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On the Exploitation of Analytic Sequences for Array Interleaving

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Abstract: This paper presents a methodology based on Almost Difference Sets (ADSs) for the design of shared aperture arrays with well behaved sidelobes. Thanks to the autocorrelation properties of ADSs, such a technique allows the design of fully interleaved arrays with predictable performances. Analytical and numerical results are provided in order to point out the computational efficiency and the effectiveness of the approach both in the linear and in the planar case.

Keywords: Interleaved Arrays, Shared Aperture Arrays, Linear Arrays, Planar Arrays, Almost Difference Sets, Sidelobe Control.

1. Introduction

Shared aperture antennas are of great interest in modern systems for communications, detection, location and remote sensing, because of the need to realize multiple functions in a limited space [1]. In this framework, aperture arrays of intermixed elements (often indicated as interleaved, interlaced or interspread arrays) provide interesting performances in terms of hardware complexity, aperture efficiency, and flexibility [1]. In order to overcome their drawbacks in terms of gain and PSL control [2], stochastic optimization methods or hybrid methodologies [1-4] have been successfully applied. However, stochastic optimization approaches can be computationally inefficient when dealing with large apertures and *a-priori* estimates of the expected performances are usually not available [1].

In this paper, the problem of designing an equally-weighted fully-interleaved array is addressed to provide design guidelines to be employed when, whether by choice or by necessity, a computationally efficient and sub-optimal solution with predictable performances is preferred to a random or a stochastically-optimized design. Towards this end, the synthesis of interleaved arrays is faced with an innovative approach that exploits the so-called Almost Difference Sets (ADSs), and its performances are numerically analyzed. ADSs are binary sequences characterized by a three-level autocorrelation [5] which have been recently used to the design of thinned arrays with predictable sidelobes [7]. As regards their applicability in interleaved arrangements, it is to be noticed that the complementary of an ADS is still an ADS [8]. Such property suggests the design of ADS-based interleaved arrangements by simply associating each subarray to the 0s (or 1s) of an ADS sequence (e.g. taken from [9]). From the previous consideration and already published results on ADSs, the main expected advantages of interleaved array designs based on such sequences are the following: (a) design simplicity and computational efficiency, as ADS designs do not require any optimization; (b) high PSL performances, as a consequence of their radiating properties [9]; remarkable flexibility, since several tradeoff solutions can be obtained from a single ADS design, thanks to the fact that a cyclic shift of an ADS is still an ADS both in the linear and in the planar case [7,8,11]. Such expected properties are analytically discussed in the following, and several linear and planar interleaved designs are provided in order to numerically assess the performances of the proposed methodology.

2. Almost Difference Sets: Definition, Properties and Associated Thinned Arrays

Let us consider a planar uniform lattice of $N = P \times Q$ positions spaced by $s_x \times s_y$ wavelengths ($Q=1$ corresponds to the linear case). A thinned array with K_I active elements defined on such an aperture will

exhibit an array factor equal to $S_I(u, v) = \sum_{p=0}^{P-1} \sum_{q=0}^{Q-1} w_I(p, q) \exp[i2\pi(ps_x u + qs_y v)]$, where $w_I(p, q) \in \{0, 1\}$

($p = 0, \dots, P-1$, $q = 0, \dots, Q-1$) and $\sum_{p=0}^{P-1} \sum_{q=0}^{Q-1} w_I(p, q) = K_I$. By exploiting the technique outlined in [7], the ADS-

associated thinned array is defined as follows:

$$w_I(p, q) = \begin{cases} 1 & \text{if } (p, q) \in D_I \text{ (} p \in D_I \text{ in the linear case)} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where D_I is a (N, K_I, Λ_I, t_I) -ADS, i.e. a particular subset $D_I = \{a_i \in Z^N; i = 0, \dots, K_I - 1\}$ of an Abelian group Z^N of order N for which the multiset $Q = \{q_j = a_h - a_l, a_h \neq a_l, j = 0, \dots, K_I(K_I - 1) - 1\}$ contains t_I nonzero elements of Z^N exactly Λ_I times, and the remaining $N - 1 - t_I$ exactly $\Lambda_I + 1$ times [7] (ADSs represent therefore a generalization of DSs [7]). By simple manipulations, it turns out that the power pattern of the ADS-based finite linear array satisfies the following

