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A LINEAR MULTI-USER DETECTOR FOR STBC MC-CDMA
SYSTEMS BASED ON THE ADAPTIVE IMPLEMENTATION OF
THE MINIMUM-CONDITIONAL BIT-ERROR-RATE CRITERION
AND ON GENETIC ALGORITHM-ASSISTED MMSE CHANNEL
ESTIMATION

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A Linear Multi-User Detector for STBC MC- CDMA Systems based on the Adaptive Implementation of the Minimum-Conditional Bit- Error-Rate Criterion and on Genetic Algorithm- assisted MMSE Channel Estimation

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Abstract

The implementation of efficient baseband receivers characterized by affordable computational load is a crucial point in the development of transmission systems exploiting diversity in different domains. In this paper, we are proposing a linear multi-user detector for MIMO MC-CDMA systems with Alamouti's Space-Time Block Coding, inspired by the concept of Minimum Conditional Bit-Error-Rate (MCBER) and relying on Genetic-Algorithm (GA)-assisted MMSE channel estimation. The MCBER combiner has been implemented in adaptive way by using Least-Mean-Square (LMS) optimization. Firstly, we shall analyze the proposed adaptive MCBER MUD receiver with ideal knowledge of Channel Status Information (CSI). Afterwards, we shall consider the complete receiver structure, encompassing also the non-ideal GA-assisted channel estimation. Simulation results evidenced that the proposed MCBER receiver always outperforms state-of-the-art receiver schemes based on EGC and MMSE criterion exploiting the same degree of channel knowledge (i.e. ideal or estimated CSI).

I. Introduction

Future mobile communications standards will be required to provide high performance in terms of data-rate, capacity and quality of service. Several non-conventional transmission solutions based on single-carrier and multi-carrier modulations (DS-CDMA, OFDM, MC-CDMA, etc.) have been proposed in order to achieve such ambitious targets. In such a perspective, Space-Time-Block-Coding (STBC) MIMO multicarrier CDMA techniques have been regarded by recent literature among the most promising technologies for future cellular standards [1]. STBC MC-CDMA systems can jointly exploit diversity in space and frequency domains in order to counteract multipath fading effects and, therefore, to provide performances very close to the single-user bound [1]. As it happens in Single-Input Single-Output (SISO) MC-CDMA, multi-user detection (MUD) plays a key role in improving system performances also in the STBC MIMO MC-CDMA case. Theoretically optimum maximum likelihood detection is characterized by an unsustainable computational complexity (exponentially growing with the number of users). For this reason, sub-optimal MUD strategies should be investigated. MMSE-MUD techniques have been proposed for STBC MC-CDMA systems in [2]. Although MMSE technique is a good choice in order to provide a simple implementation of adaptive receivers (Least-Mean-Square – LMS – and Recursive-Least-Squared – RLS – implementations are allowed), it is intrinsically suboptimal because it minimizes the Mean Squared Error between the received signal and the noiseless signal pattern. This results in a maximization of the Signal-to-Noise plus Interference Ratio (SINR), rather than in the minimization of the Bit Error Rate (BER) that is the expected target of an efficient receiver. The Minimum Bit Error Rate (MBER) criterion for multi-user detection [3] has been successfully applied to DS-CDMA ([4]), MC-CDMA ([5]) and SDMA ([6]). The potential advantages of implementing linear MBER MUD in STBC MC-CDMA context can be easily explained. STBC MC-CDMA is a transmission methodology where diversity is obtained both in space and in frequency domain. The diversity gain, increased with respect to the conventional SISO case, is obtained at the price of an increased system complexity. In our opinion, the exploitation of the “full potential” of STBC MC-CDMA techniques can be obtained only by means of optimized multi-user detection approaches. In such a perspective, the investigation of MBER strategies can be regarded as a step-ahead towards the computationally-affordable signal detection optimization also in the STBC MC-CDMA case. The application of MBER reception to MIMO MC-CDMA is not straightforward and should be investigated by carefully taking into account tight requirements in terms of ease of implementation and reduced computational effort.

In this paper, we are proposing a multi-user detector for STBC MC-CDMA transmission systems based on the Minimum Conditional Bit Error Rate (MCBER). Such a criterion has been successfully applied in [7] to DS/CDMA systems and in [8] to SISO MC-CDMA systems. MCBER is a slight modification of the original MBER criterion that allows reducing the computational complexity to a linear order with the number of users without significant losses in BER performances. The practical implementation of the proposed MCBER detector relies on an adaptive optimization strategy based on the concept of deterministic gradient. In particular, we considered a Least Mean Square (LMS) algorithm [9] to derive the MCBER solution in numerical iterative form. Such a choice has been motivated by the necessity of testing a computationally-tractable algorithm like gradient descent, widely employed in practical MUD implementations, e.g. in DS/CDMA MMSE-MUD [10], MC-CDMA MMSE-MUD [11], and MBER MC-CDMA MUD [5]. The STBC scheme considered in our work is the well-known Alamouti's scheme [12] characterized by ease of concept and implementation, computational efficiency and near-optimality when employed in multipath fading channels. The implementation and testing of the proposed MCBER detector has been performed into two steps:

- i) A first step considering the implementation of an MCBER receiver exploiting the ideal availability of Channel State Information (CSI). This step is aimed at testing how MCBER performances are close to single-user bound;
- ii) A second step aimed at testing a more realistic receiver structure encompassing non-ideal channel knowledge and, therefore, the presence of a channel estimator in the receiver chain.

The channel estimation strategy adopted is the other point of innovation proposed in this paper. Searching for reliable and computationally-efficient adaptive channel estimation, we decided to propose a novel Genetic Algorithm (GA)-assisted semi-adaptive MMSE channel estimation. Having a look to recent literature, we can list some methodologies proposed for channel estimation in STBC systems. In [13], MMSE channel estimation is considered for single-carrier STBC, using the direct inversion of signal correlation matrices. Pilot extraction and Wiener interpolation are proposed in [14] in order to estimate the channel path gains, whereas in [15] Xu and Wang considered the use of sphere decoding to implement joint ML channel estimation and data detection in single-carrier narrowband STBC. In [16], a subspace semi-blind algorithm has been proposed in order to estimate the channel condition in multi-user STBC MC-CDMA systems. At our best knowledge, genetic algorithms have not yet been used for channel estimation neither in single-carrier nor in multicarrier STBC systems.

The idea of exploiting a GA-based optimizer in order to implement MMSE channel estimation has been suggested by the promising results achieved in [17], where a semi-adaptive GA-based approach has been successfully applied to MMSE linear multi-user detection in SISO MC-CDMA systems. GA-based optimization applied to MMSE MUD evidenced a behavior very close to optimality (represented by MMSE MUD with ideal CSI knowledge) with a computational load fully comparable with adaptive MMSE receivers based on gradient descent methodologies (LMS and RLS). The key features of GA-based MMSE optimization observed in [17] are the improved capability of fast convergence towards optimal solution and the reduced sensitivity to parameterization.

