Advancements in Near-Field Antenna Characterization: A Compressive Sensing Perspective

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1 Test Case 2: *AUT* with a magnitude failure and phase shift affecting a single row; incremented failure ranges to build the over-complete basis ($\nu^{(s)} \in [0.0,\, 1.0]\,,\ F^{(s)} = 7$ and $\gamma^{(s)} \in [-\pi,\, \pi]\,,\ P^{(s)} = 5$)

1.1 Failures of the *AUT* 3rd row ($\nu^{(3)} = 0.45,\, \gamma^{(3)} = \frac{\pi}{3}$)

Parameters

Gold Antenna (Without Defects)

- Geometry : Planar array of microstrip patches on the (x, y) plane;
- Working Frequency : $f = 3.6\,[GHz]$ ($\lambda = 83.27 \times 10^{-3}\,[m]$ in free space);
- Substrate (PEC-backed) :
  - Dimensions : infinite;
  - Relative Permittivity : $\varepsilon_{r,sub} = 4.7$;
  - Loss Tangent : $tan\,\delta_{sub} = 0.014$;
  - Thickness : $h_{sub} = 0.019\,[\lambda]\ (1.6\,[mm])$;
- Microstrip patches :
  - Dimensions : $l_x \approx 0.22\,[\lambda]\ (18.16\ [mm]),\ l_y \approx 0.33\,[\lambda]\ (27.25\ [mm])$;
  - Feeding : pin-fed;
- Spacing between elements : $d_x = d_y = \frac{\lambda}{2}$;
- Number of elements in each row : $N_x = 6$;
- Number of elements in each column : $N_y = 10$;
- Total number of elements : $N = (N_x \times N_y) = 60$;
- Total size of the antenna : $L_x = 5\,[\lambda],\ L_y = 9\,[\lambda]$;
- Element excitations : $w_n^{(s)} = 1.0 + j0.0,\ n = 1, ..., N^{(s)},\ s = 1, ..., S$;

Antenna Under Test (AUT - With Defects)

1. Failures of the excitation magnitude of the 3^{rd} row;
   - Failure factor of the elements in the 3^{rd} row ($s = 3$) : $\nu^{(3)} = 0.45$;
2. Failures of the excitation phase of the 3^{rd} row;

- Phase shift of the elements in the 3^{rd} row $(s = 3)$: $\gamma^{(3)} = \frac{\pi}{3}\,[rad]$;

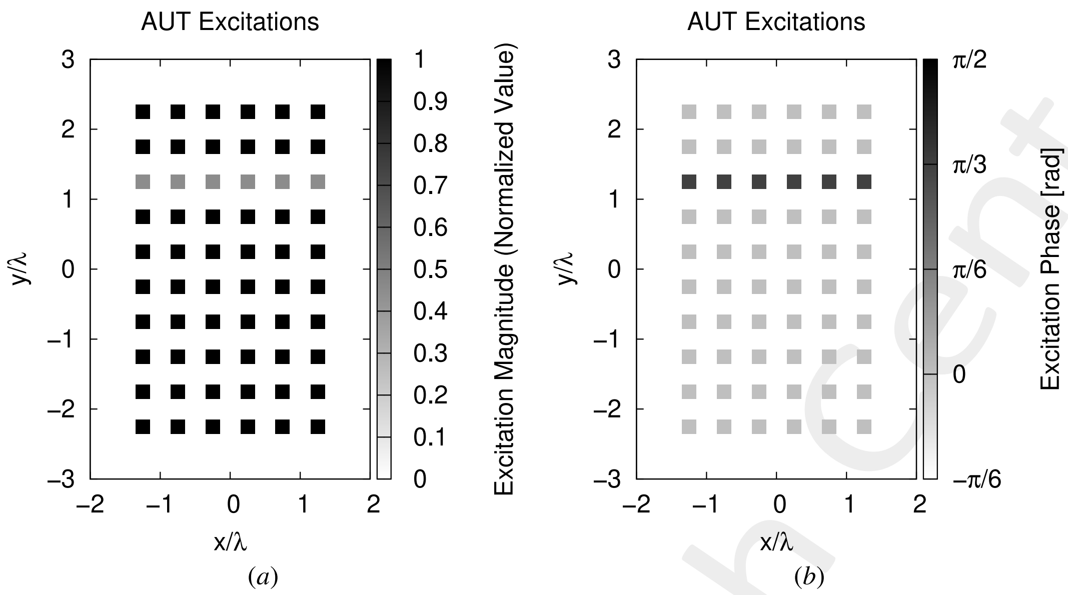


Figure 1: (a) Magnitude of the element excitations in the *AUT* $(\nu^{(3)} = 0.45)$, (b) phase of the element excitations in the *AUT* $(\gamma^{(3)} = \frac{\pi}{3}\,[rad])$.

Measurement Set-Up

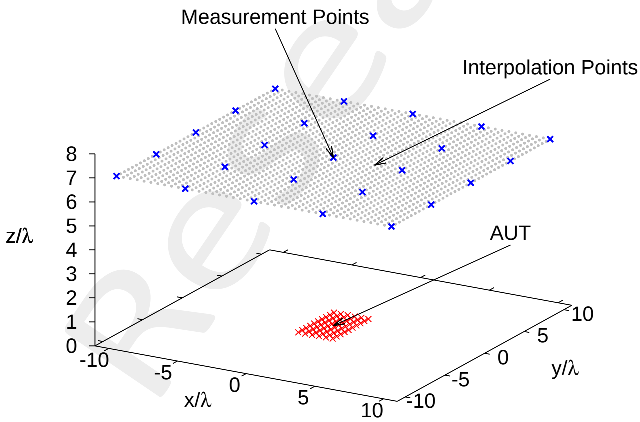


Figure 2: Disposition of the interpolation points $(T = 1681)$ and of the measurement points $(M = 25)$ in the near-field region of the *AUT*

- Type of measurements : near-field;
- Height of the measurement region : $H = 7\,[\lambda]$;
- Interpolation points :
  - Number of points : $T = 41 \times 41 = 1681$;
  - Coordinates : $x_t \in [-10, 10]\,[\lambda],\ y_t \in [-10, 10]\,[\lambda],\ z_t = H\,[\lambda],\ t = 1, ..., T$;
  - Interpolation step : $\Delta^{int}_{x/y} = 0.5\,[\lambda]$;
- Measurement points :

- Coordinates : $x_m^{meas} \in [-10,\, 10]\,[\lambda],\; y_m^{meas} \in [-10,\, 10]\,[\lambda],\; z_m^{meas} = H\,[\lambda],\; m = 1, ..., M;$
- Number of points : $M_{x/y} = 5 \rightarrow M = 25;$
- Measurement step : $\Delta_{x/y}^{meas} = 5\,[\lambda]$
- Ratio between number of measurements and total number of elements : $(M/N) = 0.42;$

Measurement-by-Design Technique

- Number of generated bases : $B = 20;$
- Bases $b = 1, ..., 10$: magnitude failures in each row ($s = 1, ..., 10$)
  - **Failure factor of the elements :** $\nu^{(s)} \in [0.0,\, 1.0],\; s = 1, ..., 10;$
  - Number of simulated failure factors : $F^{(s)} = 7,\; s = 1, ..., 10;$
- Bases $b = 11, ..., 20$: phase failures in each row ($s = 1, ..., 10$)
  - **Phase shift of the elements :** $\gamma^{(s)} \in [-\pi,\, \pi]\,[rad],\; s = 1, ..., 10;$
  - Number of simulated phase shifts: $P^{(s)} = 5,\; s = 1, .., 10;$
- Threshold on the singular values magnitude (normalized) : $\eta = -40\,[dB];$
- Total number of simulated AUT configurations : $K = S \times (F^{(s)} + P^{(s)}) = 10 \times (7 + 5) = 120;$

Dimension of the Over-Complete Basis

The dimension of the over-complete basis is

$$Q = 40$$

This number is given by the sum of the vectors belonging to the two considered bases:

1. Magnitude failures : $Q_1, ..., Q_{10} = 2;$
2. Phase failures : $Q_{11}, ..., Q_{20} = 2.$

Alternative (*BCS*) MbD parameters

- Toleration factor for *BCS* solver: $Tolerance = 1 \times 10^{-8};$
- Initial noise variance for *BCS* solver: $\eta_0^{opt_1} = 10^{-2}$ and $\eta_0^{opt_2} = 5 \times 10^{-4}$. This values have been obtained as a result of a calibration procedure;

Original (*OMP*) MbD parameters

- Max. number of iterations of the OMP algorithm : $I = \{1;\, 2;\, 3;\, ...;\, 10\};$

- Selected iteration to report the results: $I = 6$; this choice is justified by the fact that at this iteration the *OMP* algorithm reaches the best near field error as shown in the following Fig. 3.

