Efficient Near Field Antenna Analysis using Compressive Sensing

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1 Test Case 3 : *AUT* without failures ($\nu^{(s)} \in [0.0,\, 0.5]\,,\ F^{(s)} = 7$ and $\gamma^{(s)} \in [0,\, \frac{\pi}{4}]\,,\ P^{(s)} = 5$)

Parameters

Gold Antenna (Without Defects)

- Geometry : Planar array of microstrip patches on the (x, y) plane;
- Working Frequency : $f = 3.6\,[GHz]$ ($\lambda = 83.27 \times 10^{-3}\,[m]$ in free space);
- Substrate (PEC-backed) :
  - Dimensions : infinite;
  - Relative Permittivity : $\varepsilon_{r,sub} = 4.7$;
  - Loss Tangent : $tan\,\delta_{sub} = 0.014$;
  - Thickness : $h_{sub} = 0.019\,[\lambda]\ (1.6\,[mm])$;
- Microstrip patches :
  - Dimensions : $l_x \approx 0.22\,[\lambda]\ (18.16\ [mm]),\ l_y \approx 0.33\,[\lambda]\ (27.25\ [mm])$;
  - Feeding : pin-fed;
- Spacing between elements : $d_x = d_y = \frac{\lambda}{2}$;
- Number of elements in each row : $N_x = 6$;
- Number of elements in each column : $N_y = 10$;
- Total number of elements : $N = (N_x \times N_y) = 60$;
- Total size of the antenna : $L_x = 5\,[\lambda],\ L_y = 9\,[\lambda]$;
- Element excitations : $w_n^{(s)} = 1.0 + j0.0,\ n = 1, ..., N^{(s)},\ s = 1, ..., S$;

Antenna Under Test (AUT - With Defects)

In this test case the *AUT* is set equal to the gold antenna, meaning that the antenna under measurement have no defects.

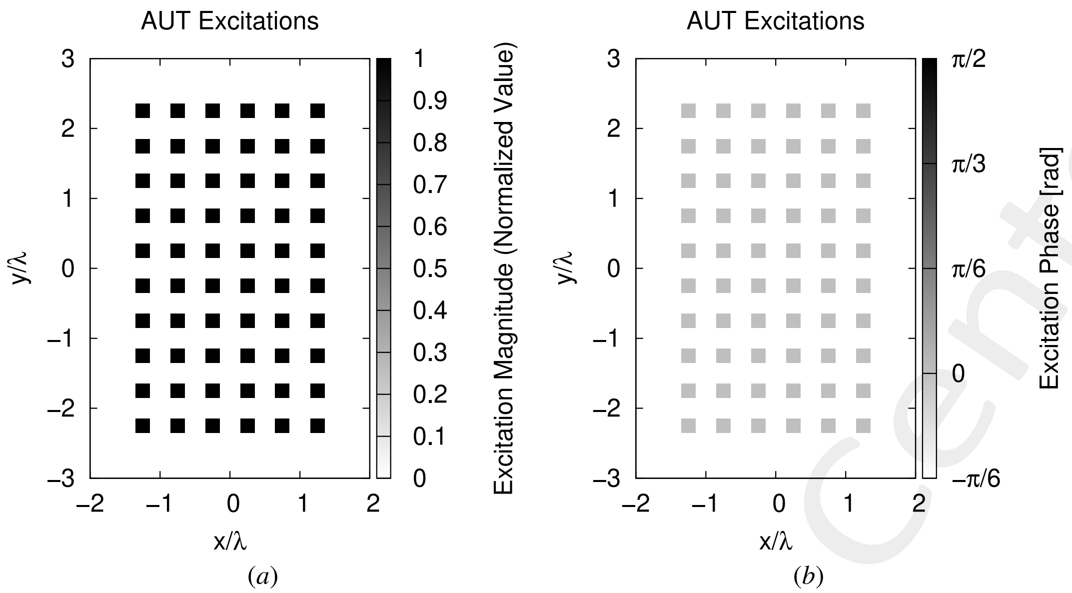

Figure 1: (a) Magnitude of the element excitations in the *AUT* , (b) phase of the element excitations in the *AUT* .

Measurement Set-Up

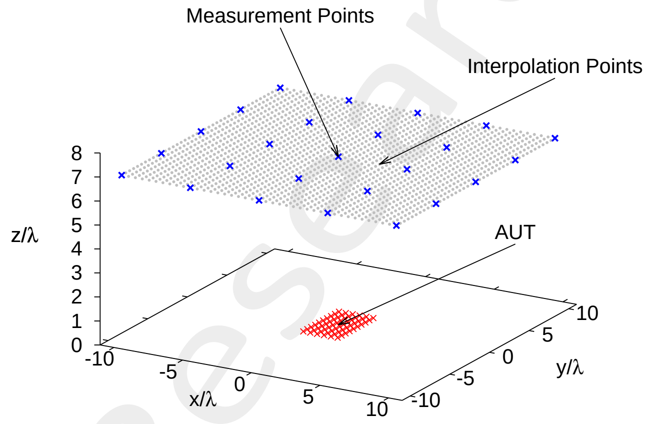

Figure 2: Disposition of the interpolation points ($T = 1681$) and of the measurement points ($M = 25$) in the near-field region of the *AUT*

- Type of measurements : near-field;
- Height of the measurement region : $H = 7\,[\lambda]$;
- Interpolation points :
  - Number of points : $T = 41 \times 41 = 1681$;
  - Coordinates : $x_t \in [-10,\, 10]\,[\lambda],\;\; y_t \in [-10,\, 10]\,[\lambda],\;\; z_t = H\,[\lambda],\;\; t = 1, ..., T$;
  - Interpolation step : $\Delta^{int}_{x/y} = 0.5\,[\lambda]$;
- Measurement points :
  - Coordinates : $x^{meas}_m \in [-10,\, 10]\,[\lambda],\;\; y^{meas}_m \in [-10,\, 10]\,[\lambda],\;\; z^{meas}_m = H\,[\lambda],\;\; m = 1, ..., M$;
  - Number of points : $M_{x/y} = 5 \rightarrow M = 25$;

- Measurement step : $\Delta_{x/y}^{meas} = 5\,[\lambda]$
- Ratio between number of measurements and total number of elements : $(M/N) = 0.42$;

Measurement-by-Design Technique

- Number of generated bases : $B = 20$;
- Bases $b = 1, ..., 10$: magnitude failures in each row $(s = 1, ..., 10)$
  - Failure factor of the elements : $\nu^{(s)} \in [0.0,\, 0.5],\;\; s = 1, ..., 10$;
  - Number of simulated failure factors : $F^{(s)} = 7,\; s = 1, ..., 10$;
- Bases $b = 11, ..., 20$: phase failures in each row $(s = 1, ..., 10)$
  - Phase shift of the elements : $\gamma^{(s)} \in [0,\, \frac{\pi}{4}]\,[rad],\;\; s = 1, ..., 10$;
  - Number of simulated phase shifts: $P^{(s)} = 5,\;\; s = 1, .., 10$;
- Threshold on the singular values magnitude (normalized) : $\eta = -40\,[dB]$;
- Total number of simulated AUT configurations : $K = S \times (F^{(s)} + P^{(s)}) = 10 \times (7 + 5) = 120$;

Dimension of the Over-Complete Basis

The dimension of the over-complete basis is

$$Q = 40$$

This number is given by the sum of the vectors belonging to the two considered bases:

1. Magnitude failures : $Q_1, ..., Q_{10} = 2$;
2. Phase failures : $Q_{11}, ..., Q_{20} = 2$.

Alternative (*BCS*) MbD parameters

- Toleration factor for *BCS* solver: $Tolerance = 1 \times 10^{-8}$;
- Initial noise variance for *BCS* solver: $\eta_0^{opt_1} = 10^{-2}$ and $\eta_0^{opt_2} = 5 \times 10^{-4}$. This values have been obtained as a result of a calibration procedure;

Original (*OMP*) MbD parameters

- Max. number of iterations of the OMP algorithm : $I = \{1;\, 2;\, 3;\, ...;\, 10\}$;
- Selected iteration to report the results: $I = 2$; this choice is justified by the fact that at this iteration the *OMP* algorithm reaches the best near field error as shown in the following Fig. 3.

