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January 2011

Technical Report # DISI-11-166

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Introduction

In recent years, several works have been devoted to show the potentialities of time-modulated arrays (TMAs) [1][2]. It is now well-established that TMAs synthesize low-sidelobe average patterns by using a set of simple on-off radio-frequency (RF) switches that modulate the static excitation weights. Moreover, a fast and simple control of the radiation pattern is enabled by just modifying the pulse sequence of RF switches. By exploiting this latter property, it would be of great interest for practical array applications the possibility of reconfiguring the pattern just modifying a reduced number of control elements [3][4] in order to simplify the overall feeding network architecture. Towards this end, this work presents an innovative two-step strategy for the synthesis of sub-arrayed time-modulated antenna arrays where RF switches are used at the sub-array level. The problem is formulated as the retrieval of the "best" sub-arraying configuration and pulse sequence generating a desired pattern at the working frequency with minimum power losses in the harmonic radiations (called *sideband radiation* - SR) generated by the periodic modulation of the static array excitations. At the first step, the sub-array configuration is determined by means of a "pulse matching" approach that exploits the guidelines of the Border Element Method (BEM) presented in [5] and recognized for its effectiveness in dealing with both small and large arrays. At the second step, a strategy based on a Particle Swarm Optimizer (PSO) has been used to determine the pulse sequence minimizing the amount of the power losses where an effective analytical relationship [6] has been taken into account to count the energy in the SR.

Mathematical Formulation

The array factor of a uniform linear array with N elements placed along the x-axis and grouped into Q time-modulated sub-arrays is mathematically expressed as

$$AF(t,\vartheta) = e^{j\omega_0 t} \sum_{n=1}^{N} \sum_{q=1}^{Q} \alpha_n \delta_{nq} U_q(t) e^{j\beta\left[n-\left(\frac{N+1}{2}\right)\right] d \sin(\vartheta)} \quad (1)$$

where ω_0 is the working angular frequency, $\underline{A} = \{\alpha_n, n = 1,...,N\}$, is the set of static excitation weights, $\beta = \omega_0 / c$, c being the speed of light in vacuum, d is the inter-element distance, and ϑ is the angle measured from broadside. Moreover, $\delta_{nq} = 1$ when $c_n = q$ and $\delta_{nq} = 0$ otherwise, $C = \{c_n \in [0, Q], n = 1,...,N\}$, being integer values identifying the sub-array configuration. In (1), $\underline{U} = \{U_q(t), q = 1,...,Q\}$, is a set of periodic rectangular pulse functions of period T_p modeling the on-off behavior of the RF switches such that

$$U_q(t) = \begin{cases} 1 & 0 \le t < \tau_q \\ \\ 0 & t \ge \tau_q \end{cases} \tag{2}$$

τ_q being the (normalized) duration of the "on" state, called *switch-on time*, of the q-th switch. By expanding (2) in its Fourier series, Equation (1) turns out being composed by and infinite number of harmonic contributions [6] where it is possible to distinguish the interested radiation at ω_0

$$AF_0(\vartheta) = \sum_{n=1}^{N} \sum_{q=1}^{Q} \alpha_n \delta_{nq} \tau_q e^{j\beta\left[n - \left(\frac{N+1}{2}\right)\right] d \sin(\vartheta)} \tag{3.a}$$

from an infinite number of (undesired) harmonic radiations generated at multiple of $\omega_p = 1/T_p$

$$AF_{SR}(\vartheta) = \sum_{h=-\infty, h\neq 0}^{+\infty} e^{j(\omega_0 + h\omega_p)t} \sum_{n=1}^{N} \sum_{q=1}^{Q} \alpha_n \delta_{nq} u_{qh} e^{j\beta\left[n - \left(\frac{N+1}{2}\right)\right] d \sin(\vartheta)} \tag{3.b}$$

