Compressive Sensing Based Synthesis of Multi-Beam Phased Array for Air Traffic Control

P. Rocca, N. Anselmi, M. A. Hannan and A. Massa

Contents

1 Numerical Assessment - Linear Array Synthesis

1.1 $P = 2\,,\, \theta_1 = 90.75\,deg,\, \theta_2 = 96.57\,deg$

The test case has been performed using the parameter below.

Parameter	*Values*					
γ_a	0.100	0.309	0.954	2.947	9.103	
	10.985	33.932	104.811	323.746	1000.000	
γ_b	0.0001	0.00054	0.00295	0.01600	0.08685	
	0.47149	2.55955	13.89495	75.43120	100.00000	
Pattern Samples, K	22	24	26	28	30	32
	34	36	38	40	42	44
Aperture Samples, N	500	600	700	800	900	1000
Noise Variance, σ	0.000010	0.000031	0.000095	0.000295	0.000910	
	0.002812	0.008685	0.026827	0.082864	0.100000	

Table I: Simulation Parameters

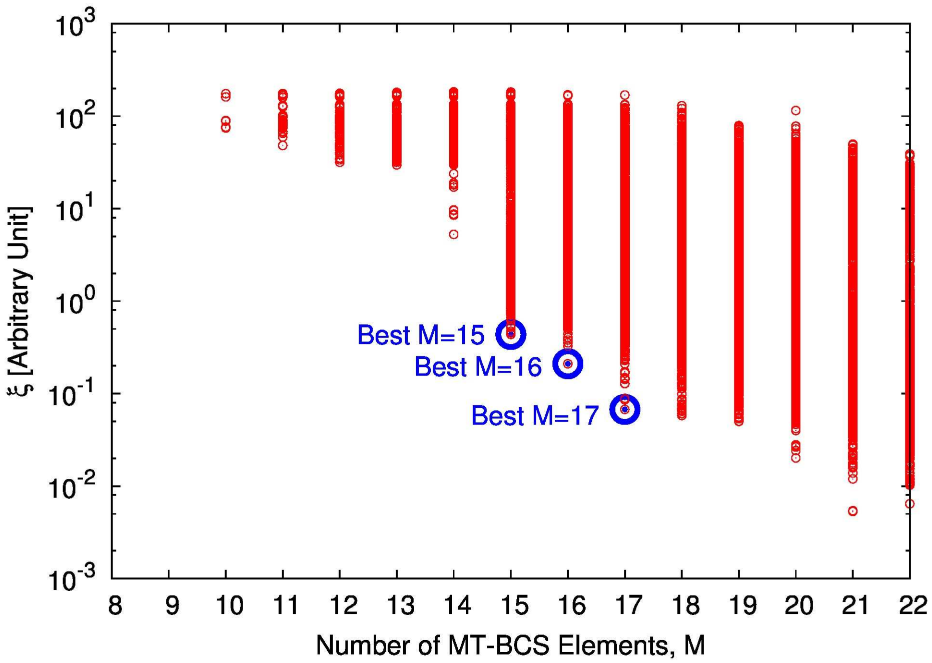


Figure 1: Output solutions from MT-BCS procedure

*Fig.*1 shows the solutions of the MT-BCS procedure having number of elements $M \leq 22$. The y-axis represents the mean error between the reference power patterns and the power pattern at the output of the MT-BCS.

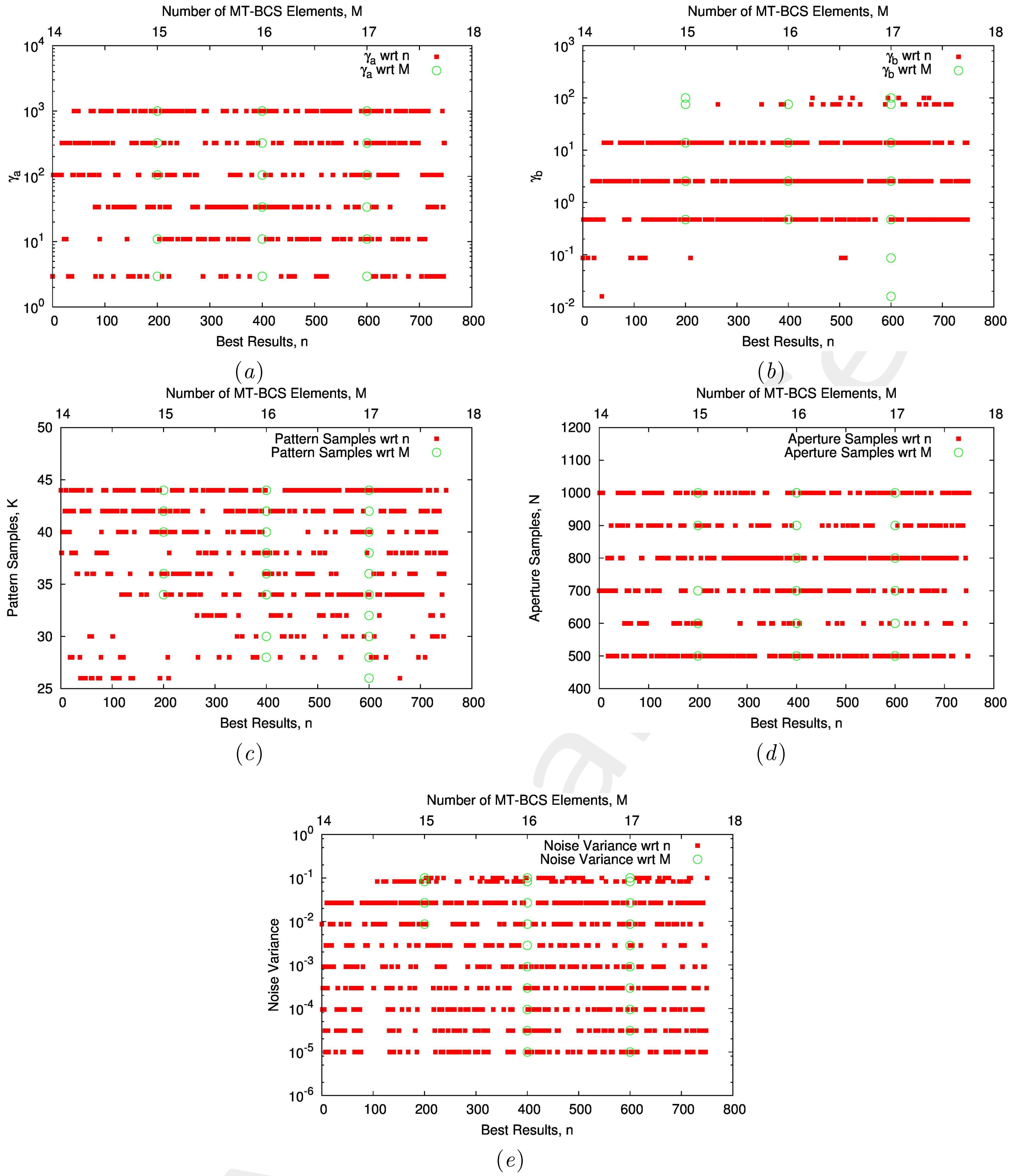


Figure 2: Parameters distribution, (a) γ_a, (b) γ_b, (c) *Pattern Samples,* (d) *Aperture Samples,* (e) *Noise Variance*

Fig.2 shows parameter distributions as function of "Best results" (the ones with error value under $8.0 \times 10^{-1}$and number of MT-BCS elements lower than 19, sorted by error value) and number of elements.

1.1.1 Solution: Best $M = 15$

γ_a	γ_b	*Pattern Samples [K]*	*Aperture Samples [N]*	*Noise Variance*
323.746	13.89495	42	1000	0.026827

Table II: MT-BCS Input Parameters

In *Tab.II* are reported MT-BCS parameters of solution Best $M = 15$. *Fig.3* shows elements positions of MT-BCS sparse linear array obtained from solution $M = 15$.

