A Modular Approach for the Design of Phased Arrays Based on Self-Replicating L-Shaped Tiles

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2020/12/07

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1 Mathematical formulation

Let us consider a rectangular $M \times N$ planar array, where each element represents an isotropic radiator, equally spaced by d_x and d_y along the x and y axis respectively. The array is paritioned into tiles gathering contiguous clusters of elements over single amplifiers. Assuming that the array is partitioned into Q<M×N tiles, the far field pattern can be computed as

$$E(u,v) = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{q=1}^{Q} a_q \delta_{c_{mn}q} e^{j\frac{2\pi}{\lambda}(d_x u + d_y v)} \tag{1}$$

where $\underline{a} = \{a_q;\, q = 1,\ldots,Q\}$, q=1,...,Q are the tiles amplification coefficients, λ is the wavelength, while $u = \sin(\theta)\cos(\phi)$ and $v = \sin(\theta)\sin(\phi)$, (θ,ϕ) being the angular direction. In Eq. 1 the tiling configuration is defined by the vector $\underline{c} = \{c_{mn};\, m = 1,\ldots,M; n = 1,\ldots,N\}$ which associates to each array element the tile index $c_{mn} \in [1,\ldots,Q]$, $\delta_{c_{mn}q}$ being the Kronecker delta function, ($\delta_{c_{mn}q} = 1$ if the (m,n)-th element belongs to the (q)-th cluster, otherwise $\delta_{c_{mn}q} = 0$). Given a specific tiling of the array, the values of the amplifiers gains are obtained from a set of reference excitations amplitudes $\underline{a}^{ref} = \{a^{ref}_{mn}; m = 1,\ldots,M; n = 1,\ldots,N\}$ as in the following:

$$a_q = \frac{1}{\gamma_q}\sum_{m=1}^{M}\sum_{n=1}^{N} a^{ref}_{mn}\delta_{c_{mn}q}\,,\, q = 1,\ldots,Q$$

where $\gamma_q = \sum_{m=1}^{M}\sum_{n=1}^{N}\delta_{c_{mn}q}$ is the number of elements grouped by the (q)-th tile. The goal of the method is to achieve a radiated power pattern as compliant as possible to achieve a radiated power pattern as compliant as possible to a power pattern mask $\psi(u,v)$, while using the lowes number of tiles. The discrepancy between $\psi(u,v)$ and the radiated power pattern $P(u,v) = |E(u,v)|^2$ is given by the mask matching metric defined as:

$$\Gamma(u,v) = \int_{\Omega}[P(u,v) - \psi(u,v)]H\{P(u,v) - \psi(u,v)\}\mathrm{d}u\mathrm{d}v$$

To perform this task the chosen geometry for the tiles is the *L-tromino* (i.e., 3 cell L-shaped polyominoes). L-tromino can be tessellated using $\sigma = 4$ smaller L-trominos.

The rep-tiling procedure can be divided into three macro steps:

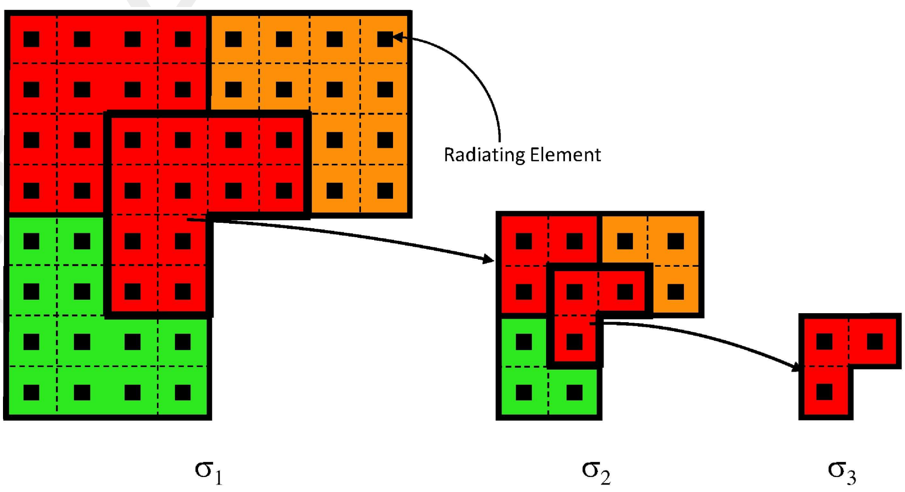

Figure 1: Three tiles levels, σ_1 with $\gamma_1 = 48$, σ_2 with $\gamma_2 = 12$ and, σ_3 with $\gamma_3 = 3$ elements

Step 1: The whole array is tiled using $Q^{(i)}|_{i=1}$ large tiles, grouping γ_i elements, i being the recursive step index ($\gamma_q^{(i)}|_{i=1} = 3 \times \sigma^{i_{max}-1}$, $q = 1, \dots, Q^{(i)}$, to allow i_{max} recursive steps for each cluster). The use of large tiles allows to exhaustivelu search the optimal configuration $\underline{c}_{opt}^{(i)}$ and tiles amplitudes $\underline{a}_{opt}^{(i)}$, solving the following minimization problem

$$(\underline{c}_{opt}^{(i)}, a_{opt}^{(i)}) = \text{arg min}[\Gamma(u, v; \underline{c}; \underline{a})]$$

The optimization problem is solved trough a nested optimization approach, exploiting the Algoritm-X for the generation of the whole set of existing L-tromino tilings, and using (2) for the computation of the clusters amplitude coefficients.

Step 2 If $\Gamma > \Gamma_{max}$ or $Q < Q_{max}$ the tiling step is incremented [$i \leftarrow (i+1)$] and a priority function is defined for each tile of the current iterative step as

$$\xi_q = \frac{1}{\xi_{max}} \sum_{m=1}^{M} \sum_{n=1}^{N} |\text{Re}\{a_{mn}^{ref} - a_q\} + j\text{Im}\{a_{mn}^{ref} - a_q\}| \delta_{c_{mn}q} \, , \, q = 1, \dots, Q^{(i)}$$

where ξ_{max} is the normalization factor. This function is used to evaluate amplitudes quantization errors with respect to the reference distribution for each tile.

Step 3 The priority values $\xi_q^{(i)}$, $q = 1, \dots, Q^{(i)}$ are sorted in a descending order, and the first K tiles corresponding to the first K sorted $\xi_q^{(i)}$ values are re-tiled using the respective smaller rep-tiles.

At this point if $\Gamma < \Gamma_{max}$and $Q < Q_{max}$ the second and third step are iterated until the requirement is met or the maximum number of clusters is reached, or alternatively when all clusters have reached the i_{max}level of clustering.

2 Array 6×9

L Tromino Tiling

Parameters

- Number of elements: 6×9 elements array, grouped in 18 clusters of 3
- Number of rows: 6
- Number of columns: 9
- Samples: $u \rightarrow 384$, $v \rightarrow 384$
- Evaluated tilings: T =3412
- Elements spacing: $dx = dy = 0.5\lambda$

The cost function only considers the mask matching.