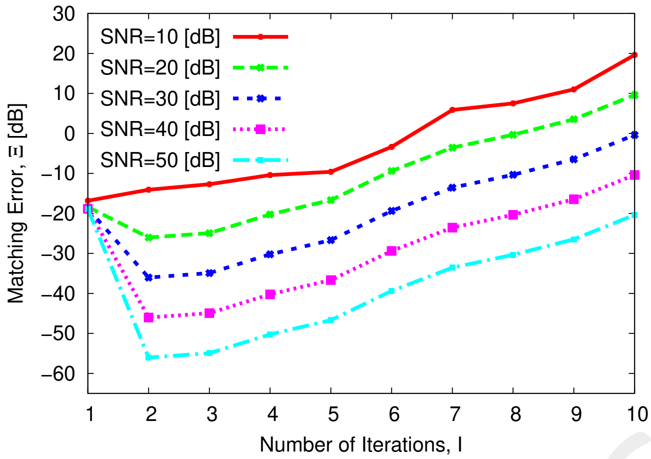

Figure 3: Behaviour of the near-field matching error versus the number of *OMP* iterations, I.

Noise

- SNR on the measured data : $SNR = \{50;\ 40;\ 30;\ 20;\ 10\}\ [dB]$;
- Noise seed : $Noise\_Seed = 11$.

1.1 Comparison between original (*OMP*) and alternative (*BCS*) MbD

Near-Field Error

The comparison, in terms of near field error, between the original (*OMP*) and the alternative (*BCS*) MbD is reported in the following Fig. 4.

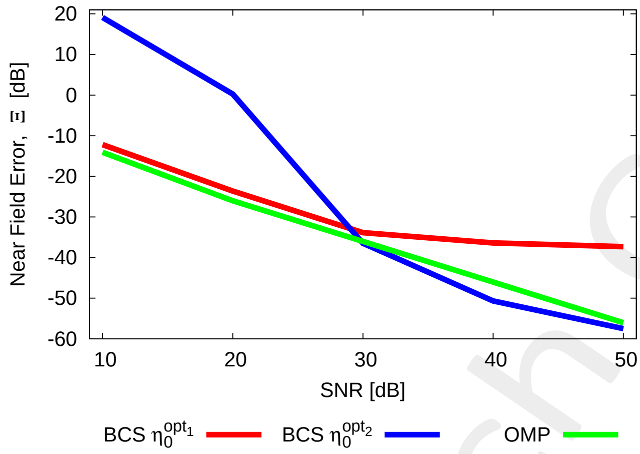

Figure 4: Near Field Error comparison between original (*OMP*) and alternative (*BCS*) MbD for different SNR values.

$SNR\,[dB]$	*Near Field Error,* Ξ $[dB]$		
	BCS		OMP
	$\eta_0^{opt_1}$	$\eta_0^{opt_2}$	
50	-37.32	-57.48	-56.03
40	-36.39	-50.69	-46.03
30	-33.88	-36.48	-36.03
20	-23.62	0.24	-26.03
10	-12.19	19.12	-14.08

Table I: Near Field Errors obtained by the original (*OMP*) and alternative (*BCS*) MbD

Observations

- In this test case, the *OMP* algorithm performs well being able to achieve an error $\Xi < -20\,[dB]$ already for $SNR = 20\,[dB]$. If compared to the *BCS* solver, the *OMP* results are very close to those of the *BCS* with $\eta_0^{opt_1}$ for $SNR \leq 30\,[dB]$ and of the BCS with $\eta_0^{opt_2}$ for $SNR \geq 30\,[dB]$;
- About the *BCS* algorithm:
  - using $\eta_0^{opt_1}$, the *BCS* solver obtains an error that comparable to that of the *OMP* algorithm for $SNR \leq 30\,[dB]$ but then its error remains stable while the *OMP* error decreases;
  - using $\eta_0^{opt_2}$, the *BCS* algorithm obtains results which are the worst for $SNR \leq 30\,[dB]$ but the best, and close to those of the *OMP* algorithm, for $SNR \geq 30\,[dB]$.

Estimated Near-Field

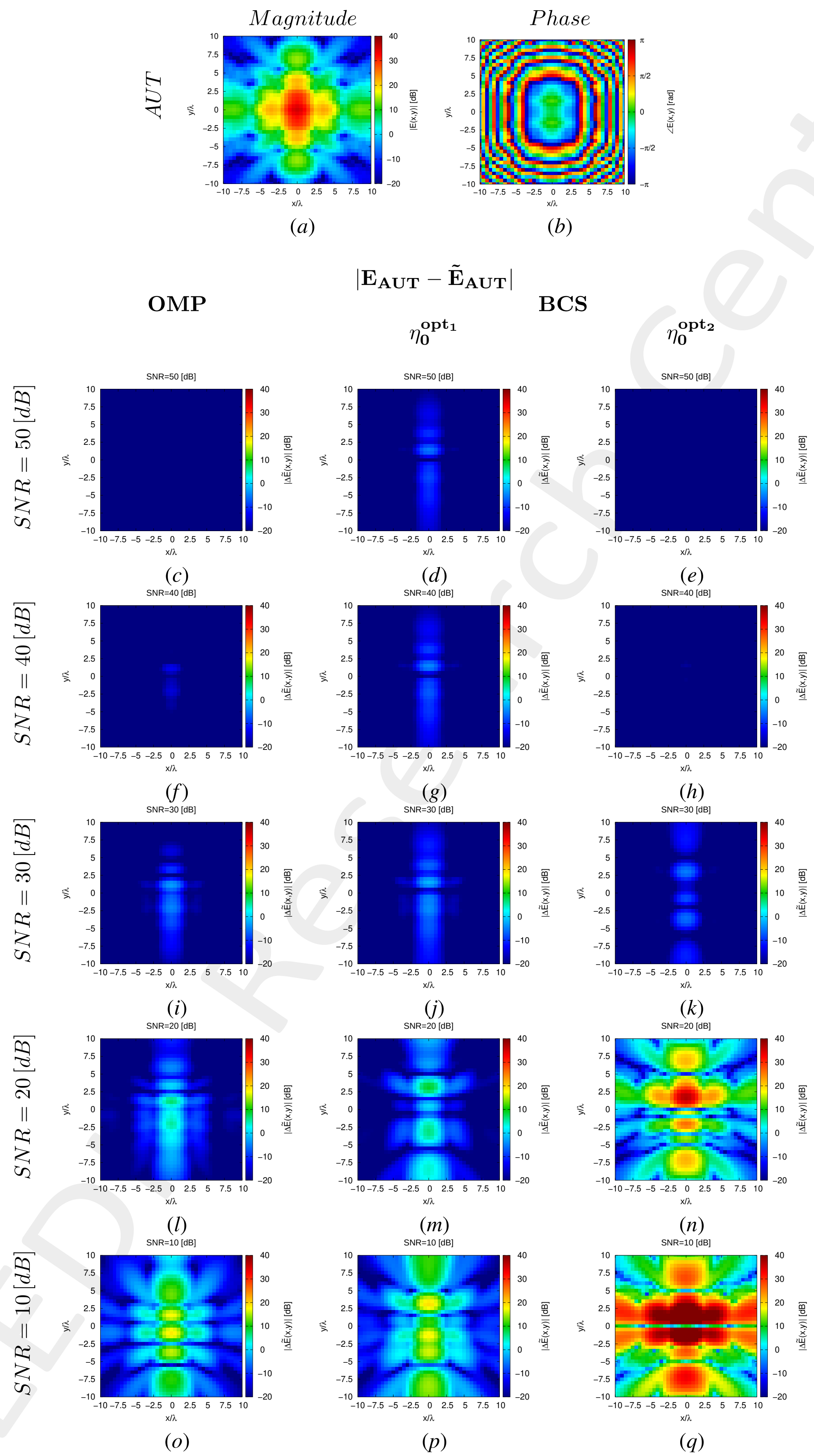

Figure 5: Magnitude difference between the actual and estimated $2-D$ near-field pattern when processing noisy measurements at different SNRs.