The paper is organized as follows: Section II will contain the description of the system model. Section III will explain the theoretical MCBER MUD criterion. Section III will detail the implementation of the proposed MCBER detector exploiting ideal CSI knowledge. Section IV is devoted at describing the GA-assisted channel estimation strategy adopted and, therefore, the complete MCBER MUD architecture. Section V will show some selected simulation results aimed at assessing the effectiveness of the proposed approach by means of a thorough comparison with other state-of-the-art receivers. Moreover, some considerations about computational complexity and parameterization both of MCBER reception and GA-assisted channel estimation will be sketched at the end of Section V. Finally, paper conclusions are drawn in Section VI.

II. STBC MC-CDMA System Description

In the present dealing, we consider a synchronous multiuser MIMO MC-CDMA system based on Alamouti's Space Time Block Coding [12]. A block scheme of the considered STBC MC-CDMA transmission system is drawn in Fig.1. Two consecutive data symbols of the generic user k ($k = 0, \dots, K - 1$) $[a_k^1, a_k^2]$ are mapped to two transmitting antennas according to the code matrix Φ_k , whose elements are given by [12]:

$$[\Phi_k] = \frac{1}{\sqrt{2}} \begin{bmatrix} a_k^1 & -a_k^{2*} \\ a_k^2 & a_k^{1*} \end{bmatrix} \quad (1)$$

This matrix represents the Alamouti's STBC block. Alamouti's scheme exhibits some clear advantages. The classical antenna diversity approach considers the utilization of multiple antennas at the receiver side and a single antenna at the transmitter side. As result, the receiver becomes larger and more expensive. This is the reason why since many years antenna diversity has been exploited only by base stations in the uplink [12]. Essentially, Alamouti's STBC scheme makes available a space diversity gain also for mobile terminals (therefore in the

downlink) only by exploiting transmit diversity. The most economic and advantageous Alamouti's configuration considers two transmit antennas (installed at base station or access point) and a single antenna mounted at the receiver side [12]. This is the configuration considered in our paper, where the focus is on the development of cost-effective mobile terminals. The encoder outputs are transmitted during two consecutive transmission periods, i.e.: T_1, T_2 . During the first transmission period T_1 , two generic BPSK symbols a_k^1 and a_k^2 are sent to two separate I-FFT-based multicarrier spreading blocks using a unique Hadamard-Walsh sequence $\mathbf{c}_k \triangleq \{c_k(n) \ n = 0, \dots, N-1\}$, where N is the length of the I-FFT and therefore the number of subcarriers employed for spreading. Finally, the RF converted Multicarrier Spread Spectrum (MC-SS) signals are simultaneously transmitted by antenna A and antenna B, respectively. In the same way, during the successive transmission period T_2 , the symbol $-a_k^{2*}$ is transmitted by antenna A and the symbol a_k^{1*} is transmitted by antenna B, respectively. The superscript operator $*$ means here the complex conjugate operation.

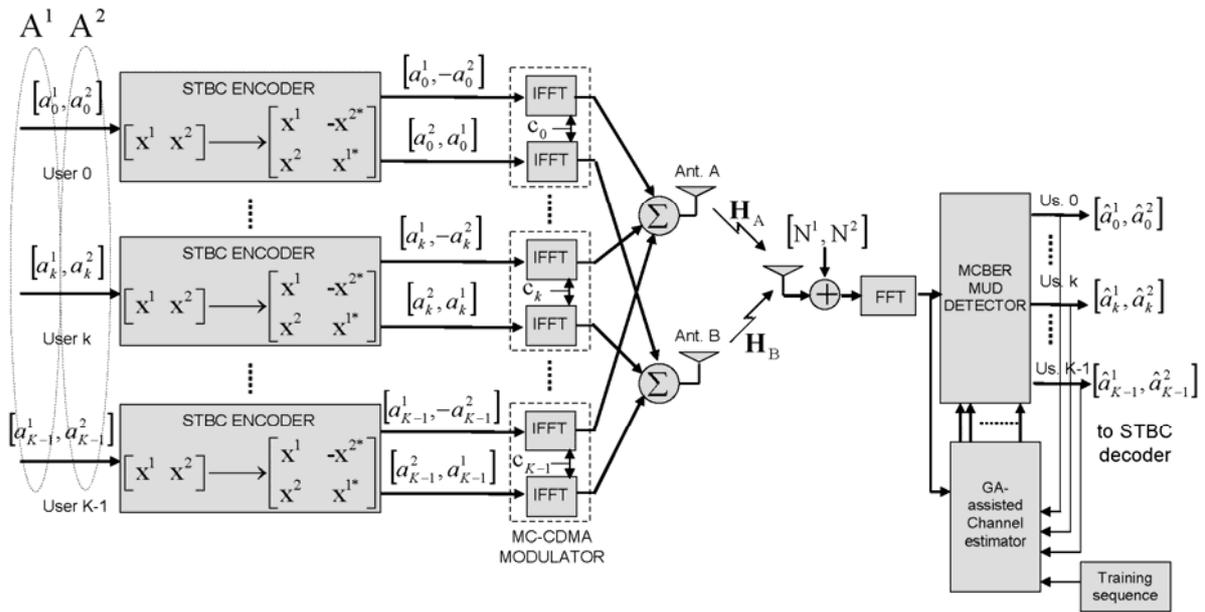


Figure 1. The considered STBC MC-CDMA system (complete block diagram)

In order to make the notation more compact in the multi-user case, we define two vectors of symbols: $\mathbf{A}^1 \triangleq [a_0^1, a_1^1, \dots, a_{K-1}^1]^T$ and $\mathbf{A}^2 \triangleq [a_0^2, a_1^2, \dots, a_{K-1}^2]^T$ and the orthonormal code matrix \mathbf{C} as:

$$\mathbf{C} = \begin{bmatrix} c_0(0) & c_1(0) & \cdots & c_{K-1}(0) \\ c_0(1) & c_1(1) & \cdots & c_{K-1}(1) \\ \vdots & \vdots & \cdots & \vdots \\ c_0(N-1) & c_1(N-1) & \cdots & c_{K-1}(N-1) \end{bmatrix} \quad (2)$$

The received signal samples acquired at two consecutive symbol periods after the FFT-based coherent demultiplexing can be expressed as follows:

$$\begin{cases} \mathbf{R}^1 = \mathbf{H}_A \mathbf{C} \mathbf{A}^1 + \mathbf{H}_B \mathbf{C} \mathbf{A}^2 + \mathbf{N}^1 \\ \mathbf{R}^2 = -\mathbf{H}_A \mathbf{C} (\mathbf{A}^2)^* + \mathbf{H}_B \mathbf{C} (\mathbf{A}^1)^* + \mathbf{N}^2 \end{cases} \quad (3)$$

where \mathbf{R}^1 and \mathbf{R}^2 are a $N \times 1$ vectors, $\mathbf{N}^j = [\eta_0, \eta_1, \dots, \eta_{N-1}]^T$ (with $j \in \{1, 2\}$) is the Additive White Gaussian Noise (AWGN) vector (all vector components are independent and identically-distributed with zero mean and variance σ^2), and $\mathbf{H}_{ant} = \text{diag}\{h_0^{ant}, h_1^{ant}, \dots, h_{N-1}^{ant}\}$ (with $ant \in \{A, B\}$) is the $N \times N$ diagonal channel matrix, where h_n^{ant} is the complex channel coefficient related to subcarrier n and to the transmit antenna ant . We reasonably assume that fading is flat over each subcarrier and almost time-invariant during two consecutive transmission periods (i.e.: the coherence time is much greater than the symbol period).