Results

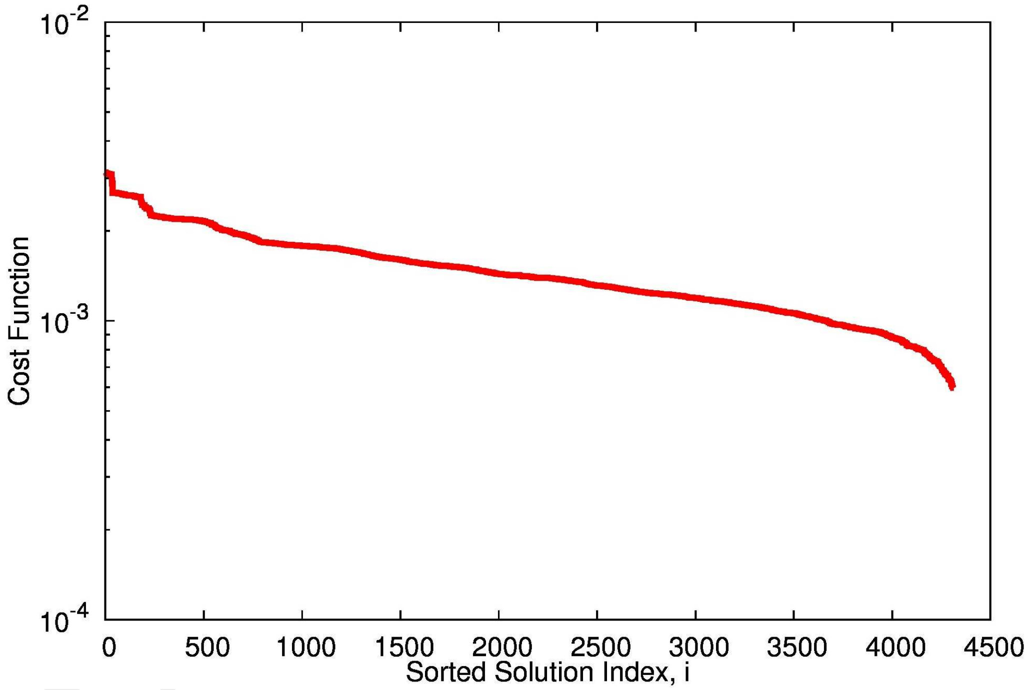

Figure 2: Solution index vs Cost function.

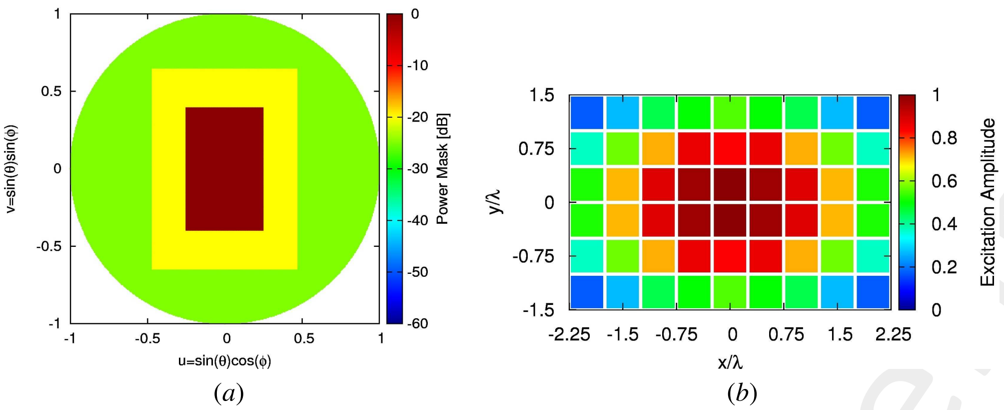

Figure 3: (*a*) Mask used for the computation of the cost function (*b*) Reference amplitudes.

