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SYNTHESIS OF MONOPULSE ANTENNAS THROUGH  
THE ITERATIVE CONTIGUOUS PARTITION METHOD

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# Synthesis of Monopulse Antennas through the Iterative Contiguous Partition Method

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The synthesis of “optimal” compromise sum and difference patterns subject to arbitrary sidelobe bounds is addressed by means of a simple and effective sub-arraying technique based on the optimal excitation method. The obtained results positively compare with those from state-of-the-art methods both in terms of performances and computational indexes.

*Introduction:* In designing monopulse radar systems, the synthesis of both a sum pattern and a difference pattern that satisfy some specifications [e.g., narrow beamwidth, low side-lobe-level (SLL), etc..] is required. In order to avoid an expensive implementation of independent feed networks, compromise solutions based on sub-arraying techniques have been successfully proposed. In such a framework, two different methodological approaches might be recognized. The former [1] is aimed at determining the “best compromise” difference pattern close as much as possible to the optimum in the Dolph-Chebyshev sense [2] (i.e., narrowest first null beamwidth and largest normalized difference slope on the boresight for a specified sidelobe level). The other reformulates the original synthesis problem in an optimization one. As far as the “optimization” techniques are concerned, [3]-[5] contemporarily optimize the clustering into subarrays and their weights according to the following rationale “for a given beamwidth, find the subarray configuration and the coefficients of the subarray sum signals such that the maximum SLL is minimized.” On the contrary, in [6], a hybrid

approach is used for pursuing the following task: “find the subarray configuration and the coefficients of the subarray sum signals such that the corresponding radiation pattern has a null with the maximum possible slope in a given direction, while being bounded by an arbitrary function elsewhere.”

In the framework of optimal matching techniques, this contribution considers a new approach for synthesizing best compromise patterns with SLL control. Towards this end, exploiting the property that the partition minimizing the distance between optimal and synthesized difference excitations is a contiguous partition (CP), the CP method (CPM) determines the difference solution close to the optimal Dolph-Chebyshev pattern with SLL under the user-defined threshold.

*Description of the CPM:* With reference to a linear uniform array of  $N = 2M$  elements, let us consider sum and difference patterns generated by means of symmetric,  $S = \{s_m = s_{-m}; m = 1, \dots, M\}$ , and an anti-symmetric,  $D = \{d_m = -d_{-m}; m = 1, \dots, M\}$ , real excitations set, respectively. Because of the symmetry properties and according to the guidelines of sub-arraying techniques, the sum pattern is obtained by assuming ideal excitations,  $s_m = \phi_m$  [7][8][9], while difference excitations are synthesized as  $d_m = \phi_m (\delta_{c_m q} \rho_q)$ ,  $Q$  being the number of sub-arrays,  $\rho_q$  is the weight of the  $q$ -th sub-array,  $\delta_{c_m q} = 1$  if  $c_m = q$  and  $\delta_{c_m q} = 0$  otherwise, and  $c_m$  is the sub-array index of the  $m$ -th array element.

To obtain the best compromise difference excitations (i.e., a set of excitations giving a pattern as close as possible to the ideal one in the Dolph-Chebyshev sense that satisfies at the same time a constraint on the SLL), the following

procedure is performed: (1) initialize the iteration index ( $i = 0$ ). Compute the optimal sum excitations  $\Phi = \{\phi_m; m = 1, \dots, M\}$  and set the user-desired sidelobe level  $SLL^{(des)}$ . According to [10], define an optimal – in the Dolph-Chebyshev sense - difference excitations set  $\Psi^{(obj)} = \{\theta_m^{(obj)}; m = 1, \dots, M\}$  that generates a beam pattern with a sidelobe level  $SLL^{(obj)} \leq SLL^{(des)}$ . For each element of the array, compute a reference parameter (called optimal gain)  $v_m = \theta_m^{(obj)} / \phi_m$ . Sort the reference parameters in a list  $L = \{l_m; m = 1, \dots, M\}$  where  $l_k \leq l_{k+1}$ ,  $k = 1, \dots, M-1$ ,  $l_1 = \min\{v_m\}$  and  $l_M = \max\{v_m\}$ ;

