UNIVERSITY
OF TRENTO

DIPARTIMENTO DI INGEGNERIA E SCIENZA DELL'INFORMAZIONE

38123 Povo – Trento (Italy), Via Sommarive 14
http://www.disi.unitn.it

SYNTHESIS OF SUB-ARRAYED MONOPULSE PLANAR ARRAYS BY MEANS OF AN INNOVATIVE EXCITATION MATCHING METHOD

L. Manica, P. Rocca, and A. Massa

January 2011

Technical Report # DISI-11-216

SYNTHESIS OF SUB-ARRAYED MONOPULSE PLANAR ARRAYS BY MEANS OF AN INNOVATIVE MATCHING METHOD

L. Manica, P. Rocca, and A. Massa

Department of Information and Communication Technologies, University of Trento
via Sommarive 14, I-38050, Trento, Italy
phone: +390461882057, email: andrea.massa@ing.unitn.it, web: www.eledia.ing.unitn.it

Keywords: monopulse antennas, planar arrays, sub-arraying, sum-difference pattern synthesis, excitation matching, contiguous partition.

ABSTRACT

Planar antenna arrays are widely used as transmitting and receiving part of monopulse trackers because of the need to generate sum and difference radiation modes to estimate the angular location of an unknown target. In such a framework, this paper presents an innovative synthesis technique able to synthesize two orthogonal sub-arrayed difference patterns close to an optimal reference one. The proposed numerical results assess the effectiveness of the proposed approach also dealing with large arrays.

1. INTRODUCTION

The antenna device of a monopulse tracker is requested to generate different beams on the same aperture [1]. This objective might be obtained by using a linear or a planar array of antennas. However, the use of planar arrays is usually preferred to accurately estimate the flight path of the target. More in detail, a planar array of a monopulse tracker generates three different beams [2]: the *sum mode*, the *azimuth mode* (or *H-mode*), and the *elevation mode* (or *E-mode*). These field configurations are subjected to some constraints as low side lobe level (SLL) to reject undesired interferences and narrow beamwidth to enhance the resolution of the target detection. In order to compute the excitations coefficients of planar structures, analytical techniques, able to generate optimal in the Dolph-Chebyshev sense sum [3] (i.e., beamwidth between the first nulls is the narrowest possible for a specified SLL, array elements, and interelement spacing) and difference [4] (i.e., narrowest first null beamwidth and largest normalized difference slope on the boresight for a specified sidelobe level) patterns, have been developed. Nevertheless, the obtained solutions require the design of three independent feed networks (one for each mode) with usually unacceptable costs, a non-negligible circuit complexity, and the occurrence of electromagnetic interferences. Consequently, the definition of suitable compromise solutions aimed at optimizing the trade-off between feed network complexity and optimality of the synthesized patterns, has been explored. Sub-array techniques synthesize the sum mode by means of excitation coefficients analytically-computed, while the difference coefficients are obtained by grouping the array elements in sub-arrays and assigning a suitable weight to each of them. In such a fashion, the corresponding feed network is simpler and the compromise beams still fit the problem constraints. As regards to linear arrays, McNamara proposed in [5] an analytical method where, for each possible grouping, the weights are computed by means of a pseudo-inversion of an over-determined system. Since the best configuration is not *a-priori* known, the synthesis process is time-consuming due to the exhaustive search. Moreover, the matrix system is severely ill-conditioned when the number of elements increases. To overcome these drawbacks, different global optimization approaches have been proposed by Lopez *et al.* [6] and Caorsi *et al.* [7] as well as hybrid methods [8][9]. Despite the achieved improvements, global optimizers are usually computationally-expensive especially dealing with large arrays because of the exponential enlargement of the dimension of the solution space. Because of such a reason, as well, stochastic global methods deal with linear arrays and planar structures are not taken into account (except in [8] where the sub-array configuration has been *a-priori* fixed). To overcome such a drawback, an effective methodology [indicated as *Contiguous Partition* (CP) approach] has been proposed in [10][11] for linear arrays. By exploiting the relationships between the optimal sum/difference beam excitations, the solution space is considerably reduced and efficiently explored just analyzing a small sub-set of the candidate solutions and changing the sub-array memberships of the array elements. This contribution is aimed at proposing the CP method also for monopulse planar arrays. This is not a trivial task and certainly it is not a simple extension of the linear model because of the increasing of the solution space and the complexity of the planar structure.

