UNIVERSITY
OF TRENTO

DIPARTIMENTO DI INGEGNERIA E SCIENZA DELL'INFORMAZIONE

38123 Povo – Trento (Italy), Via Sommarive 14
http://www.disi.unitn.it

EFFECTIVE EXPLOITATION OF THE INFORMATION CONTENT OF NOISY DATA IN INVERSE SCATTERING PROBLEMS: A FUZZY-LOGIC-BASED STRATEGY

A. Casagranda, M. Donelli, D. Franceschini, and A. Massa

January 2011

Technical Report # DISI-11-286

EFFECTIVE EXPLOITATION OF THE INFORMATION CONTENT OF NOISY DATA IN INVERSE SCATTERING PROBLEMS: A FUZZY-LOGIC-BASED STRATEGY

Aronne Casagranda*, Massimo Donelli*, Davide Franceschini*, and Andrea Massa*
* *ELEDIA Research Group* @ DIT
University of Trento -Via Sommarive, 14 - 38050 Trento - ITALY
E-mail: {*aronne.casagranda, massimo.donelli, davide.franceschini, andrea.massa*}*@dit.unitn.it*
Web-site: *http://www.eledia.ing.unitn.it*

Abstract. *One of the main drawbacks when dealing with inverse scattering problems is their intrinsic ill-conditioned nature, emphasized by the presence of the noise in the measured data. In order to avoid/limit this drawback, this paper presents an innovative fuzzy-logic-based approach. The proposed fuzzy system allows to take into account the corrupted nature of the data in a simple and effective way. The effectiveness of the proposed inversion strategy is shown by comparing the achieved results with those obtained with state-of-the-art inverse scattering techniques.*

1. Introduction

In the framework of the microwave imaging, some of the main difficulties in the reconstruction process are certainly due to the ill-posedness of the inverse problem and to the noisy nature of the available measured data.

In practical situations, experimental and environmental noises add to the scattered signals due to the mechanical positioning of the electromagnetic field sensors or to the electromagnetic interferences in the test-chamber. These factors significantly affect the result of the electromagnetic imaging process (characterized by an high intrinsic instability), leading to inaccurate reconstructions of the scenario under test.

In the framework of spatial-domain procedures, this drawback is generally overcome by reformulating the original inverse scattering problem in the minimization of a suitable cost function. Such a function provides a measure of the fitting between the reconstructed and the available data corrupted by the noise. It is composed of two terms: the data term and the state term which depend on the scattered field in the observation domain and on the incident field in the investigation domain, respectively. Suitable multiplicative regularization parameters allows one to weight more the one or the other term, depending on the uncertainties associated with both of them.

In order to properly exploit the information-content available in the noisy data, the proposed approach avoids a direct estimation of the reliability of the scattering data (because of the cost and the complexity of such an estimate) and it takes into account the presence of the noise through a strategy based on a fuzzy logic [1]. The fuzzy system determines the values of the weighting regularization parameters estimating, in an unsupervised way, the degree of reliability of the available data.

The paper is organized as follows. In Section 2, the mathematical formulation of the inverse scattering problem is briefly resumed and a description of the fuzzy-logic-based approach is presented (Sect. 3). Section 4 shows a set of selected numerical results in order to point out the improvement brought by the fuzzy system. A final discussion is reported in Section 5.

2. Inverse Problem Formulation

Referring to a two-dimensional problem characterized by a cylindrical geometry, a set of TM plane waves ($E_v^{inc}(x,y)\hat{\mathbf{z}}$, $v = 1,..V$) successively illuminates an unknown investigation domain, whose electromagnetic parameters (or the object function $\tau(x,y)$) have to be determined. The background medium is assumed to be homogeneous (characterized by a dielectric permittivity ε_0) and lossless. The relation between dielectric properties of the investigation domain and scattered fields is mathematically described through the well-known integral inverse scattering equations [2]. Their discretized version, obtained subdividing the investigation domain by N rectangular basis functions, is reported in (1) and (3):

$$\Im_{STATE}\left\{\tau(x_n,y_n);\, E_v^{tot}(x_n,y_n)\right\} = E_v^{tot}(x_n,y_n) - j\frac{k_0^2}{4}\sum_{p=1}^{N}\tau(x_p,y_p)E_v^{tot}(x_p,y_p)G_{2D}^{v}(x_n,y_n|x_p,y_p) \tag{1}$$

where

$$G_{2D}^{v}(x_m,y_m|x_n,y_n) = \begin{cases} (j/2)\left[\pi k_0 a_p H_1^{(2)}(k_0 a_p) - 2j\right] & if\ p=n \\ (j\pi k_0 a_n/2)H_0^{(2)}(k_0\rho_{mn}^{v})J_1(k_0 a_n) & otherwise \end{cases} \tag{2}$$

and

$$\Im_{DATA}\left\{\tau(x_n,y_n);\, E_v^{tot}(x_n,y_n)\right\} = j\frac{k_0^2}{4}\sum_{n=1}^{N}\tau(x_n,y_n)E_v^{tot}(x_n,y_n)G_{2D}^{v}(x_m,y_m|x_n,y_n) \tag{3}$$

where

$$G_{2D}^{v}(x_m,y_m|x_n,y_n) = (j\pi k_0 a_n/2)H_0^{(2)}(k_0\rho_{mn}^{v})J_1(k_0 a_n) \tag{4}$$

J_1, $H_0^{(2)}$, and $H_1^{(2)}$ being the Bessel function of first kind and the 0-th and first-order Hankel function of the second kind, respectively; $E_v^{tot}(x_n,y_n)$ is the unknown electric field in the investigation domain and $\rho_{mn}^{v} = \sqrt{\left(x_{m_{(v)}} - x_n\right)^2 + \left(y_{m_{(v)}} - y_n\right)^2}$.