Estimated Coefficients

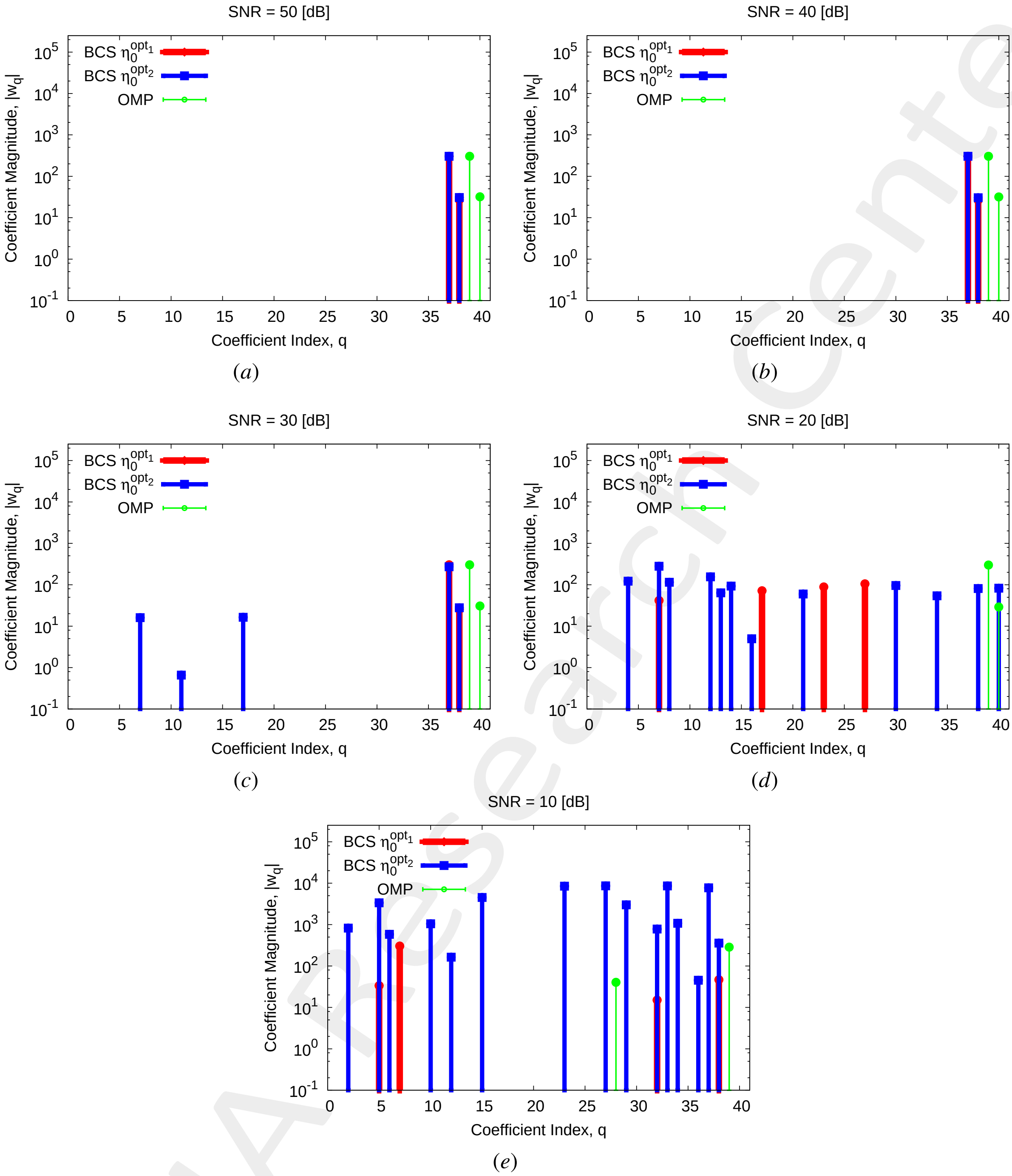

Figure 6: Coefficient comparison between original (*OMP*) and alternative (*BCS*) MbD : (*a*) $SNR = 50\,[dB]$, (*b*) $SNR = 40\,[dB]$, (*c*) $SNR = 30\,[dB]$, (*d*) $SNR = 20\,[dB]$, (*e*) $SNR = 10\,[dB]$

Observations

- The *OMP* solver selects always the same vectors (vector indexes $q = 39, 40$) for $SNR \geq 20\,[dB]$, which are related to phase failures and different from those selected by the *BCS* algorithm;
- The *BCS* algorithm presents solutions that are not sparse for low SNR values, $SNR = 20\,[dB]$ and $SNR = 10\,[dB]$, when $\eta_0^{opt_2}$ is used. Both the two *BCS* versions select two same vectors (vector index $q = 37, 38$) obtaining the same solution for $SNR \geq 40\,[dB]$, solution that is always different from that of the *OMP* solver.

1.1.1 OMP vs best BCS

The main idea of this section is to compare the performance of the *OMP* algorithm and the best *BCS* configuration.

Near-Field Error

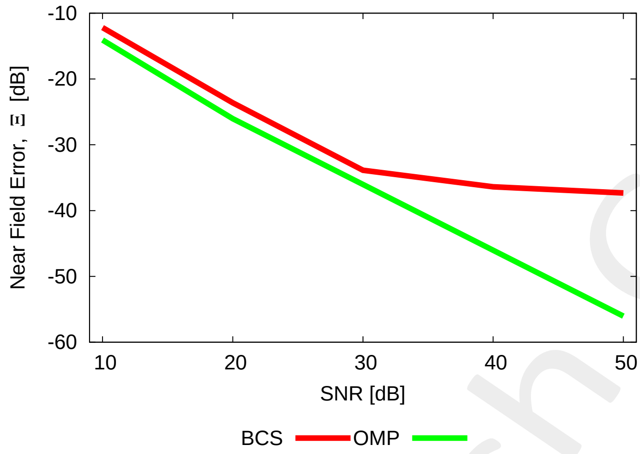

Figure 7: Near Field Error comparison between original (*OMP*) and alternative (*BCS*) MbD for different SNR values.

$SNR\,[dB]$	*Near Field Error*, Ξ $[dB]$	
	BCS	OMP
50	-37.32	-56.03
40	-36.39	-46.03
30	-33.88	-36.03
20	-23.62	-26.03
10	-12.19	-14.08

Table II: Near Field Errors obtained by the original (*OMP*) and alternative (*BCS*) MbD

Estimated Far-Field

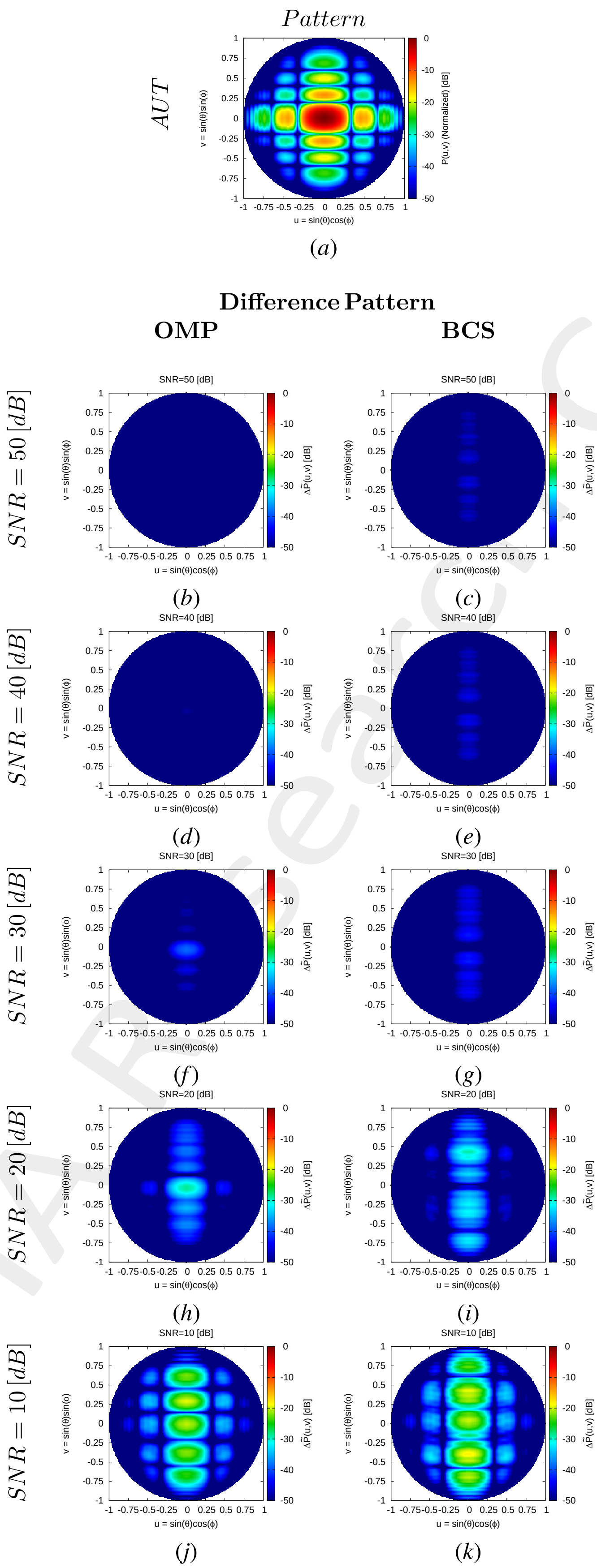

Figure 8: Difference between the actual and estimated $2-D$ far-field pattern when processing noisy measurements at different SNRs.

