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THROUGH A TWO-STEP STRATEGY

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ABSTRACT

In the following contribution an innovative two step strategy for the inversion of amplitude-only data in microwave imaging applications is presented. At the first step an inverse source problem is solved for computing the incident field in the investigation domain. Then, in the successive step, an iterative multi-resolution method combined with an evolutionary optimiser is applied in order to extract the reduced amount of information of the problem at hand. Some numerical and experimental results show the effectiveness and the limit of this technique.

1. INTRODUCTION

The diffusion of microwave imaging techniques in many different fields, such as biomedical and industrial diagnostic, is due to their capability of a qualitative and quantitative analysis of the dielectric properties of the means under test. Unfortunately, these methodologies present some drawbacks related to the ill-posedness and the highly non-linear nature of inverse scattering problems. Another non-trivial point is the amount of collectable information. Multi-illumination, multi-view and multi-frequency systems can be considered, but the achievable information still remains restricted to an upper-bound depending on the geometrical and physical characteristics of the system [1][2].

Moreover, the data acquisition requires complex and expensive hardware setups. In particular the measurement of the phase distribution turns to be critical when high frequencies are considered. As a matter of fact, holographic and interferometric techniques, generally used in optical application [3][4], allow to retrieve the phase information starting from amplitude-only data, but they require undesired additional post-processing. In order to realize a reliable and cost-effective imaging apparatus, some different strategies based on phaseless data have been developed in the last years (see [5] and references therein). Two main approaches are possible: solving the classical inverse scattering problem after the extraction of the phase information or using an ad-hoc algorithm to directly reconstruct the dielectric properties of the scatterer.

In this contribution an innovative two-step strategy, belonging to the second class, is proposed and presented in Sect. 2. In the first step (Sub-Sect. 2.1) the source is synthesized by tuning the coefficients of a linear array of equally-spaced line-sources according to the *Distributed-Cylindrical-Waves Model (DCW-Model)*. Afterward (Sub-Sect. 2.2) a multi-resolution cost functional [6] is defined on the basis of the difference between measurement and estimation of the amplitudes of the incident field and the electromagnetic field in presence of the object under test. Such functional is minimized through the Particle Swarm Optimizer [7], one of the most effective evolutionary iterative procedures. In Sect. 3 and 4 some numerical and experimental results will be analyzed in order to draw some conclusions (Sect. 5) on the effectiveness of the proposed methodology.

2. MATHEMATICAL FORMULATION

Let us consider the classical tomographic imaging configuration in which an unknown cylindrical object is located in an inaccessible investigation domain D_I and it is illuminated by a set of V TM-polarized incident electromagnetic fields at a fixed working frequency f . The aim of the proposed algorithm is the reconstruction of the object function

$$\tau(\mathbf{r}) = \varepsilon_r(\mathbf{r}) - 1 - j \frac{\sigma(\mathbf{r})}{2\pi f \varepsilon_0} \quad (\mathbf{r}) \in D_I \quad (1)$$

where $\varepsilon_r, \varepsilon_0, \sigma$ are the relative dielectric permittivity, the free space dielectric permittivity and the electric conductivity, respectively.

We assume to measure only the amplitude of the total field, $|E_{tot}^v(\mathbf{r}_{m(v)})|$, and both amplitude and phase of the incident electric field, $E_{inc}^v(\mathbf{r}_{m(v)})$, in $M^{(v)}$ points, belonging to the observation domain D_M external to D_I . The knowledge of the phase of the incident field is a realistic assumption since this measurement can be carried out once and off-line for each hardware setup. Moreover it is not so expensive because it is limited to a reduced number of points in the observation domain and therefore it does not represent a limit to the phaseless nature of the proposed technique.