$$|S_I(u_n, v_m)|^2 = F[A_I(\tau_x, \tau_y)] = F(k, l) \quad (2)$$

at fixed known sampling points $u_n = \frac{n}{Ps_x}$, $v_m = \frac{m}{Qs_y}$, where F is the Fourier transform operator and

$A_I(\tau_x, \tau_y)$ is the periodic autocorrelation function of the sequence $w_I(p, q)$ [7], which is given by

$$A_I(\tau_x, \tau_y) = \sum_{p=0}^{P-1} \sum_{q=0}^{Q-1} w_I(p, q) w_I[(p + \tau_x) \bmod P, (q + \tau_y) \bmod Q]. \quad (3)$$

and turns out to exhibit the following three-level behaviour [7]

$$A_I(\tau_x, \tau_y) = \begin{cases} K_I & \text{if } \tau_x = \tau_y = 0 \\ \Lambda_I & \text{for } t \text{ values of } (\tau_x, \tau_y) \\ \Lambda_I + 1 & \text{otherwise} \end{cases} \quad (4)$$

In [7], starting from (2), suitable bounds for the PSL of ADS-based thinned arrays, which is defined as

$$PSL = \frac{\max_{u \neq M} |S_I(u, v)|^2}{|S_I(0, 0)|^2} \quad (5)$$

(M being the mainlobe region) were derived.

3. ADS-based Interleaved Array Design

In order to introduce the exploitation of ADSs for interleaved array design, let us consider the following Theorem:

Theorem 1 [8]: if D_I is a (N, K_I, Λ_I, t_I) -ADS, then its complementary set is $D_C = Z^N \setminus D_I$ is a (N, K_C, Λ_C, t_C) -ADS with $K_C = N - K_I$, $\Lambda_C = N - 2K_I + \Lambda_I$.

Accordingly, starting from a binary ADS sequence $w_I(p, q)$ defining a thinned array on a given aperture, one can obtain a fully interleaved arrangement with similar properties by simply associating the following binary sequence to a second sub-array

$$w_C(p, q) = 1 - w_I(p, q) \quad p = 0, \dots, P-1, q = 0, \dots, Q-1 \quad (6)$$

As a consequence, the aperture efficiency of the resulting interleaved arrangement turns out to be equal to 1. In order to illustrate such property, let us consider the $(26, 13, 6, 19)$ -ADS, which is defined as [9]

$$D_I = \{0, 4, 5, 10, 12, 14, 15, 16, 17, 19, 22, 23, 25\} \quad (7)$$

The resulting linear interleaved arrangement, which is plotted in Fig. 1, confirms that each of the 26 elements composing the shared aperture is associated either to one sub-array or to the other.

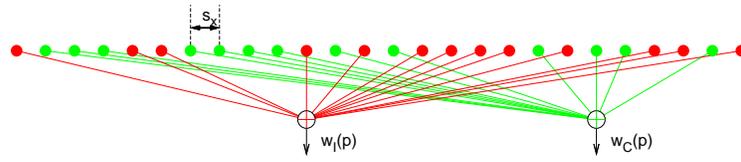


Fig. 1 – Properties of ADS-based interleaved arrangements [(26,13,6,19)-ADS]. Geometry of the ADS-based linear interleaved arrangement [shared aperture size: 12.5λ].

Similar considerations could be performed also in the planar case. As a visual example, the interleaved arrangement resulting from the (49,25,12,24)-ADS is provided in Fig. 2.