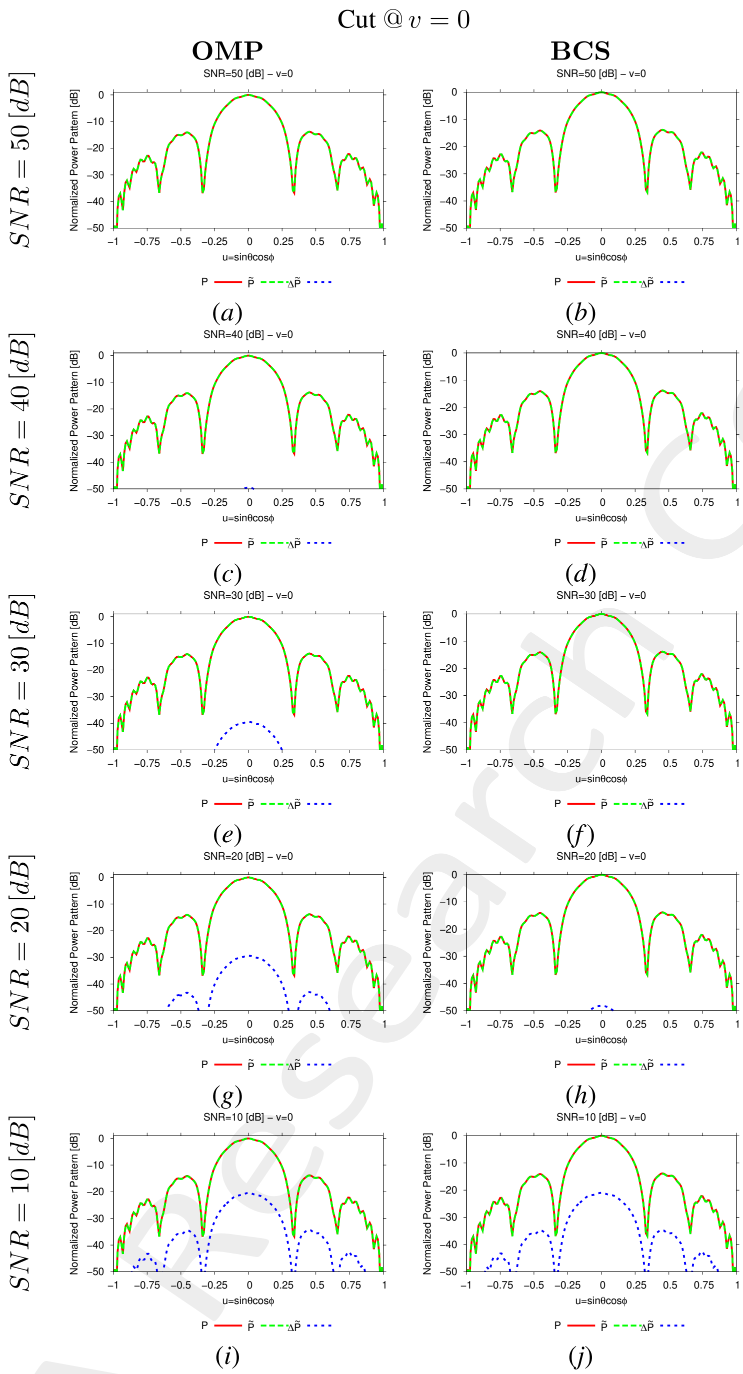

Figure 9: $1-D$ cuts of the estimated far-field pattern (obtained through near-to-far-field transformation from the estimated near-field patterns) under several noisy conditions

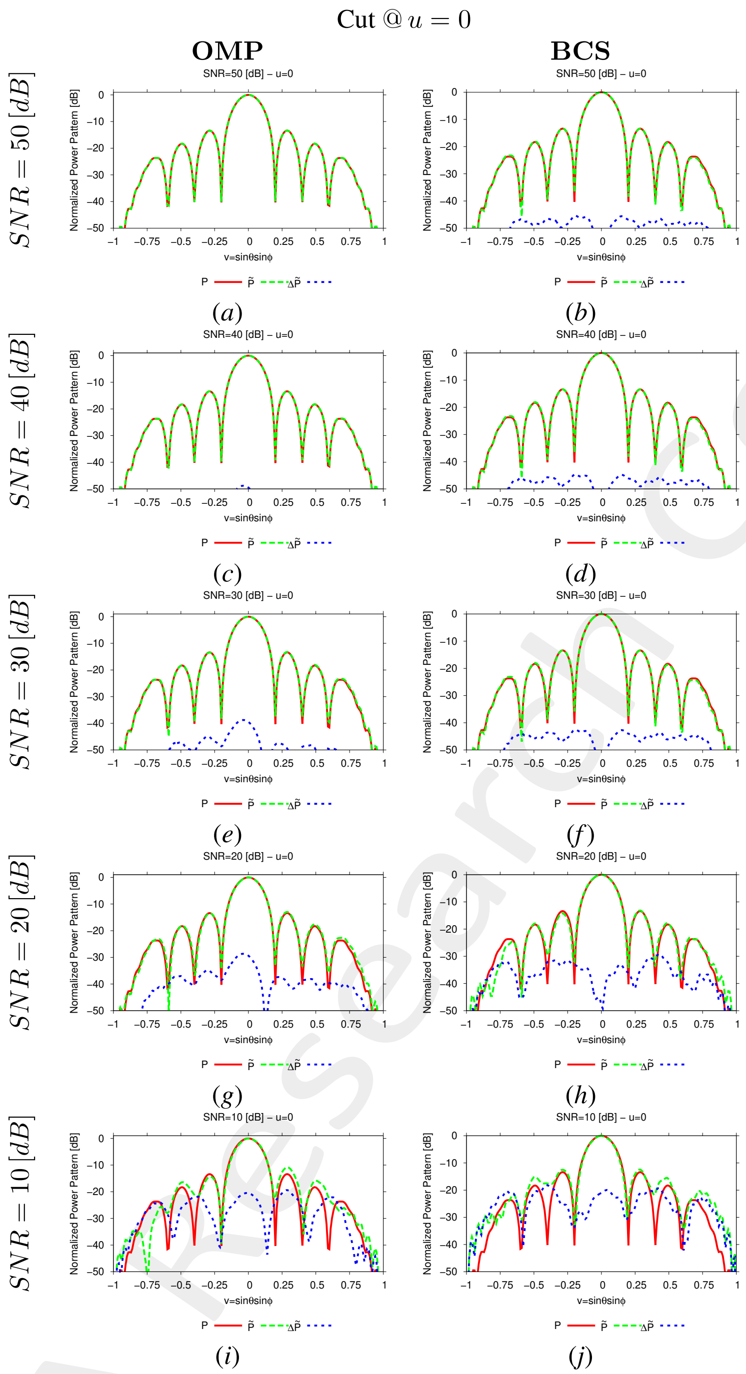

Figure 10: $1-D$ cuts of the estimated far-field pattern (obtained through near-to-far-field transformation from the estimated near-field patterns) under several noisy conditions

$SNR\,[dB]$	$Far-Field\,Error,\,\chi\,[dB]$	
	BCS	OMP
50	-38.87	-57.17
40	-38.09	-47.16
30	-35.98	-37.13
20	-25.07	-27.03
10	-13.54	-15.23

Table III: Far-field matching error between the actual and estimated AUT patterns (both obtained through near-to-far-field transformation from the corresponding near-field patterns) under several noisy conditions.