The paper is organized as follows. In Section 2, the problem is presented and mathematically formulated. Section 3 is aimed at describing the synthesis process. Selected numerical results are reported in Sect. 4 to show the effectiveness of the CP approach in synthesizing planar sub-arrayed difference patterns. Finally, some conclusions are drawn (Sect. 5).

2. MATHEMATICAL FORMULATION

Let us consider a planar array with circular boundary lying on the $xy-plane$. The coordinates of each element are $x_m = [m + 1/2\,\mathrm{sgn}(m)] \times d, m = \pm 1, \ldots, N_x$ and $y_n = [n + 1/2\,\mathrm{sgn}(n)] \times d,\ n = \pm 1, \ldots, N_y^m$ $d_x = d_y = d$ being the inter-element spacing. The array factor is given by [12]:

$$AF(\theta, \phi) = \sum_{m=-N_x}^{N_x - 1} \sum_{n=-N_y^m}^{N_y^m - 1} I_{mn} e^{j[k_x (m+1/2)d + k_y (n+1/2)d]}, \tag{1}$$

where I_{mn} are the excitations coefficients and $k_x = \frac{2\pi}{\lambda} \sin\theta \cos\phi$, $k_y = \frac{2\pi}{\lambda} \sin\theta \sin\phi$.

Dealing with the monopulse antenna system, let us assume that the sum pattern is generated by quadrantal symmetric real excitations computed by sampling the continuous Taylor's distribution [3]:

$$A = \{\alpha_{mn} = \alpha_{(-m)n} = \alpha_{m(-n)} = \alpha_{(-m)(-n)}; m = 1, \ldots, N_x; n = 1, \ldots, N_y^m\} \tag{2}$$

Concerning the optimal *H*-mode difference pattern, it is obtained by a set of anti-symmetrical real excitations obtained from the sampling of the Bayliss's distribution [4]:

$$B^H = \{\beta_{mn}^H = \beta_{(-m)n}^H = -\beta_{m(-n)}^H = -\beta_{(-m)(-n)}^H; m = 1, \ldots, N_x; n = 1, \ldots, N_y^m\} \tag{3}$$

Because of the symmetry, the optimal coefficients of the difference E-mode turn out to be

$$B^E = \{\beta_{mn}^E = -\beta_{mn}^H; m = 1, \ldots, N_x; n = 1, \ldots, N_y^m\} \tag{4}$$

It is worth noting that such optimal/reference patterns are characterized by low SLLs and narrow beamwidths. Moreover, the Taylor's pattern assures a high directivity, whereas the Bayliss's beam is characterized by a maximum normalized slope in the bore-sight direction (i.e., high sensitivity).

Since the use of three independent feed networks is unfeasible in several applications, a sub-arraying strategy aimed at defining a simplified sub-arrayed feed network (Fig. 1) is mandatory. As a consequence, the problem at hand is reformulated as follows "*for a given optimal sum (difference) mode, defining the sub-array configuration and the corresponding sub-array gains such that, the synthesized difference (sum) modes are as close as possible to the optimal ones*" [5]. Accordingly, the problem solution is univocally determined by defining the grouping matrix C^H

$$C^H = \{c_{mn}^H; m = 1, \ldots, N_x; n = 1, \ldots, N_y^m\}, \tag{5}$$

and the weight coefficients, $W = \{w_q; q = 1, \ldots, Q\}$, associated to each sub-array, $c_{mn}^H \in [1, Q]$ being the sub-array index of the array element located at the *m*-th row and *n*-th column of the array. In such a way, the sub-arrayed excitations are given by

$$B_{SA}^H = \{b_{mn}^H = \alpha_{mn} \delta(c_{mn}^H, q) w_q; m = 1, \ldots, N_x; n = 1, \ldots, N_y^m; q = 1, \ldots, Q\}, \tag{6}$$

where $\delta(c_{mn}^H, q) = 1$ if $c_{mn} = q$ and $\delta(c_{mn}^H, q) = 0$ otherwise. Moreover, $B_{SA}^E = \{b_{mn}^E = -b_{mn}^H; m = 1, \ldots, N_x; n = 1, \ldots, N_y^m\}$ according to Eq. (4).