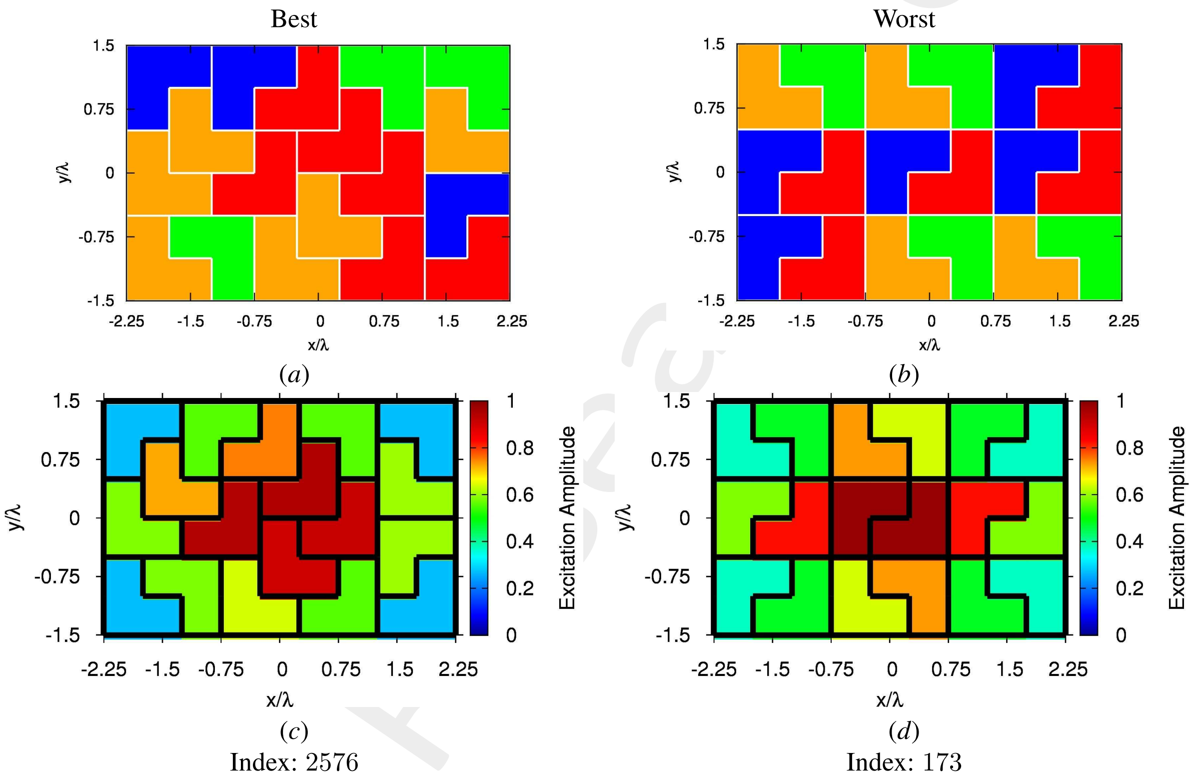

Figure 4: Numerical Assessment ($M = 6,\ N = 9$, $d = 0.5\lambda$, (θ_0,ϕ_0)=(0.0, 0.0) [deg]; $Q = 18$) - Plots of (*a*) optimal solution clustering and of the (*b*) worst solution clustering, with the respective (*c*) clustered excitations value for the best solution and (*d*) the clustered excitations for the worst performance solution.