Estimated Coefficients

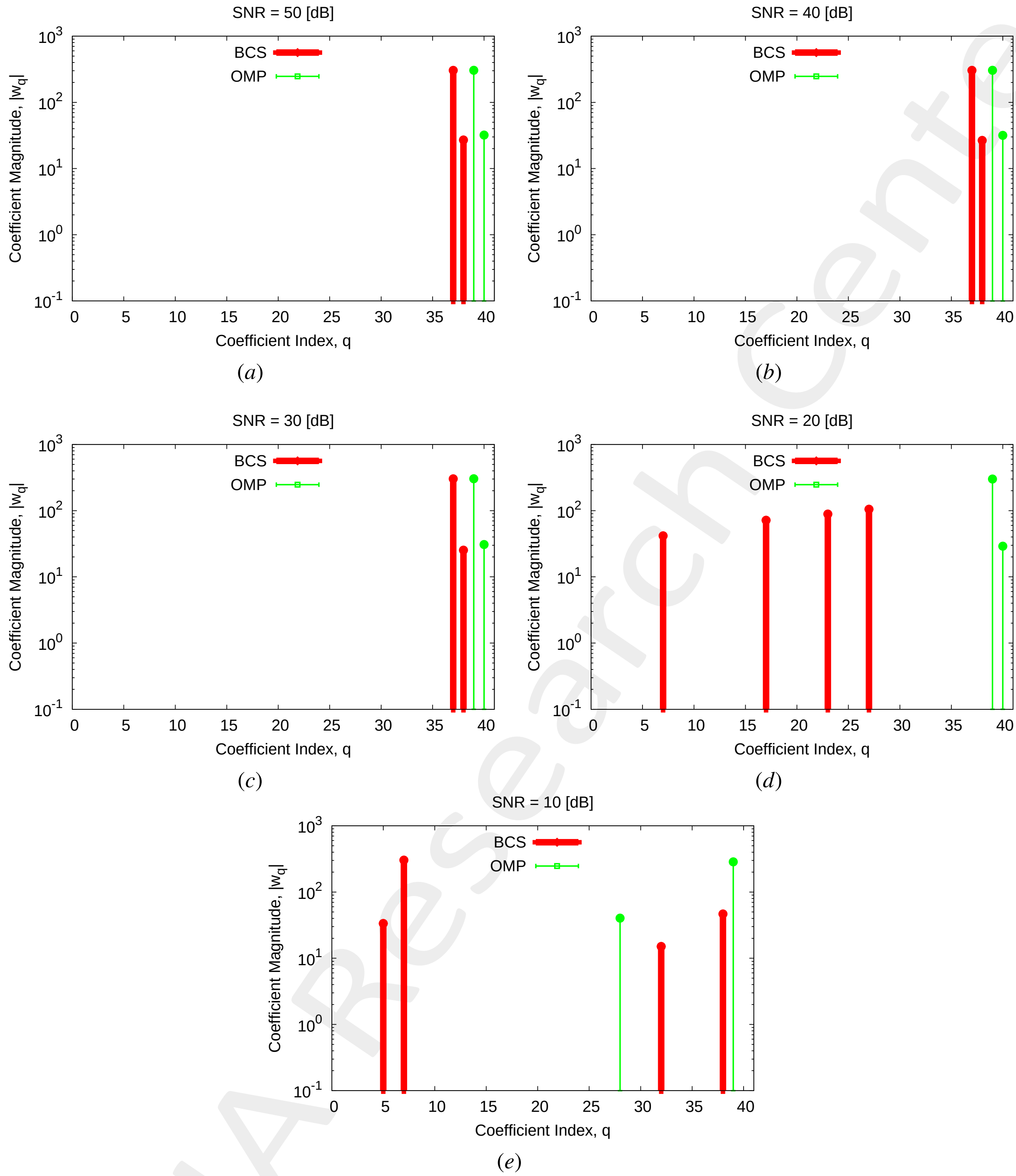

Figure 11: Coefficient comparison between original (*OMP*) and alternative (*BCS*) MbD: (*a*) $SNR = 50\,[dB]$, (*b*) $SNR = 40\,[dB]$, (*c*) $SNR = 30\,[dB]$, (*d*) $SNR = 20\,[dB]$, (*e*) $SNR = 10\,[dB]$

1.2 Incremented failure ranges to build the over-complete basis ($\nu^{(s)} \in [0.0,\ 1.0]$, $F^{(s)} = 7$ and $\gamma^{(s)} \in [-\pi,\ \pi]$, $P^{(s)} = 5$)

Note: All the simulation parameters are the same of those listed in Sez. 1, except the following ones:

Measurement-by-Design Technique

- Number of generated bases : $B = 20$;
- Bases $b = 1, ..., 10$: magnitude failures in each row ($s = 1, ..., 10$)
  - **Failure factor of the elements :** $\nu^{(s)} \in [0.0,\ 1.0]$, $s = 1, ..., 10$;
  - Number of simulated failure factors : $F^{(s)} = 7$, $s = 1, ..., 10$;
- Bases $b = 11, ..., 20$: phase failures in each row ($s = 1, ..., 10$)
  - **Phase shift of the elements :** $\gamma^{(s)} \in [-\pi,\ \pi]\,[rad]$, $s = 1, ..., 10$;
  - Number of simulated phase shifts: $P^{(s)} = 5$, $s = 1, .., 10$;
- Threshold on the singular values magnitude (normalized) : $\eta = -40\,[dB]$;
- Total number of simulated AUT configurations : $K = S \times (F^{(s)} + P^{(s)}) = 10 \times (7 + 5) = 120$;

Original (*OMP*) MbD parameters

- Max. number of iterations of the OMP algorithm : $I = \{1;\ 2;\ 3;\ ...;\ 10\}$;
- Selected iteration to report the results: $I = 2$; this choice is justified by the fact that at this iteration the *OMP* algorithm reaches the best near field error as shown in the following Fig. 12.

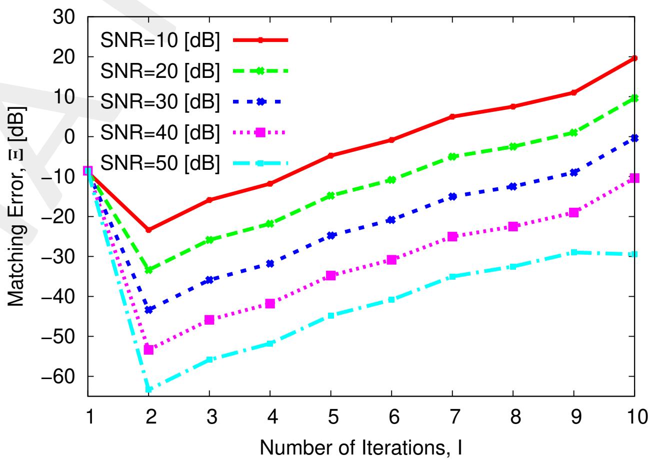

Figure 12: Behaviour of the near-field matching error versus the number of *OMP* iterations, I.

1.2.1 Comparison between original (*OMP*) and alternative (*BCS*) MbD

Near-Field Error

The comparison, in terms of near field error, between the original (*OMP*) and the alternative (*BCS*) MbD is reported in the following Fig. 13.

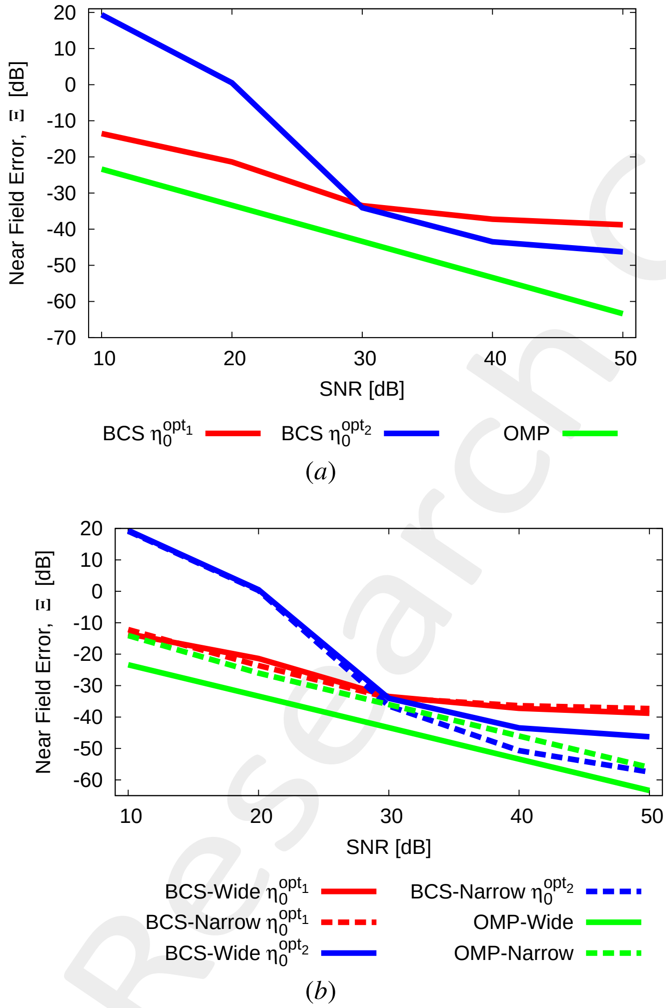

Figure 13: (*a*) Near Field Error comparison between original (*OMP*) and alternative (*BCS*) MbD for different SNR values and (*b*) comparison between the results

$SNR\,[dB]$	*Near Field Error*, Ξ $[dB]$		
	BCS		OMP
	$\eta_0^{opt_1}$	$\eta_0^{opt_2}$	
50	-38.79	-46.28	-63.37
40	-37.23	-43.47	-53.37
30	-33.47	-34.04	-43.37
20	-21.38	0.51	-33.37
10	-13.53	19.36	-23.37

Table IV: Near Field Errors obtained by the original (*OMP*) and alternative (*BCS*) MbD

Observations

Considering Fig. 13 (*a*):

- In this test case, the *OMP* algorithm outperforms the *BCS* solver whatever the *SNR* value, achieving an error $\Xi < -20\,[dB]$ even for the lower considered *SNR* value, $SNR = 10\,[dB]$;
- About the *BCS* algorithm:
  - using $\eta_0^{opt_1}$, the *BCS* solver obtains an error that is more or less $10\,[dB]$ higher than that of the *OMP* algorithm for $SNR \leq 30\,[dB]$ and $20 - 30\,[dB]$ higher for $SNR > 30\,[dB]$. However, this *BCS* version performs highly better than the other one until $SNR = 30\,[dB]$ and then the results are comparable;
  - using $\eta_0^{opt_2}$, the *BCS* algorithm obtains results which are the worst for $SNR \leq 30\,[dB]$ and a little bit better than the other *BCS* version, but always worse than those of the *OMP* algorithm.