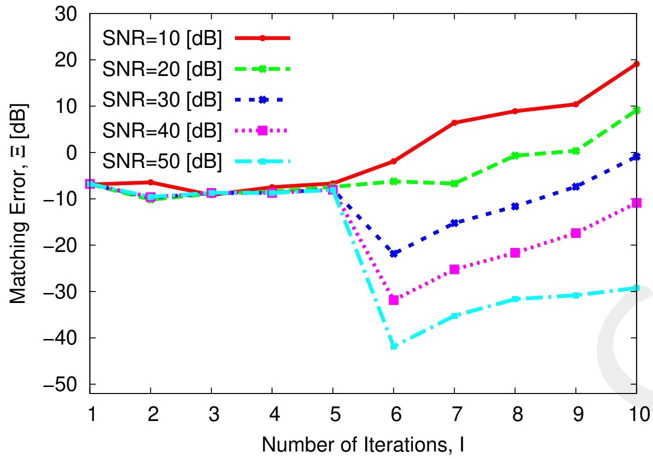


Figure 3: Behaviour of the near-field matching error versus the number of *OMP* iterations, I.

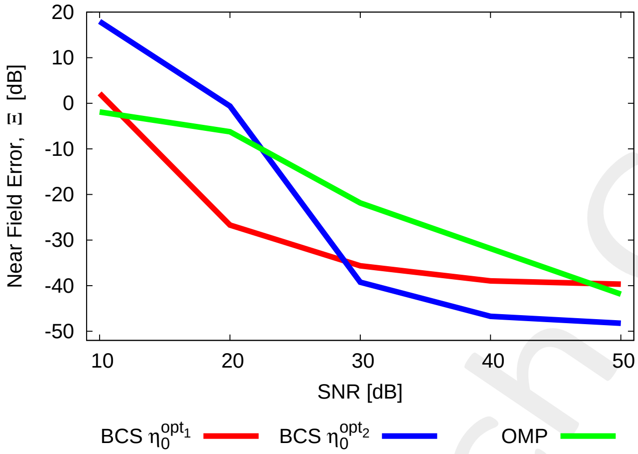
Noise

- SNR on the measured data : $SNR = \{50;\ 40;\ 30;\ 20;\ 10\}\ [dB]$;
- Noise seed : $Noise\_Seed = 11$.

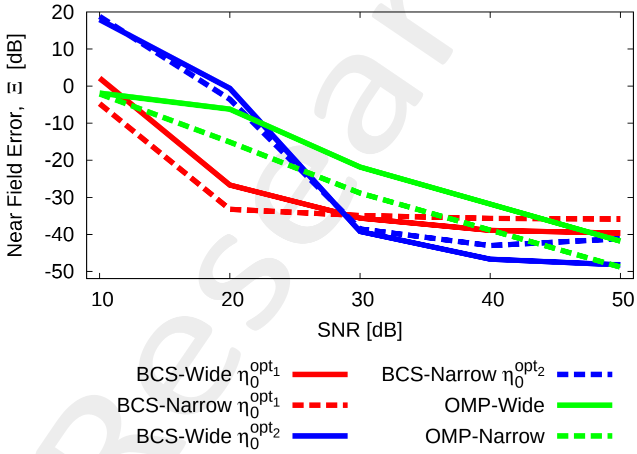
1.1.1 Comparison between original (*OMP*) and alternative (*BCS*) MbD

Near-Field Error

The comparison, in terms of near field error, between the original (*OMP*) and the alternative (*BCS*) MbD is reported in the following Fig. 4.



(*a*)



(*b*)

Figure 4: (*a*) Near Field Error comparison between original (*OMP*) and alternative (*BCS*) MbD for different SNR values and (*b*) comparison between the results

$SNR\,[dB]$	*Near Field Error,* Ξ $[dB]$		
	BCS		OMP
	$\eta_0^{opt_1}$	$\eta_0^{opt_2}$	
50	-39.67	-48.24	-41.85
40	-38.95	-46.72	-31.85
30	-35.54	-39.24	-21.85
20	-26.70	-0.61	-6.23
10	2.16	17.96	-1.90

Table I: Near Field Errors obtained by the original (*OMP*) and alternative (*BCS*) MbD

Observations

Considering Fig. 4 (*a*) :

- The *OMP* algorithm performs poorly until $SNR = 30\,[dB]$ from which starts to obtain an error $\Xi < -20\,[dB]$ which linearly decreases as the SNR value increases; nevertheless, among the used solvers and in the considered test case, it is possible to evaluate the *OMP* as the algorithm that reaches the worst result;
- about the *BCS* solver:
  - using $\eta_0^{opt_1}$, the *BCS* reaches the best results since it outperforms the other algorithm at low SNR values (i.e. $SNR < 30\,[dB]$) and presents an error comparable to the others for higher SNR values;
  - using $\eta_0^{opt_2}$, the *BCS* obtains results that are worse or comparable with the others for $SNR < 30\,[dB]$, but for $SNR \geq 30\,[dB]$ it achieves better results than the other methods, in particular at $SNR = 30\,[dB]$ it presents an error which is more or less $18\,[dB]$ lower than that of the *OMP;*

Considering Fig. 4 (b) :

- The increase of the failure ranges considered to build the over-complete basis involves a performance deterioration of the *OMP* algorithm; instead, the *BCS* solver seems to be quite insensitive to this change since the results are essentially the same.

Estimated Near-Field

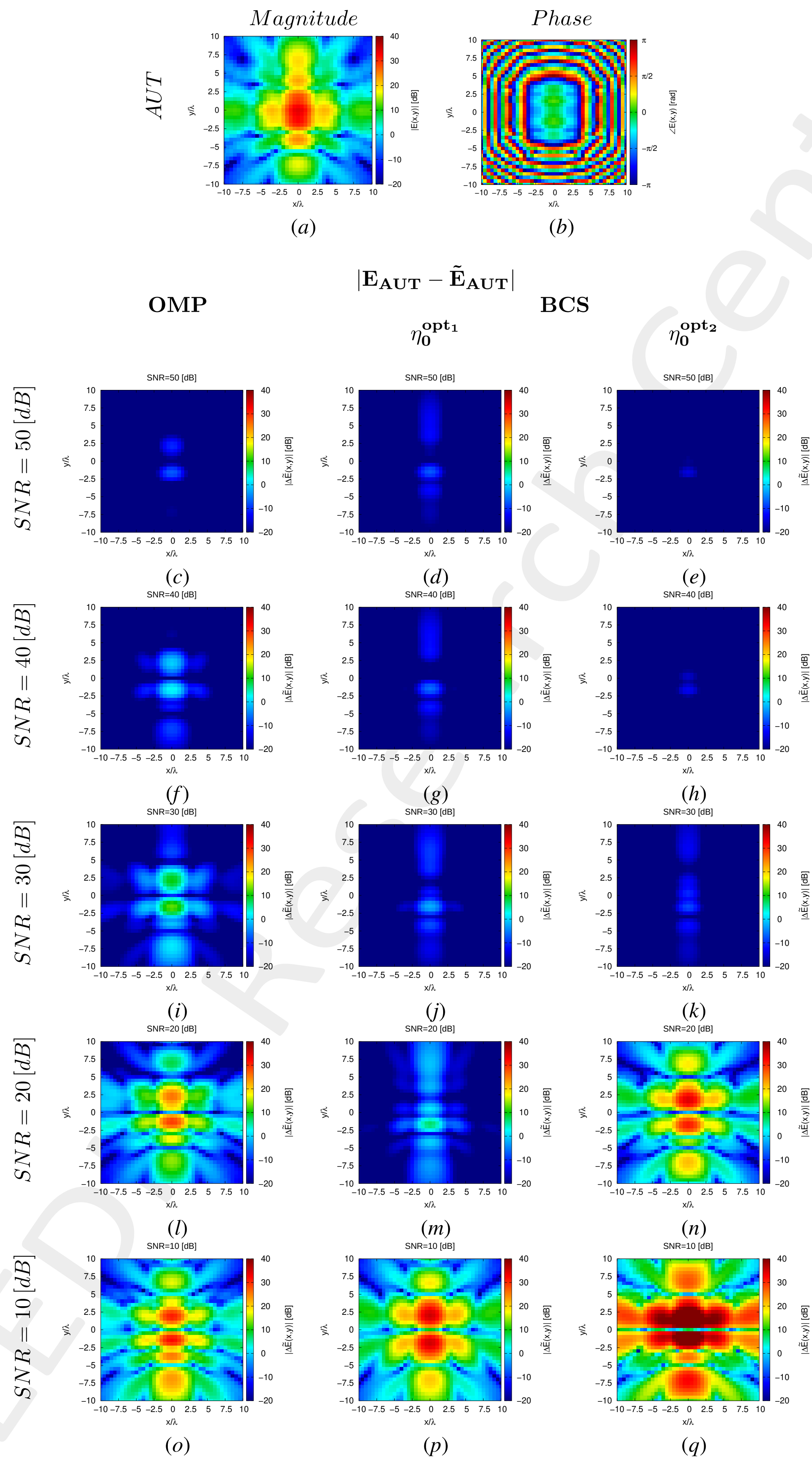


Figure 5: Magnitude difference between the actual and estimated $2-D$ near-field pattern when processing noisy measurements at different SNRs.