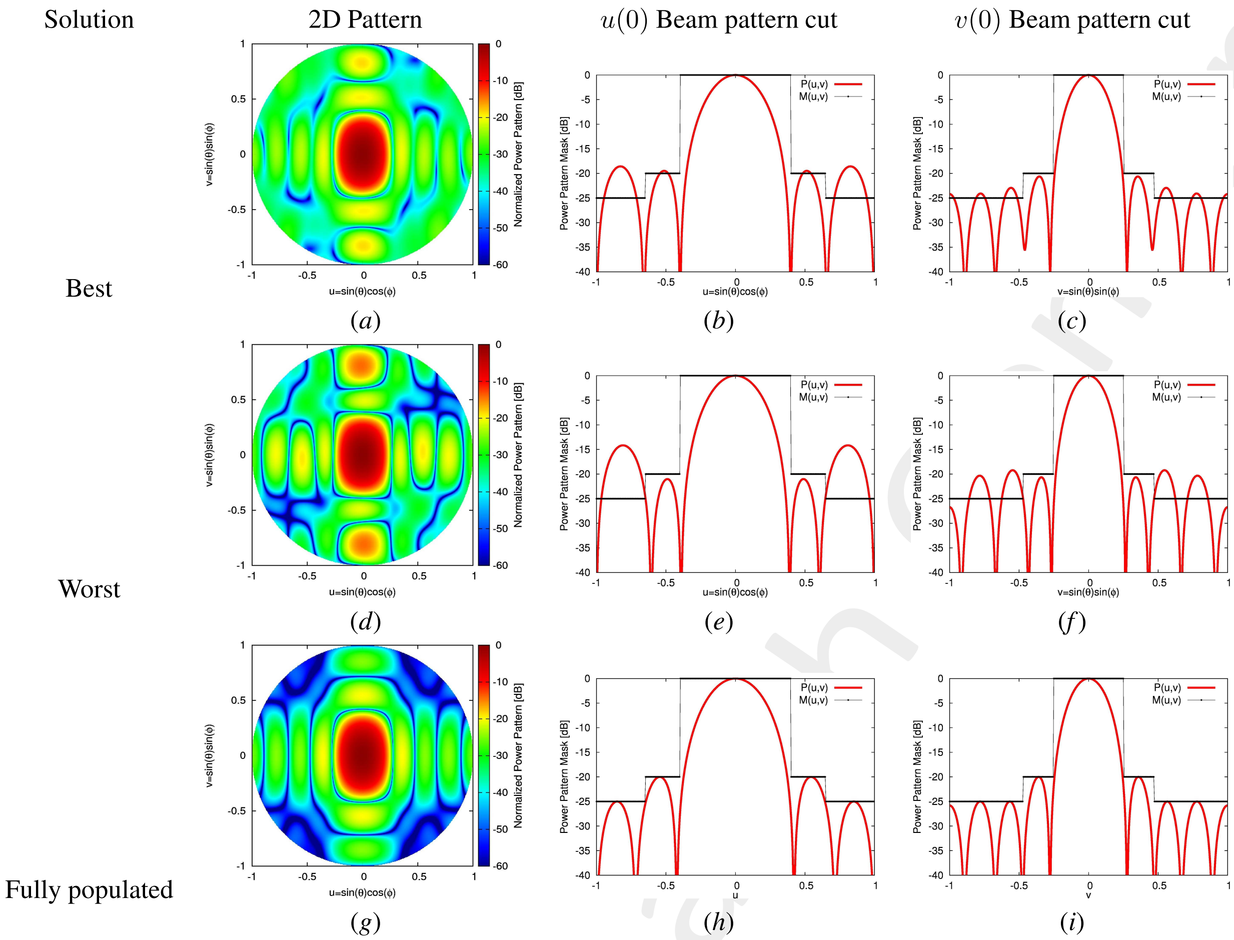

Figure 5: Numerical Assessment ($M = 6,\ N = 9$, $d = 0.5\lambda$, (θ_0,ϕ_0)=(0.0, 0.0) [deg]; $Q = 18$) - Plots of (*a*) the normalized power pattern radiated in the whole angular range ($-1 \leq u \leq 1$, $-1 \leq v \leq 1$) and along (*b*) the $\phi = 0$ [deg] and (*c*) the $\phi = 90$ [deg] planes for the best solution. Plots of (*d*) the normalized power pattern radiated in the whole angular range ($-1 \leq u \leq 1$, $-1 \leq v \leq 1$) and along (*e*) the $\phi = 0$ [deg] and (*f*) the $\phi = 90$ [deg] planes for the worst solution. Plots of (*g*) the normalized power pattern radiated in the whole angular range ($-1 \leq u \leq 1$, $-1 \leq v \leq 1$) and along (*h*) the $\phi = 0$ [deg] and (*i*) the $\phi = 90$ [deg] planes for the best solution.

Solution	SLL $[dB]$	Max. Directivity $[dBi]$	Mask Matching	HPBW (AZ) $[deg]$	HPBW (EL) $[deg]$
Best	-18.557	21.569	5.983×10^{-4}	13.03	19.13
Worst	-14.121	21.596	3.131×10^{-3}	12.64	18.64
Fully populated	-20.000	21.525	4.593×10^{-10}	13.13	20.03

Table I: Numerical Assessment ($M = 6,\ N = 9$, $d = 0.5\lambda$, (θ_0,ϕ_0)=(0.0, 0.0) [deg] - Pattern features obtained parameters.

More information on the topics of this document can be found in the following list of references.

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