



UNIVERSITY  
OF TRENTO

---

**DIPARTIMENTO DI INGEGNERIA E SCIENZA DELL'INFORMAZIONE**

---

38123 Povo – Trento (Italy), Via Sommarive 14  
<http://www.disi.unitn.it>

HYBRID APPROACH FOR SUB-ARRAYED MONOPULSE  
ANTENNA SYNTHESIS

P. Rocca, L. Manica, and A. Massa

January 2008

Technical Report # DISI-11-054



# Hybrid Approach for Sub-arrayed Monopulse Antenna Synthesis

P. Rocca, L. Manica, and A. Massa

A computationally effective hybrid approach to define the “optimal” compromise between sum and difference patterns in monopulse arrays is presented. Firstly, the partitioning into sub-arrays is performed by exploiting the knowledge of independently optimal sum and difference excitations. Then, the sub-array gains are computed by means of a gradient-based procedure, which takes advantage from the convexity of the problem at hand. Selected results are shown and compared with those from state-of-the-art methods in dealing with representative test cases.

*Introduction:* Monopulse radar systems require antennas able to generate sum and difference patterns [1]. The sum mode is used in both the transmission modality and the reception one to detect the target. The difference mode provides information on the angular position of the target. As far as the corresponding beams are concerned, both patterns should have low sidelobe levels (SLLs). Moreover, the sum pattern needs a high gain, while the difference one is required to have a null at the boresight direction with the maximum normalized slope (i.e., high sensitivity). In order to yield these features avoiding the use of a two-module feed network, sub-arraying techniques have been introduced [2]. Such approaches are aimed at defining a sub-array configuration and its gains that satisfy the user-defined requirements and guarantee a suitable trade-off between circuit complexity (i.e., costs and e.m. interferences) and pattern features. In such a framework, different stochastic procedures have been proposed where both sub-array memberships and gains [3][4] or only part of the unknowns [5][6] are

optimized. Interesting results have been reached in [6] wherein the convexity of the functional with respect to the sub-array coefficients has been profitably exploited once the sub-array membership of each array element has been determined by means of a Simulated Annealing (SA) optimization. Recently, an innovative approach, namely the contiguous partition method (CPM), has been presented in [7][12]. Such a technique implies a smaller (compared to stochastic optimization methods) computational burden thanks to a proper reduction of the solution space of the admissible aggregations of array elements.

In this letter, a hybrid approach for the solution of the “optimal” compromise problem is presented. It takes advantage from both the convex programming (CP) algorithm described in [6] and the CPM [12]. Starting from the knowledge of the optimal difference excitations [8][9] as well as from their relationships with the optimized sum coefficients [10][11], the sub-array configurations are determined as in [12]. Successively, for a given element clustering, the CP procedure is used to compute the sub-array weights.

*Hybrid-CPM for sub-arrayed monopulse antenna synthesis:* Let us consider a linear array of  $N = 2M, n = -M, \dots, -1, 1, \dots, M$  equally spaced elements. Following the guidelines of the well known excitation matching method (EMM) [2], for a set of optimal sum coefficients  $A^s = \{a_n^s\}$ , the cost function

$\Psi(C, G) = \sum_{n=1}^N |a_n^d - b_n(C, G)|^2$  to be minimized is the L2-norm distance between

the optimal difference excitations,  $A^d = \{a_n^d\}$ , and the actual ones

$B = \{b_n = \delta_{c_n, q} g_q a_n^s\}$ ,  $\delta_{c_n, q}$  being the Kronecker delta ( $\delta_{c_n, q} = 1$  if  $c_n = q$ ,  $\delta_{c_n, q} = 0$

otherwise). Moreover,  $G = \{g_q\}$  is the set of Q gains of the difference sub-arrayed

network and  $C = \{c_n\}$  defines the sub-array membership of each element. Since

sum and difference modes are characterized by symmetric ( $a_m^s = a_{-m}^s$ ) and anti-symmetric ( $a_m^d = -a_{-m}^d$  and  $b_m = -b_{-m}$ ) coefficients, only  $M$  elements are considered in the synthesis procedure.