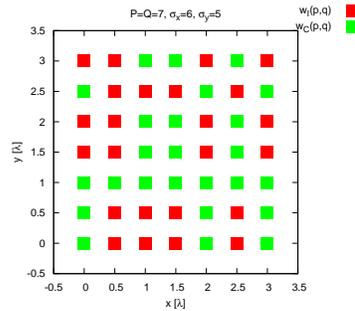


Fig. 2 – Properties of ADS-based interleaved arrangements [(49,25,12,24)-ADS]. ADS-based planar interleaved arrangement [shared aperture size: $3 \lambda \times 3 \lambda$].

Since the sub-arrays arising from the described procedure are essentially two ADS-based thinned independent arrays, several of the results already deduced for those arrangements can be extended straightforwardly. More specifically, by simple manipulations one could show that

$$A_C(\tau_x, \tau_y) = A_I(\tau_x, \tau_y) + N - 2K_I \quad (8)$$

[$A_C(\tau_x, \tau_y)$ being the cyclic autocorrelation of $w_C(p, q)$] i.e., the cyclic autocorrelations of the two ADS-based sub-arrays are equal, except for a fixed and known shift (Fig. 3), and, consequently [exploiting (2)] that

$$|S_C(\tau_x, \tau_y)|^2 = \Psi |S_I(\tau_x, \tau_y)|^2 \quad (9)$$

where $\Psi = \left(\frac{N}{K_I} - 1 \right)$, i.e. the samples of the autocorrelation pattern of the two arrays are equal, except for a

fixed and known shift factor. Due to such properties, it turns out that

1. the design of an ADS based interleaved array is computationally very efficient (it is only required to associate the 0s and 1s of known sequences to the two sub-arrays) [Eq. (6)];
2. the two resulting sub-array have very similar radiation properties when $K_I \cong K_C$ [Eq. (9)];
3. each of the resulting sub-array has a well-behaved PSL [7];
4. thanks to the cyclic shift property of ADSs [7], each ADS sequence corresponds to N different interleaved arrangements, which can provide different tradeoffs in terms of PSL on the two sub-arrays.

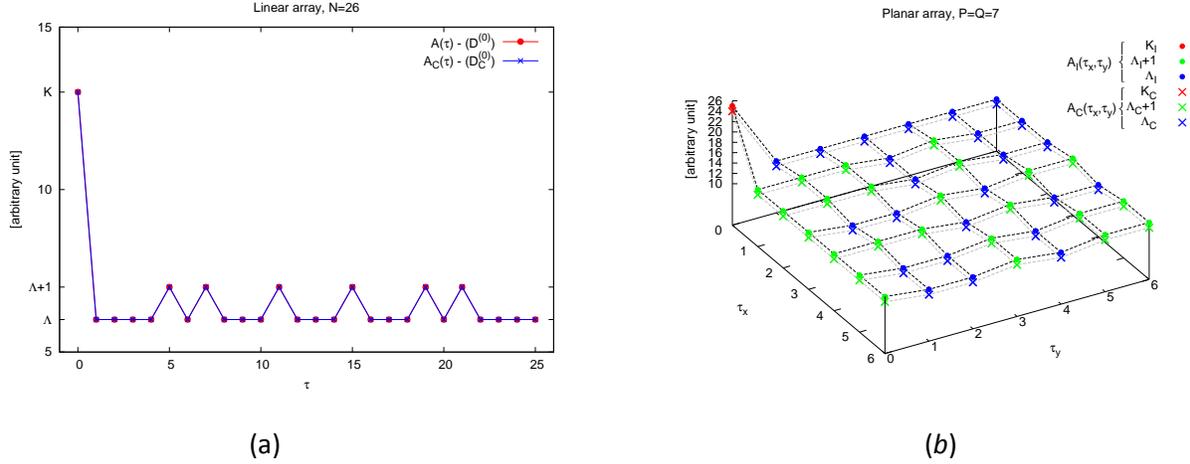


Fig. 3 – Properties of ADS-based interleaved arrangements. Autocorrelation of (a) linear ADS-based interleaved arrays ($N = 26$), and (b) planar ADS-based interleaved arrays ($P = Q = 7$).