(2) Update the iteration index ( $i \leftarrow i + 1$ ). If  $i = 1$ , then randomly generate a trial grouping  $C^{(i)} = \{c_m^{(i)}; m = 1, \dots, M\}$  corresponding to a CP,  $\Gamma_Q^{(i)}$ , of  $L$  in  $Q$  subsets  $\Gamma_Q^{(i)} = \{L_q^{(i)}; q = 1, \dots, Q\}$ . Otherwise, update the grouping vector  $C^{(i)}$  by deriving a new CP starting from the previous one  $\Gamma_Q^{(i-1)}$  and just modifying the subarray membership of the subset border elements ( $b_m = l_m \in L_q^{(i)}$  such that  $l_{m-1} \in L_{q-1}^{(i)}$  and/or  $l_{m+1} \in L_{q+1}^{(i)}$ ,  $q \in [1, Q]$ );

(3) Compute the set of weights  $P^{(i)} = \{p_q^{(i)} = \delta_{c_m q} e_m^{(i)}; q = 1, \dots, Q\}$ , where

$$e_m^{(i)} = \frac{\sum_{r=1}^M \delta_{c_s q} v_r}{\sum_{r=1}^M \delta_{c_s q}}.$$

Evaluate the closeness of the  $i$ -th trial solution

$D^{(i)} = \{d_m^{(i)}; m = 1, \dots, M\}$  (or  $\{C^{(i)}, P^{(i)}\}$ ) to the reference  $\Psi^{(obj)}$  by computing the

$$\text{cost function value } \Xi^{(i)} = \sum_{m=1}^M |v_m - e_m^{(i)}|^2.$$

Moreover, compute the achieved

sidelobe level  $SLL^{(i)} = SLL\{D^{(i)}\}$ . Update the “optimal” value of the cost

( $\Xi_{opt}^{(i)} = \Xi^{(i)}$ ) as well as the optimal set of coefficients ( $D_{opt}^{(i)} = D^{(i)}$ ) and set

$$SLL_{opt} = SLL^{(i)} \text{ if } \Xi^{(i)} < \Xi_{opt}^{(i-1)};$$

(4) If the maximum number of iterations ( $i = I$ ) or a stationary condition [i.e.,

$$\left( \left| I_{win} \Xi_{opt}^{i-1} - \sum_{j=1}^{I_{win}} \Xi_{opt}^j \right| / \Xi_{opt}^i \right) \leq \eta \text{ and } SLL_{opt} \leq SLL^{(des)}, I_{win} \text{ and } \eta \text{ being a fixed}$$

number of iterations and an assigned threshold, respectively] is reached, then

stop the process and return the final solution  $D_{opt} = D_{opt}^{(i)}$  ( $i = I_{opt}$ ). Otherwise,

go to step (2);

*Numerical Validation:* As test cases, let us consider some situations ( $Q = 4, 6, 8$ ) already tackled in [5][6] and concerned with a  $M = 20$  linear array with inter-element spacing  $d = \lambda/2$  when the sum pattern excitations have been fixed to produce a Dolph-Chebyshev pattern with  $SLL = -20 \text{ dB}$ . Moreover the desired sidelobe level has been set to  $SLL^{(des)} = -20 \text{ dB}$  and the CPM has been used for minimizing the  $SLL_{opt}$ . The obtained results are shown in Fig. 1 ( $Q = 4$ ,  $I_{opt} = 2$ ), Fig. 2 ( $Q = 6$ ,  $I_{opt} = 2$ ), Fig. 3 ( $Q = 8$ ,  $I_{opt} = 3$ ) and compared in terms of SLL value with other existing techniques in Tab. I. As it can be noticed, although we are not exactly optimizing the same parameter as in [5][6 – Tab. II], the proposed approach outperforms other state-of-the-art approaches in a non-negligible fashion.

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**Figure captions:**

Fig. 1 - Comparison between synthesized difference patterns ( $Q = 4$ )

- CPM [ $SLL^{(obj)} = -35\text{ dB}$ ]
- - - Hybrid Approach
- ..... DE Approach

Fig. 2 - Comparison between synthesized difference patterns ( $Q = 6$ )

- CPM [ $SLL^{(obj)} = -45\text{ dB}$ ]
- - - Hybrid Approach
- ..... DE Approach

Fig. 3 - Comparison between synthesized difference patterns ( $Q = 8$ )

- CPM [ $SLL^{(obj)} = -45\text{ dB}$ ]
- - - Hybrid Approach
- ..... DE Approach