III. The Minimum Conditional BER Criterion for Multi-User Detection in STBC MC-CDMA

In this section, we shall derive an analytical expression for the error probability of a STBC MC-CDMA system using generic linear combining at the receiver side and, consequently the Minimum Conditional Bit-Error-Rate criterion for multi-user detection. A generic linear multiuser detector generates two decision variables based on the linear combination of the received signal samples. Thus, the decision variables for the user k , denoted by $x_{k,1}$ and $x_{k,2}$, can be expressed as follows:

$$\begin{cases} x_{k,1} = \mathbf{w}_k^{1*} \mathbf{R}^1 + \mathbf{w}_k^2 \mathbf{R}^{2*} \\ x_{k,2} = \mathbf{w}_k^{2*} \mathbf{R}^1 - \mathbf{w}_k^1 \mathbf{R}^{2*} \end{cases} \quad (4)$$

where $\mathbf{w}_k^1 = [w_k^1(0), w_k^1(1), \dots, w_k^1(N-1)]$ and $\mathbf{w}_k^2 = [w_k^2(0), w_k^2(1), \dots, w_k^2(N-1)]$ are the vectors of receiver gains employed by the generic k -th user in order to recombine the baseband output of the FFT-based demodulation stage. The two bits contained in a single STBC block (corresponding to the BPSK symbols $[a_k^1, a_k^2]$) can be estimated by observing the real part of $x_{k,1}$ and $x_{k,2}$, respectively. In our dealing, we are considering the employment of a digital

BPSK modulation with real antipodal symbols. The choice of BPSK has been motivated by the fact that such a modulation allows us to write an exact analytical expression of the bit-error-rate at the output of the linear MUD combiner. Other modulations, like M-PSK and M-QAM require approximations that can sound very complex from a mathematical viewpoint or can be quite imprecise (in fact, the BPSK choice is common in literature about MBER and MCBER detection. It is employed, for instance, in [6-8]).

Under such hypothesis and substituting (3) in (4), we can write the real parts of $x_{k,1}$ and $x_{k,2}$ as follows:

$$\begin{cases} \Re(x_{k,1}) = \Re\left\{\left(\mathbf{w}_k^1\right)^* \mathbf{H}_A \mathbf{C} \mathbf{A}^1 + \left(\mathbf{w}_k^1\right)^* \mathbf{H}_B \mathbf{C} \mathbf{A}^2 + \left(\mathbf{w}_k^1\right)^* \mathbf{N}^1 - \mathbf{w}_k^2 \mathbf{H}_A^* \mathbf{C} \mathbf{A}^2 + \mathbf{w}_k^2 \mathbf{H}_B^* \mathbf{C} \mathbf{A}^1 + \mathbf{w}_k^2 \left(\mathbf{N}^2\right)^*\right\} \\ \Re(x_{k,2}) = \Re\left\{\left(\mathbf{w}_k^2\right)^* \mathbf{H}_A \mathbf{C} \mathbf{A}^1 + \left(\mathbf{w}_k^2\right)^* \mathbf{H}_B \mathbf{C} \mathbf{A}^2 + \left(\mathbf{w}_k^2\right)^* \mathbf{N}^1 + \mathbf{w}_k^1 \mathbf{H}_A^* \mathbf{C} \mathbf{A}^2 - \mathbf{w}_k^1 \mathbf{H}_B^* \mathbf{C} \mathbf{A}^1 - \mathbf{w}_k^1 \left(\mathbf{N}^2\right)^*\right\} \end{cases} \quad (5)$$

Conditioned on the transmitted bit vectors \mathbf{A}^1 and \mathbf{A}^2 , the random variables $\Re(x_{k,1})$ and $\Re(x_{k,2})$ are Gaussian-distributed with mean values:

$$\begin{cases} \mu_{k,1} = \Re\left\{\left(\mathbf{w}_k^1\right)^* \mathbf{H}_A \mathbf{C} \mathbf{A}^1 + \left(\mathbf{w}_k^1\right)^* \mathbf{H}_B \mathbf{C} \mathbf{A}^2 - \mathbf{w}_k^2 \mathbf{H}_A^* \mathbf{C} \mathbf{A}^2 + \mathbf{w}_k^2 \mathbf{H}_B^* \mathbf{C} \mathbf{A}^1\right\} \\ \mu_{k,2} = \Re\left\{\left(\mathbf{w}_k^2\right)^* \mathbf{H}_A \mathbf{C} \mathbf{A}^1 + \left(\mathbf{w}_k^2\right)^* \mathbf{H}_B \mathbf{C} \mathbf{A}^2 + \mathbf{w}_k^1 \mathbf{H}_A^* \mathbf{C} \mathbf{A}^2 - \mathbf{w}_k^1 \mathbf{H}_B^* \mathbf{C} \mathbf{A}^1\right\} \end{cases} \quad (6)$$

and variances:

$$\begin{cases} \sigma_{k,1}^2 = \sigma^2 \left(\|\mathbf{w}_k^1\|^2 + \|\mathbf{w}_k^2\|^2 \right) \\ \sigma_{k,2}^2 = \sigma^2 \left(\|\mathbf{w}_k^1\|^2 + \|\mathbf{w}_k^2\|^2 \right) \end{cases} \quad (7)$$

To make the derivation of the probability of error easier, we can refer to the so-called sign-adjusted decision variables [6] defined as follows: $x_{k,1}^S \triangleq a_k^1 \Re(x_{k,1})$ and $x_{k,2}^S \triangleq a_k^2 \Re(x_{k,2})$. These new decision variables, conditioned on the transmitted bit vectors \mathbf{A}^1 and \mathbf{A}^2 , are Gaussian-distributed as well with mean values $a_k^1 \mu_{k,1}$ and $a_k^2 \mu_{k,2}$ and variances $\sigma_{k,1}^2$ and $\sigma_{k,2}^2$, respectively. Thus, the probability to have an error in a STBC block, conditioned on \mathbf{A}^1 and \mathbf{A}^2 , can be written as follows:

$$P_{E/A^1, A^2} = \Pr\left(x_{k,1}^S < 0, x_{k,2}^S > 0\right) + \Pr\left(x_{k,1}^S > 0, x_{k,2}^S < 0\right) + \Pr\left(x_{k,1}^S < 0, x_{k,2}^S < 0\right) \quad (8)$$

As $x_{k,1}^S$ and $x_{k,2}^S$ are independent random variables, (8) becomes:

$$P_{E/A^1, A^2} = Q\left(\frac{\mu_{k,1}^S}{\sigma_{k,1}}\right) + Q\left(\frac{\mu_{k,2}^S}{\sigma_{k,2}}\right) - Q\left(\frac{\mu_{k,1}^S}{\sigma_{k,1}}\right) Q\left(\frac{\mu_{k,2}^S}{\sigma_{k,2}}\right) \quad (9)$$

where $Q(x)$ is the conventional Gaussian error function. Assuming that the two symbols contained in a STBC block have the same probability of error, the third term in (9) is negligible with respect to the first two ones. Thus, an approximation of (9) is given by:

$$P_{E/A^1, A^2} \cong \hat{P}_{E/A^1, A^2} = Q\left(\frac{\mu_{k,1}^S}{\sigma_{k,1}}\right) + Q\left(\frac{\mu_{k,2}^S}{\sigma_{k,2}}\right) \quad (10)$$