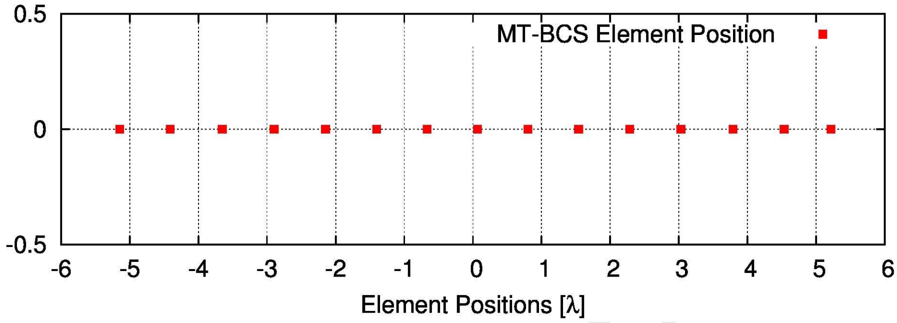
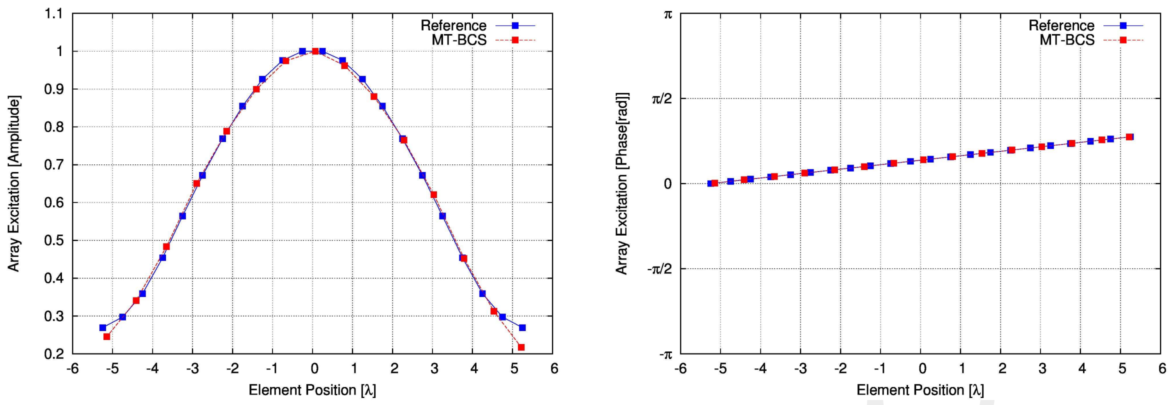


Figure 3: MT-BCS array elements positions

Pattern $\theta_1 = 90.75\, deg$



(a) (b)

Figure 4: Array excitations, (a)Amplitudes, (b)Phases

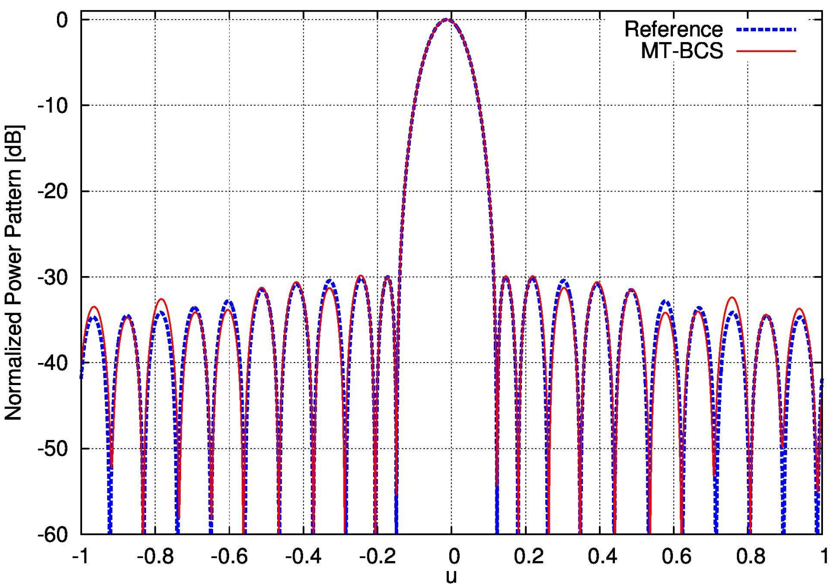


Figure 5: Power pattern comparison

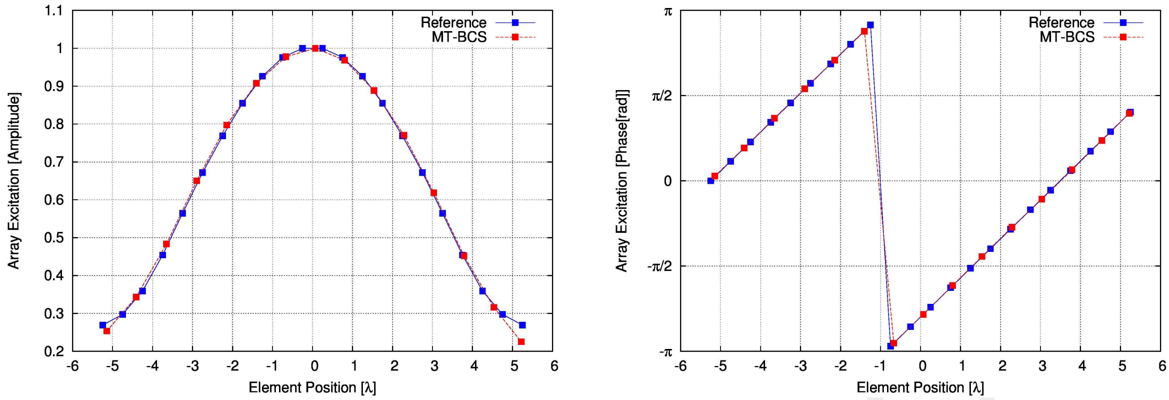
Fig.4(a), *4*(b) show comparison between Reference and MT-BCS amplitudes and phases excitations respectively. *Fig.5* shows the comparison between Reference and MT-BCS power pattern for the steereng angle $\theta_1 = 90.75$ degrees.

	$SLL\,[dB]$	$D\,[dB]$	$HPBW\,[deg]$	M	err_1
$Reference$	-29.99	12.76	5.82	22	$-$
$MT-BCS$	-29.81	12.75	5.84	15	5.1194727×10^{-1}

Table III: Pattern $\theta_1$performance

In *Tab.*III comparison between Reference and MT-BCS power pattern performance parameters.

Pattern $\theta_2 = 96.57\, deg$



(a) (b)

Figure 6: Array excitations, (a)Amplitudes, (b)Phases

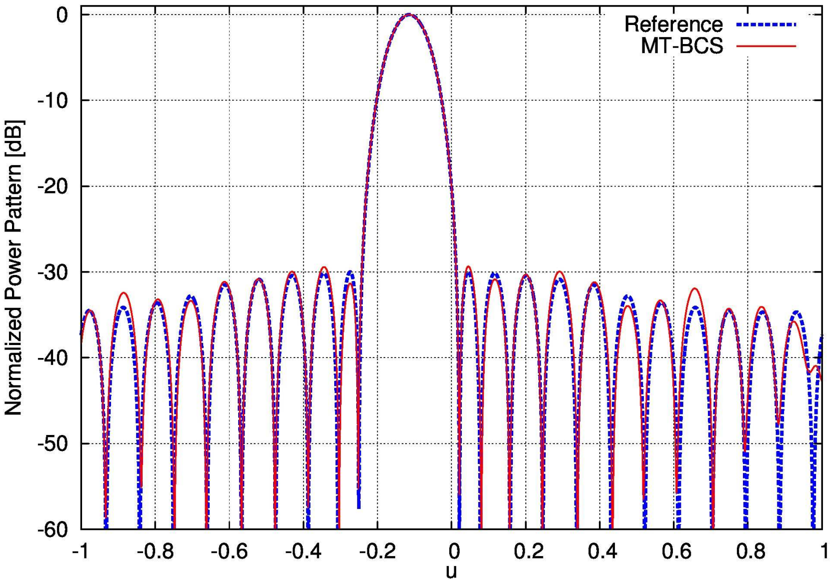


Figure 7: Power pattern comparison

Fig.6(a), $6(b)$ show comparison between Reference and MT-BCS amplitudes and phases excitations respectively. *Fig.7* shows the comparison between Reference and MT-BCS power pattern for the steereng angle $\theta_2 = 96.57$ degrees.