Estimated Coefficients

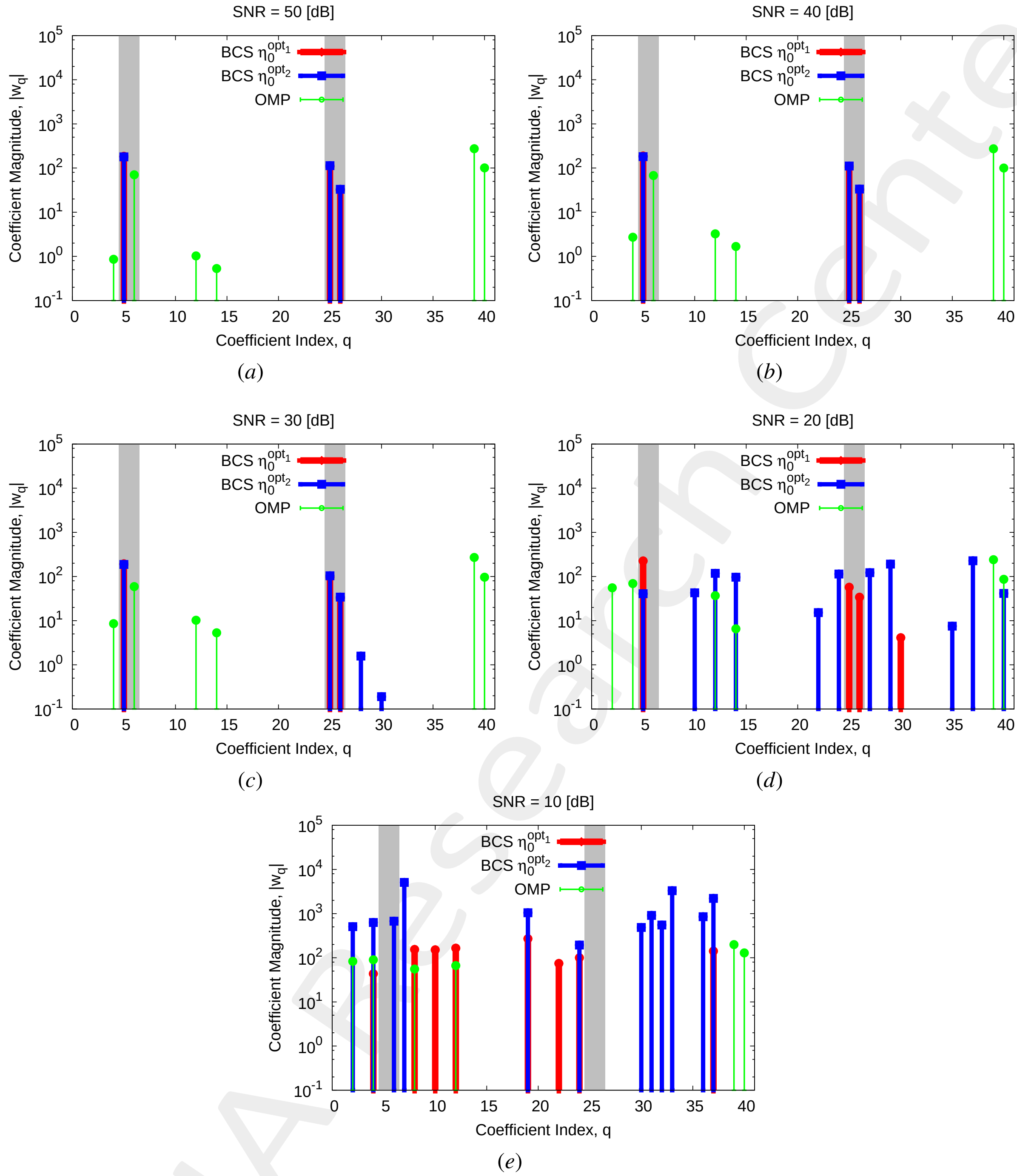


Figure 6: Coefficient comparison between original (*OMP*) and alternative (*BCS*) MbD : (*a*) $SNR = 50\,[dB]$, (*b*) $SNR = 40\,[dB]$, (*c*) $SNR = 30\,[dB]$, (*d*) $SNR = 20\,[dB]$, (*e*) $SNR = 10\,[dB]$

Observations

- The *OMP* algorithm is able to select only the vector concerning the magnitude failure affecting the 3^{rd} row of the *AUT* for $SNR \geq 30\,[dB]$ and none of the vectors associated to the phase failure. Moreover, the *OMP* chooses the same vectors for $SNR \geq 30\,[dB]$;
- The *BCS* algorithm obtains solutions which are not much sparse when the $SNR \in [10,\,20]$, in particular when $\eta_0^{opt_2}$ is used, but become sparse for $SNR \geq 30\,[dB]$; furthermore, the *BCS* correctly detects both magnitude and

phase failures, at first with low precision (i.e. for $SNR = 20\,[dB]$) and then with very high exactness, especially when $\eta_0^{opt_1}$ is used.

1.1.2 OMP vs best BCS

The main idea of this section is to compare the performance of the *OMP* algorithm and the best *BCS* configuration.

Near-Field Error

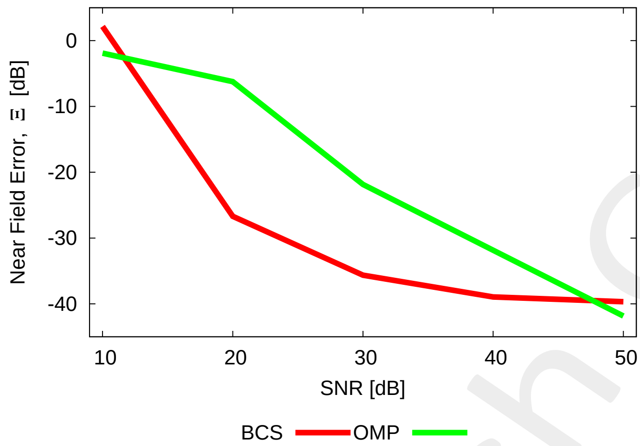


Figure 7: Near Field Error comparison between original (*OMP*) and alternative (*BCS*) MbD for different SNR values.

$SNR\,[dB]$	*Near Field Error*, Ξ $[dB]$	
	BCS	OMP
50	-39.67	-41.85
40	-38.95	-31.85
30	-35.54	-21.85
20	-26.70	-6.23
10	2.16	-1.90

Table II: Near Field Errors obtained by the original (*OMP*) and alternative (*BCS*) MbD