By considering that the 2^{2K} possible transmitted bit vectors are independent and equiprobable, the average probability of error for the k -th user can be written as:

$$\hat{P}_E^{AV}(\mathbf{w}_k^1, \mathbf{w}_k^2) = \frac{1}{2^{2K}} \sum_{\forall A^1} \sum_{\forall A^2} \hat{P}_{E/A^1, A^2} \quad (11)$$

The couple of receiver gains vectors $(\mathbf{w}_k^1, \mathbf{w}_k^2)$ minimizing the average probability of error shown in (11) practically implement the ideal MBER detection criterion for the considered STBC MC-CDMA system. The computational complexity of this detector is exponential in the number of users $O(2^{2K})$, so its practical application becomes unfeasible as K increases. However, the computational burden of the MBER criterion can be reduced by minimizing the *conditional probability of error* instead of the average probability of error [7][8]. The conditional probability of error can be expressed as follows:

$$\hat{P}_E^{COND}(\mathbf{w}_k^1, \mathbf{w}_k^2) = \frac{1}{4} \sum_{\forall a_k^1} \sum_{\forall a_k^2} \hat{P}_{E/a_k^1, a_k^2} \quad (12)$$

The BER is conditioned in (12) only to the symbols not transmitted by the user of interest in its own STBC block, and is thus averaged with respect to the possible combinations of these particular user's symbols. In our case, the Alamouti's block is 2x2-sized; therefore the number of possible symbol combinations is $2^2=4$. Dua, Desai and Mallik claim in [7] that the Minimum Conditional BER (MCBER) adaptive MUD has a convergence rate comparable to MBER-MUD with a minor degradation in terms of bit-error-rate and affordable computational burden linearly increasing with users' number. For this reason, the MCBER criterion can be regarded as an effective and feasible solution also for multi-user detection in STBC-MIMO MC-CDMA systems. Formally, the following optimization problem must be solved:

$$[\mathbf{w}_k^1, \mathbf{w}_k^2]^{opt} = \arg \min_{\mathbf{w}_k^1, \mathbf{w}_k^2} \left\{ \hat{P}_E^{COND}(\mathbf{w}_k^1, \mathbf{w}_k^2) \right\} \quad (13)$$

In [6], it is pointed out that no closed-form expression for MBER solution has to be found. The same fact is verified for MCBER solution. For this reason, a numerical solution has to be investigated. A possible solution is to exploit the Least Mean Square (LMS) algorithm based on the concept of gradient descent [9][13]. LMS updating of the receiver weights is done

iteratively along the negative gradient of the error probability surface, along both directions \mathbf{w}_k^1 and \mathbf{w}_k^2 . The updating rule at i -th iteration is given by:

$$\begin{cases} \mathbf{w}_k^1(i+1) = \mathbf{w}_k^1(i) - \lambda \cdot \nabla_1 \left(\hat{P}_E^{COND}(\mathbf{w}_k^1, \mathbf{w}_k^2) \right)(i) \\ \mathbf{w}_k^2(i+1) = \mathbf{w}_k^2(i) - \lambda \cdot \nabla_2 \left(\hat{P}_E^{COND}(\mathbf{w}_k^1, \mathbf{w}_k^2) \right)(i) \end{cases} \quad (14)$$

where λ is the step-size parameter and ∇_1 and ∇_2 represent the gradient along the two directions \mathbf{w}_k^1 and \mathbf{w}_k^2 , respectively. For the first iteration (namely: $i=0$), the following initialization has been chosen: $\mathbf{w}_k^1(0) = \mathbf{w}_k^2(0) = \mathbf{c}_k$. This is a reasonable choice, commonly used in the literature dealing with adaptive detection of MC-CDMA signals (see e.g. [5] and [11]). The LMS implementation of the MCBER detector can be obtained straightforwardly by using the updating rule of (14) combined with the following gradient expressions (mathematical details are omitted for sake of brevity):

$$\begin{cases} \nabla_1 \left(\hat{P}_E^{COND}(\mathbf{w}_k^1, \mathbf{w}_k^2) \right) = -\frac{2}{\sqrt{2\pi}} \frac{1}{4} \sum_{\forall a_k^1} \sum_{\forall a_k^2} \left\{ e^{-\frac{\mu_{k,1}^2}{2\sigma_{k,1}^2}} \frac{a_k^1}{\sigma_{k,1}} \left[\mathbf{H}_A \mathbf{C} \tilde{\mathbf{A}}^1 + \mathbf{H}_B \mathbf{C} \tilde{\mathbf{A}}^2 - \mu_{k,1} \frac{\mathbf{w}_k^1}{\|\mathbf{w}_k^1\|^2 + \|\mathbf{w}_k^2\|^2} \right] + \right. \\ \left. + e^{-\frac{\mu_{k,2}^2}{2\sigma_{k,2}^2}} \frac{a_k^2}{\sigma_{k,2}} \left[\mathbf{H}_A \mathbf{C} \tilde{\mathbf{A}}^2 - \mathbf{H}_B \mathbf{C} \tilde{\mathbf{A}}^1 - \mu_{k,2} \frac{\mathbf{w}_k^1}{\|\mathbf{w}_k^1\|^2 + \|\mathbf{w}_k^2\|^2} \right] \right\} \\ \nabla_2 \left(\hat{P}_E^{COND}(\mathbf{w}_k^1, \mathbf{w}_k^2) \right) = -\frac{2}{\sqrt{2\pi}} \frac{1}{4} \sum_{\forall a_k^1} \sum_{\forall a_k^2} \left\{ e^{-\frac{\mu_{k,1}^2}{2\sigma_{k,1}^2}} \frac{a_k^1}{\sigma_{k,1}} \left[\mathbf{H}_B \mathbf{C} \tilde{\mathbf{A}}^1 - \mathbf{H}_A \mathbf{C} \tilde{\mathbf{A}}^2 - \mu_{k,1} \frac{\mathbf{w}_k^2}{\|\mathbf{w}_k^1\|^2 + \|\mathbf{w}_k^2\|^2} \right] + \right. \\ \left. + e^{-\frac{\mu_{k,2}^2}{2\sigma_{k,2}^2}} \frac{a_k^2}{\sigma_{k,2}} \left[\mathbf{H}_B \mathbf{C} \tilde{\mathbf{A}}^2 + \mathbf{H}_A \mathbf{C} \tilde{\mathbf{A}}^1 - \mu_{k,2} \frac{\mathbf{w}_k^2}{\|\mathbf{w}_k^1\|^2 + \|\mathbf{w}_k^2\|^2} \right] \right\} \end{cases} \quad (15)$$

In this way, we can just compute the statistical mean over data symbols of the user of interest. As far as data symbols transmitted by users $\bar{k} = 0, \dots, K-1$ $\bar{k} \neq k$ are concerned, we define the vectors $\tilde{\mathbf{A}}^j \triangleq [\tilde{a}_0^u, \dots, a_k^u, \dots, \tilde{a}_{K-1}^u]$ $u \in \{1, 2\}$ in which the element \tilde{a}_k^u is the symbol decision performed by user \bar{k} . The choice of exploiting the symbol decisions taken by users $\bar{k} \neq k$ has been considered in order to improve the convergence of the adaptive optimization algorithm (as noted in [8]).