	$SLL\,[dB]$	$D\,[dB]$	$HPBW\,[deg]$	M	err_2
$Reference$	-29.99	12.76	5.86	22	$-$
$MT-BCS$	-29.35	12.75	5.87	15	3.6014238×10^{-1}

Table IV: Pattern θ_2 performance

In *Tab.*IV comparison between Reference and MT-BCS power pattern performance parameters.

1.1.2 MT-BCS errors

In *Tab.*V are reported the error values. In particular:

- I: total number of reference power pattern $[I]$;
- ξ: total error of the entire procedure;
- err_i $(i = 1, \ldots, I)$: error on sigular reference and MT-BCS pattern pair.

Solution	err_1	err_2
Best $M = 15$	5.1194727×10^{-1}	3.6014238×10^{-1}

Solution	I	ξ
Best $M = 15$	2	4.360448×10^{-1}

Table V: MT-BCS errors

1.2 $P = 3\,,\ \theta_1 = 84.93\,deg,\ \theta_2 = 90.75\,deg,\ \theta_3 = 96.57\,deg$

The test case has been performed using the parameters below.

Parameter	*Values*					
γ_a	0.100	0.309	0.954	2.947	9.103	
	10.985	33.932	104.811	323.746	1000.000	
γ_b	0.0001	0.00054	0.00295	0.01600	0.08685	
	0.47149	2.55955	13.89495	75.43120	100.00000	
Pattern Samples, K	22	24	26	28	30	32
	34	36	38	40	42	44
Aperture Samples, N	500	600	700	800	900	1000
Noise Variance, σ	0.000010	0.000031	0.000095	0.000295	0.000910	
	0.002812	0.008685	0.026827	0.082864	0.100000	

Table VI: Simulation Parameters

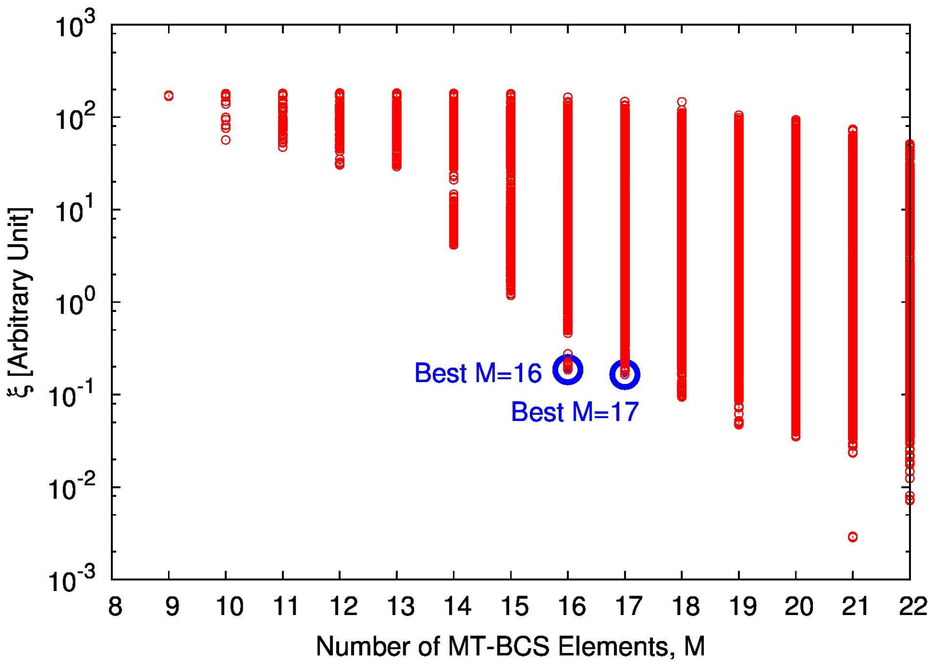


Figure 8: Output solutions from MT-BCS procedure

*Fig.*8 shows the solutions of the MT-BCS procedure having number of elements $M \leq 22$. The y-axis represents the mean error between the reference power patterns and the power pattern at the output of the MT-BCS.

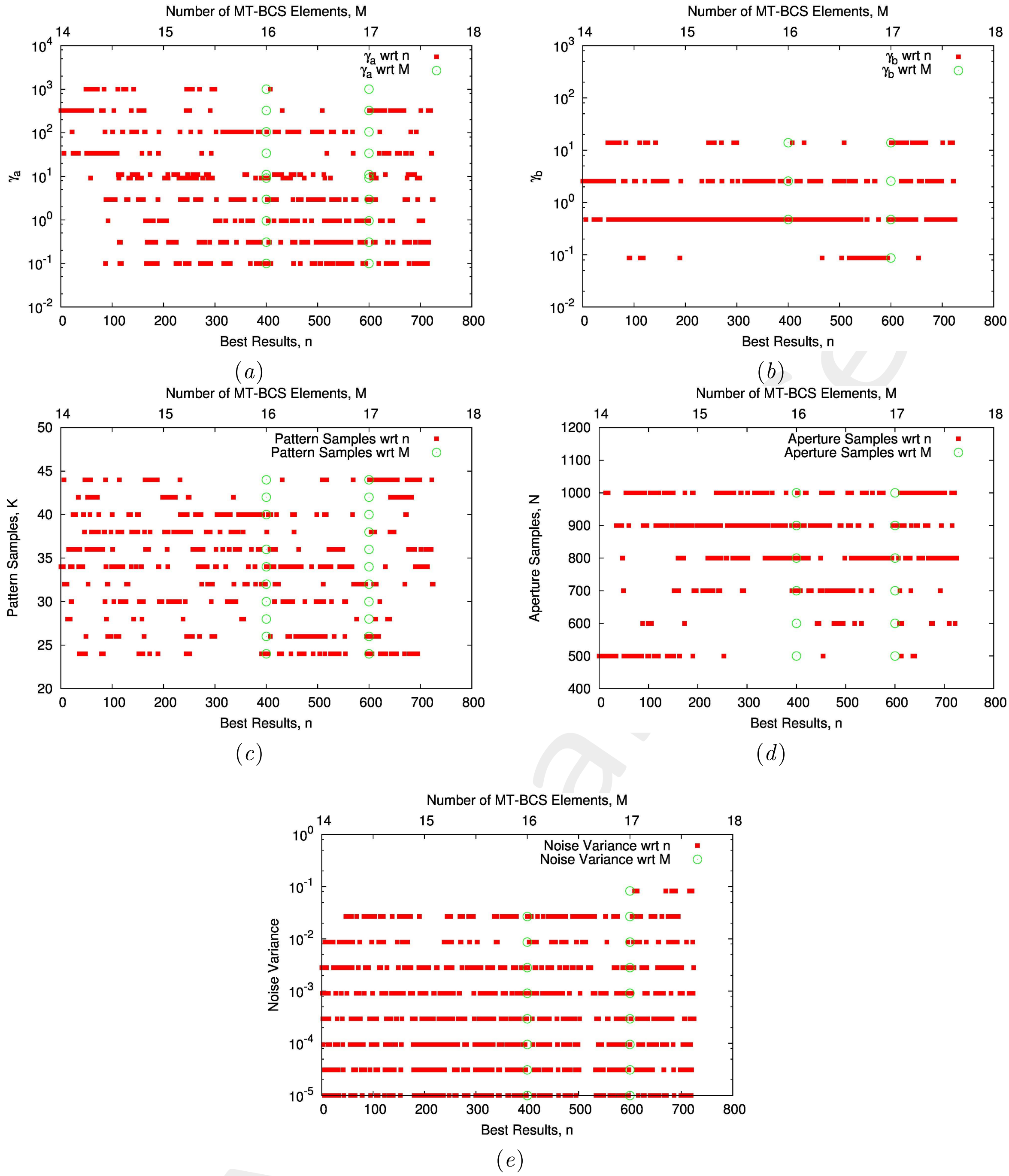


Figure 9: Parameters distribution, (a) γ_a, (b) γ_b, (c) *Pattern Samples,* (d) *Aperture Samples,* (e) *Noise Variance*

Fig.9 shows parameter distributions as function of "Best Results" (the ones with error value under $8.0 \times 10^{-1}$and number of MT-BCS elements lower than 18, sorted by error value) and number of elements.