Estimated Far-Field

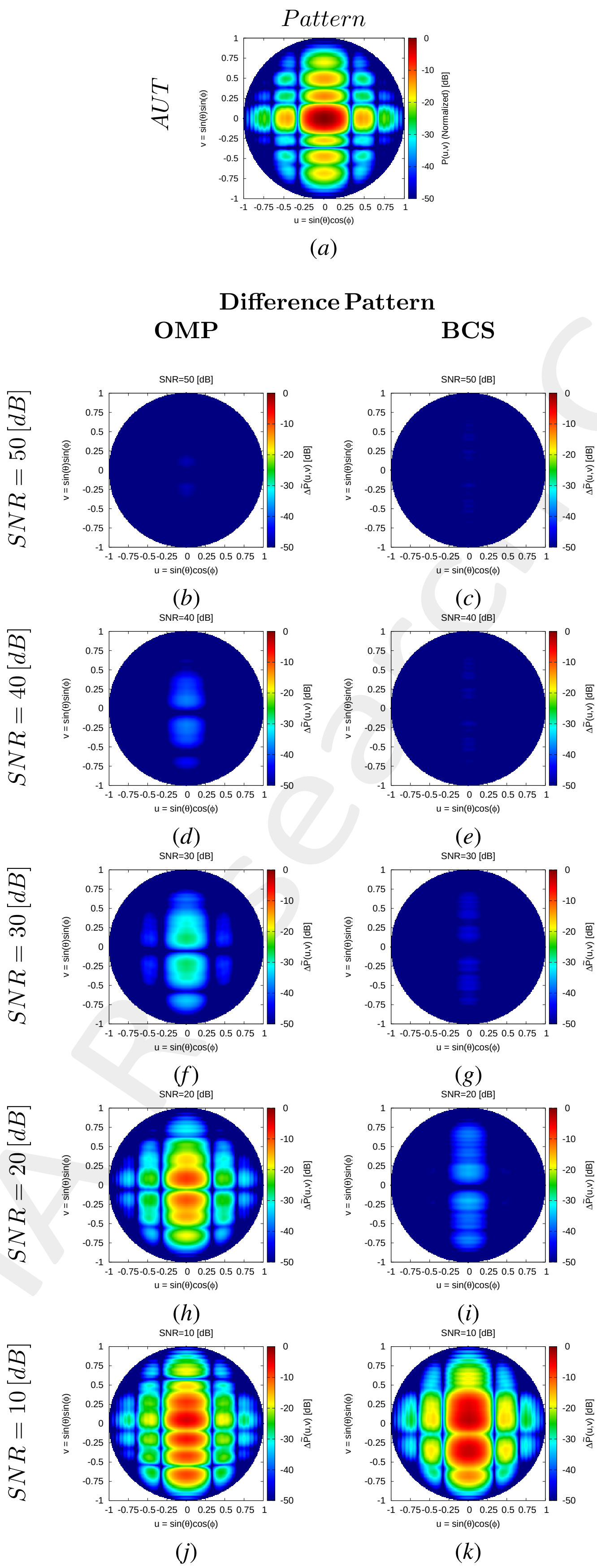


Figure 8: Difference between the actual and estimated $2-D$ far-field pattern when processing noisy measurements at different SNRs.

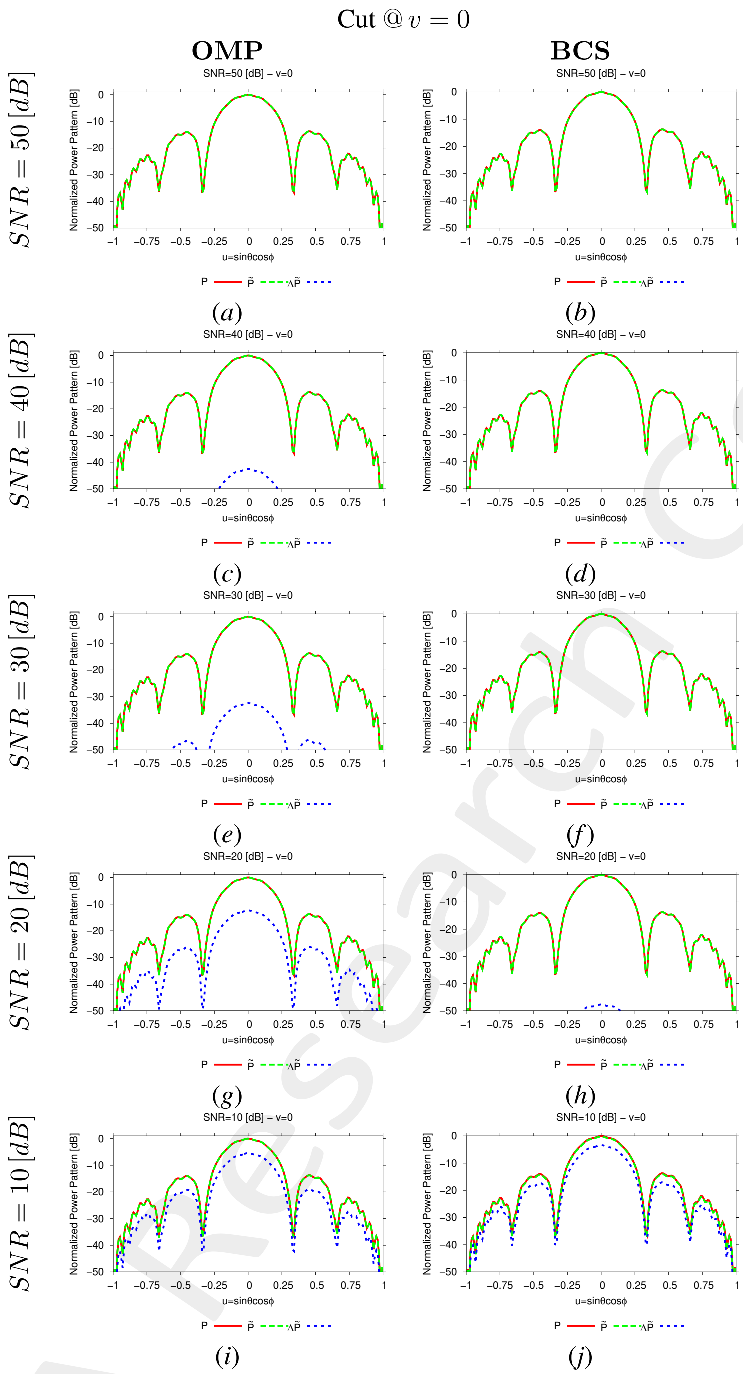


Figure 9: $1-D$ cuts of the estimated far-field pattern (obtained through near-to-far-field transformation from the estimated near-field patterns) under several noisy conditions

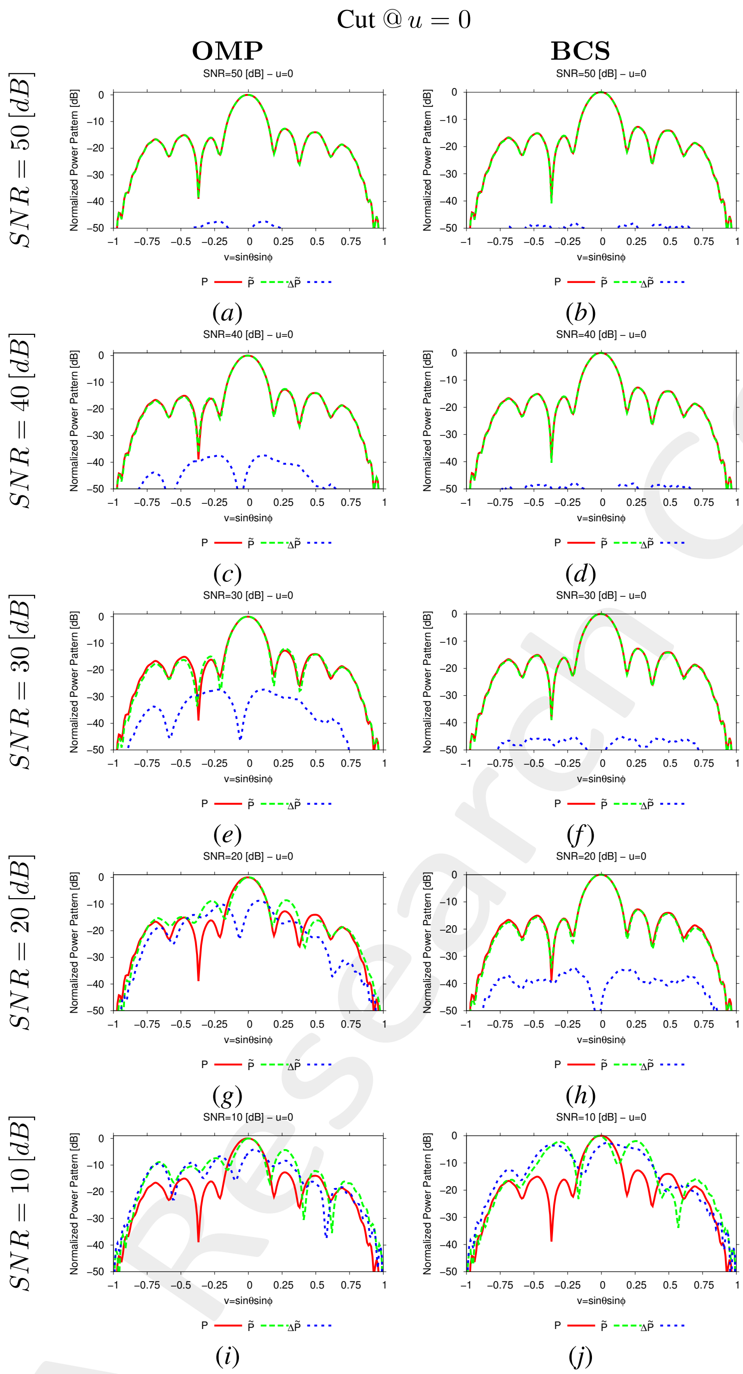


Figure 10: $1-D$ cuts of the estimated far-field pattern (obtained through near-to-far-field transformation from the estimated near-field patterns) under several noisy conditions

$SNR\,[dB]$	$Far - Field\,Error,\,\chi\;[dB]$	
	BCS	OMP
50	-41.27	-42.80
40	-40.91	-32.77
30	-38.28	-22.65
20	-29.17	-6.15
10	1.59	-1.44

Table III: Far-field matching error between the actual and estimated AUT patterns (both obtained through near-to-far-field transformation from the corresponding near-field patterns) under several noisy conditions.