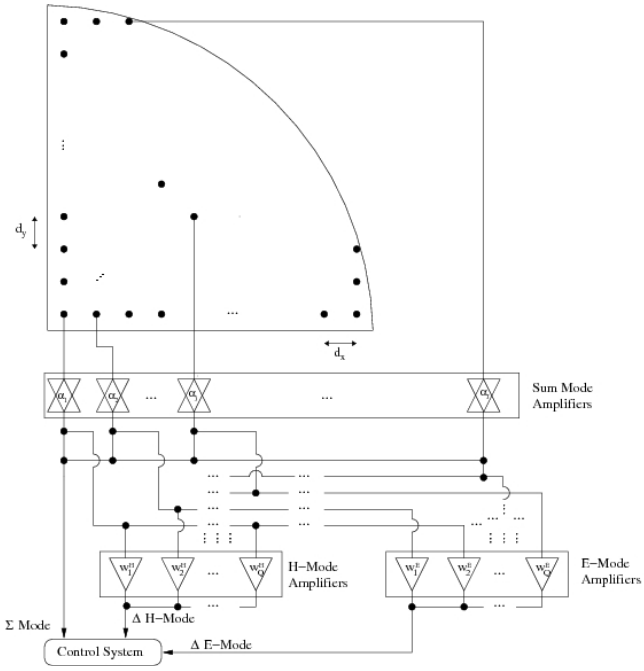

Figure 1: Sub-arrayed antenna feed network

In order to define C_{opt}^{H} and W^{opt}, let us consider the guidelines of the synthesis approach presented in [10] and concerned with linear arrays.

3. SYNTHESIS PROCEDURE

In order to quantify the closeness of a trial solution to an optimal one, let us define the following cost function

$$\Psi\left(C^{H}\right)=\frac{1}{N_x}\sum_{m=1}^{N_x}\frac{1}{N_y^m}\sum_{n=1}^{N_y^m}\left|\beta_{mn}^{H}-b_{mn}^{H}\right|^2 . \tag{7}$$

After simple algebra and Eqs. (6), it results that

$$\Psi\left(C^{H}\right)=\sum_{q=1}^{Q}\frac{1}{N_x}\sum_{m=1}^{N_x}\frac{1}{N_y^m}\sum_{n=1}^{N_y^m}\alpha_{mn}^2\left|\left[\gamma_{mn}-g_{mn}\left(C^{H}\right)\right]\right|^2 \tag{8}$$

where

$$\gamma_{mn}=\frac{\beta_{mn}}{\alpha_{mn}}, m=1,\ldots,N_x; n=1,\ldots,N_y^m \tag{9}$$

are real coefficients indicated as *target gains*. Moreover, the actual sub-array coefficients g_{mn}, called *estimated gains*, are defined taking into account the conclusions drawn from the Fisher’s work [13]:

$$g_{mn}\left(C^{H}\right)=\frac{\sum_{m=1}^{N_x}\sum_{n=1}^{N_y^m}\alpha_{mn}^2\delta\left(c_{mn}^{H},q\right)\gamma_{mn}}{\sum_{m=1}^{N_x}\sum_{n=1}^{N_y^m}\alpha_{mn}^2\delta\left(c_{mn}^{H},q\right)} \tag{10}$$

and the corresponding sub-array weights turn out to be

$$w_q = \delta\left(c_{mn}^H, q\right) g_{mn}\left(C^H\right); m = 1, \dots, N_x; \quad n = 1, \dots, N_y^m; \; q = 1, \dots, Q \quad . \tag{11}$$

The minimization of the cost function (8) is carried out by exploiting the procedure presented in [10][11] and customized to planar monopulse antennas as detailed in the following by pointing out the main differences with respect to the case of linear arrays. Once the optimal gains γ_{mn} are computed, they are sorted to obtain the ordered list $L = \{l_j; j = 1, \dots, J\}$, $J = \frac{1}{4}\sum_{m=1}^{N_x} N_y^m$ where $l_i \le l_{i+1}, i = 1, \dots, J-1$, $l_1 = \min_{mn}\{\gamma_{mn}\}$ and $l_J = \max_{mn}\{\gamma_{mn}\}$. Then, L is partitioned in Q parts to define a suitable *contiguous partition* (CP) (i.e., a sub-array configuration which is characterized by Q convex sets of the list L) fitting the problem requirements. As a matter of fact, Fisher proved in [13] that the solution minimizing (12) is a contiguous partition.