From *Fig.9* it is possible to notice that there are no "Best Results " with $M = 15$ number of elements.

1.2.1 Solution: Best $M = 16$

γ_a	γ_b	*Pattern Samples [K]*	*Aperture Samples [N]*	*Noise Variance*
323.746	0.47149	44	500	0.008685

Table VII: MT-BCS Input Parameters

In *Tab.VII* are reported MT-BCS parameters of solution Best $M = 16$. *Fig.10* shows elements positions of MT-BCS sparse linear array obtained from solution Best $M = 16$.

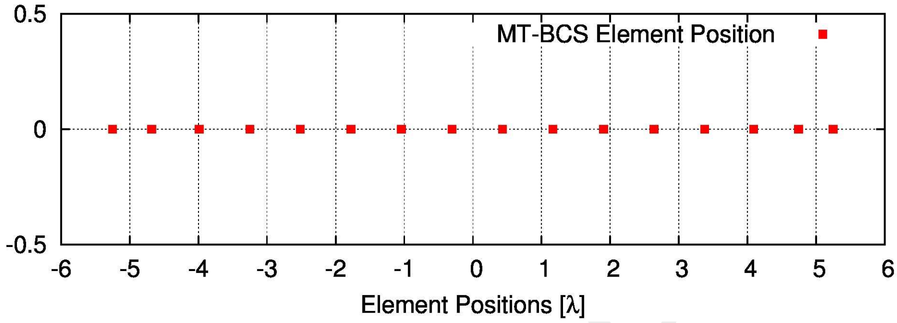
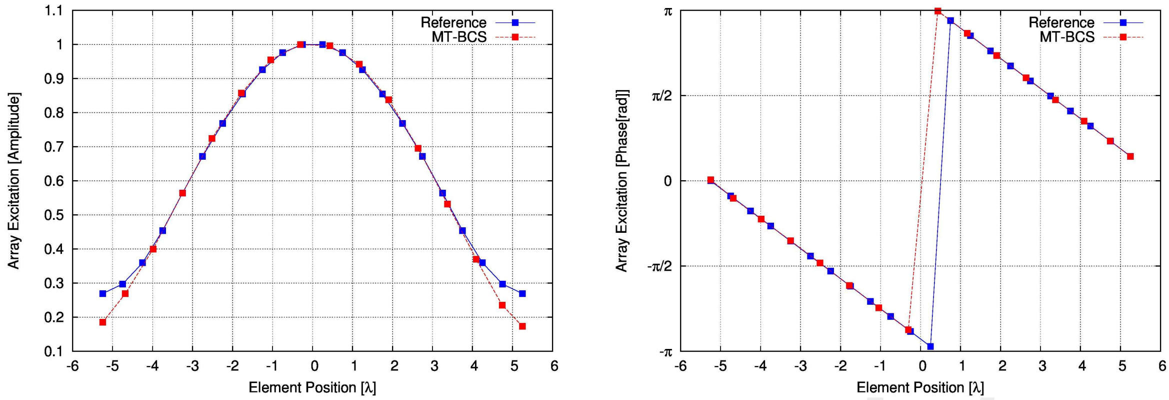


Figure 10: MT-BCS array elements positions

Pattern $\theta_1 = 84.93\,deg$



(a) (b)

Figure 11: Array excitations, (a)Amplitudes, (b)Phases

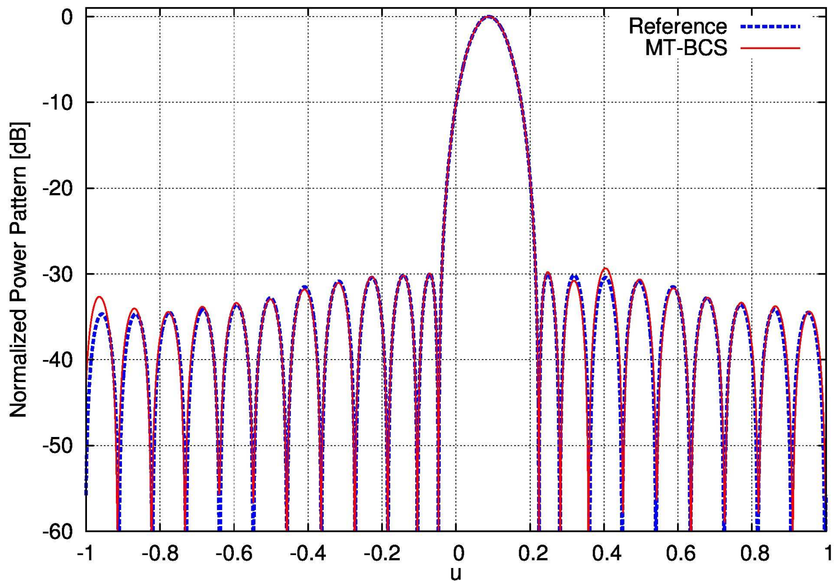


Figure 12: Power pattern comparison

Fig.11(a), *11*(b) show comparison between Reference and MT-BCS amplitudes and phases excitations respectively.

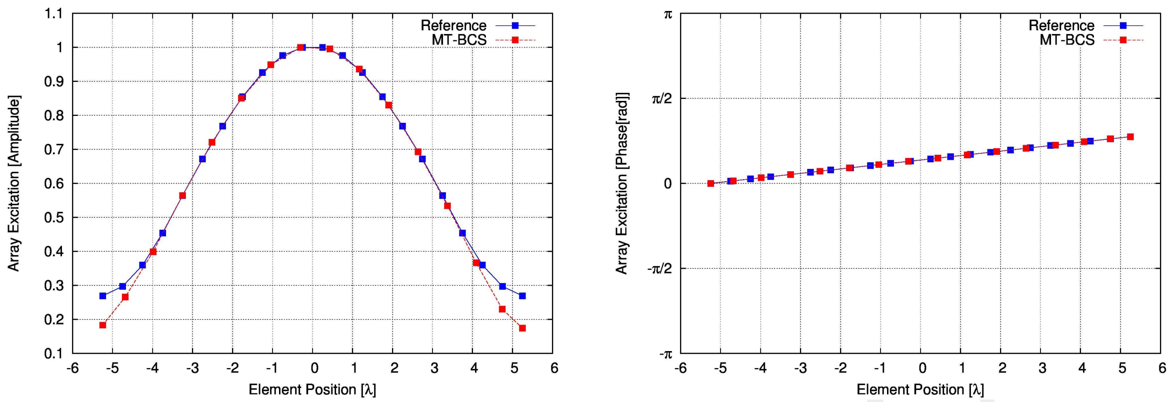
Fig.12 shows the comparison between Reference and MT-BCS power pattern for the steereng angle $\theta_1 = 84.93$ degrees.

	$SLL\,[dB]$	$D\,[dB]$	$HPBW\,[deg]$	M	err_1
$Reference$	-29.99	12.77	5.84	22	$-$
$MT-BCS$	-29.34	12.76	5.85	16	1.7800032×10^{-1}

Table VIII: Pattern $\theta_1$performance

In *Tab.*VIII comparison between Reference and MT-BCS power pattern performance parameters.