In [6], it has been shown that the functional  $\Psi$  is convex with respect to  $G$  for a given clustering  $C$ , while it is not convex (i.e., local minima exist) with respect to  $C$ . On the other hand, by exploiting the knowledge of the optimal excitations of the difference beam [8][9], the method proposed in [7] has strongly reduced the solution space to a limited number of sub-array configurations. As a consequence, the occurrence of sub-optimal solutions has been reduced and the convergence of the sub-arraying process improved.

As in [7], let us consider the following cost function

$$\hat{\Psi}^{CPM}(C, G) = \sum_{m=1}^M \left| a_m^s \left( \frac{a_m^d}{a_m^s} - \delta_{c_m q} g_q(C^{CPM}) \right) \right|^2 \quad \text{that defines an optimal gain matching}$$

problem, where  $v_m = a_m^d / a_m^s, m = 1, \dots, M$  and  $g_q^{CPM} = g_q(C^{CPM}), q = 1, \dots, Q$  are the reference and the CPM sub-array gains, respectively. Once the sub-array configuration  $C^{opt}$ , which minimizes  $\hat{\Psi}^{CPM}$ , has been computed by means of the CPM [12], the optimal set of gains  $G^{opt}$  is determined as in [6]. More in detail,  $G^{opt}$  is equal to the set  $G$  that minimizes the functional  $d \operatorname{Re}\{AF^d(\vartheta)\} / d\vartheta$  at boresight ( $\vartheta = \pi/2$ ) subject to  $d \operatorname{Im}\{AF^d(\vartheta)\} / d\vartheta|_{\vartheta=\pi/2} = 0$  and some constraints on the SLL [i.e.,  $|AF^d(\vartheta)|^2 \leq UB(\vartheta)$ ]. Moreover,  $AF^d$  is the array factor of the difference beam and  $UB(\vartheta)$  is a non-negative function that defines the upper bound of the pattern sidelobes. Furthermore,  $Re$  and  $Im$  denotes the real part and the imaginary one, respectively.

*Numerical assessment:* Let us consider three benchmarks presented in [2][6] and still considered in [7][12] for comparison purposes. These test cases are concerned with a uniform ( $d = \lambda/2$ ) linear array of  $N = 20$  elements.

In the first two examples, the sum coefficients  $A^s$  have been fixed to a Villeneuve distribution [13] with a pattern characterized by  $SLL = -25 \text{ dB}$ . The results obtained with the Hybrid-CPM are compared with those of [2] and [7] in terms of SLL reduction for a fixed beamwidth. In particular, the cases of  $Q = 3$  sub-arrays and with  $Q = 5$  are shown in Fig. 1 and Fig. 2, respectively. The last experiment deals with  $Q = 8$  partitions by considering a set  $A^s$  that generates a Dolph-Chebyshev pattern [11] with  $SLL = -20 \text{ dB}$ . The synthesized pattern is shown in Fig. 3. For completeness, the values of the SLL are summarized in Tab. I.

It is worth to note that the proposed method slightly outperforms the bare CPM in correspondence with a limited number (e.g.,  $Q = 3$ ) of sub-arrays [Fig. 1]. On the other hand, more significant improvements with respect state-of-the-art techniques are obtained for the configurations with  $Q = 5$  and  $Q = 8$ .

*Conclusion:* A hybrid approach to define the “optimal” compromise between sum and difference patterns in monopulse arrays has been presented. Once the sub-array aggregations have been defined with the CPM, the sub-array gains are computed by means of a CP algorithm. A set of representative results and comparisons assesses the effectiveness, but also limitations, of the approach.