Although the number of candidate sub-array configurations reduces from $U = Q^J$ to $U^{(ess)} = \binom{J-1}{Q-1}$ (i.e., the number of contiguous partitions of *L*), the dimension of the solution space for planar architectures still remain very large. Therefore, the solution space (i.e., the whole set of CPs) is coded into a suitable graph to minimize the storage costs as well as to facilitate the sampling of the space of admissible solutions. As a matter of fact, the use of the tree-based representation of the linear case [11] would require a non-negligible amount of computer memory and a redundant representation. The graph is composed by Q rows and J columns. The *q*-th row is related to the *q*-th sub-array ($q = 1, \dots, Q$), whereas the *j*-th column ($j = 1, \dots, J$) maps the l_j-th element of L. A path ζ of the graph codes a compromise solution and it is constituted by a set of J vertexes, $\{v_j; j = 1, \dots, J\}$, connected by $J-1$ links, $\{t_j; n = 1, \dots, J-1\}$. The graph is explored to determine the optimal compromise configuration (or optimal path) that minimizes (8). Towards this end, a trial path is iteratively updated just modifying the membership of the *border vertexes* (i.e., an internal vertex with at most one of this adjacent vertexes that belongs to a different row of the graph) until the termination criterion expressed in terms of maximum number of iterations or minimum value of the cost function.

4. NUMERICAL RESULTS

In order to give some indications on the effectiveness of the proposed approach, let us consider a set of representative results. The numerical validation is concerned with a planar array lying on the $xy-$plane with circular boundary and radius $R = 5\lambda$. The radiating elements are supposed to be isotropic and located on a 20×20 grid with inter-element spacing equal to $d = \lambda/2$. The sum excitations have been set to obtain the Taylor pattern shown in Fig. 2 with transition index $\overline{n} = 6$ and $SLL = -35\ [dB]$. The target H-mode excitations have been chosen to generate a Bayliss pattern with $\overline{n} = 6$ and $SLL = -40\,[dB]$ and the number of sub-arrays has been set to $Q = 4$.

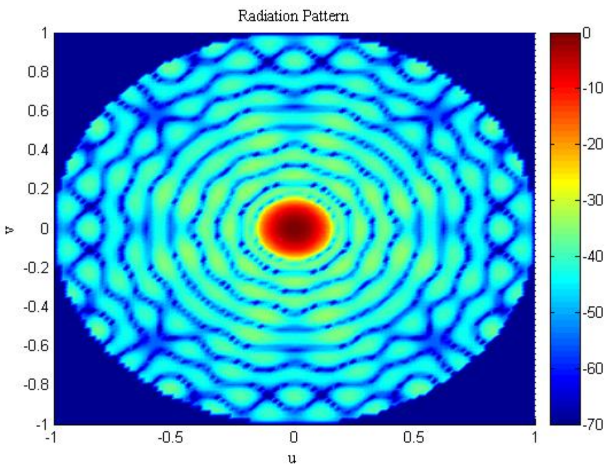

Figure 2: Taylor sum pattern – dB scale

The results of the synthesis process are shown in Fig. 3 and Fig. 4 for the H-mode and the E-mode, respectively. Moreover, Table I summarizes the main features of the synthesized pattern. A comparison with the characteristics of the reference/optimal pattern is given, as well.

	3dB BW [u]	*3dB BW [v]*	*Max SLL [dB]*
CPM	0.0752	0.1029	–30.55
Bayliss	0.0748	0.1008	–37.50

Table I: Pattern features (H-mode)

In order to further point out the reliability of the proposed approach, Figures 5 and 6 show two cuts of the synthesized difference pattern as compared to the reference one.