Pattern $\theta_2 = 90.75\,deg$



(a) (b)

Figure 13: Array Excitations, (a)Amplitudes, (b)Phases

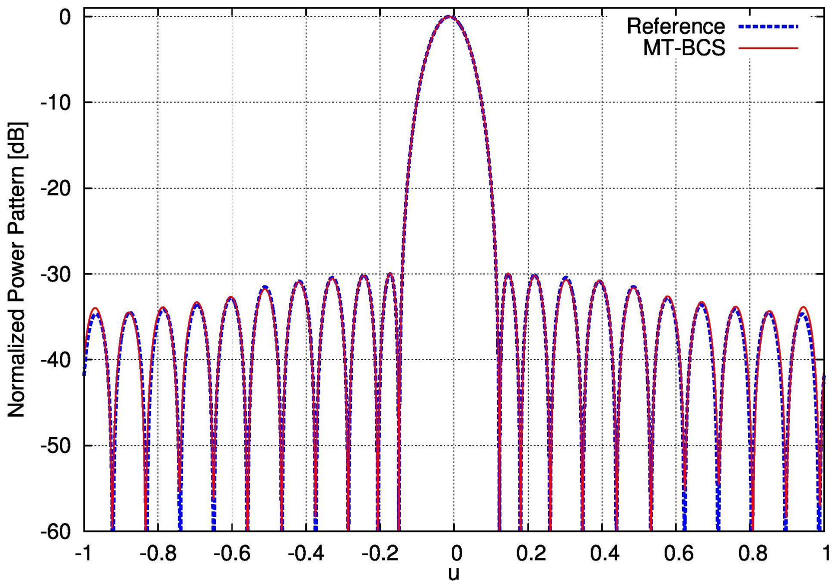


Figure 14: Power pattern comparison

Fig.13(a), *13*(b) show comparison between Reference and MT-BCS amplitudes and phases excitations respectively.

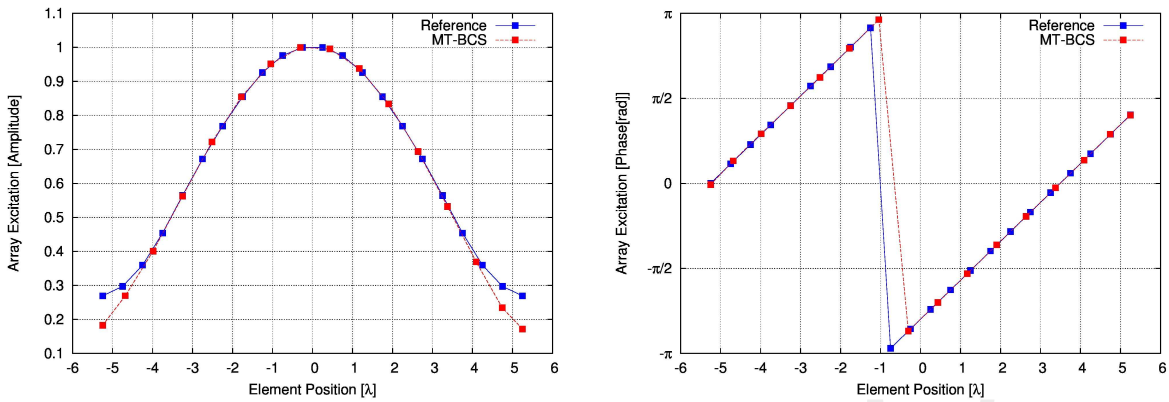
Fig.14 shows the comparison between Reference and MT-BCS power pattern for the steereng angle $\theta_2 = 90.75$ degrees.

	$SLL\,[dB]$	$D\,[dB]$	$HPBW\,[deg]$	M	err_2
$Reference$	-29.99	12.77	5.82	22	$-$
$MT-BCS$	-29.92	12.76	5.83	16	1.9301757×10^{-1}

Table IX: Pattern $\theta_2$performance

In *Tab.*IX comparison between Reference and MT-BCS power pattern performance parameters.

Pattern $\theta_3 = 96.57\,deg$



(a) (b)

Figure 15: Array Excitations, (a)Amplitudes, (b)Phases

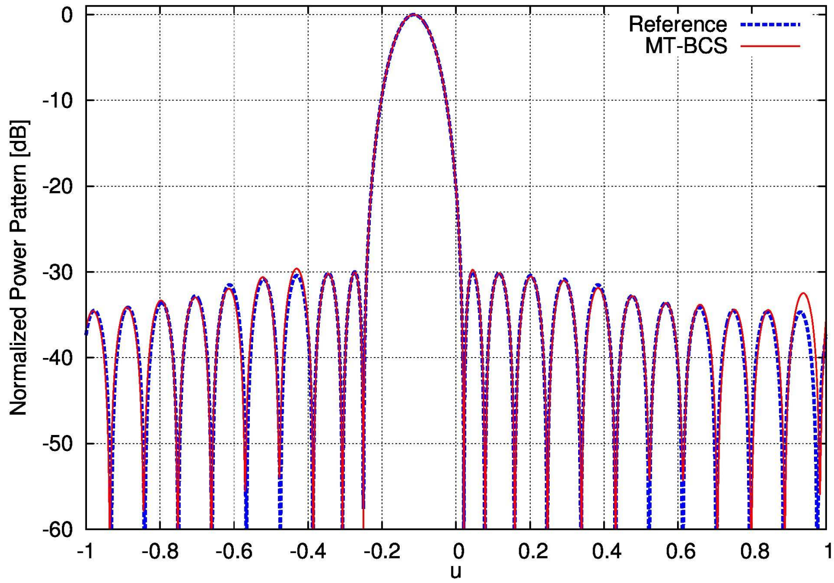


Figure 16: Power pattern comparison

Fig.15(a), *15*(b) show comparison between Reference and MT-BCS amplitudes and phases excitations respectively.

Fig.16 shows the comparison between Reference and MT-BCS power pattern for the steereng angle $\theta_3 = 96.57$ degrees.

	$SLL\,[dB]$	$D\,[dB]$	$HPBW\,[deg]$	M	err_3
$Reference$	-29.99	12.77	5.86	22	$-$
$MT-BCS$	-29.61	12.76	5.87	16	1.8657449×10^{-1}

Table X: Pattern $\theta_3$performance

In *Tab.*X comparison between Reference and MT-BCS power pattern performance parameters.

1.2.2 MT-BCS errors

In *Tab.*XI are reported the error values. In particular:

- I: total number of reference power pattern $[I]$;
- ξ: total error of the entire procedure;
- err_i $(i = 1, \dots, I)$: error on sigular reference and MT-BCS pattern pair.

Solution	err_1	err_2	err_3
Best $M = 16$	1.7800032×10^{-1}	1.9301757×10^{-1}	1.8657449×10^{-1}

Solution	I	ξ
Best $M = 16$	3	1.858641×10^{-1}

Table XI: MT-BCS errors

1.3 $P = 4\,,\,\theta_1 = 79.06\,deg,\,\theta_2 = 84.93\,deg,\,\theta_3 = 90.75\,deg,\,\theta_3 = 96.57\,deg$

The test case has been performed using the parameters below.