The relation between unknowns ($\tau(\mathbf{r})$ and $E_{tot}^v(\mathbf{r})$) and data is expressed by the Eqs. 2 and 3

$$\begin{aligned} |E_{tot}^v(\mathbf{r}_{m(v)})| = \\ \left| E_{inc}^v(\mathbf{r}_{m(v)}) + j\omega\mu_0 \int_{D_I} \tau(\mathbf{r}') E_{tot}^v(\mathbf{r}') G(\mathbf{r}_{m(v)}/\mathbf{r}') d\mathbf{r}' \right| \end{aligned} \quad (2)$$

$$|E_{inc}^v(\mathbf{r})| = \left| E_{tot}^v(\mathbf{r}) - j\omega\mu_0 \int_{D_I} \tau(\mathbf{r}') E_{tot}^v(\mathbf{r}') G(\mathbf{r}/\mathbf{r}') d\mathbf{r}' \right| \quad (3)$$

in which $G(\mathbf{r}/\mathbf{r}')$ is the free space green function.

In Eq. 3 the knowledge of the amplitude of the incident field in the investigation domain is necessary, but it is supposed to measure the incident field only in the observation domain. In the first step (Sub-Sect. 2.1) this information is reconstructed through a suitable synthesis of the electromagnetic source and in the second one (Sub-Sect. 2.2) the system of Eqs. 2-3 it is solved.

2.1. Step 1

The complexity and sometimes the unfeasibility of the direct measurement of the amplitude of the incident field in the investigation domain $|E_{inc}^v(\mathbf{r})|$ requires the development of a suitable model in order to synthesize the real source and evaluate it numerically.

Toward this end the *DCW* Model is considered. Accordingly, the antenna is modeled by means of a linear array of W equally spaced line-sources and therefore the electric field can be expressed in the following form

$$E_{inc}^{v, syn}(\mathbf{r}) = -\frac{k_0^2}{8\pi f \epsilon_0} \sum_{w=1}^W A_w H_0^{(2)}(k_0 d_w) \quad (4)$$

where d_w is the Euclidean distance between the position of w -th element of the array and \mathbf{r} , k_0 is the free-space wavenumber, $H_0^{(2)}$ is the 0-th order second-kind Hankel function and A_w is the unknown weighting coefficient related to the w -th element of the array. For a finite number of points we can re-write Eq. 4 in a discretized form as follows

$$[E_{inc}^{v, syn}] = [H^v][A] \quad (5)$$

Successively we can determine the optimal configuration of the vector $\bar{\mathbf{A}} = \{A_w; w=1, \dots, W\}$ minimizing the differences between the measurements of the incident field and the synthesized values in the observation domain D_M as follows

$$\bar{\mathbf{A}}_{opt} = \arg \left\{ \min_{\bar{\mathbf{A}}} \left(\frac{\sum_{v=1}^V \sum_{m(v)=1}^{M^{(v)}} \|E_{inc}^{v, meas}(\mathbf{r}_{m(v)}) - E_{inc}^{v, syn}(\mathbf{r}_{m(v)})\|^2}{\sum_{v=1}^V \sum_{m(v)=1}^{M^{(v)}} \|E_{inc}^{v, meas}(\mathbf{r}_{m(v)})\|^2} \right) \right\} \quad (6)$$

by computing the pseudo-inverse of the matrix $[H^v]$ using the well-know *Singular-Value-Decomposition* technique and evaluating the parameters that provide the optimal matching through the following matrix equation

$$[A_{opt}] = [H^v]^+ [E_{inc}^{v,meas}] \quad (7)$$

According to Eq. 4 we can obtain the electric field in every point of the investigation domain. As can be noticed in Eq. 3, only the amplitude of the synthesized field is used and this assumption guarantees a reduction of the noise introduced by the synthesis.

2.2. Step 2

The amount of information content of the data in inverse scattering problems is limited. Moreover, it is further reduced considering amplitude-only data. Therefore, in order to efficiently use the available information, the iterative multi-scaling approach has been introduced [6] and customized for phaseless data. The considered multi-resolution approach allows us to control the dimension of the search space and to improve the quality of the reconstructed profile. In Fig. 1 a flow chart of the algorithm is presented. After the characterization of the source model discussed in Sub-Sect. 2.1, the iterative procedure of the multi-resolution technique is initialised assuming a uniform distribution of the unknowns whose number is chosen on the basis of the available information and according to [1][2]. After evaluating the distribution of the incident field in the investigation domain, the system of Eqs. 2, 3 is recast to the minimization of an appropriate cost function. According to the IMSA [6], the resolution is progressively improved in the *Regions of Interest* where the object is supposed to be located. As a matter of fact such procedure can be obtained defining a multi-resolution cost function