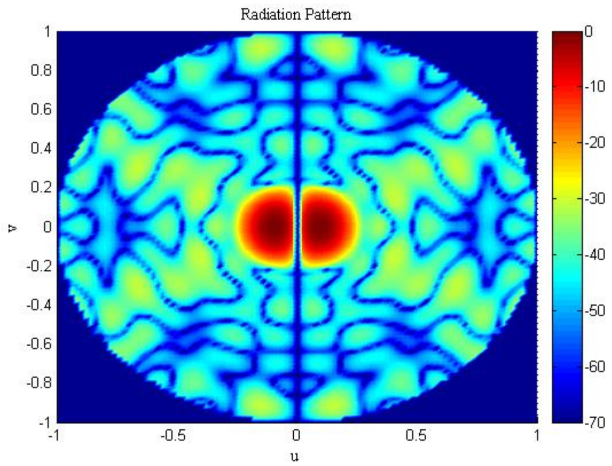

Figure 3: sub-arrayed H difference mode – dB scale

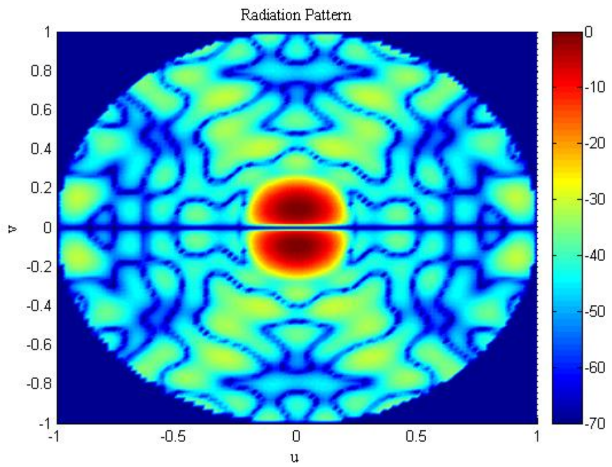

Figure 4: sub-arrayed E difference mode – dB scale

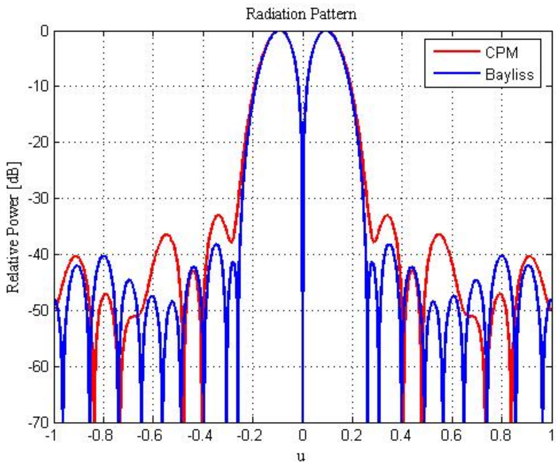

Figure 5: H-mode for v=0 – Optimal and Subarrayed Pattern

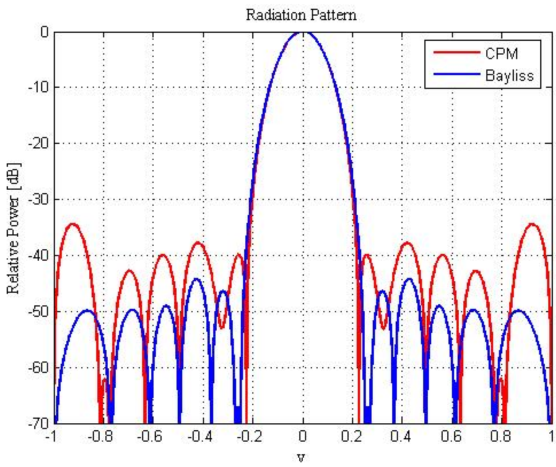

Figure 6: H-mode for u=0 – Optimal and Subarrayed Pattern

On the other, it is worth to focus on the computational issues concerned with the planar CP method. Table II gives some indications on the efficiency of the approach in determining the compromise solution characterized by the pattern characteristics in Tab. I. As far as the total time is concerned, only the CPU time needed to complete the path search has been reported, while the the CPU costs to compute the sum and difference excitations have been neglected (few seconds).