Parameter	*Values*					
γ_a	0.100	0.309	0.954	2.947	9.103	
	10.985	33.932	104.811	323.746	1000.000	
γ_b	0.0001	0.00054	0.00295	0.01600	0.08685	
	0.47149	2.55955	13.89495	75.43120	100.00000	
Pattern Samples, K	22	24	26	28	30	32
	34	36	38	40	42	44
Aperture Samples, N	500	600	700	800	900	1000
Noise Variance, σ	0.000010	0.000031	0.000095	0.000295	0.000910	
	0.002812	0.008685	0.026827	0.082864	0.100000	

Table XII: Simulation Parameters

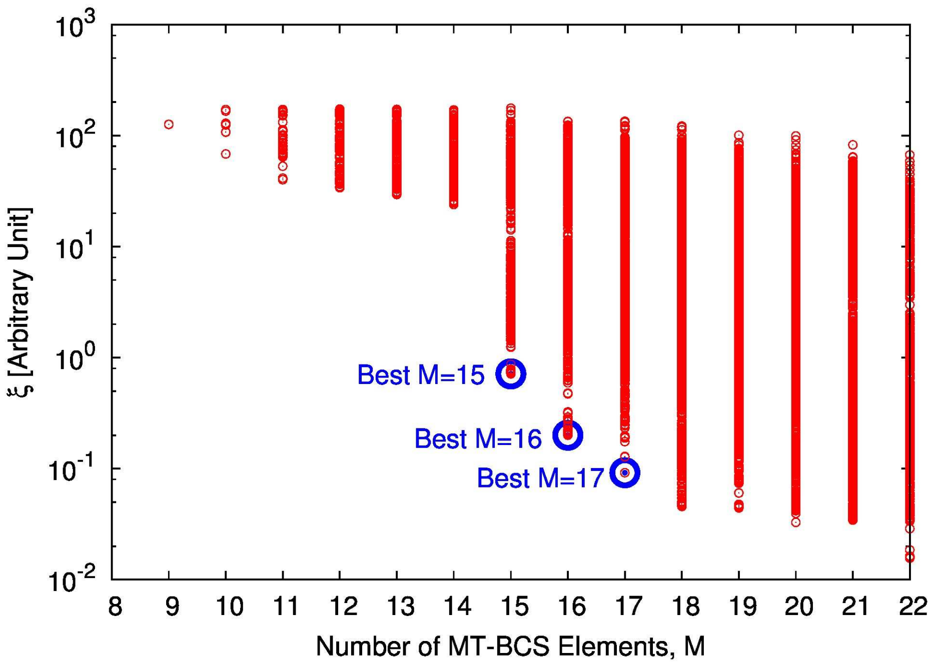


Figure 17: Output solutions from MT-BCS procedure

*Fig.*17 shows the solutions of the MT-BCS procedure having number of elements $M \leq 22$. The y-axis represents the mean error between the reference power patterns and the power pattern at the output of the MT-BCS.

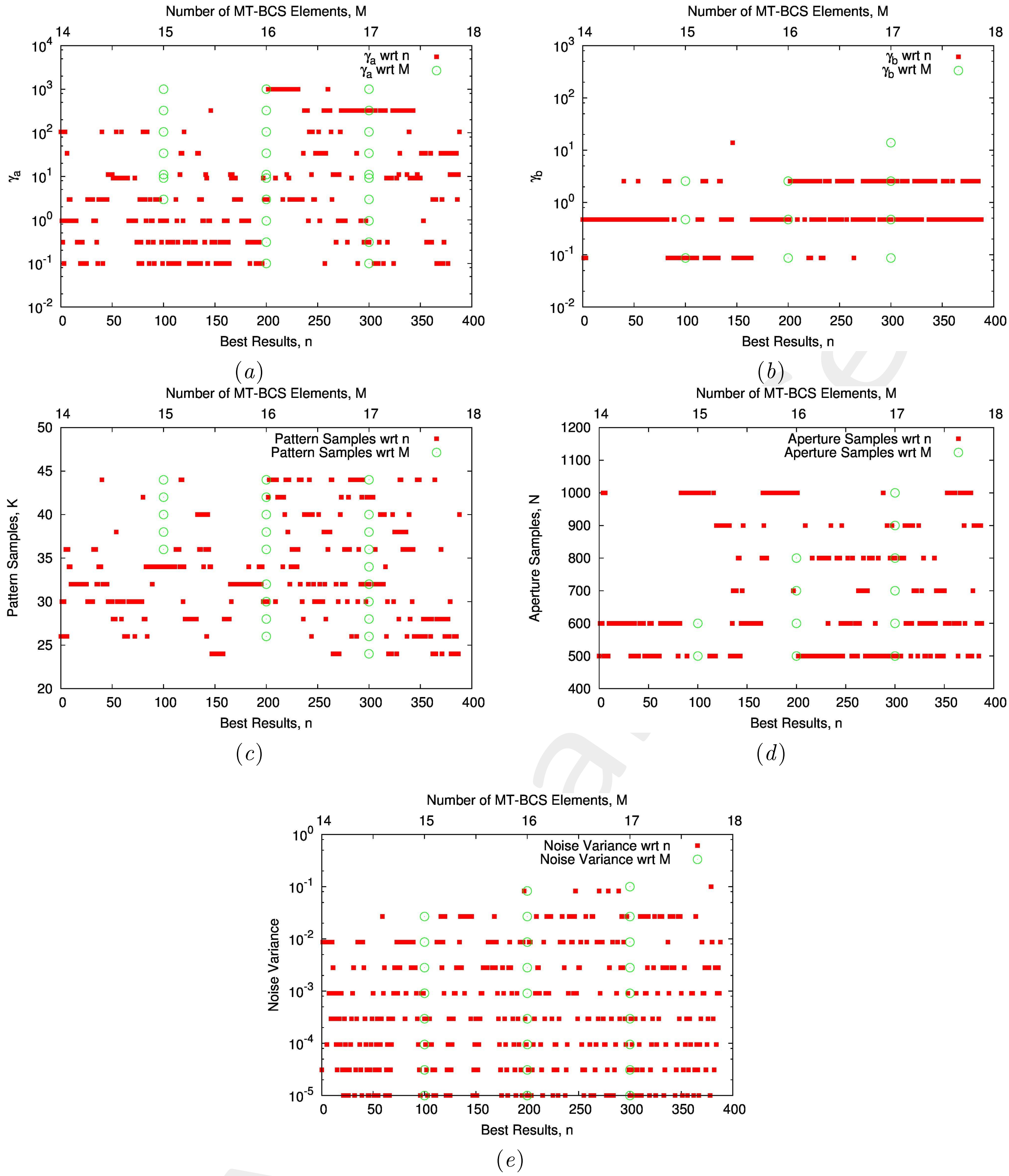


Figure 18: Parameters distribution, (a) γ_a, (b) γ_b, (c) *Pattern Samples,* (d) *Aperture Samples,* (e) *Noise Variance*

Fig.18 shows parameter distributions as function of "Best Results" (solutions with error value under $8.0 \times 10^{-1}$and number of MT-BCS elements lower than 18, sorted by error value) and number of elements.

1.3.1 Solution: Best $M = 15$

γ_a	γ_b	*Pattern Samples [K]*	*Aperture Samples [N]*	*Noise Variance*
104.811	0.47149	40	600	0.00091

Table XIII: MT-BCS Input Parameters

In *Tab.XIII* are reported MT-BCS parameters of solution Best $M = 15$. *Fig.19* shows elements positions of MT-BCS sparse linear array obtained from solution Best $M = 15$.

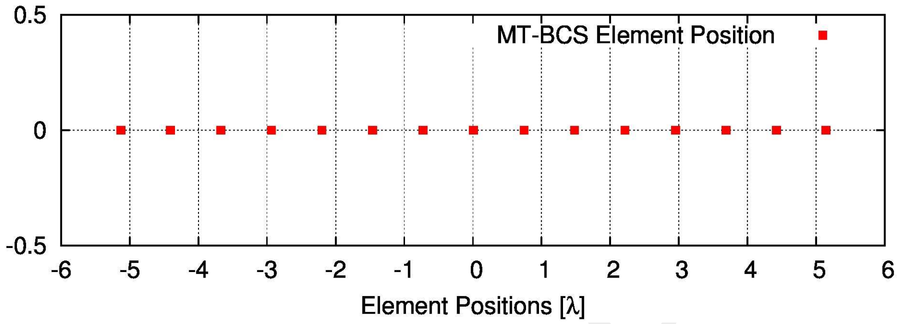
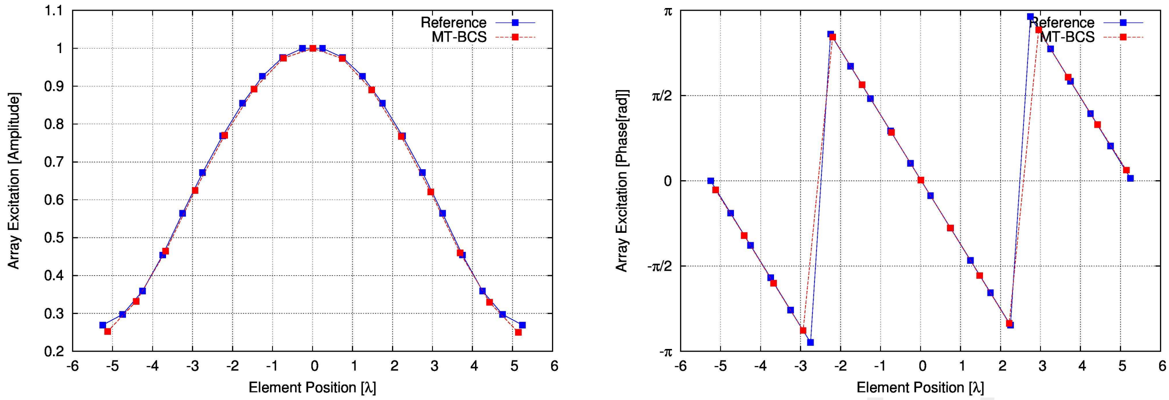