$$\Phi_{IMSA-PD} = \Phi_{IMSA-PD-State} + \Phi_{IMSA-PD-Data} \quad (8)$$

$$\Phi_{IMSA-PD-State} = \frac{\sum_{v=1}^V \sum_{r=1}^s \sum_{n(r)=1}^{N(r)} \|E_{inc}^{v,syn}(\mathbf{r}_{n(r)}) - |E_{inc}^{v,rec}(\mathbf{r}_{n(r)})\|^2}{\sum_{v=1}^V \sum_{r=1}^s \sum_{n(r)=1}^{N(r)} |E_{inc}^{v,syn}(\mathbf{r}_{n(r)})|^2} \quad (9)$$

$$\Phi_{IMSA-PD-Data} = \frac{\sum_{v=1}^V \sum_{m(v)=1}^{M(v)} \|E_{tot}^{v,meas}(\mathbf{r}_{m(v)}) - |E_{tot}^{v,rec}(\mathbf{r}_{m(v)})\|^2}{\sum_{v=1}^V \sum_{m(v)=1}^{M(v)} |E_{tot}^{v,meas}(\mathbf{r}_{m(v)})|^2} \quad (10)$$

where the term $\Phi_{IMSA-PD-State}$ is related to the difference between the amplitude of the synthesized and reconstructed incident field in the investigation domain and the term $\Phi_{IMSA-PD-Data}$ to the difference between the amplitude of the measured and reconstructed total field.

In Eqs. 9, 10 the reconstructed electric fields are evaluated as follows

$$\left| E_{tot}^{v,rec}(\mathbf{r}_{m(v)}) \right| = \left| E_{inc}^{v,meas}(\mathbf{r}_{m(v)}) + \sum_{t=0}^{s-1} \sum_{q(t)=1}^{N(t)} \left\{ \omega_{q(t)}^{(s)} [\tau(\mathbf{r}_{q(t)}) E_{tot}^{v,rec}(\mathbf{r}_{q(t)}) G(\mathbf{r}_{m(v)}/\mathbf{r}_{q(t)})] \right\} \right| \quad (11)$$

$$\left| E_{inc}^{v,rec}(\mathbf{r}_{n(r)}) \right| = \left| E_{tot}^{v,rec}(\mathbf{r}_{n(r)}) - \sum_{t=0}^{s-1} \sum_{q(t)=1}^{N(t)} \left\{ \omega_{q(t)}^{(s)} [\tau(\mathbf{r}_{q(t)}) E_{tot}^{v,rec}(\mathbf{r}_{q(t)}) G(\mathbf{r}_{n(r)}/\mathbf{r}_{q(t)})] \right\} \right| \quad (12)$$

where the weighting function $\omega_{q(t)}^{(s)}$ assumes 0 or 1 value on the basis of the information collected in the previous iterations [6].