## References:

- 1 SKOLNIK, M.I.: 'Radar handbook, 2nd ed.' (McGraw-Hill, New York, 1990)
- 2 MCNAMARA, D.A.: 'Synthesis of sub-arrayed monopulse linear arrays through matching of independently optimum sum and difference excitations', IEE Proc. H, 1988, 135, (5), pp. 371-374
- 3 LOPEZ, P., RODRIGUEZ, J.A., ARES, F., and MORENO, E.: 'Subarray weighting for difference patterns of monopulse antennas: joint optimization of subarray configurations and weights', IEEE Trans. Antennas Propagat., 2001, 49, (11), pp. 1606-1608
- 4 CAORSI, S., MASSA, A., PASTORINO, M., and RANDAZZO, A.: 'Optimization of the difference patterns for monopulse antennas by a hybrid real/integer-coded differential evolution method', IEEE Trans. Antennas Propagat., 2005, 53, (1), pp. 372-376
- 5 ARES, F., RODRIGUEZ J.A., MORENO, E., and RENGARAJAN, S.R.: 'Optimal compromise among sum and difference patterns', J. Electromagn. Waves Applicat., 1996, 10, (11), pp. 1543-1555
- 6 D'URSO, M., ISERNIA, T., and MELIADO, E. F.: 'An effective hybrid approach for the optimal synthesis of monopulse antennas', IEEE Trans. Antennas Propagat., 2007, 55, (4), pp. 1059 - 1066
- 7 MANICA, L., ROCCA, P., MARTINI, A., and MASSA, A.: 'An innovative approach based on a tree-searching algorithm for the optimal matching of independently optimum sum and difference excitations', IEEE Trans. Antennas Propagat. (in press)
- 8 BAYLISS, E.T.: 'Design of monopulse antenna difference patterns with low sidelobes', Bell System Tech. Journal, 1968, 47, pp. 623-640
- 9 MCNAMARA, D.A.: 'Direct synthesis of optimum difference patterns for discrete linear arrays using Zolotarev distribution', IEE Proc. H, 1993, 140, (6), pp. 445-450
- 10 TAYLOR, T.T.: 'Design of line-source antennas for narrow beam-width and low side lobes', IRE Trans. Antennas Propagat., 1955, (3), pp. 16-28
- 11 DOLPH, C.L.: 'A current distribution for broadside arrays which optimises the relationship between beam width and sidelobe level', IRE Proc., 1946, (34), pp. 335-348
- 12 ROCCA, P., MANICA, L., and MASSA, A.: 'Synthesis of monopulse antennas through iterative contiguous partition method', Electronics Letters, 2007, 43, (16), pp. 854-856
- 13 VILLENEUVE, A.T.: 'Taylor patterns for discrete arrays', IEEE Trans. Antennas Propagat., 1984, AP-32, pp. 1089-1093

**Authors' affiliations:**

P. Rocca, L. Manica, and A. Massa  
Department of Information and Communication Technology,  
University of Trento, Via Sommarive 14, 38050 Trento – ITALY

**Corresponding Author:**

A. Massa  
ELEDIA Research Group  
Department of Information and Communication Technology  
University of Trento  
Via Sommarive 14  
38050 Trento  
ITALY  
E-mail: [andrea.massa@ing.unitn.it](mailto:andrea.massa@ing.unitn.it)

**Figure captions:**

Fig. 1 - ( $Q = 3$ ) Compromise difference patterns obtained with:

— Hybrid CPM  
- - - CPM  
· · · · · EMM

Fig. 2 - ( $Q = 5$ ) Compromise difference patterns obtained with:

— Hybrid CPM  
- - - CPM  
· · · · · EMM

Fig. 3 - ( $Q = 8$ ) Compromise difference patterns obtained with:

— Hybrid CPM  
- - - CPM  
· · · · · Hybrid SA

Figure 1

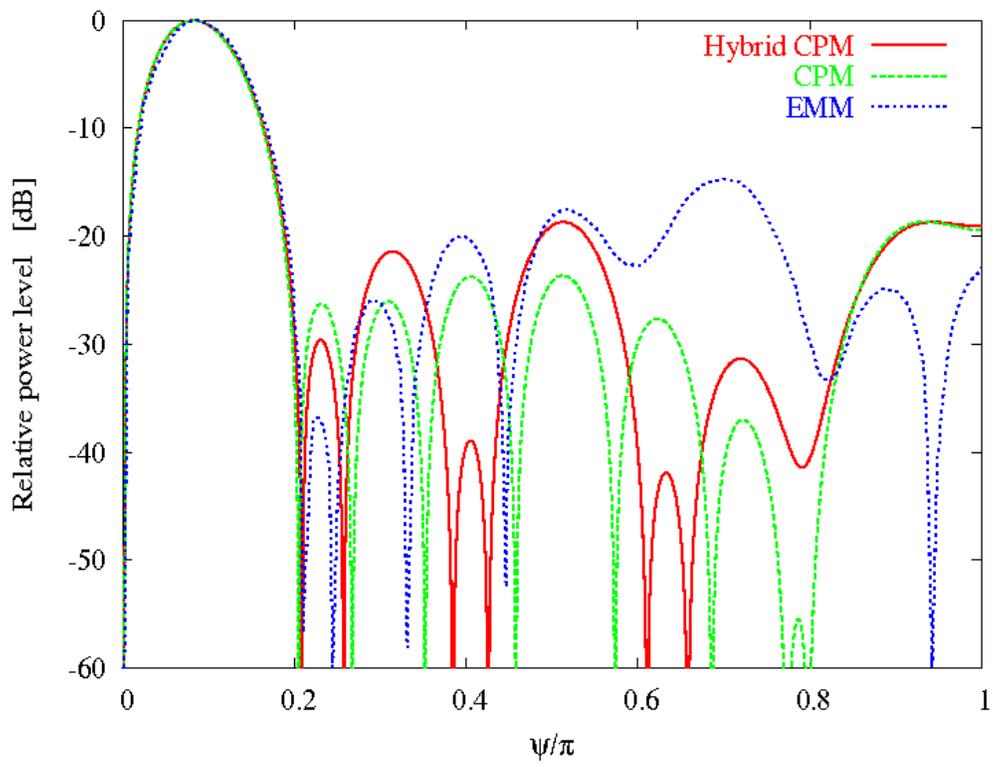


Figure 2

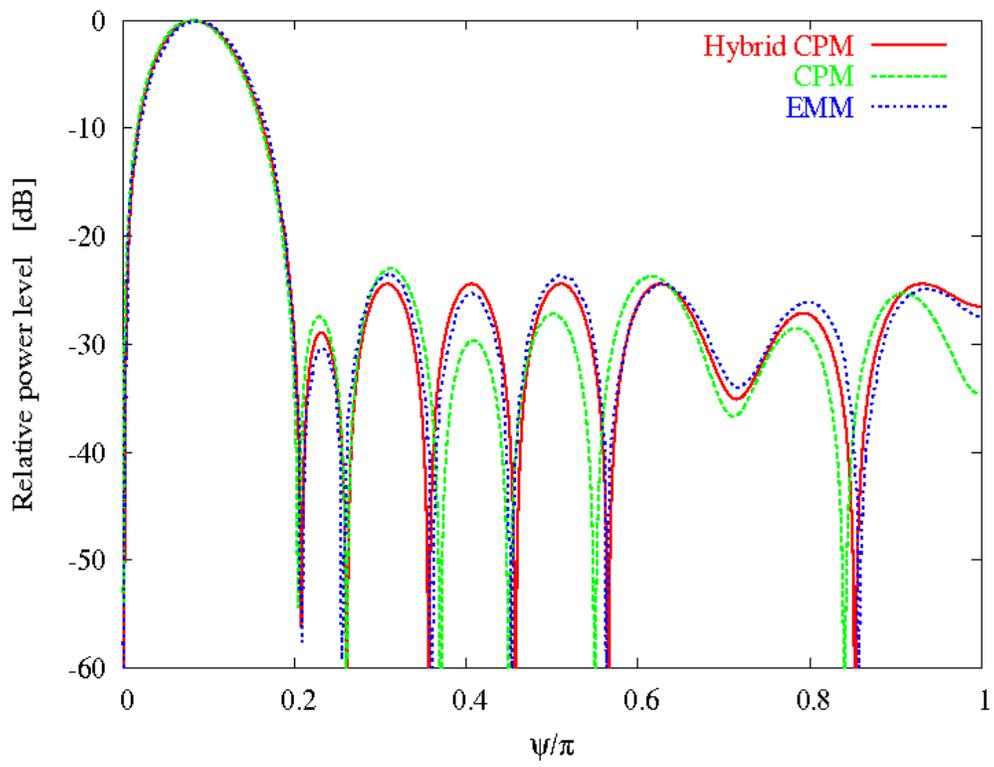


Figure 3

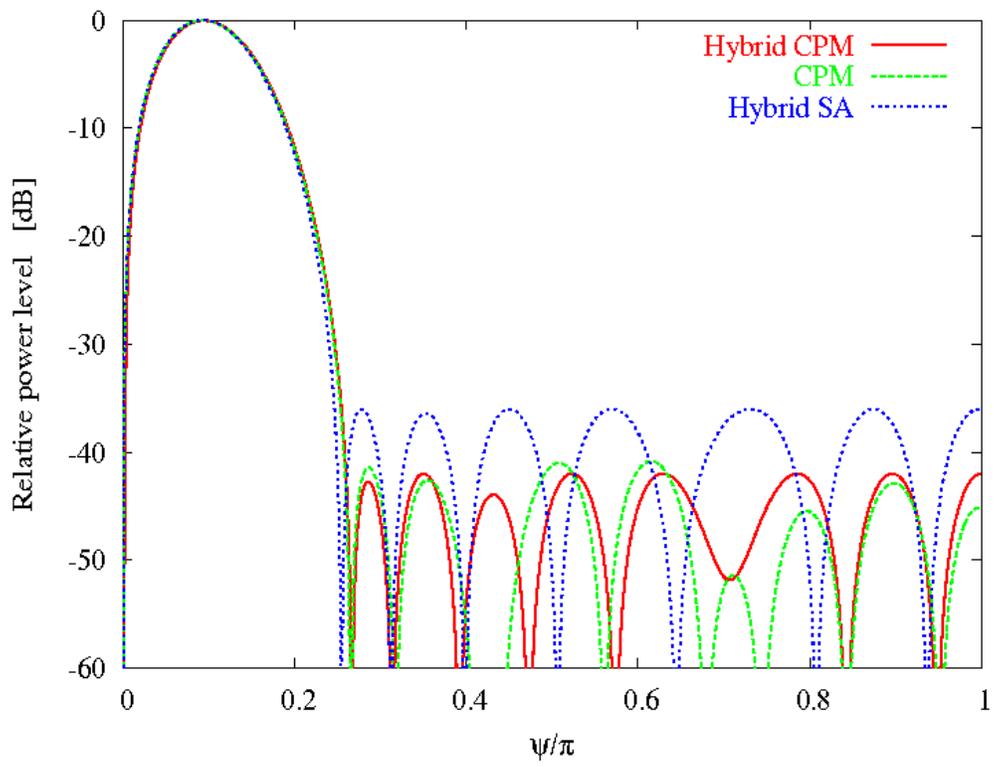


Table I

	<i>CPM</i>	<i>Hybrid CPM</i>	<i>EMM</i>	<i>Hybrid SA</i>
<b>Q=3</b>	- 18.63	- 18.80	- 14.70	-
<b>Q=5</b>	- 23.00	- 24.40	- 23.40	-
<b>Q=8</b>	- 40.85	- 42.00	-	- 36.50