Figure 19: MT-BCS array elements positions

Pattern $\theta_1 = 79.06\,deg$



(a) (b)

Figure 20: Array excitations, (a)Amplitudes, (b)Phases

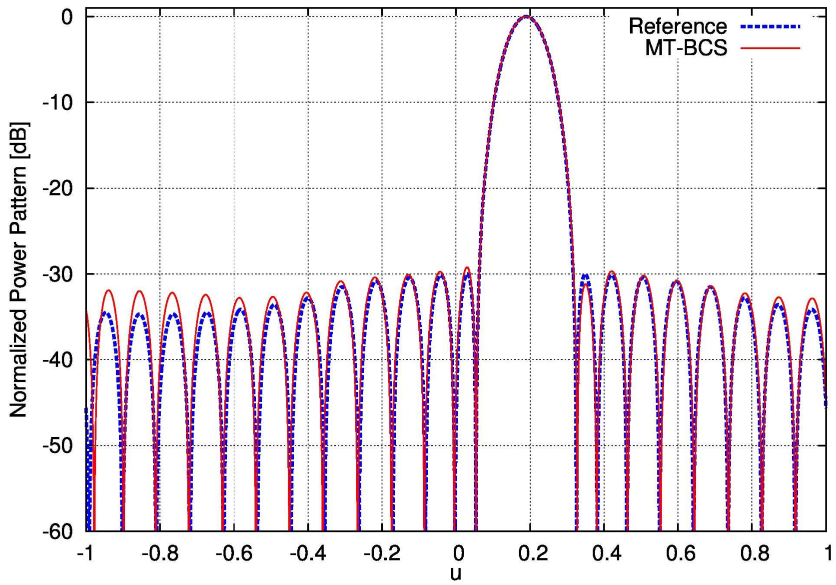


Figure 21: Power pattern comparison

Fig.20(a), *20*(b) show comparison between Reference and MT-BCS amplitudes and phases excitations respectively.

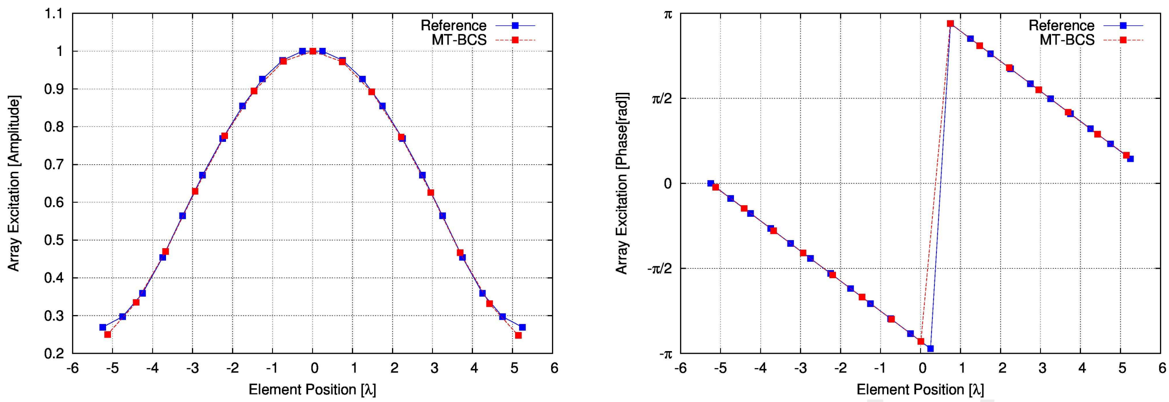
Fig.21 shows the comparison between Reference and MT-BCS power pattern for the steereng angle $\theta_1 = 79.06$ degrees.

	$SLL\,[dB]$	$D\,[dB]$	$HPBW\,[deg]$	M	err_1
$Reference$	-29.99	12.76	5.93	22	$-$
$MT-BCS$	-29.22	12.74	5.97	15	8.2470542×10^{-1}

Table XIV: Pattern $\theta_1$performance

In *Tab.*XIV comparison between Reference and MT-BCS power pattern performance parameters.

Pattern $\theta_2 = 84.93\,deg$



(a) (b)

Figure 22: Array Excitations, (a)Amplitudes, (b)Phases

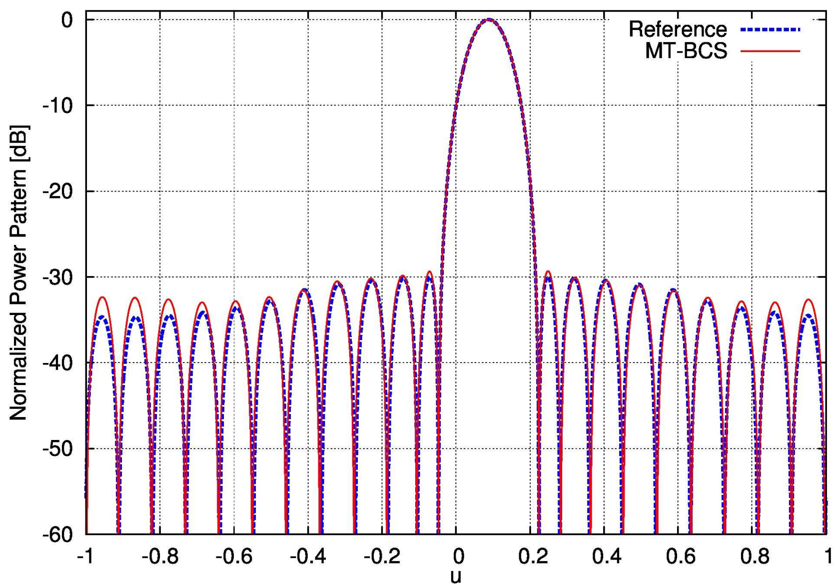


Figure 23: Power pattern comparison

Fig.22(a), *22*(b) show comparison between Reference and MT-BCS amplitudes and phases excitations respectively.

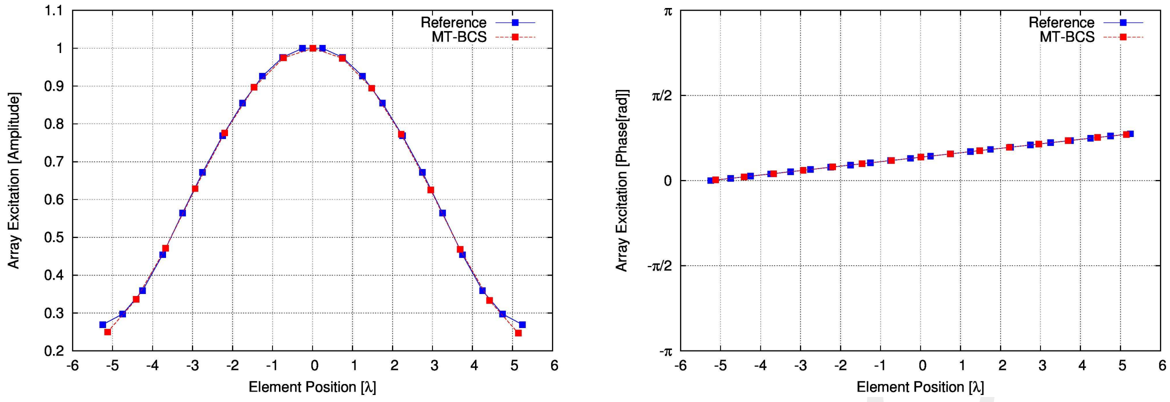
Fig.23 shows the comparison between Reference and MT-BCS power pattern for the steereng angle $\theta_2 = 84.93$ degrees.