Considering Fig. 13 (b):

- The increase of the failure ranges to build the over-complete basis does not have a great impact on the results; in particular:
  - the *OMP* presents a performance improvement of about $7\,[dB]$ with respect to the same test case without enlarged ranges;
  - The BCS achieves the same result when $\eta_0^{opt_1}$ is used and slightly ($\sim\ 8\,[dB]$) worse results for $SNR \geq 40\,[dB]$ when $\eta_0^{opt_2}$ is used.

Estimated Near-Field

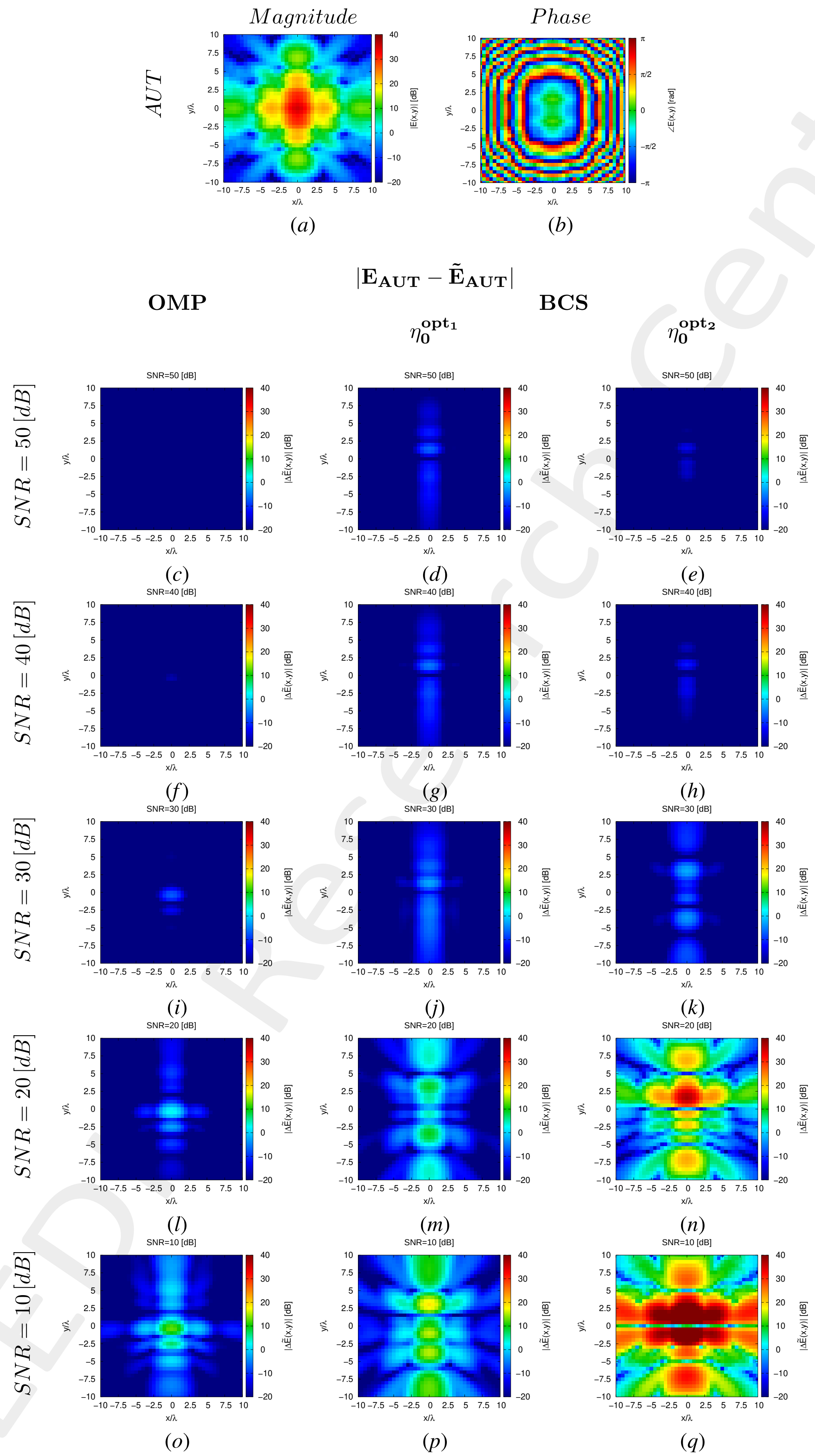

Figure 14: Magnitude difference between the actual and estimated $2-D$ near-field pattern when processing noisy measurements at different SNRs.

Estimated Coefficients

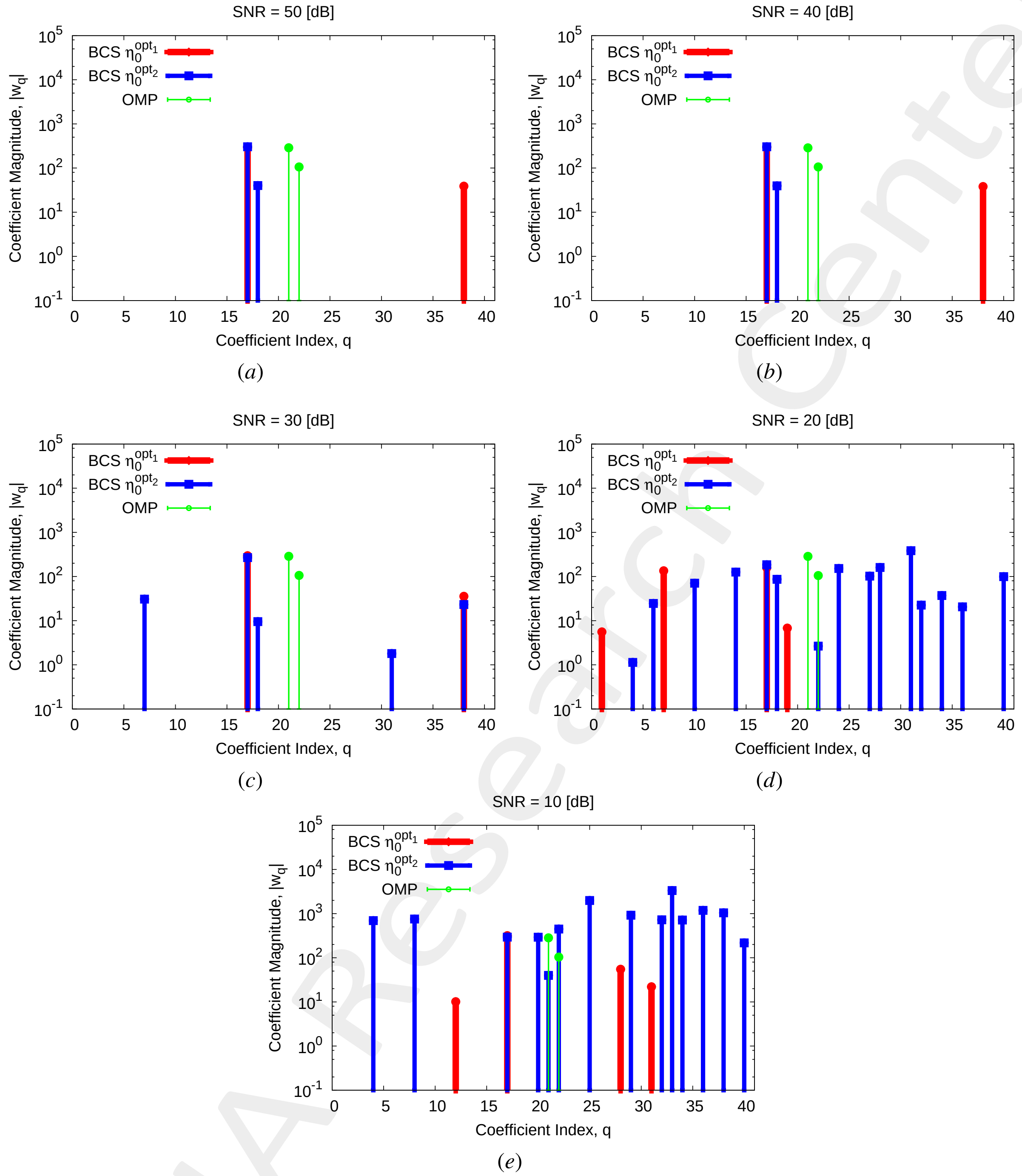

Figure 15: Coefficient comparison between original (*OMP*) and alternative (*BCS*) MbD : (*a*) $SNR = 50\,[dB]$, (*b*) $SNR = 40\,[dB]$, (*c*) $SNR = 30\,[dB]$, (*d*) $SNR = 20\,[dB]$, (*e*) $SNR = 10\,[dB]$

Observations

- The *OMP* solver selects always the same vectors (vector indexes $q = 21,\ 22$), independently from the SNR value, which are related to phase failures and different from those selected by the *BCS* algorithm;
- The *BCS* algorithm presents solutions that are not sparse for low SNR values, $SNR = 20\,[dB]$ and $SNR = 10\,[dB]$, when $\eta_0^{opt_2}$ is used. Both the two *BCS* versions select always a same vector (vector index $q = 17$), but the overall vector selection is different.

