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COMPROMISE PATTERN SYNTHESIS BY MEANS OF OPTIMIZED  
TIME-MODULATED ARRAY SOLUTIONS

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# **Compromise Pattern Synthesis by means of Optimized Time-Modulated Array Solutions**

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## **Introduction**

Originally, the use of time-modulated arrays was mainly devoted to the synthesis of low and ultra-low sidelobes arrays for the detection of radar signals [1]. Successively, only a few works have considered the use of time-modulated arrays for other applications (e.g., wireless communications [2]), although the first work proposing the use of time as an additional degree of freedom dates back to the 1950s [3]. Such an event has been probably caused by the main drawback of time-modulation that is the unavoidable presence of undesired sideband radiations (SRs) in the radiated field due to the periodic commutation of the RF switches between the on and off states. A detailed analysis on such a topic has been recently presented in [4], where a close-form relationship quantifying the power wasted in SR has been obtained.

In the last decades, thanks to the growing computational capabilities of modern PC, some approaches based on evolutionary algorithms have been proposed for the reduction of the losses due to SRs in time-modulated architectures. More specifically, the differential evolution (DE) algorithm [5] and the simulated annealing (SA) [6] have been considered. Despite the successful results, the potentialities of time-modulated array have been only partially investigated. As a matter of fact, a few works have been devoted to extend the possibilities of applying the time-modulation to array antenna synthesis. In [7], the problem concerned with the synthesis of sum and difference patterns has been discussed. Moreover, different switching strategies and their effects on SR have been shown in [8].

In this paper, the potentialities of time-modulation are further investigated in the framework of compromise array synthesis as an alternative solution to standard compromise methods (see [9] and the reference therein). More in detail, starting from a set of static excitations aimed at synthesizing a user-defined sum pattern at the carrier frequency, a compromise difference beam is generated through a suitable sub-arraying pattern matching procedure [9] by optimizing the pulse durations at the input ports of each sub-array. Successively, the unavoidable SR at the harmonic frequencies is minimized by applying a particle swarm optimization to set the switch-on time instants of each time sequence.

## Mathematical Formulation

Let us consider a linear array of  $N$  equi-spaced elements laying on the  $z$ -axis. The static array excitations  $\alpha_n, n=0, \dots, N-1$  are fixed to generate a sum pattern whose array factor is expressed as

$$F_{\Sigma}(\theta, \alpha_n) = \sum_{n=0}^{N-1} \alpha_n \exp(jnkd \cos \theta) \quad (1)$$

where  $k = \frac{2\pi}{\lambda_0}$  is the background wave-number,  $d$  is the inter-element spacing, and  $\theta$  is the angular direction with respect to the array axis. The compromise difference beam is then generated by aggregating the array elements into  $Q$  sub-arrays and enforcing a periodic on-off sequence  $U_q(t), q=1, \dots, Q$  to the output signal of each sub-array. The rectangular pulse function  $U_q(t)$  has period equal to  $T_p$  and it holds true that  $U_q(t) = 1$  for  $t_q^1 \leq t \leq t_q^2$  (with  $0 \leq t_q^1 \leq t_q^2 \leq T_p$ ) and  $U_q(t) = 0$ . Moreover,  $T_p \gg T_0 = \frac{2\pi}{\omega_0}$ . Since  $U_q(t), q=1, \dots, Q$ , is periodic, it can be represented through its Fourier series

$$U_q(t) = \sum_{h=-\infty}^{\infty} u_{hq} \exp(jh\omega_p t) \quad (2)$$

where  $\omega_p = \frac{2\pi}{T_p}$ . Accordingly, the array factor of the difference pattern turns out being the summation of an infinite number of harmonics [4] where the pattern at the carrier angular frequency  $\omega_0$  is equal to

$$F_{\Delta}^{(0)}(\theta, c_n, \tau_q) = \sum_{n=0}^{N-1} \alpha_n \sum_{q=1}^Q \delta_{c_n q} \tau_q \exp(jnkd \cos \theta) \quad (3)$$

where

$$\tau_q = u_{0q} = \frac{1}{T_p} \int_0^{T_p} U_q(t) dt = \frac{t_q^2 - t_q^1}{T_p}. \quad (4)$$

and  $\delta_{c_n q}$  is the Kronecker delta, being  $\delta_{c_n q} = 1$  if  $c_n = q$ , and  $\delta_{c_n q} = 0$  otherwise, where  $c_n \in [1; Q]$ ,  $n=0, \dots, N-1$ , are integer values identifying the sub-array membership of the array elements. Differently, the pattern of the harmonic radiations (i.e.,  $h \neq 0$ ) is given by

$$F_{\Delta}^{(h)}(\theta, c_n, u_{hq}) = \sum_{h=-\infty}^{\infty} \exp[j(h\omega_p + \omega_0)t] \sum_{n=0}^{N-1} \alpha_n \sum_{q=1}^Q \delta_{c_n q} u_{hq} \exp(jnkd \cos \theta). \quad (5)$$

where the Fourier coefficients  $u_{hq}$  are function of the switch-on and switch-off instants  $t_q^1$  and  $t_q^2$  or equivalently of the switch-on instants and the pulse duration  $\tau_q$  thanks to (4). It is worth noting from (3), that it is enough to define the values  $\tau_q$ ,  $q = 1, \dots, Q$  and  $c_n$ ,  $n = 0, \dots, N-1$ , to synthesized the compromise pattern at the carrier frequency. Hence, according to the guidelines of [9], the following cost function is minimized