IV. The proposed GA-Assisted Channel Estimation

In a realistically-implemented receiver, the channel matrices \mathbf{H}_A and \mathbf{H}_B present in (15) are not deterministically known, but they should be replaced by their estimates $\hat{\mathbf{H}}_A$ and $\hat{\mathbf{H}}_B$. In this paper, we substantially considered an MMSE channel estimation targeted to MIMO STBC systems. In this paper, we modified the MMSE approach proposed in [13] for the STBC MC-CDMA system with Alamouti's coding described in Section II. In particular, we have to minimize with respect to the estimated channel matrices the following MSE metric:

$$J(\hat{\mathbf{H}}_A, \hat{\mathbf{H}}_B) = \left\| \mathbf{R}^1 - \hat{\mathbf{H}}_A \mathbf{C} \mathbf{A}^1 - \hat{\mathbf{H}}_B \mathbf{C} \mathbf{A}^2 \right\|^2 + \left\| \mathbf{R}^2 + \hat{\mathbf{H}}_A \mathbf{C} (\mathbf{A}^2)^* - \hat{\mathbf{H}}_B \mathbf{C} (\mathbf{A}^1)^* \right\|^2 \quad (16)$$

This is not a trivial task from a computational viewpoint. State-of-the-art methodologies for channel estimation in STBC systems are substantially based on the insertion of training sequences and on the inversion of signal covariance matrices [13-14]. If we applied such methodologies to the case of multi-user STBC MC-CDMA we should compute the inverse of big covariance matrices, growing more and more with the users' number K . In [15], a novel sphere decoding algorithm is proposed to solve the problem of joint ML channel estimation and data detection in STBC systems. A lattice decoder, namely sphere decoder, is employed to jointly detect data symbols and to estimate the channel. In [16] second-order statistics and subspace decompositions are employed to estimate the channel matrix in the Alamouti's STBC MC-CDMA case. These kinds of methodologies are surely interesting, but conceptually complex and computationally hard. Therefore, they are not very suitable for practical implementation over state-of-the-art signal processing platforms.

Adaptive optimization methodologies based on steepest descent [18], like Least Mean Square (LMS) and Recursive Least Square (RLS) optimization are commonly employed in real-world applications. These approaches are very efficient from a computational point of view and mathematically elegant. Unfortunately, their performances and convergence rates are strongly influenced by the choice of the LMS/RLS updating parameters. These weakpoints can hinder the employment of adaptive MMSE multi-user detection in time-varying fading channels, making them more suitable for static channels [11].

In this paper, we are considering a GA-assisted MMSE channel estimation, following a strategy similar to the one successfully applied in [17] to linear MUD in the single-carrier MC-CDMA case. Genetic algorithms have a 20-years history of successful applications in

telecommunications, signal processing and electromagnetic fields due to some basic features, useful to solve complex problems with reasonable computational effort [19]:

- 1) The convergence to the optimal solution is theoretically guaranteed (provided that a proper parameterization of the GA procedure is set), avoiding that solution be trapped in local minima;
- 2) The GA-based procedure can dynamically adapt itself to time-varying system conditions, because a new population of individuals is computed at each new generation.

Standard GA implementations [19] represent feasible solutions as a set of individuals (called population). The cost function to be minimized (or maximized) is called *fitness function* (in our case, the fitness function is the metric of (16)). At each iteration (namely: generation), the genetic operators of *crossover* and *mutation* are applied on selected chromosomes with probability α and γ respectively, in order to generate new solutions belonging to the search space. The population generation terminates when a satisfactory solution has been produced or when a fixed number of generations have been completed.

The block diagram of the MCBER MUD receiver with GA-assisted channel estimation is shown in Fig. 2. In order to make MMSE channel estimation robust and adaptive with respect to channel variations, we adopted a GA-assisted MMSE strategy articulated into two steps:

- 1) *Training-aided step*: during this step, an L bit-length binary training sequence $\underline{\hat{a}}_k = [\hat{a}_k^1, \dots, \hat{a}_k^L]$ is transmitted in form of header for each user k . The bits of the training sequence are organized in $L/2$ consecutive pairs, each one corresponding to a pair of symbols transmitted in two consecutive signaling periods. In such a way, the vectors of known bits $\hat{\mathbf{A}}^1(j) = \{\hat{a}_k^{2j-1} \mid k = 1, \dots, K, j = 1, \dots, L/2\}$ and $\hat{\mathbf{A}}^2(j) = \{\hat{a}_k^{2j} \mid k = 1, \dots, K, j = 1, \dots, L/2\}$ are employed to compute the MSE metric (j is the index of the signaling period). The training step is repeated with a period approximately equal to the coherence time of the channel. The GA optimizer computes at each signaling period the estimated channel matrices using a selected parameterization in terms of generation number G_{Tr} , population size P_{Tr} , crossover and mutation probabilities α_{Tr} and γ_{Tr} respectively. The footer Tr means that the GA parameterization is related to the training step.
- 2) *Decision-directed adaptive step*: the output of the training step is the channel matrices $(\hat{\mathbf{H}}_A)^{Tr}$ and $(\hat{\mathbf{H}}_B)^{Tr}$ obtained by a GA-based optimizer parameterized in such a way to

“learn” the channel in reliable way. During a coherence time period, the stochastic values of the channel coefficients acting over each subcarrier are strongly correlated. By this, a decision-directed adaptive updating step should be reasonably forecast (see e.g. [17]). In the present dealing, the decision-directed updating step is performed by the GA, working with a different parameterization. The GA-based updating procedure is initialized by the solution computed during the training-aided step, i.e.: $(\hat{\mathbf{H}}_A)^{Tr}$ and $(\hat{\mathbf{H}}_B)^{Tr}$. During a symbol period a single iteration is performed by the GA and a single generation of individuals is produced. The symbols employed in this step to fill the data vectors \mathbf{A}^1 and \mathbf{A}^2 are the estimated symbols decided at the previous signaling period. In particular, we have: $\tilde{\mathbf{A}}^1(j) = \{\tilde{a}_k^1(j-1)k = 1, \dots, K\}$ and $\tilde{\mathbf{A}}^2(j) = \{\tilde{a}_k^2(j-1)k = 1, \dots, K\}$. In such a step, crossover and mutation operators don't work, because only a GA generation runs. This updating procedure is “light”, but this is reasonable because only small variations of the channel amplitude and phase are to be tracked during the coherence period. Moreover, in such a way, the effects of possible symbol errors on channel estimation are conveniently reduced. In order to make clearer to reader our approach, we can summarize the whole GA-assisted channel estimation procedure as follows:

- i) At time $t=0$ the GA-based procedure is initialized by a constant-value population. In particular, the identity matrices have been chosen for initialization.
- ii) The training-aided step begins. The L -bit known training sequence is transmitted and the estimated channel matrices are computed by minimizing the cost function of (16). The GA parameterization is chosen as: generation number = G_{Tr} , population size = P_{Tr} , crossover probability = α_{Tr} and mutation probability = γ_{Tr} .
- iii) The training-aided step ends with the computation of the channel matrices at the time $t=LT+\varepsilon T$ (ε is the execution time of the GA-based optimization procedure expressed in number of bit periods). Now, the GA switches to the decision-directed adaptive modality.
- iv) At the beginning of the adaptive step, the GA is initialized with the channel matrices computed at the end of ii), and the GA parameters are reassigned as follows: generation number $G_D=1$, population size P_D , crossover probability $\alpha_D=0$ and mutation probability $\gamma_D=0$. The footer D means that the GA parameterization is related now to the decision-directed adaptive step. The GA procedure produces a single population of individuals that are quite close to the one chosen during the previous signaling interval. Such kind of

population is stochastically generated in Gaussian way, imposing to the Gaussian generator an updating standard deviation σ_{up} that actually is a system parameter. Such a parameter is linked to the Doppler spread and to the signal-to-noise ratio. An explicit mathematical link is very difficult to be obtained, but, as thumb rule, we can say that it needs to be increased as SNR and Doppler spread increase.