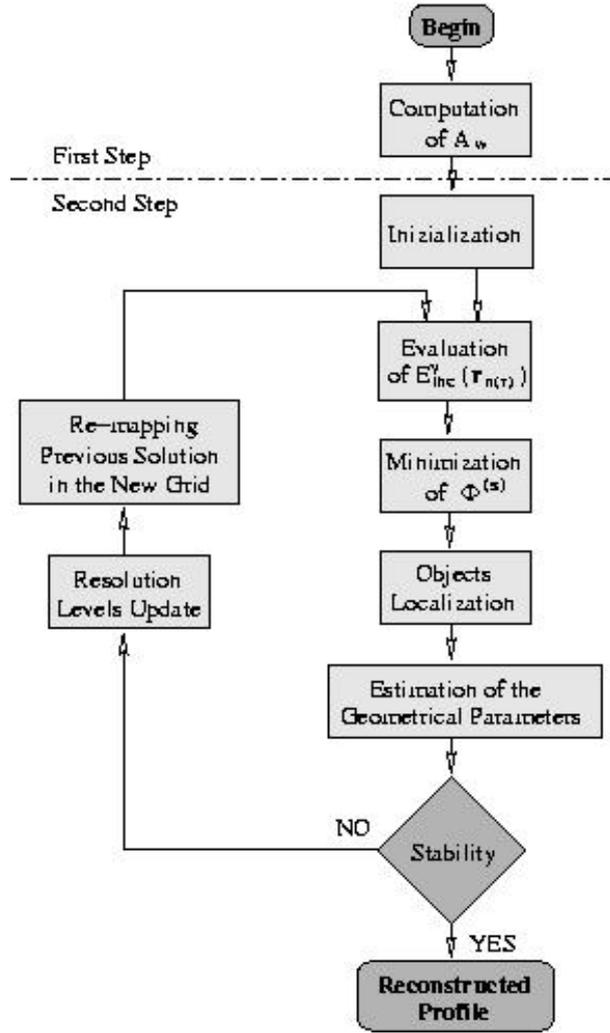


Figure 1. Schematic representation of the algorithm

In order to reduce the local minima problem, the cost function of Eq. 8 is minimized using one of the most effective evolutionary technique: the Particle Swarm Optimiser (for a detailed description see [7]). Finally, the multi-resolution procedure is iterated until a stationary condition is reached [6].

3. NUMERICAL VALIDATION

A preliminary analysis has been carried out through a synthetical test case. A homogeneous dielectric square cylinder

$L_{obj} = \frac{\lambda}{2}$ sided with object function $\tau_{obj} = 1.0$ is located in the center of a square investigation domain $L_I = 2\lambda$

sided. The scenario is illuminated by means of $V = 32$ plane waves and $M^{(v)} = 32$ equally-spaced measurements points are displaced in the observation domain on a circle $R_M = 5\lambda$ in radius in order to collect the scattered radiation.

As far as the reconstruction process is concerned the following configuration of PSO parameters is selected according to the indications in literature [7][8][9]: constant inertial weight $\omega = 0.4$, acceleration coefficients $C_1 = C_2 = 2.0$,

swarm dimension $I = \frac{5}{100}U$, being U the number of unknowns.

At the first step of the proposed approach, the coefficients A_w are computed and a good matching between actual and synthesized values of the incident electric field in the observation domain is achieved both in amplitude (Fig. 2) and in phase (Fig. 3).

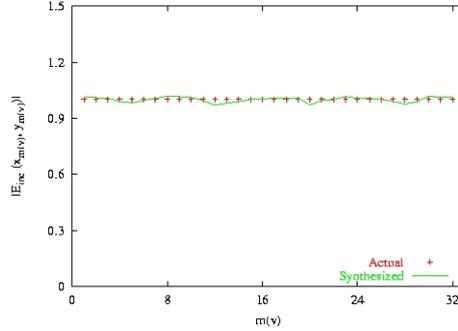


Figure 2. Matching between synthesized and actual values of the amplitude of the incident field in D_M

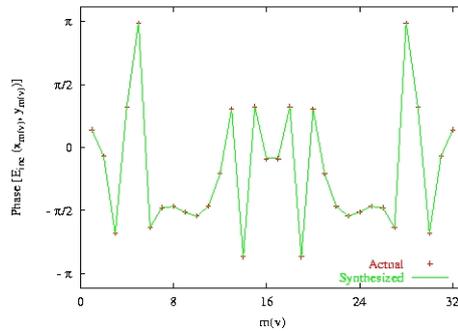


Figure 3. Matching between synthesized and actual values of the phase of the incident field in D_M

Such values are exploited during the object function reconstruction process (second step) in which the phaseless iterative multi-scaling approach is applied. As shown in Fig. 4, the reconstructed profile is quite close to the actual one (dashed line) both qualitatively (localization, dimension and shape) and quantitatively (value of the object function).