4. Numerical Results

In order to numerically point out the characteristics and advantages discussed above of ADS-based interleaved arrays, let us consider as a first example the design of an ADS-based linear interleaved array. In Figure 4(a) the PSL of the two interleaved arrangements obtained by cyclically shifting a reference ADS, i.e. using

$$D_I^{(\sigma)} = \{(a_I + \sigma) \bmod N; a_I \in Z^N; i = 0, \dots, K_I - 1\} \quad (10)$$

$$D_C^{(\sigma)} = \{(a_C + \sigma) \bmod N; a_C \in Z^N; i = 0, \dots, K_C - 1\} \quad (11)$$

(where $\sigma = 0, \dots, N-1$) is reported for three representative examples of ADSs taken from [9] ($N = 60, 88, 108, \nu = \frac{K_I}{N} = 0.5, \eta = \frac{t}{N-1} = 0.25$). As expected, different tradeoff designs can be obtained from each ADS [Fig. 4(a)].

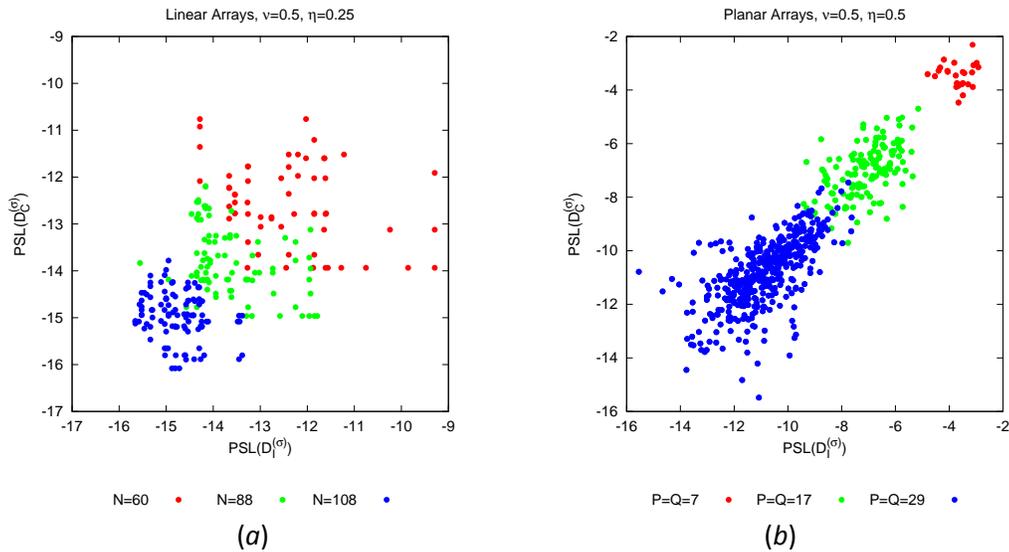


Fig. 4 – Numerical validation. Behaviour of $PSL\{D_C^{(\sigma)}\}$ vs. $PSL\{D_I^{(\sigma)}\}$ for (a) linear interleaved arrangements with $N = 60, 88, 108$; (b) planar interleaved arrangements with $P = Q = 7, 17, 29$.

Moreover, it is seen that the PSL performances increase with the array size (as it happens for ADS-based thinned arrays [7]), for both interleaved sub-arrays. By analyzing the behavior of the beam pattern obtained for a representative case (Fig. 5 - $N = 88$, $\sigma = 23$), one can also observe that the power patterns of the two fully-interleaved sub-arrays are very similar: this is a consequence of the fact that both sub-arrays show a power pattern which complies with Eq. (2) (Fig. 5), and therefore have to assume the same values for $u_n = \frac{n}{P S_x}$.