where u_{qh}, $q = 1,...,Q$, $|h| = 1,...,\infty$, is the *h*-th coefficient of the Fourier expansion. For a given the set of static element excitations, $\underline{A}$, the proposed approach is aimed at determining the sub-array configuration, $\underline{C}$, and the pulse sequence modulating the sub-arrays, $\underline{U}$, synthesizing a pattern at the center frequency as close as possible to a desired one, $AF_{ref}(\vartheta)$, while minimizing the power losses in the SR (3.b). Towards this end, the problem is dealt with as a two-step procedure. First, the generation of the desired pattern at ω_0 is carried out by exploiting the BEM to define the two sets $\underline{A}$ and $\underline{C}$. More specifically, the problem at hand is recast as a pulse matching problem where the following cost function

$$\Psi_0(\underline{C}) = \frac{1}{N} \sum_{n=1}^{N} \left| \chi_n - \sum_{q=1}^{Q} \delta_{nq} \tau_q \right|^2 \tag{4}$$

is minimized according to [5]. In (4), χ_n, $n = 1,...,N$, is the *n*-th reference excitation of the set synthesizing $AF_{ref}(\vartheta)$. Moreover, the values of the switch-on times are analytically computed as $\tau_q = \sum_{n=1}^{N} \delta_{nq} \chi_n \Big/ \sum_{n=1}^{N} \delta_{nq}$, $q = 1,...,Q$.

To encompass the optimization of the SR, the PSO is used in the second step to minimize

$$\Psi_{SR}(\underline{\tau}) = w_{SR} \frac{P_{SR}}{P_{tot}} + w_{BW} \frac{\left|BW_{ref} - BW_0\right|^2}{\left|BW_{ref}\right|^2} + w_{SLL} \frac{\left|SLL_{ref} - SLL_0\right|^2}{\left|SLL_{ref}\right|^2} \tag{5}$$

starting from the solution yielded at the previous step. In (5), $\underline{\tau} = \{\tau_q(t), q = 1,\ldots,Q\}$ and P_{SR} is the amount of power losses of $AF_{SR}(\vartheta)$ analytically computed as [6]

$$P_{SR}(\underline{\tau}) = \sum_{n=1}^{N} \tau_n (1-\tau_n) + \sum_{n,m=1,n\neq m}^{N} \{\mathrm{sinc}[\beta d(m\text{-}n)]\}(\tau_{mn} - \tau_m \tau_n) \tag{6}$$

where $\tau_{mn}=\tau_n$ is $\tau_n \leq \tau_m$ and $\tau_{mn}=\tau_m$ otherwise. Moreover, P_{tot} is the total power radiated by the antenna and the other two terms on the rigth-side of (6) are introduced to keep the pattern synthesized at ω_0 close to the reference one, BW_{ref}, BW_0 and SLL_{ref}, SLL_0 being the $-3dB$ beamwidth and the sidelobe level of the reference and current solution, respectively.

Numerical Results

To evaluate the reliability and potentialities of the proposed approach, a comparison with the test case in [8] solved by means of a Simulated Annealing (SA) based approach has been analyzed. Towards this purpose, the same linear array with $N = 30$ with $d = 0.7\lambda_0$ (λ_0 being the free space wavelength) and uniform static excitations, $\alpha_n = 1$, $n = 1,\ldots,N$, is considered. A Zolotarev difference pattern with the same sidelobe level of the solution achieved in [8] has been chosen as reference for the BEM. Moreover, the same number of switches has been used (i.e., $Q = 8$), as well. The sub-array aggregation and the switch-on times obtained at the end of the two steps are given in Tab. I ($c_n = 0$ and $c_n = -$ mean that the *n*-th element is not modulated and not used, respectively).

BEM	$\underline{C}$	123400004433211112334400004321
	$\underline{\tau}$	0.24, 0.52, 0.73, 0.90
BEM-PSO	$\underline{C}$	- 010000000110 - - - - 011000000010 -
	$\underline{\tau}$	0.73

Tab. I – Optimized subarray aggregation and switch-on times.

The pattern radiated at ω_0 is shown in Fig. 1 and pictorially compared with the SA solution.