- v) The decision-directed updating step ends at the time $t=LT+\varepsilon T+W_{coh}T$, where W_{coh} is the coherence time-window of the channel (expressed in number of bits). The GA is re-initialized with the channel matrices computed at the end of the coherence time-window, and re-parameterized in order to start again with the training-aided step ii).

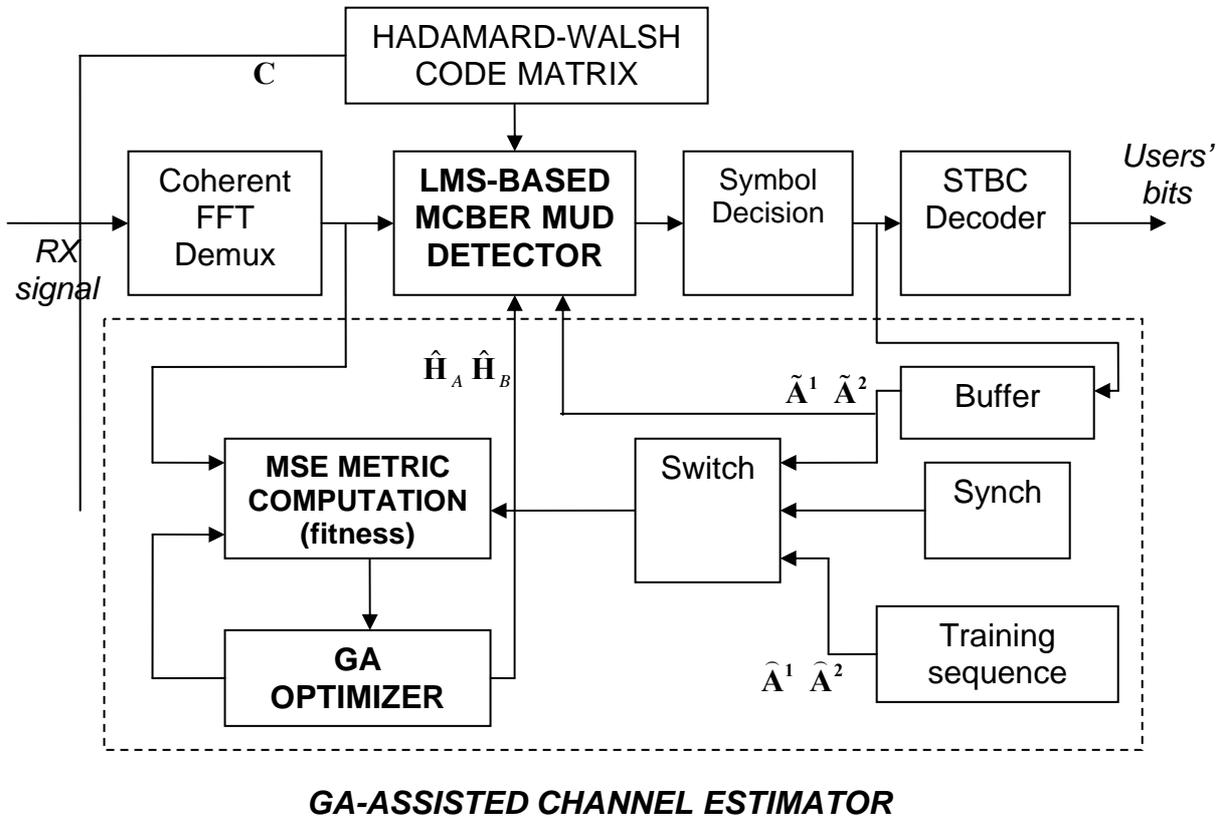


Figure 2. Block diagram of the MCBER receiver supported by GA-assisted MMSE channel estimation

V. Experimental Results

A. Simulation results

The performances of the proposed LMS-based MCBER detector are evaluated by means of intensive simulation trials (performed in MATLAB-SIMULINK 7.1 environment) in a Rayleigh fading channel fixing the following parameters: number of subcarriers $N=8$, transmission data rate $r_b=1024$ Kb/s, coherence bandwidth of the channel 2.1 MHz, Doppler

spread of the channel 100 Hz. We considered two test cases: the most theoretical case related to the MCBER detector exploiting the ideal CSI knowledge and the more realistic case related to the MCBER detector supported by the GA-assisted channel estimation. In such a way, we shall be able to test the effectiveness of the channel estimation strategy adopted together with the impact of non-ideal channel estimation on MCBER performances.

In order to verify the effectiveness of the proposed approach, we considered other state-of-the-art receivers for comparison, namely:

- The ideal MMSE MUD receiver [2][16] exploiting ideal CSI knowledge;
- The LMS adaptive implementation of MMSE receiver shown in [2] and [11];
- The single-user Equal-Gain Combining (EGC) receiver considered in [2];
- The MMSE MUD receiver supported by the GA-assisted channel estimation.

As lower bound, we considered the BER curve obtained by Maximal Ratio Combining (MRC) detection in the single-user case (i.e., the optimal single-user detection, supposing the absence of multi-user interference). In our simulations, we considered three different scenarios including $K=2$, $K=4$, and $K=6$ users. The GA optimizer has been parameterized as follows: i) training-aided step $G_{Tr}=30$, $P_{Tr}=30$, $\alpha_{Tr}=0.9$, $\gamma_{Tr}=0.01$, $L=32$ ii) decision-directed adaptive step $P_D=10$. As the Jakes channel model has been adopted for simulation, the channel coherence time window approximately equals about 1800 bits. Therefore the overhead due to the insertion of the training sequence equals to less than 1.8%.

The corresponding BER curves vs. E_b/N_0 are shown for all the tested receivers in Fig. 3, Fig. 4, and Fig. 5, respectively. It can be seen that in all scenarios the proposed LMS-based MCBER detector with and without ideal CSI knowledge clearly outperforms both EGC detector and LMS-based MMSE adaptive detector that exhibit a nasty error floor as the number of users increases. Moreover, the proposed MCBER detector exploiting ideal CSI knowledge yields performances that are better than those ones of ideal MMSE detector. Such a last improvement is clearly evident for $K=2$ and $K=4$ users; whereas it becomes slighter for $K=6$ users. In general, for an increasing number of users, BER curves related to ideal MMSE and MCBER tend to become closer to each other and more distant with respect to the single-user bound. Such behaviour is not unexpected. As number of users K increases, the global detection noise (including AWGN and multi-user interference) is getting more and more Gaussian-distributed and, therefore, optimizing the receiver with respect to the MMSE provides very close results to optimizing on BER. Moreover, the single-user bound curve depicted in Fig.3-5 is a lower bound also on theoretically optimum ML detection as it doesn't take into account the

presence of the multi-user interference. In case of increasing number of users, it is getting more and more difficult for a linear receiver to approach the single-user bound.