U	U^{ess}	*Iteration time [sec]*	*Iterations*	*Total Time [sec]*
4.92×10^{37}	3003	0.0044	3	0.0132

Table II: Computational Indexes

As expected, the computational burden is very limited as thanks to both the reduction of the solution space (3003 admissible solutions vs. 4.92×10^{37}) and the effective sampling of its graph representation.

5. CONCLUSIONS

In this paper, an effective procedure for the synthesis of sub-arrayed planar arrays has been presented. The approach is based on an optimal excitations matching technique and the compromise solution has been determined through an iterative minimization of a suitable cost function aimed at quantifying the distance between the compromise pattern and the reference one. Thanks to the effective exploitation of the contiguous partition paradigm and a suitable graph-based representation of the solution space, the approach demonstrated a good accuracy in reproducing target patterns as well as a significant computational efficiency. Such properties suggest an implementation of the CP-based approach where the simultaneous generation of multiple beams on the same antenna aperture is needed. On the other hand, the method seems to be a good candidate to deal with time-varying scenarios and applications thanks to its real-time performance.

REFERENCES

[1] M. I. Skolnik, RADAR Handbook. McGraw-Hill, 1990.

[2] P. W. Hannan, "Optimum feeds for all three modes of a monopulse antenna I: theory," *Trans. IRE Antennas Propagat.,* vol. 9, pp. 444-454, May 1961.

[3] T. T. Taylor, "Design of circular apertures for narrow beamwidth and low sidelobes," *Trans. IRE Antennas Propagat.*, vol. 8, pp. 17-22, Jan. 1960.

[4] E. T. Bayliss, "Design of monopulse antenna difference patterns with low sidelobes," *Bell System Tech. Journal*, vol. 47, pp. 623-640, 1968.

[5] D. A. McNamara, "Synthesis of sub-arrayed monopulse linear arrays through matching of independently optimum sum and difference excitations," *IEE Proc. H*, vol. 135, pp. 371-374, May 1988.

[6] P. Lopez, J. A. Rodriguez, F. Ares, and E. Moreno, "Subarray weighting for difference patterns of monopulse antennas: joint optimization of subarray configuration and weights," *IEEE Trans. Antennas Propagat.*, vol. 49, pp. 1606-1608, Nov. 2001.

[7] S. Caorsi, A. Massa, M. Pastorino, and A. Randazzo, "Optimization of the difference patterns for monopulse antennas by an hybrid real/integer-coded differential evolution method," *IEEE Trans. Antennas Propagat.*, vol. 53, pp. 372-376, Jan. 2005.

[8] F. Ares, S. R. Rengarajan, J. A. Rodriguez, and E. Moreno, "Optimal compromise among sum and difference patterns through sub-arraying," in Proc. IEEE Antennas and Propagation Symposium 1996, Baltimore, Maryland USA, July 21-26. 1996, pp. 1142-1145.

[9] M. D'Urso, T. Isernia, and E. F. Meliado', "An effective hybrid approach for the optimal synthesis of monopulse antennas," *IEEE Trans. Antennas Propagat.*, vol. 55, pp. 1059-1066, Apr.. 2007.

[10] P. Rocca, L. Manica and A. Massa, "Synthesis of monopulse antenna through iterative contiguous partition method," *Electron. Lett.,* vol. 43, pp. 854-856, Aug. 2007.

[11] L. Manica, P. Rocca, A. Martini, and A. Massa, "An innovative approach based on a tree-searching algorithm for the optimal matching of independently optimum sum and difference excitations," *IEEE Trans. Antennas Propagat.*, vol. 56, pp. 58-66, Jan. 2008.

[12] R. E. Hodges and Y. Rahmat-Samii, "On sampling continuous aperture distributions for discrete planar arrays," *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 1499-1508, Nov. 1996.

[13] W. D. Fisher. "On grouping for maximum homogeneity," *American Statistical Journal*, pp. 789-798, 1958.