1.2.2 OMP vs best BCS

The main idea of this section is to compare the performance of the *OMP* algorithm and the best *BCS* configuration.

Near-Field Error

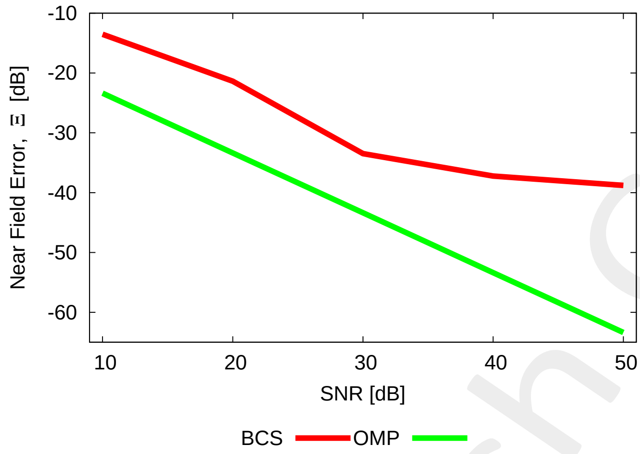

Figure 16: Near Field Error comparison between original (*OMP*) and alternative (*BCS*) MbD for different SNR values.

$SNR\,[dB]$	*Near Field Error,* Ξ $[dB]$	
	BCS	OMP
50	-38.79	-63.37
40	-37.23	-53.37
30	-33.47	-43.37
20	-21.38	-33.37
10	-13.53	-23.37

Table V: Near Field Errors obtained by the original (*OMP*) and alternative (*BCS*) MbD

Estimated Far-Field

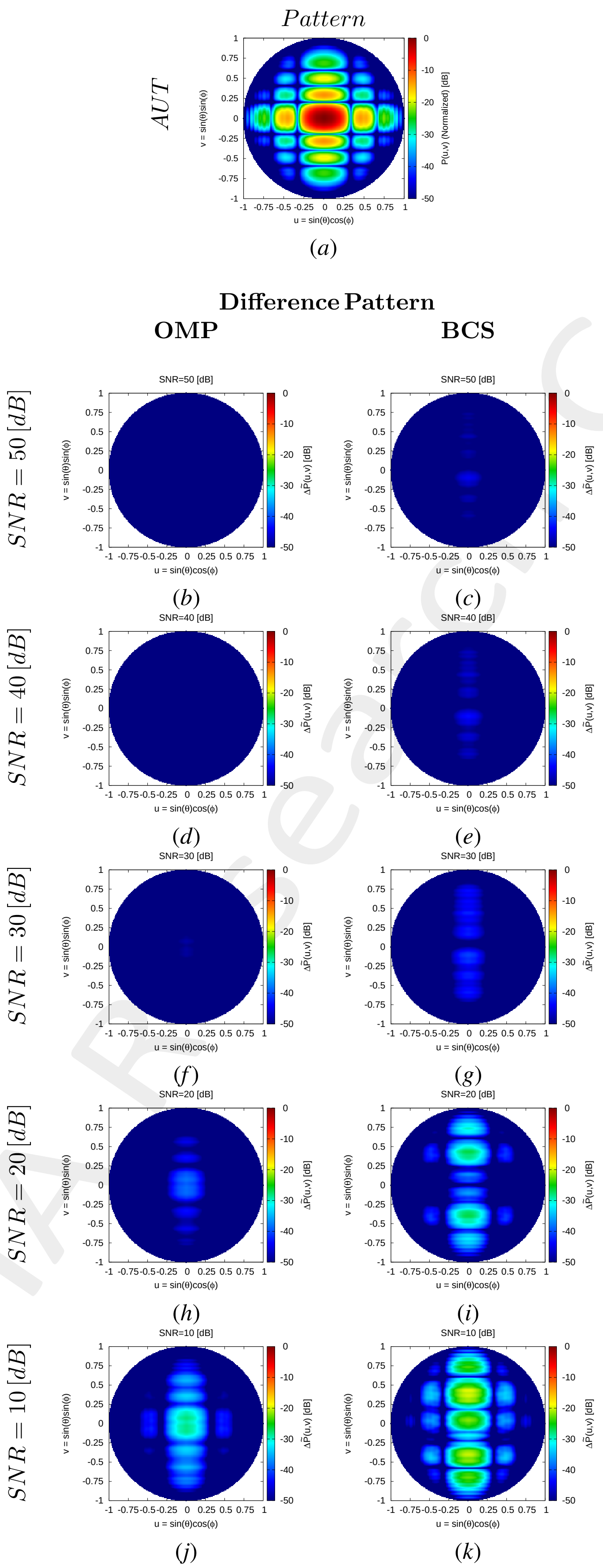

Figure 17: Difference between the actual and estimated $2-D$ far-field pattern when processing noisy measurements at different SNRs.

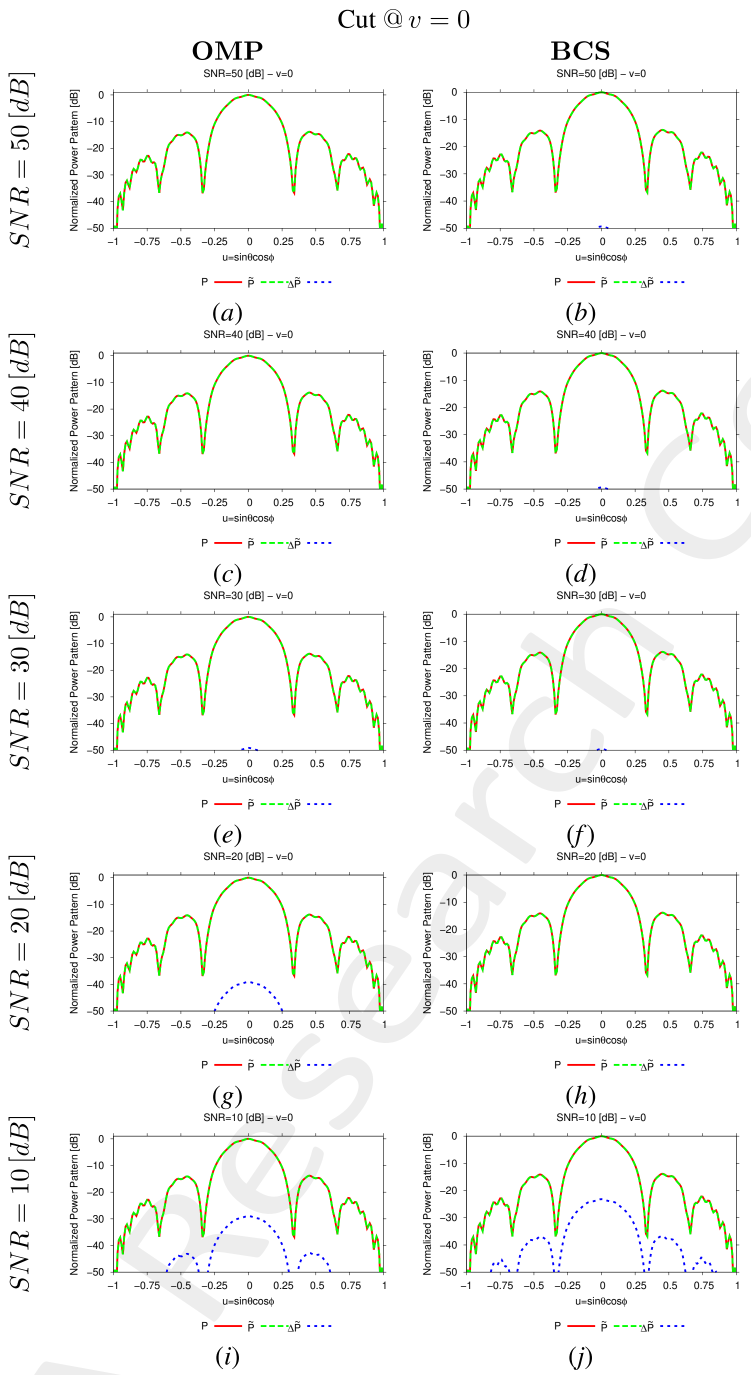

Figure 18: $1-D$ cuts of the estimated far-field pattern (obtained through near-to-far-field transformation from the estimated near-field patterns) under several noisy conditions