Estimated Coefficients

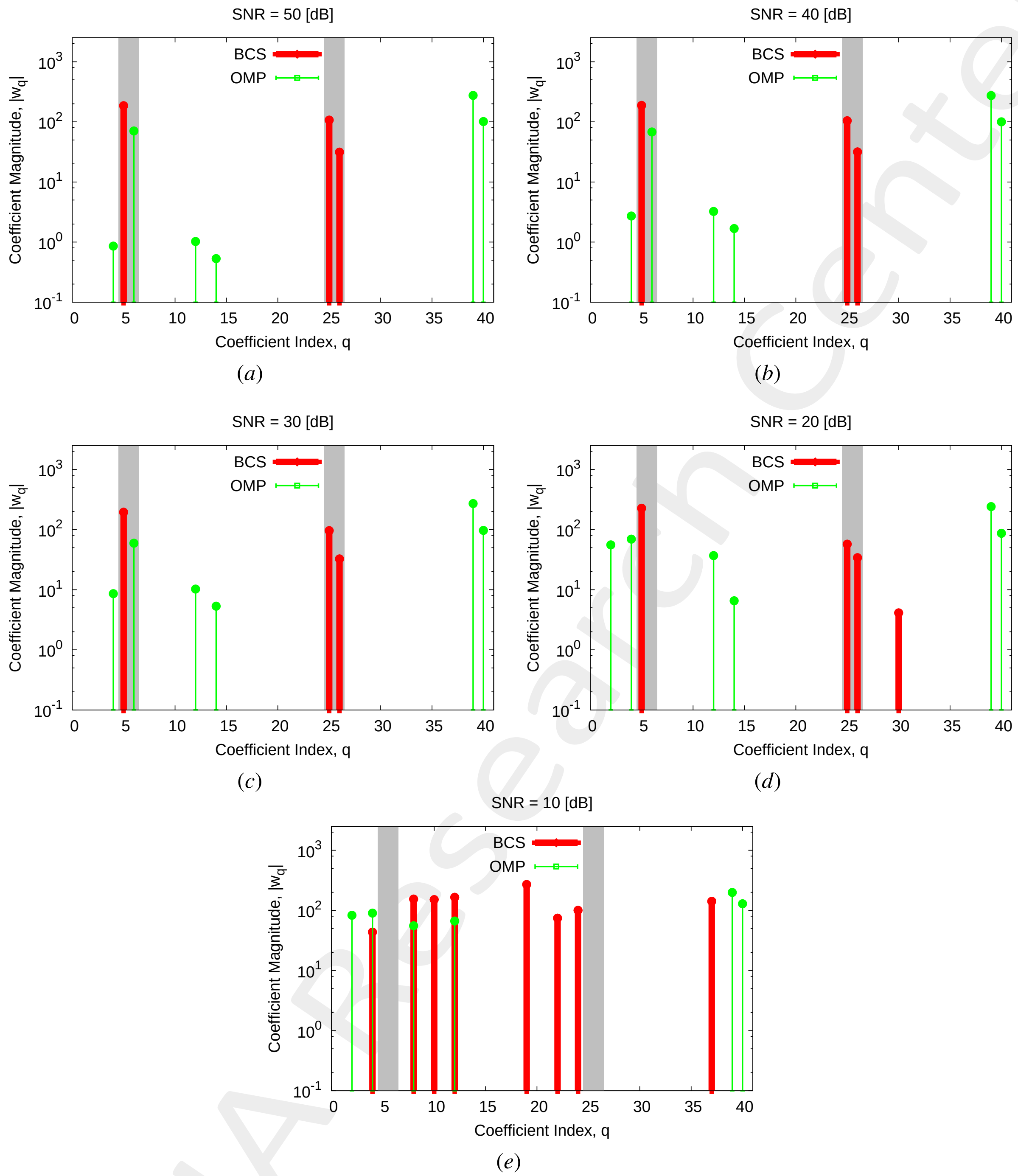


Figure 11: Coefficient comparison between original (*OMP*) and alternative (*BCS*) MbD: (*a*) $SNR = 50\,[dB]$, (*b*) $SNR = 40\,[dB]$, (*c*) $SNR = 30\,[dB]$, (*d*) $SNR = 20\,[dB]$, (*e*) $SNR = 10\,[dB]$

1.2 Failures of the *AUT* 9^{th} row ($\nu^{(9)} = 0.45$, $\gamma^{(9)} = \frac{\pi}{3}$)

Note: The simulation parameters are the same of those listed in previous section, except for the following ones:

- Index of the failed (subarray) row : $s = 9$;

Original (*OMP*) MbD parameters

- Max. number of iterations of the OMP algorithm : $I = \{1;\ 2;\ 3;\ ...;\ 10\}$;
- Selected iteration to report the results: $I = 10$; this choice is justified by the fact that at this iteration the *OMP* algorithm reaches the best near field error as shown in the following Fig. 12.

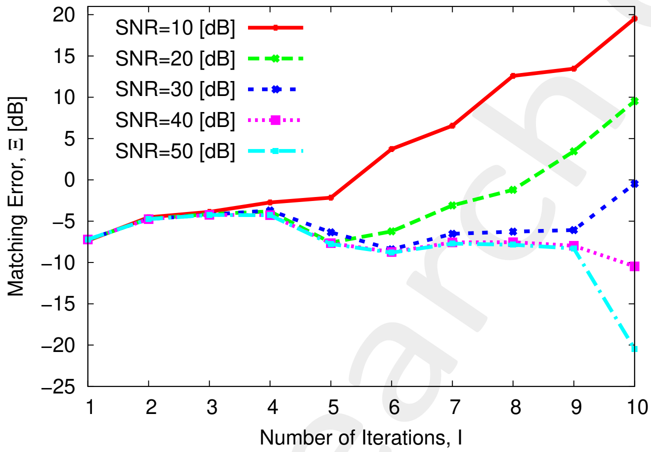
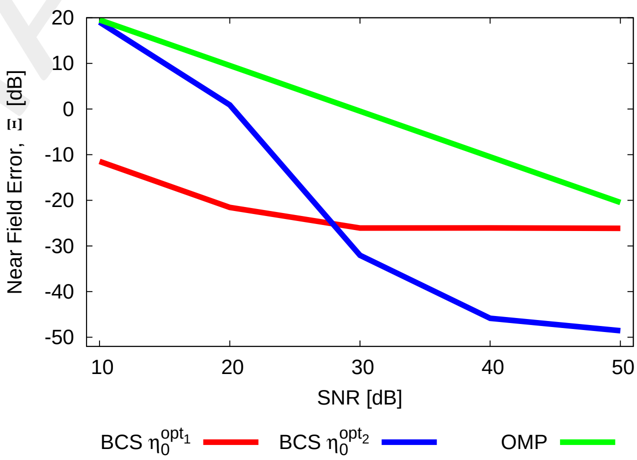


Figure 12: Behaviour of the near-field matching error versus the number of *OMP* iterations, I.

1.2.1 Comparison between original (*OMP*) and alternative (*BCS*) MbD

Near-Field Error

The comparison, in terms of near field error, between the original (*OMP*) and the alternative (*BCS*) MbD is reported in the following Fig. 13.