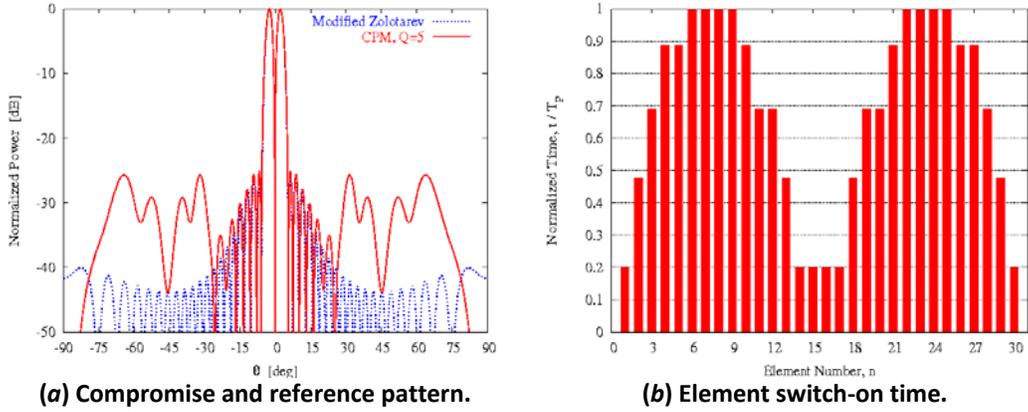


Figure 1.

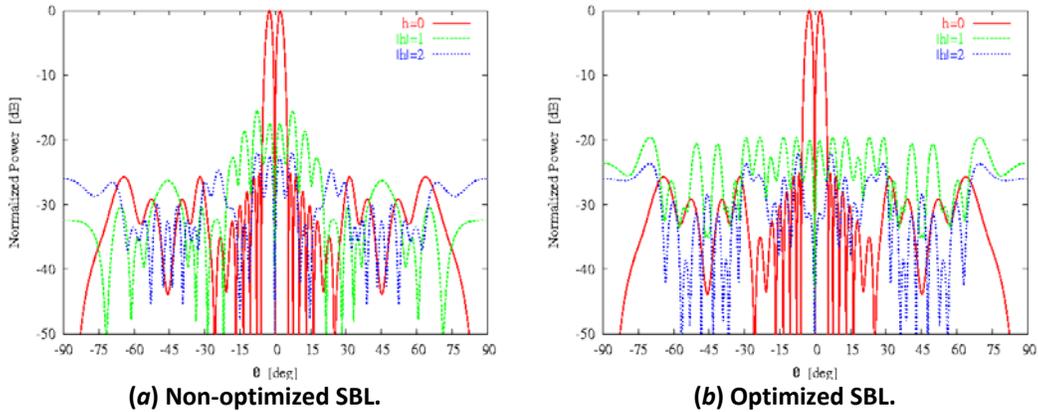


Figure 2 – Normalized power pattern.

$$\Psi^{(0)}(c_m, \tau_q) = \frac{1}{N} \sum_{n=0}^{N-1} \left\| \alpha_n \left( \frac{\beta_n}{\alpha_n} - \sum_{q=1}^Q \delta_{c_n q} \tau_q \right) \right\|^2. \quad (5)$$

where  $\beta_n$ ,  $n = 0, \dots, N-1$ , is set of reference excitation coefficients computed according to classical methods (e.g., Bayliss). Then, in order to reduce the interferences due to SR, only the values  $t_q^1$ ,  $q = 1, \dots, Q$ , are optimized since the

$\tau_q$ ,  $q=1,\dots,Q$  are fixed to those computed in (5). Accordingly, a PSO-based strategy is used and a proper functional  $\Psi^{(h)}(\mathbf{t}_q^1)$  is minimized measuring the mismatch between the actual sideband level and the desired one.

### Numerical Results

For representative purposes, let us consider the synthesis of a  $N=30$  array antenna. Starting from a set of static excitations affording a Villeneuve sum pattern with  $SLL=-20\text{ dB}$  and  $\bar{n}=3$ , the compromise difference pattern synthesized through the proposed approach is shown in Fig. 1(a) together with the reference difference pattern (Modified Zolotarev difference pattern) when  $Q=10$ . The values of  $\tau_q$ ,  $q=1,\dots,Q$  and  $c_n$ ,  $n=0,\dots,N-1$  are shown in Fig. 1(b), as well. The harmonic frequencies before and after the optimization by means of the PSO strategy are shown in Fig. 2(a) and Fig. 2(b), respectively. It is worth noting how the level of the higher harmonics is reduced.

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