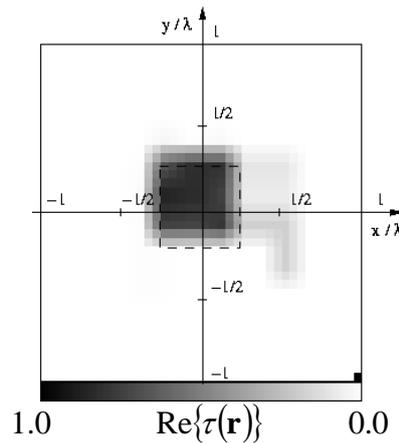


Figure 4. Reconstructed profile of the real part of the object function

4. EXPERIMENTAL VALIDATION

In order to further assess the robustness and the effectiveness of the algorithm, a dataset of experimental measurements has been considered. Such a database has been kindly provided by M. Saillard and K. Belkebir (for details see [10]). In particular, the test case concerning the so-called “*dielTM_dec8f.exp*” scattering configuration is considered. It refers to a

homogeneous dielectric cylinder ($\tau_{obj} = 2.0 \pm 0.3$) $R_{obj} = 1.5 \times 10^{-2} [mt]$ in radius located in a square investigation domain $L_t = 3.0 \times 10^{-1} [mt]$ sided at position ($x_{obj} = 0.0$; $y_{obj} = -3.0 \times 10^{-2} [mt]$). All $V = 36$ different views and $M^{(v)} = 49$ measurement points of the single frequency dataset at $f = 1 [GHz]$ have been considered.

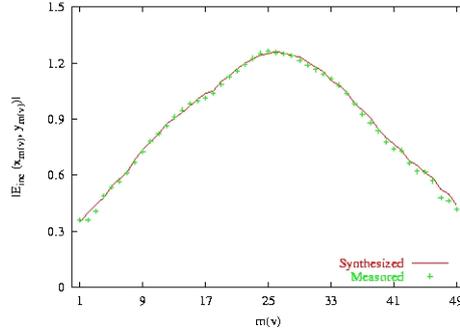


Figure 5. Matching between synthesized and measured values of the amplitude of the incident field in D_M

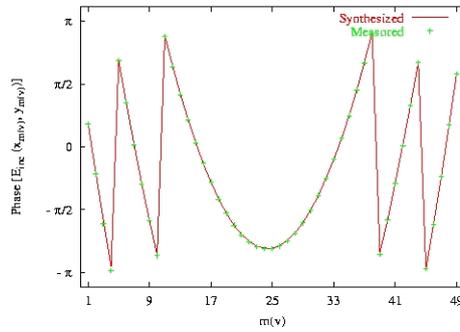


Figure 6. Matching between synthesized and measured values of the amplitude of the incident field in D_M

First, Figs. 5 and 6 shows that the synthesis of the source presents a good agreement between measured and synthesized values of amplitude and phase of the electric incident field. Moreover, also the phaseless reconstruction of Fig. 7 is satisfactory. However, from a quantitative point of view, some artifacts around the actual object can be noticed and the dielectric properties are under-estimated. On the contrary, from a qualitative point of view, the object is well-localized.

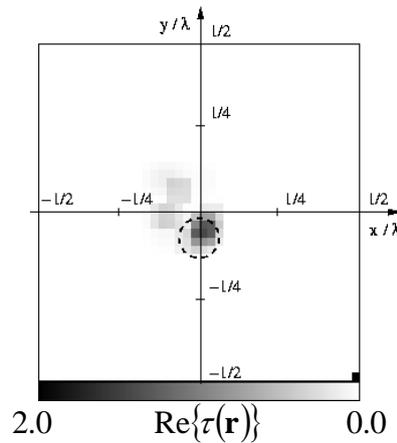


Figure 7. Reconstructed profile of the real part of the object function

5. CONCLUSIONS

In this contribution an innovative two-step strategy has been presented and its performances analysed both with numerical and experimental data. The results point out the effectiveness of the approach and the feasibility of the inversion of amplitude-only data without the need of any expensive post processing of the data in order to retrieve the phase distribution. As a matter of fact, the *DCW*-Model provides an accurate representation of the incident electric field and in the second step, thanks to the iterative multi-scaling approach, the algorithm has shown a good capability of exploiting the reduced amount of information available through the phaseless measurements. Moreover introducing the Particle Swarm Optimiser the proposed method turns out to be effective also in reconstructing experimental data.

6. REFERENCES

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