(a)

Figure 13: (a) Near Field Error comparison between original (*OMP*) and alternative (*BCS*) MbD for different SNR values

$SNR\,[dB]$	*Near Field Error,* Ξ $[dB]$		
	BCS		OMP
	$\eta_0^{opt_1}$	$\eta_0^{opt_2}$	
50	-26.12	-48.57	-20.47
40	-26.04	-45.85	-10.47
30	-26.07	-32.03	-0.47
20	-21.56	0.89	9.53
10	-11.46	19.04	19.53

Table IV: Near Field Errors obtained by the original (*OMP*) and alternative (*BCS*) MbD

Observations

- The *OMP* algorithm performs poorly whatever the SNR value and among the used solvers in the considered test case, it is possible to evaluate the *OMP* as the algorithm that reaches the worst result since it is not able to go below an error $\Xi \simeq 20\,[dB]$;
- about the *BCS* solver:
  - using $\eta_0^{opt_1}$, the *BCS* reaches the best results for low SNR values (i.e. $SNR < 30\,[dB]$) and presents an error $\Xi \simeq 26\,[dB]$ for higher SNR values, which is a better result if compared to that of *OMP* solver but not if compared to the other *BCS* version;
  - using $\eta_0^{opt_2}$, the *BCS* obtains results that are between those of the *OMP* and the other *BCS* for $SNR < 30\,[dB]$, but for $SNR \geq 30\,[dB]$ it outperforms the other methods, in particular at $SNR \geq 40\,[dB]$ it presents an error which is more than $20\,[dB]$ lower than the best of the others;

Estimated Near-Field

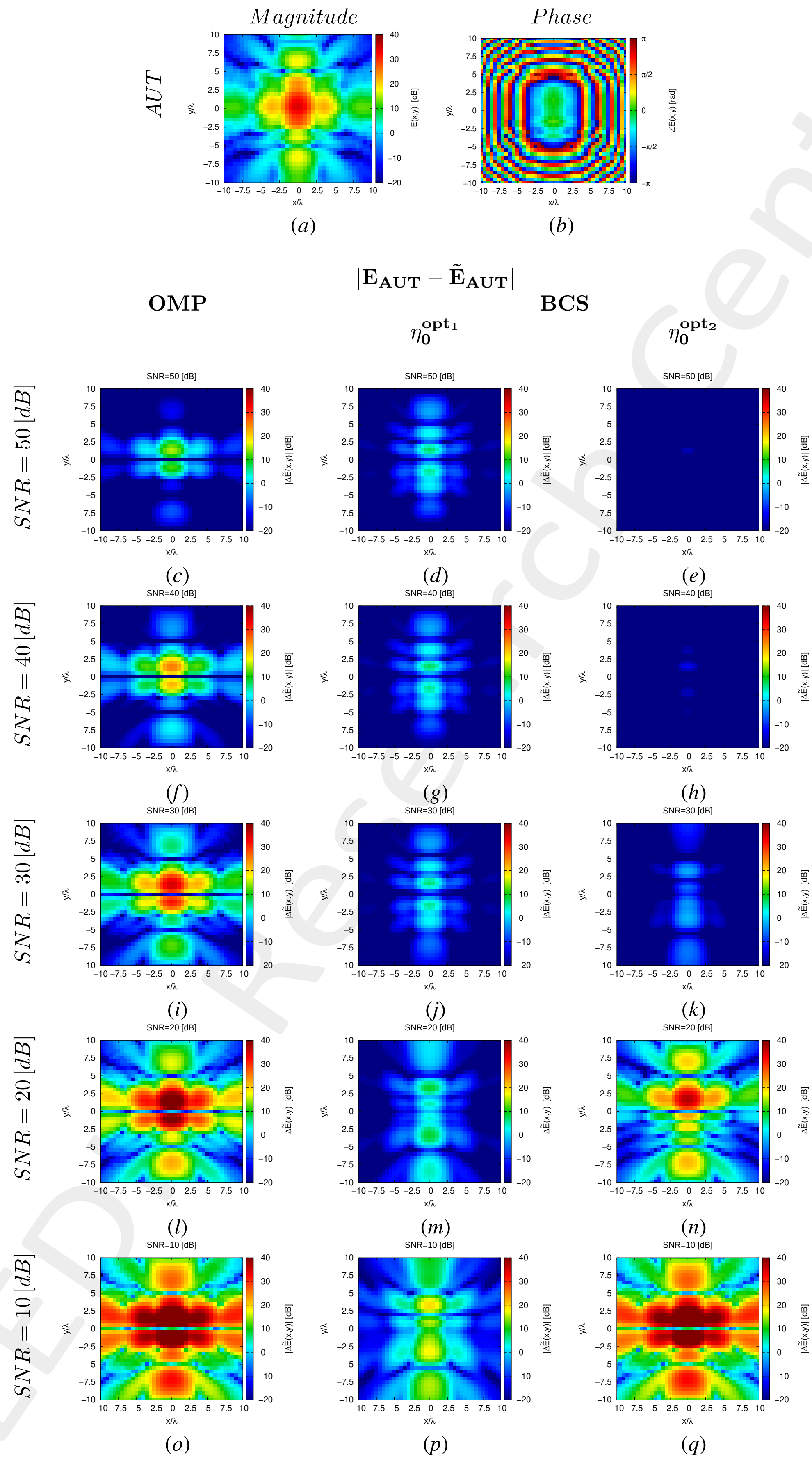


Figure 14: Magnitude difference between the actual and estimated $2-D$ near-field pattern when processing noisy measurements at different SNRs.

Estimated Coefficients

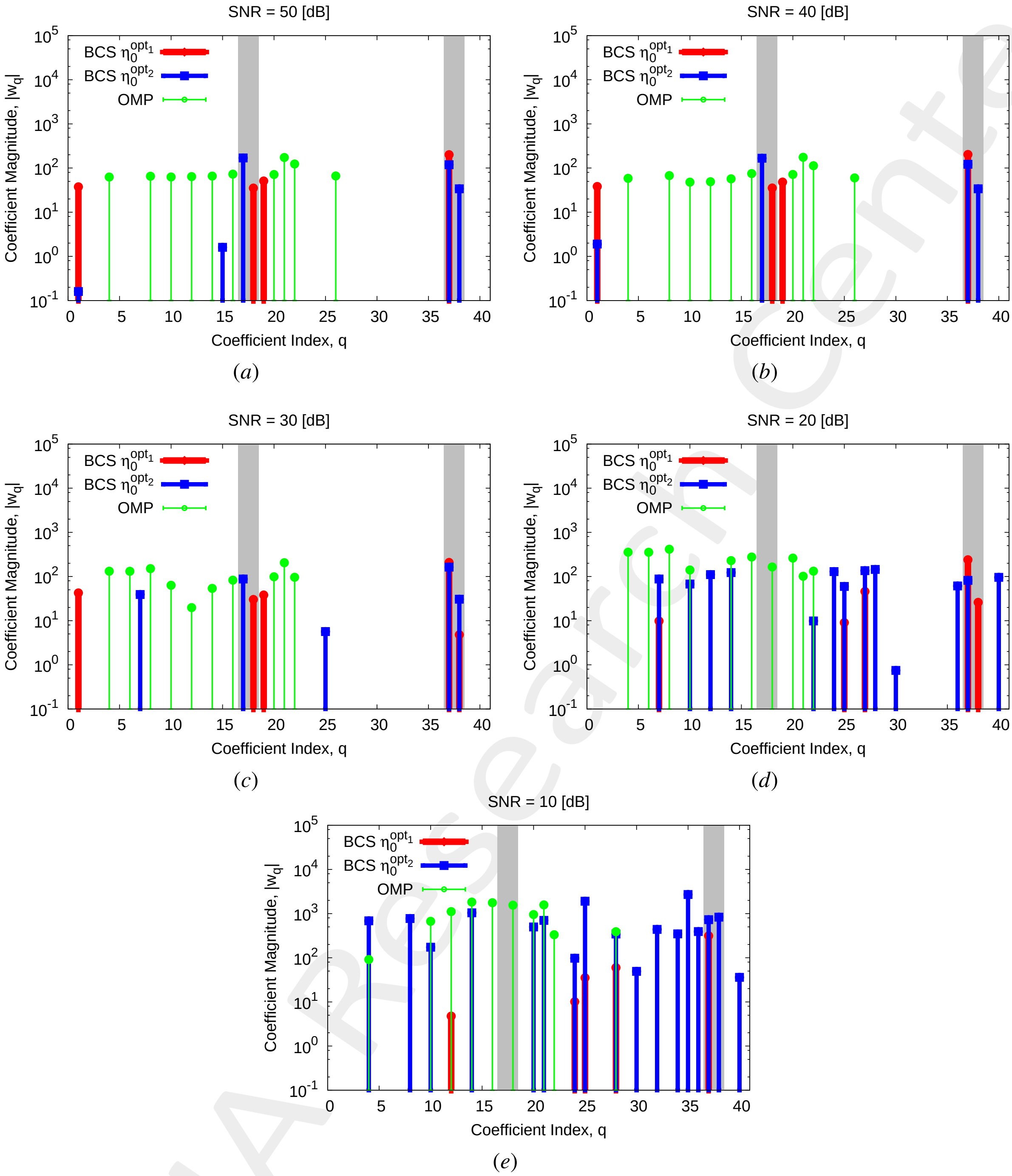


Figure 15: Coefficient comparison between original (*OMP*) and alternative (*BCS*) MbD : (*a*) $SNR = 50\,[dB]$, (*b*) $SNR = 40\,[dB]$, (*c*) $SNR = 30\,[dB]$, (*d*) $SNR = 20\,[dB]$, (*e*) $SNR = 10\,[dB]$

Observations

- The *OMP* algorithm is not able to select the vectors concerning the failures affecting the 9^{th} row of the *AUT* for $SNR \geq 20\,[dB]$ while for $SNR < 20\,[dB]$ it identifies the magnitude failure. In general, the *OMP* tends to select more vectors associated to magnitude failures rather than those related to phase failures;
- The *BCS* algorithm obtains solutions which are not much sparse when the $SNR = 10\,[dB]$, in particular when $\eta_0^{opt_2}$ is used, but become sparse for $SNR \geq 30\,[dB]$; furthermore, the *BCS* correctly detects both magnitude

and phase failures, at first with low precision (i.e. for $SNR = 30\,[dB]$) and then with very high exactness for $SNR \geq 40\,[dB]$, independently what value of η_0 is used.