Through a multi-view/multi-illumination measurement system the data of the inverse scattering problem are acquired. The scattered electric field $E_v^{scatt}(x_{m_{(v)}}, y_{m_{(v)}})$ is collected at each view, $v = 1,...,V$, in $M_{(v)}$ measurement points equally-spaced along a circular observation domain. Moreover, the incident field $E_v^{inc}(x,\, y)$ is measured, without the scatterer, in the investigation domain.

Then, the object function $\tau(x_n,y_n)$ as well as $E_v^{tot}(x_n,y_n)$ are retrieved after the minimization of a suitable cost function defined, according to the fuzzy-logic approach, in Section 3.

3. The Fuzzy-Logic-Based Approach

The fuzzy-logic system acts between the data acquisition and the definition of the cost function to be minimized, by determining the values of a set of weighting coefficients (α_v^m and β_v^n, $m_{(v)} = 1,...,M_{(v)}$, $v = 1,...,V$, $n = 1,...,N$) that express the reliability of each sample of the measured scattered and incident field. Consequently, the arising cost function turns out to be

$$\Phi\left\{\tau(x_n,y_n); E_v^{tot}(x_n,y_n)\right\} = \frac{\sum_{v=1}^{V}\sum_{m=1}^{M_{(v)}}\alpha_{\mathbf{v}}^{\mathbf{m}}\left|E_v^{scatt}(x_m,y_m) - \Im_{DATA}\left\{\tau(x_n,y_n);E_v^{tot}(x_n,y_n)\right\}\right|^2}{\sum_{v=1}^{V}\sum_{m=1}^{M_{(v)}}|E_v^{scatt}(x_m,y_m)|^2} + \frac{\sum_{v=1}^{V}\sum_{n=1}^{N}\beta_{\mathbf{v}}^{\mathbf{n}}\left|E_v^{inc}(x_n,y_n) - \Im_{STATE}\left\{\tau(x_n,y_n);E_v^{tot}(x_n,y_n)\right\}\right|^2}{\sum_{v=1}^{V}\sum_{n=1}^{N}|E_v^{inc}(x_n,y_n)|^2} \tag{5}$$

In order to define the weighting coefficients by taking into account the presence of the noise on the measurement data, the effectiveness of a fuzzy-logic strategy can be fully exploited.

Firstly, the scattering data are normalized ("*normalization*" step) by means of the computation of the following coefficients

$$\eta_v^m = \frac{\left|\frac{E_v^{scatt}(x_m,y_m)}{E_v^{tot}(x_m,y_m)}\right|}{max\left\{\eta_v^m\right\}} \qquad \xi_v^n = \frac{\left|E_v^{inc}(x_n,y_n)\right|}{max\left\{\xi_v^n\right\}} \qquad v=1,...,V;\ m_{(v)}=1,...,M_{(v)} \tag{6}$$

Then, the "*fuzzyfication*" step follows. Let us define a set of heuristically-defined rules $\Re$, composed by a set of *antecedents* (Λ) and relative *consequences* ($\mathcal{C}$). The system inputs (η_v^m, ξ_v^n) are transformed into their fuzzy counterparts ($\varphi\left(\eta_m^v\right)$, $\varphi\left(\xi_n^v\right)$) by associating to each sample a gaussian membership function [1]. Such a process allows to activate a rule according to the degree of similarity between $\varphi(\bullet)$ and the antecedent of the rule in Λ. An activated rule is characterized by an activation value that determines the degree of truth of every consequence of $\mathcal{C}$. Finally a *defuzzifier* provides the reliability index (α_m^v, β_n^v) by considering the composition of the activated consequences of $\mathcal{C}$.
Successively, the cost function (where each measured data contributes according to own reliability index) is minimized by means of a suitable gradient-based minimization algorithm [3].

4. Numerical Results

To assess the effectiveness of the proposed fuzzy-based methodology, the reference square ($1.2\,\lambda_0$-sided) cylindrical profile ($\tau = 1.0 + j0.0$) has been considered (Fig. 1(a)). The investigation domain ($4\,\lambda_0$ in side), discretized in $N = 100$ square cells, has been illuminated through a set of $V = 8$ TM plane waves. The inversion data, characterized by a white gaussian noise with $SNR = 10\,dB$, have been collected in $M_{(v)} = 35\,(v = 1,..,V)$ measurement points.
For comparison purposes, the results when the fuzzy system is considered have been compared to those of a standard approach. The quality improvement in the final reconstruction can be clearly observed in Fig. 1(c) (with respect to Fig. 1(b) where the profile retrieved with the standard procedure is shown). For completeness, Figure 2 shows a comparison between the decreasing of the cost function during the iterative minimization process with and without the fuzzy system.