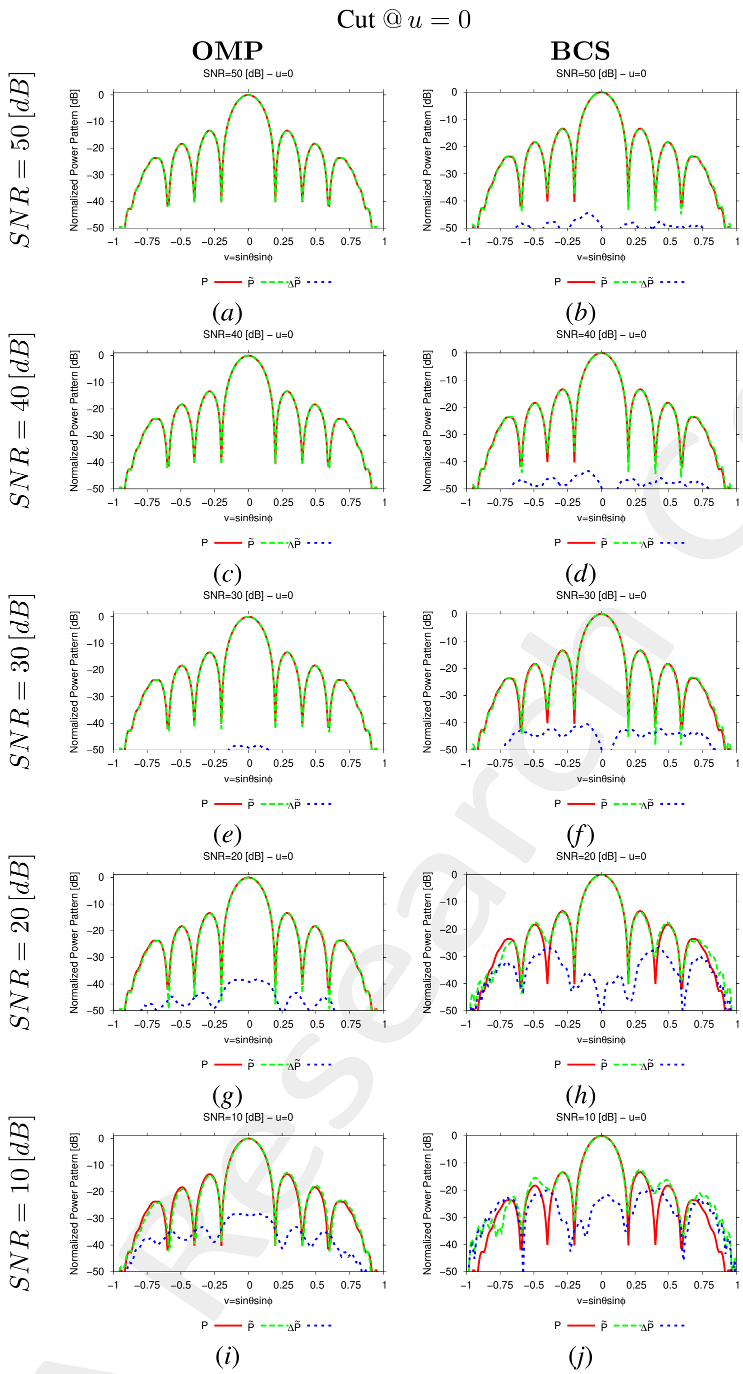

Figure 19: $1-D$ cuts of the estimated far-field pattern (obtained through near-to-far-field transformation from the estimated near-field patterns) under several noisy conditions

$SNR\,[dB]$	$Far - Field\,Error,\ \chi\ [dB]$	
	BCS	OMP
50	-39.86	-64.36
40	-38.35	-54.36
30	-34.70	-44.34
20	-22.90	-34.30
10	-14.86	-24.18

Table VI: Far-field matching error between the actual and estimated AUT patterns (both obtained through near-to-far-field transformation from the corresponding near-field patterns) under several noisy conditions.

Estimated Coefficients

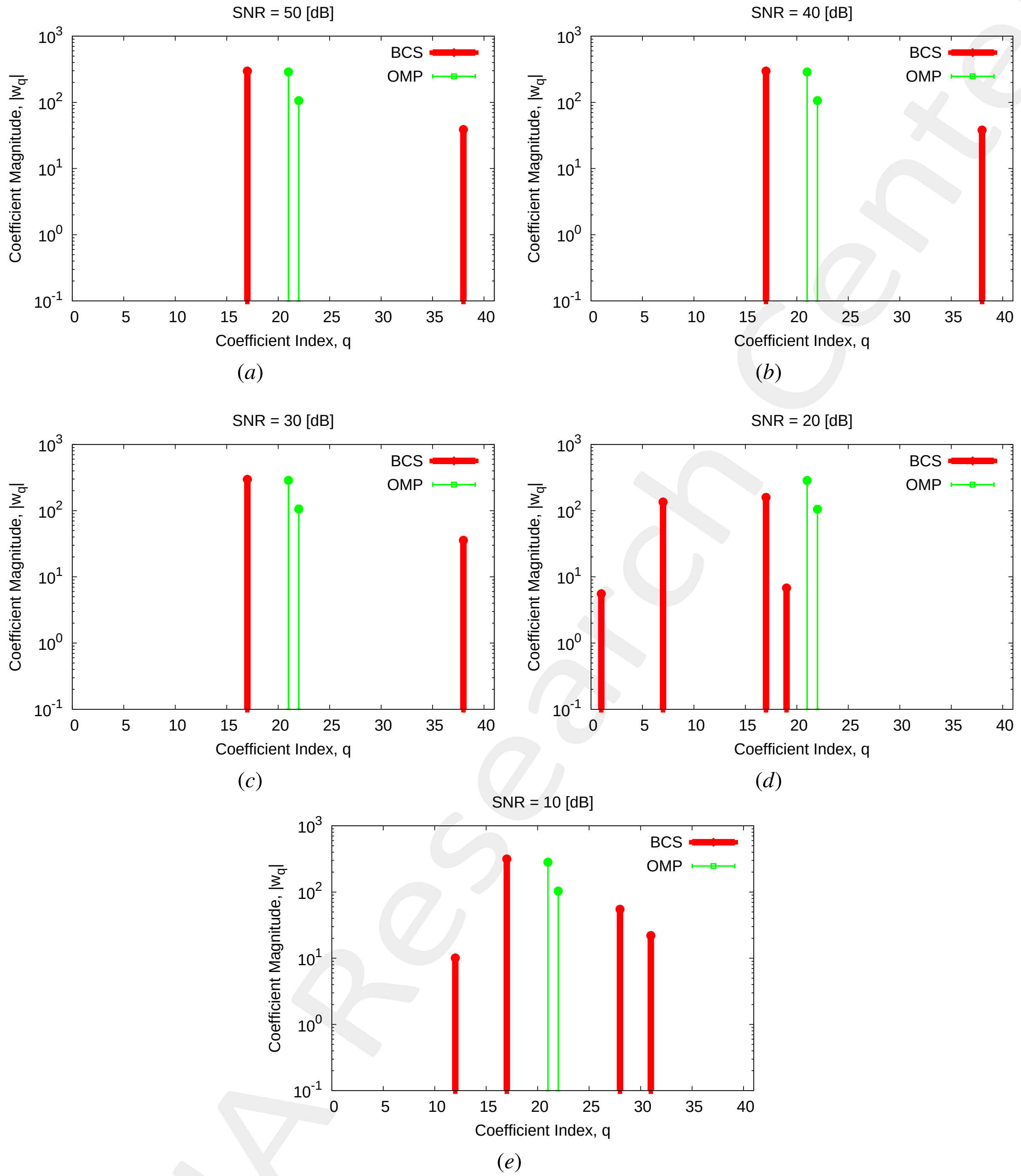

Figure 20: Coefficient comparison between original (*OMP*) and alternative (*BCS*) MbD: (*a*) $SNR = 50\,[dB]$, (*b*) $SNR = 40\,[dB]$, (*c*) $SNR = 30\,[dB]$, (*d*) $SNR = 20\,[dB]$, (*e*) $SNR = 10\,[dB]$

More information on the topics of this document can be found in the following list of references.

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