1.2.2 OMP vs best BCS

The main idea of this section is to compare the performance of the *OMP* algorithm and the best *BCS* configuration.

Near-Field Error

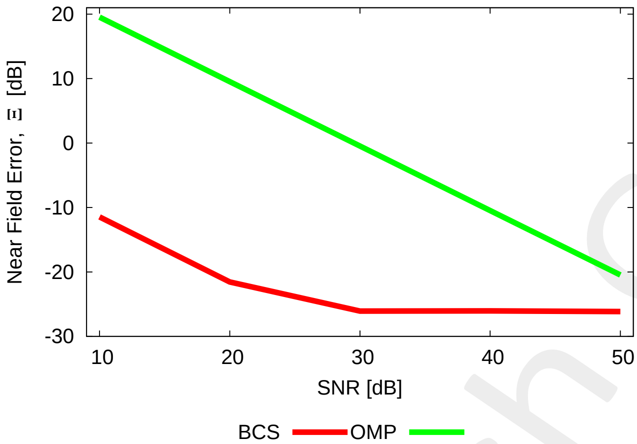


Figure 16: Near Field Error comparison between original (*OMP*) and alternative (*BCS*) MbD for different SNR values.

$SNR\,[dB]$	*Near Field Error,* Ξ $[dB]$	
	BCS	OMP
50	-26.12	-20.47
40	-26.04	-10.47
30	-26.07	-0.47
20	-21.56	9.53
10	-11.46	19.53

Table V: Near Field Errors obtained by the original (*OMP*) and alternative (*BCS*) MbD

Estimated Far-Field

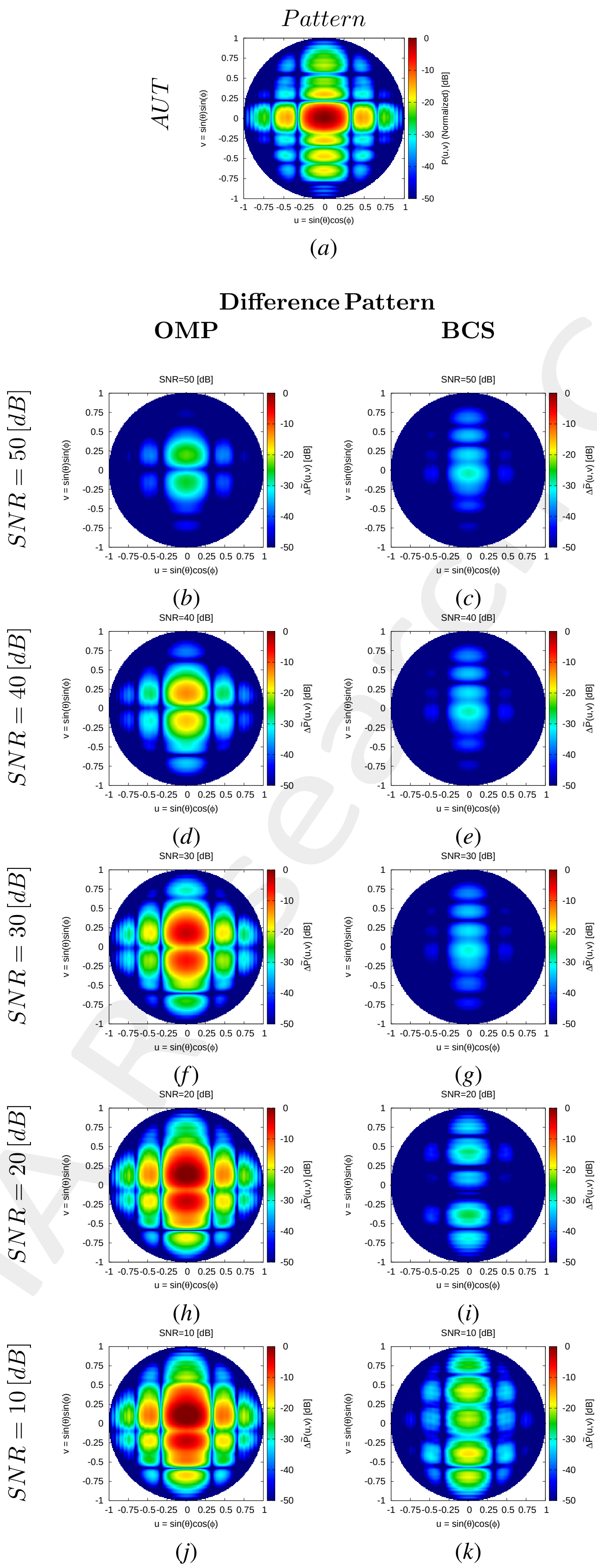


Figure 17: Difference between the actual and estimated $2-D$ far-field pattern when processing noisy measurements at different SNRs.

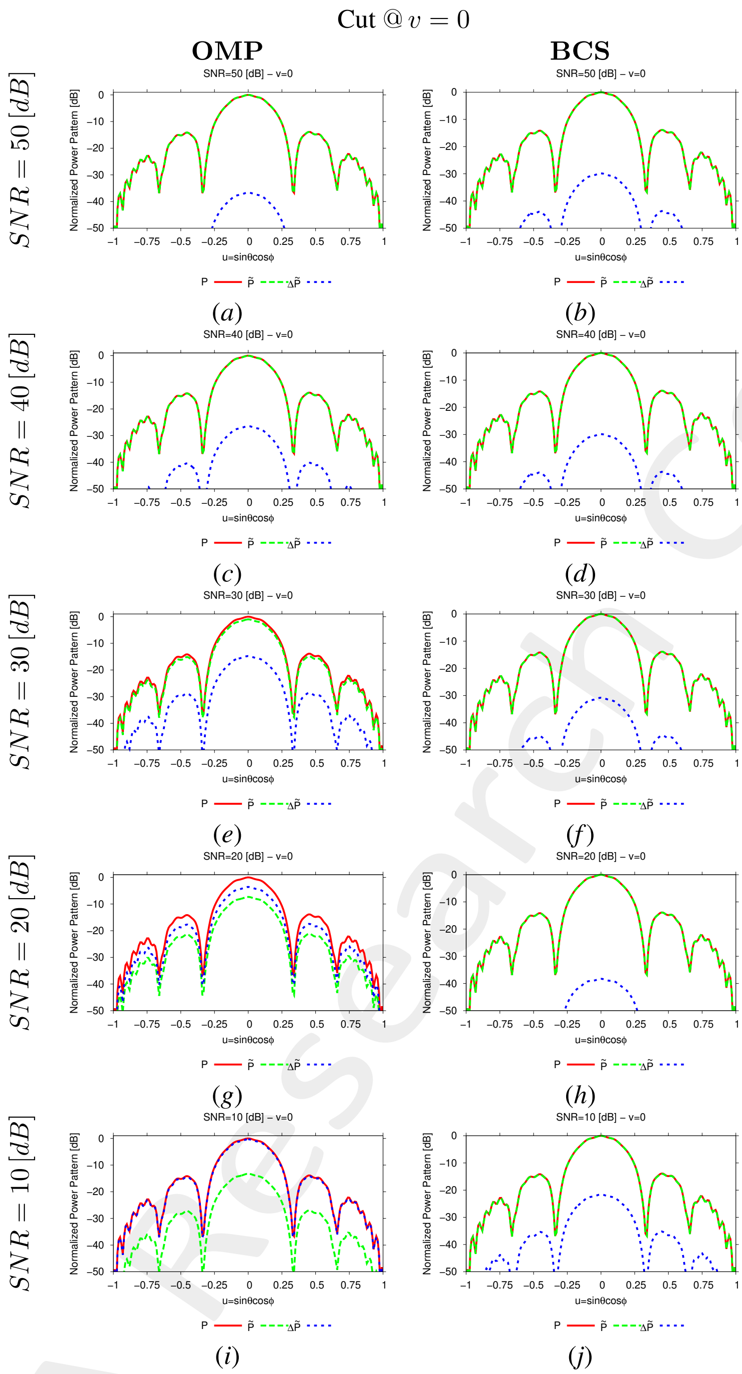


Figure 18: $1-D$ cuts of the estimated far-field pattern (obtained through near-to-far-field transformation from the estimated near-field patterns) under several noisy conditions