5. Conclusions

In this paper, an innovative approach to inverse scattering problems has been presented. The approach allows a simple and effective introduction of the information on the reliability of collected noisy data. Such an information is introduced into the cost function to be minimized through suitable regularization coefficients determined by means of a fuzzy-logic system. The advantages of the fuzzy-logic-based strategy over standard inversion approaches have been preliminarily assessed by considering a set of numerical experiments.

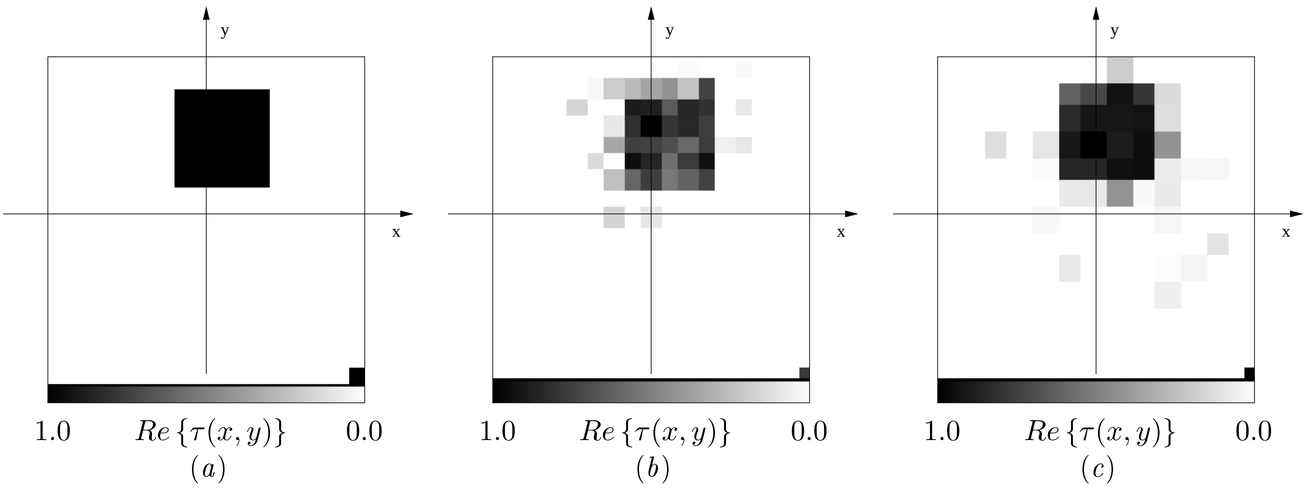

Figure 1. Actual profile (a). Reconstructed profile obtained with (b) the Standard Technique and (c) with the Fuzzy Strategy.

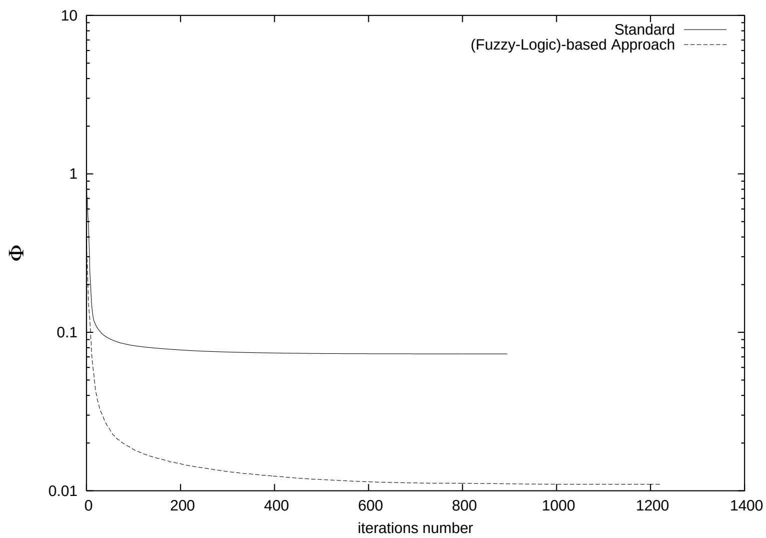

Figure 2. Behavior of the cost function

References

[1] L. A. Zadeh, "Fuzzy Sets", *Information and Control*, vol. 8, pp. 338-353, 1965.

[2] S. Caorsi, M. Donelli, D. Franceschini, A. Massa, and M. Pastorino, "Microwave imaging of two-dimensional scatterers using an iterative multi-scaling methodology," *in Proc. Jina2002*, pp. 293-296, vol. 2, Nice, France, 12-14 November 2002.

[3] H. Harada, D. J. Wall, T. T. Takenaka, and T. Tanaka, "Conjugate gradient method applied to inverse scattering problems," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 784-792, 1995.