Focusing our attention on the effectiveness of the proposed GA-assisted channel estimation methodology, we can see from Fig.3-5 that the MCBER detector supported by non-ideal channel estimation performs very close to MCBER detector exploiting the ideal channel knowledge for $K=2$ and $K=4$ users. The effects of non-ideal channel estimation are more evident for higher number of users ($K=6$) and therefore in the presence of higher level of multi-user interference.

In our opinion, the most relevant fact able at confirming the correctness of our analysis is that the proposed MCBER detector always outperforms MMSE MUD when working together in the same conditions of “channel knowledge”.

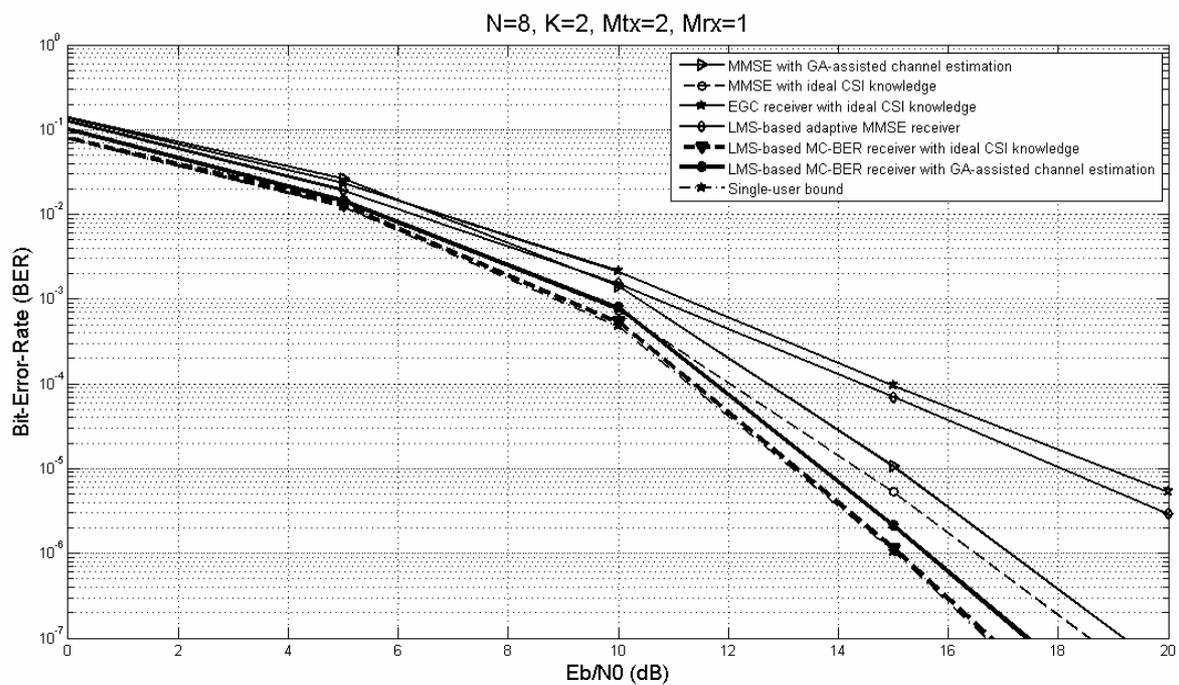


Figure 3. BER performance yielded by the LMS-based MCBER (with ideal CSI knowledge and GA-assisted channel estimation), EGC (with ideal CSI knowledge), LMS-based MMSE and ideal MMSE (with ideal CSI knowledge and GA-assisted channel estimation) multi-user detectors for $N=8$ subcarriers and $K=2$ users

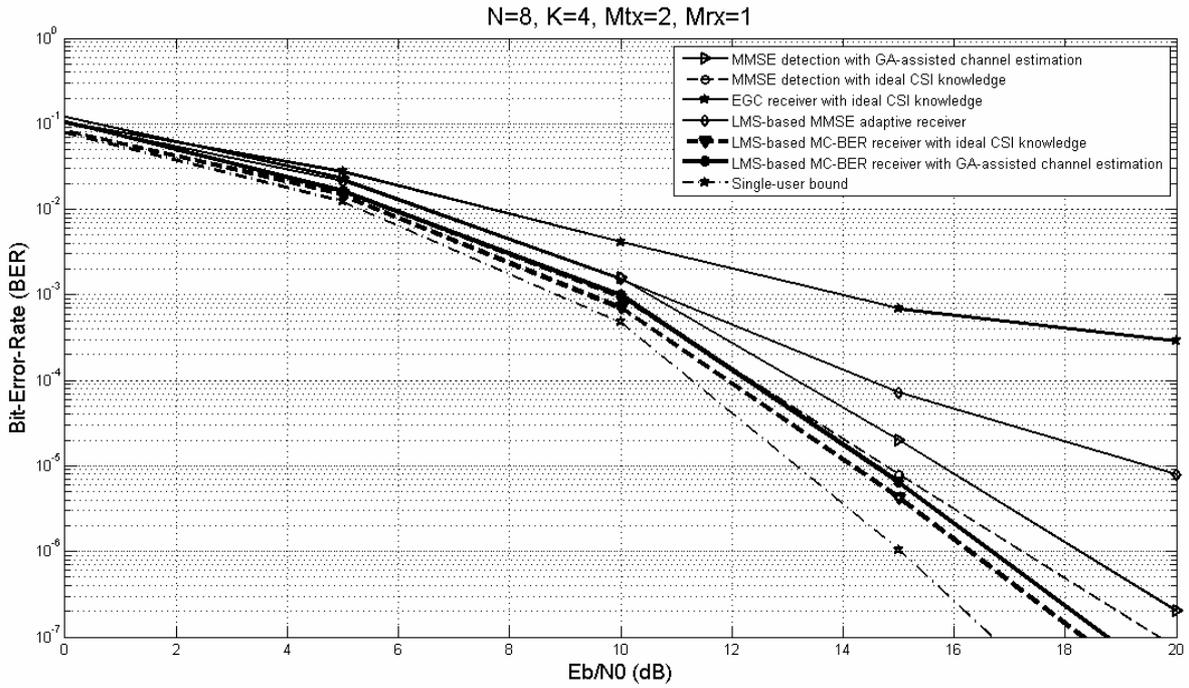


Figure 4. BER performance yielded by the LMS-based MCBER (with ideal CSI knowledge and GA-assisted channel estimation), EGC (with ideal CSI knowledge), LMS-based MMSE and ideal MMSE (with ideal CSI knowledge and GA-assisted channel estimation) multi-user detectors for $N=8$ subcarriers and $K=4$ users