	$SLL\,[dB]$	$D\,[dB]$	$HPBW\,[deg]$	M	err_2
$Reference$	-29.99	12.77	5.84	22	$-$
$MT-BCS$	-29.31	12.74	5.88	15	6.596669×10^{-1}

Table XV: Pattern $\theta_2$performance

In *Tab.*XV comparison between Reference and MT-BCS power pattern performance parameters.

Pattern $\theta_3 = 90.75\,deg$



(a) (b)

Figure 24: Array Excitations, (a)Amplitudes, (b)Phases

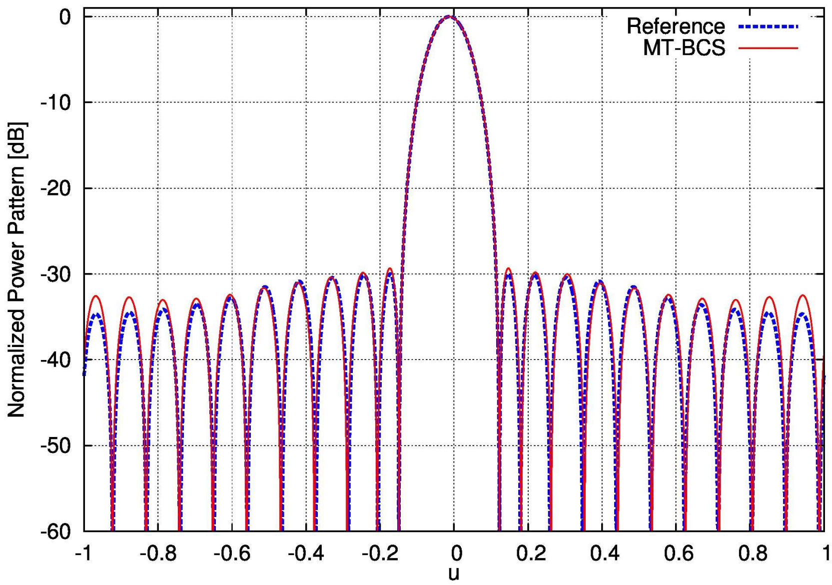


Figure 25: Power pattern comparison

Fig.24(a), *24*(b) show comparison between Reference and MT-BCS amplitudes and phases excitations respectively.

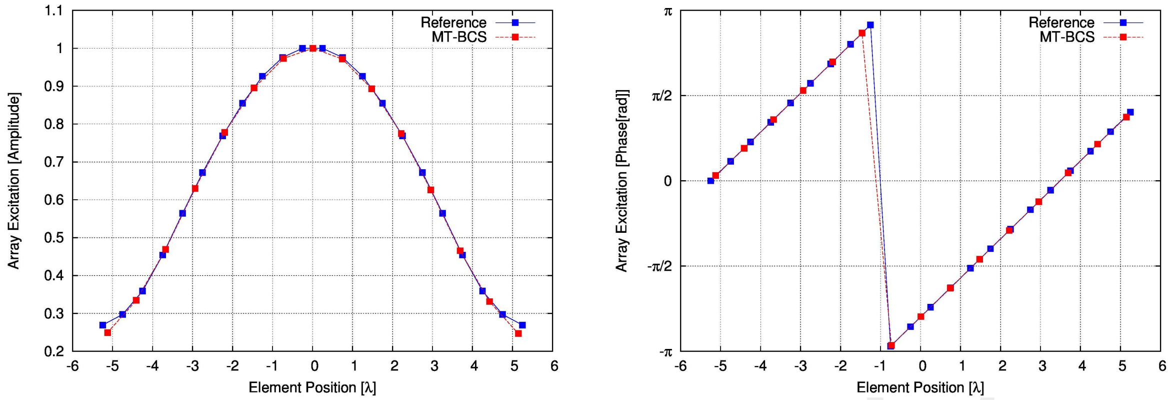
Fig.25 shows the comparison between Reference and MT-BCS power pattern for the steereng angle $\theta_3 = 90.75$ degrees.

	$SLL\,[dB]$	$D\,[dB]$	$HPBW\,[deg]$	M	err_3
$Reference$	-29.99	12.77	5.82	22	$-$
$MT-BCS$	-29.36	12.74	5.85	15	6.5778869×10^{-1}

Table XVI: Pattern $\theta_3$performance

In *Tab.*XVI comparison between Reference and MT-BCS power pattern performance parameters.

Pattern $\theta_4 = 96.57\, deg$



(a) (b)

Figure 26: Array Excitations, (a)Amplitudes, (b)Phases

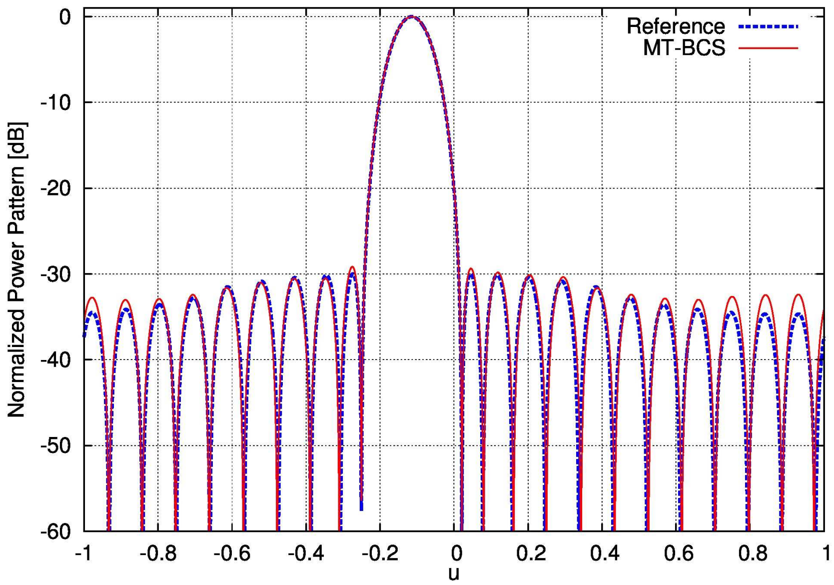


Figure 27: Power pattern comparison

Fig.26(a), *26*(b) show comparison between Reference and MT-BCS amplitudes and phases excitations respectively.

Fig.27 shows the comparison between Reference and MT-BCS power pattern for the steereng angle $\theta_4 = 96.57$ degrees.

	$SLL\,[dB]$	$D\,[dB]$	$HPBW\,[deg]$	M	err_4
$Reference$	-29.99	12.77	5.86	22	$-$
$MT-BCS$	-29.16	12.74	5.89	15	7.1513563×10^{-1}

Table XVII: Pattern $\theta_4$performance

In *Tab.*XVII comparison between Reference and MT-BCS power pattern performance parameters.

1.3.2 MT-BCS errors

In *Tab.*XVIII are reported the error values. In particular:

- I: total number of reference power pattern $[I]$;
- ξ: total error of the entire procedure;
- err_i $(i = 1, \ldots, I)$: error on sigular reference and MT-BCS pattern pair.

Solution	err_1	err_2	err_3	err_4
Best $M = 15$	8.247054×10^{-1}	6.596669×10^{-1}	6.577887×10^{-1}	7.151356×10^{-1}

Solution	I	ξ
Best $M = 15$	4	7.143242×10^{-1}

Table XVIII: MT-BCS errors

More information on the topics of this document can be found in the following list of references.

References

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