Figure 1

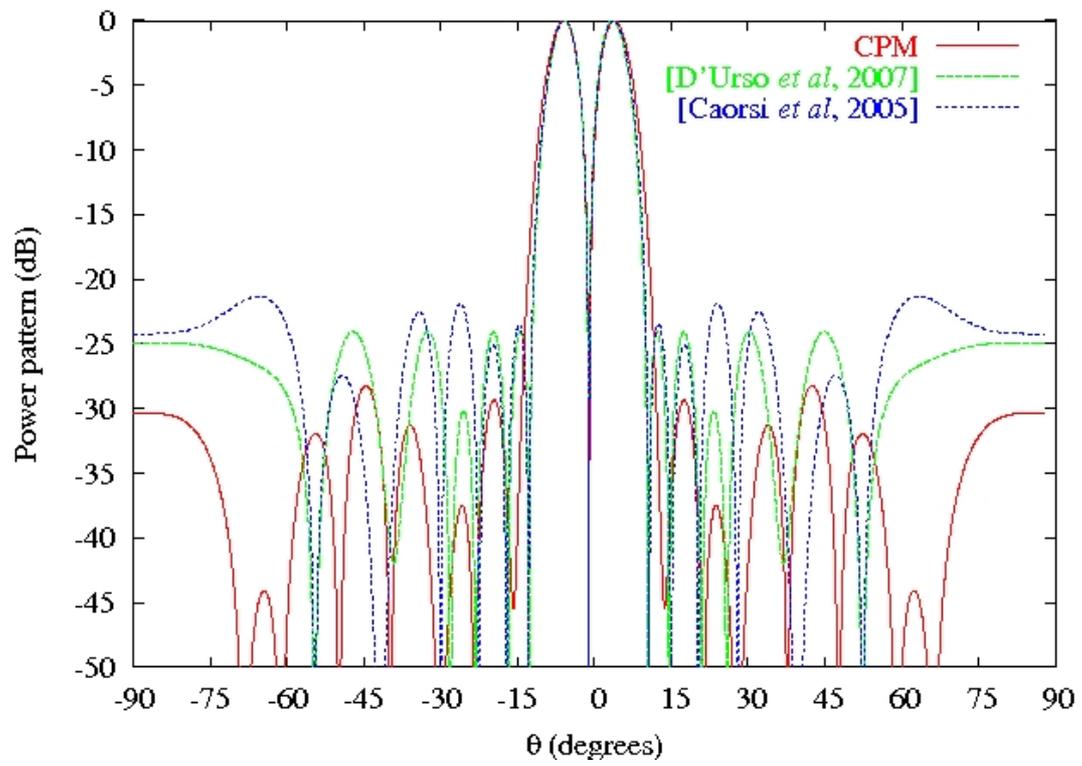


Figure 2

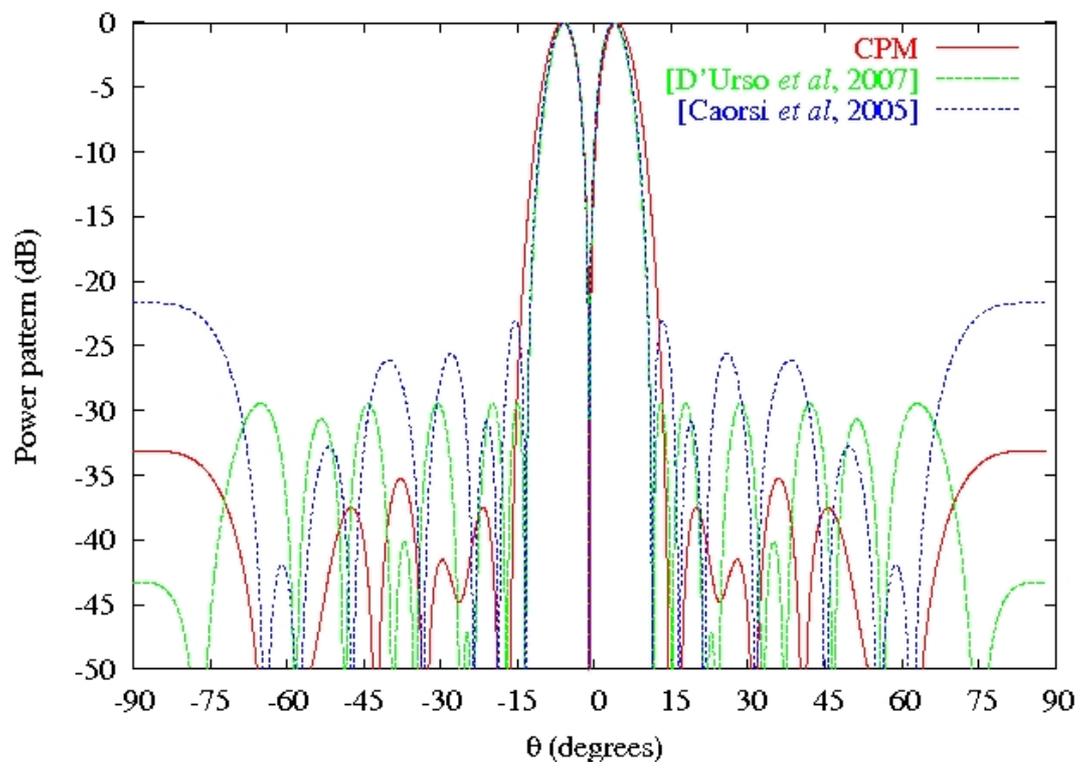


Figure 3

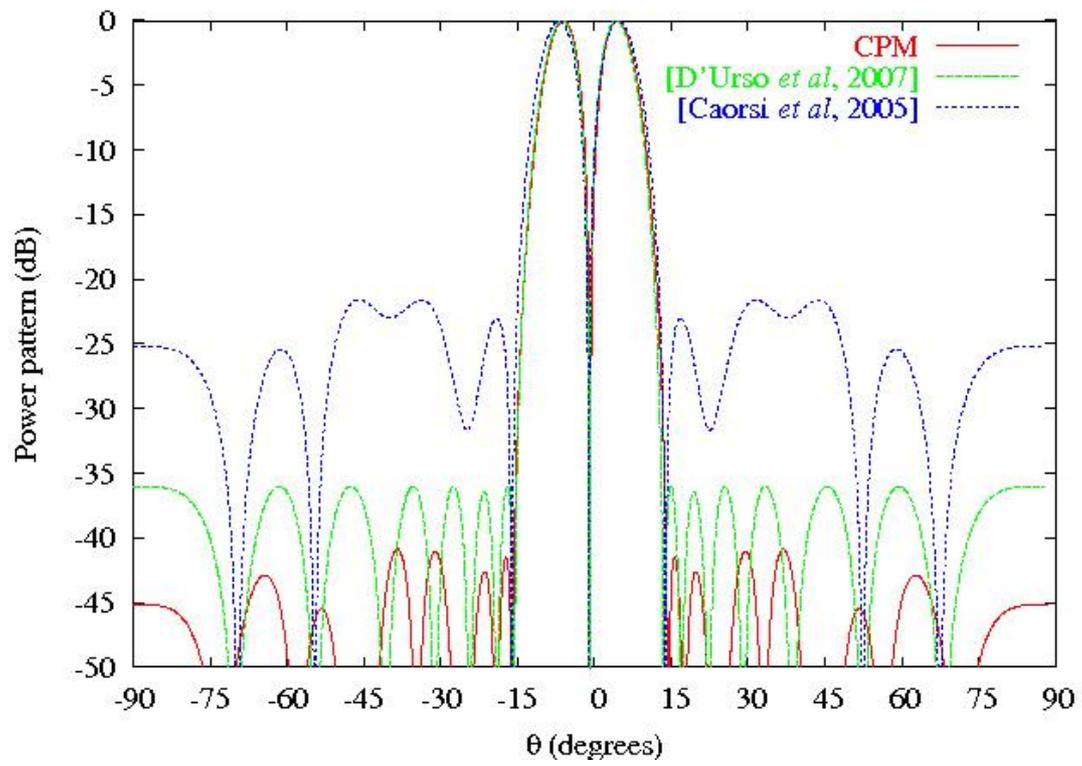


Table I

	<b>Q = 4</b>	<b>Q = 6</b>	<b>Q = 8</b>
<b><i>CPM</i></b>	- 28.23	- 33.00	- 40.85
<b><i>Hybrid Approach</i></b>	- 25.00	- 30.00	- 36.50
<b><i>DE Approach</i></b>	- 21.30	- 21.66	- 21.59