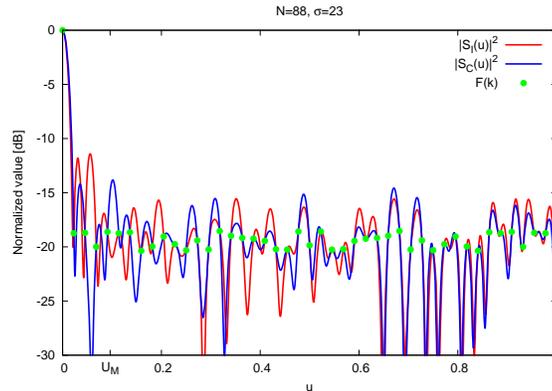


Fig. 5 – Numerical validation. Power patterns of a representative tradeoff ADS-based linear arrangement for $N = 88$, and predicted samples $F(k)$.

Similar observations hold true also for the planar case [Fig. 4(b) - $P = Q = 7, 17, 29$; $\nu = 0.5$, $\eta = 0.5$]. From the reported results, one can notice that the PSL of the two ADS-based interleaved sub-arrays is similar for each considered shift, as it happened in the linear case [Fig. 4(b) vs. Fig. 4(a)]. This is also confirmed by the plots of the power pattern of two representative ADS planar interleaved arrays (Fig. 6 - $P = Q = 29$). As it can be noticed, the two sub-arrays exhibit a close radiation pattern. Moreover, as expected from the previous Sections, the sidelobes exhibit a quite regular behavior (i.e. similar ripple levels for each sidelobe). Such a property, which can be also noticed in the linear case (Fig. 5), is a consequence of the ADS sampling properties [Eq. (2)], and is similar to the radiation properties of thinned arrays based on ADSs.

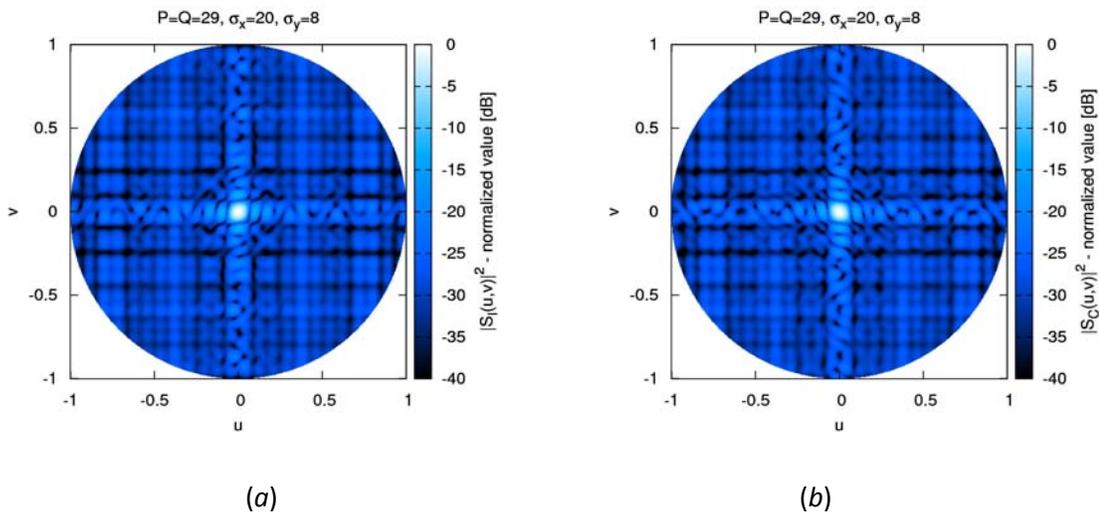


Fig. 6 – Numerical validation. Power patterns of a representative tradeoff ADS-based linear arrangement for $P = Q = 29$.

4. Conclusions

In this paper, an ADS-based methodology has been proposed for interleaving equally weighted linear arrays operating on the same frequency band. Such a deterministic approach is not aimed at obtaining optimal arrays, but rather at providing guidelines for the efficient design of shared aperture arrangements with predictable performances. The obtained results have pointed out the efficiency, effectiveness, simplicity and reliability of ADS-based shared aperture arrangements.

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