It is worth noticing that the power losses are reduced of more than 13% of P_{tot} when using the PSO approach. Figure 2 illustrates the arising behavior of the peak level of the sideband radiation, called *sideband level* (SBL). It turns out to be reduced of more than $8dB$ from $SBL_{BEM} = -16.42\,dB$ to $SBL_{BEM-PSO} = -24.81dB$. As for the array architecture, Table I indicates that the solution synthesized by means of the BEM-PSO requires only 2 switches, one for each half the array.

Conclusions

In this paper, an innovative two-step PSO-based approach has been presented for the synthesis of sub-arrayed time-modulated arrays. First, the BEM have been used to determine the “best” sub-array configuration, while the PSO has been successively exploited to optimize the pulse sequence in order to minimize the power losses in SR.

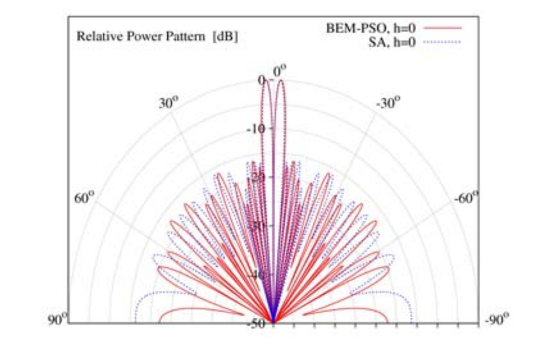

Fig.1 – Pattern at the central frequency.

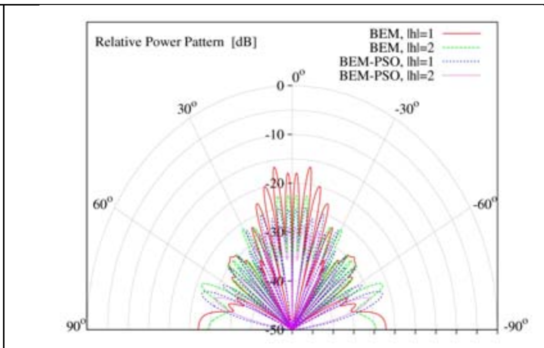

Fig. 2 – Harmonic radiations, |h|=1,2.

References

[1] Y. Chen, S. Yang, and Z. Nie, “Synthesis of satellite footprint patterns from time-modulated planar arrays with very low dynamic range ratios,” *Int. J. Numer. Model.*, vol. 21, pp. 493-506, 2008.

[2] A. Tennant and B. Chambers, “Time-switched array analysis of phase-switched screens,” *IEEE Trans. Antennas Propag.*, vol. 57, no. 3, pp. 808-812, Mar. 2009.

[3] R. L. Haupt, “Optimized weighting of uniform subarrays of unequal size,” *IEEE Trans. Antennas Propag.*, vol. 55, no. 4, pp. 1207-1210, Apr. 2007.

[4] L. Manica, P. Rocca, and A. Massa, “Design of subarrayed linear and planar array antennas with SLL control based on an excitation matching approach,” *IEEE Trans. Antennas Propag.*, vol. 57, no. 6, pp. 1684-1691, Jun. 2009.

[5] L. Manica, P. Rocca, A. Martini, and A. Massa, “An innovative approach based on a tree-searching algorithm for the optimal matching of independently optimum sum and difference excitations,” *IEEE Trans. Antennas Propag.*, vol. 56, no. 1, pp. 58-66, Jan. 2008.

[6] J. C. Brégains, J. Fondevila, G. Franceschetti, and F. Ares, “Signal radiation and power losses of time-modulated arrays,” *IEEE Trans. Antennas Propag.*, vol. 56, no. 6, pp. 1799-1804, Jun. 2008.

[7] P. Rocca, M. Benedetti, M. Donelli, D. Franceschini, and A. Massa, “Evolutionary optimization as applied to inverse scattering problems,” Inverse Problems, vol. 25, no. 12, pp. 1-41, Dec. 2009.

[8] J. Fondevila, J. C. Brégains, F. Ares, and E. Moreno, “Application of time modulation in the synthesis of sum and difference patterns by using linear arrays,” *Microw. Opt. Technol. Lett.*, vol. 48, no. 5, pp. 829-832, May 2006.