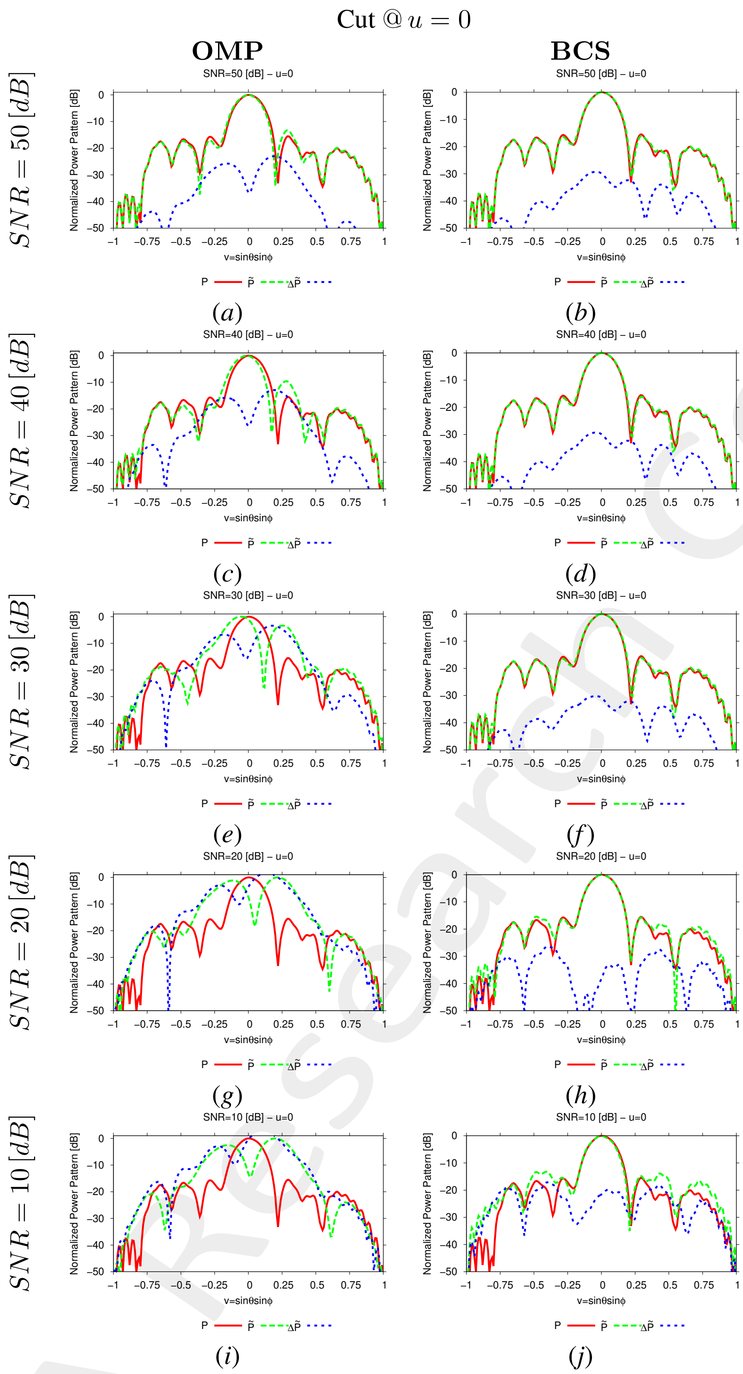


Figure 19: $1-D$ cuts of the estimated far-field pattern (obtained through near-to-far-field transformation from the estimated near-field patterns) under several noisy conditions

$SNR\,[dB]$	$Far-Field\,Error,\,\chi\,[dB]$	
	BCS	OMP
50	-26.45	-20.61
40	-26.36	-10.55
30	-26.50	-1.08
20	-23.01	3.67
10	-12.86	4.52

Table VI: Far-field matching error between the actual and estimated AUT patterns (both obtained through near-to-far-field transformation from the corresponding near-field patterns) under several noisy conditions.

Estimated Coefficients

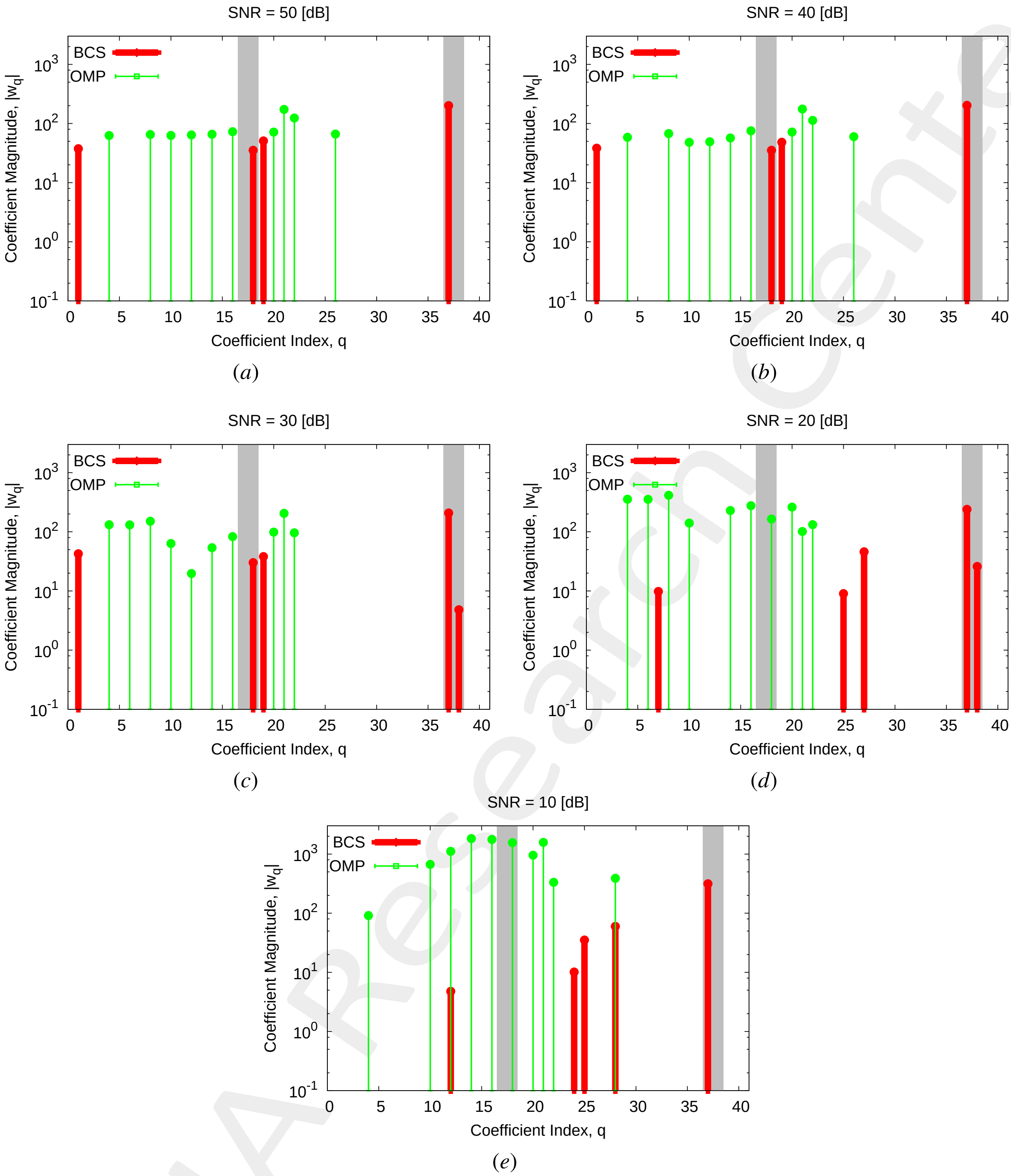


Figure 20: Coefficient comparison between original (*OMP*) and alternative (*BCS*) MbD: (*a*) $SNR = 50\,[dB]$, (*b*) $SNR = 40\,[dB]$, (*c*) $SNR = 30\,[dB]$, (*d*) $SNR = 20\,[dB]$, (*e*) $SNR = 10\,[dB]$

More information on the topics of this document can be found in the following list of references.

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