. Antennas Propagat.*, vol. 52, pp. 1386-1397, 2004.
7. M. Benedetti, M. Donelli, and A. Massa, "Multicrack Detection in Two-Dimensional Structures by means of GA-based strategies", *IEEE Trans. Antennas Propagat.*, vol. 55, pp. 205-215, 2007.