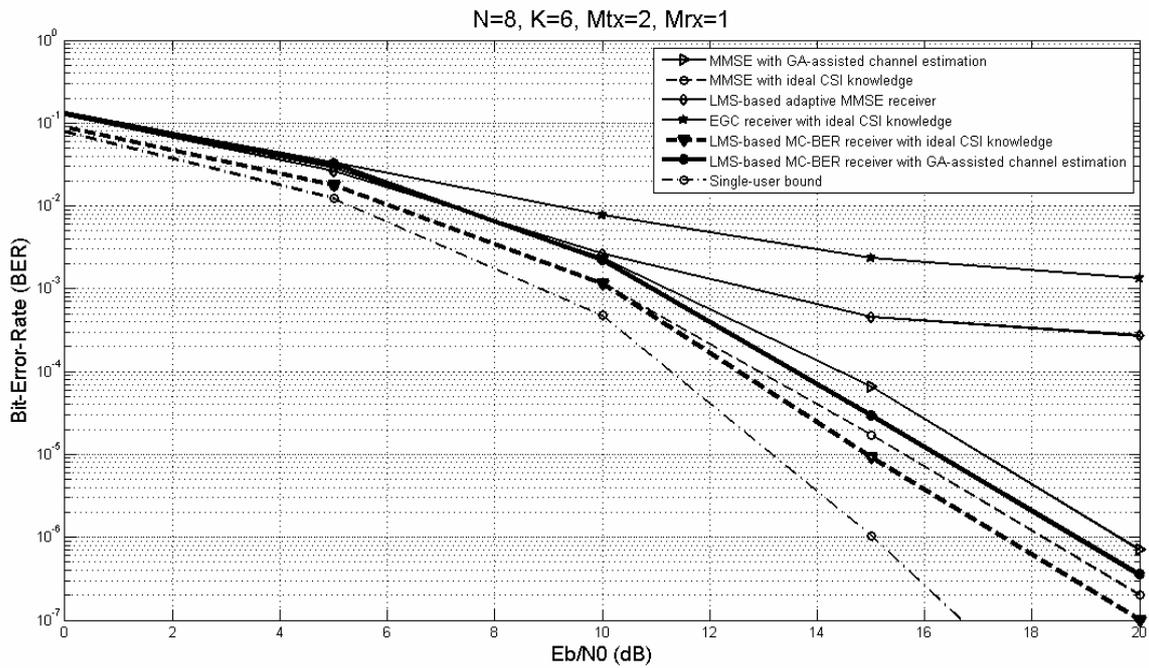


Figure 5. BER performance yielded by the LMS-based MCBER (with ideal CSI knowledge and GA-assisted channel estimation), EGC (with ideal CSI knowledge), LMS-based MMSE and ideal MMSE (with ideal CSI knowledge and GA-assisted channel estimation) multi-user detectors for $N=8$ subcarriers and $K=6$ users

B. Computational complexity considerations and algorithmic parameterization

As far as computational issues are concerned, current literature claims that MCBER criterion has a computational order which is linear with the number of users [7]. In such a case, the computational effort would be reduced with respect to the ideal MMSE detector (which is $O(K^3)$ [2][8][16][20]) and it would be comparable with the LMS-based adaptive implementation of MMSE (which is linear again). In our opinion, the reduction of the computational complexity is one of the main advantages yielded by the proposed approach. In fact, the MCBER criterion is theoretically closer to optimality than MMSE and simulation results shown in Fig.3-5 confirm this claim. But the developed adaptive MCBER-MUD algorithm is also less demanding from a computational viewpoint than ideal MMSE, although requiring the same knowledge of the channel state information.

About computational complexity of the GA-assisted MMSE channel estimator we can say that GA requires a number of elementary operations to derive a solution that is equal to $\nu_{op} = (\alpha + \gamma)GP$ [19]. Thus, the computational burden required during the training-aided step is of the order of $L \cdot K \cdot N \cdot G_{Tr} \cdot P_{Tr}$ elementary operations to be executed during an execution time window εT , where $\varepsilon > 1$. The value assigned to ε mainly depends on the computational power of the signal-processing device employed. During the decision-directed adaptive step, the computational requirement of the GA is reduced to $K \cdot M \cdot P_D$ elementary operations to be executed during a signalling period T . Such a computational requirement is comparable with that one involved by state-of-the-art STBC channel estimation algorithms [13-16].

To conclude this section, we provide now some notes about algorithmic parameterization. The step-size parameter λ of both LMS-based algorithms (MMSE and MCBER) has been chosen empirically for each scenario in order to minimize the overall BER over the various E_b/N_0 values. From the parameters selection phase, we noted that LMS-based MCBER detector is characterized by a reduced sensitivity to parameterization with respect to state-of-the-art LMS-based MMSE MUD. Indeed, fixing the number of users K , the step-size λ is substantially invariant with respect to E_b/N_0 values. On the other hand, LMS-based MMSE multi-user detector would require a different value of λ for each E_b/N_0 in order to provide satisfactory BER performances.

As far as the parameterization of the GA-based optimizer is concerned we must say that formal methodologies targeted to find an optimal parameterization of genetic procedures are not available. In literature, there are only some interesting heuristic analysis like the one proposed by Tsoi in [21]. So GA parameters has been selected by means of explicitly-devoted simulation trials, performed by keeping into account the major guidelines pointed out in [21] that basically are these two ones: a) the population size should be sufficiently large in order to have a conveniently- dimensioned space search, b) the number of generations should be appropriately assigned in dependence of the population size. In fact, in case of large population, too strict limit for the search time can force algorithm to stop without having enough time to realize its search possibility. At the end of the simulation trials devoted to parameterization, we found numerical values for generation number, population size, crossover and mutation probabilities (shown in sub-section A) that have assured the best tradeoff between achieved results and computational load. In particular, we assumed an intermediate E_b/N_0 equal to 15 dB as reference value, and we derived by simulations the best parameterization for this value. Then, we observed by other simulations that the GA parameterization chosen for the reference E_b/N_0 is “very close to the best” also for higher and lower E_b/N_0 . The reduced sensitivity to parameterization of GA procedures, already observed in [17] is again confirmed.

VI. Conclusions

In this paper, a multi-user detection strategy inspired to the concept of Bit-Error-Rate minimization has been investigated for MIMO MC-CDMA systems using Alamouti’s space-time block coding. In particular, we considered a Minimum Conditional Bit-Error-Rate (MCBER) MUD receiver in order to reduce the computational complexity without significant performance degradations with respect to the formal MBER criterion. In the perspective of a real receiver deployment, an adaptive LMS-based implementation of the MCBER detector has been proposed supported by a robust and computationally-affordable MMSE channel estimation assisted by a genetic optimizer. The proposed MCBER approach always allows improving BER performances with respect to other state-of-the-art linear detectors working with the same degree of channel knowledge. It is worth noting that the performance improvement with respect to MMSE-MUD strategies is achieved by spending a reduced computational effort, linearly increasing with the number of users. Numerical results evidenced that BER curves of MCBER and ideal MMSE are getting closer as the number of users approaches the maximum allowable value. This behaviour is intrinsic to the linear multi-user detection and fully motivated by the nature of the MCBER criterion adopted.

Future research activities might concern the utilization of non-conventional optimization strategies (e.g. GA, Particle Swarm Optimization – PSO, etc.) to provide a numerical solution to the MCBER problem instead of the proposed LMS-based solution.

VII. References

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