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# Compressive Sensing Based Minimum-Complexity Failure Correction in Linear Arrays

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# 1 Numerical Results - Non-Iterative MCFC

## 1.1 Test case 1—Dolph-Chebyshev, $N = 20$ , SLL = $-20$ [dB], faulty element 3

### 1.1.1 Goal of the analysis

The goal of test case 1 is that of **demonstrating the functionality** of the MFC method developed. We expect the corrected pattern to have a lower SLL of the faulty pattern, but higher than the original pattern.

### 1.1.2 Parameters

The array considered in test case 1 has the following properties

- Number of array elements:  $N = 20$
- Tapering: Dolph-Chebyshev, SLL =  $-20$  [dB]
- Damaged element indexes set:  $\Omega = \{3\}$
- Number of faulty elements:  $D = 1$
- Damaged element excitation:  $\mathbf{w}_{\text{corr,immut}} = [0]$

Figure 1 shows the original excitations and the damaged ones.

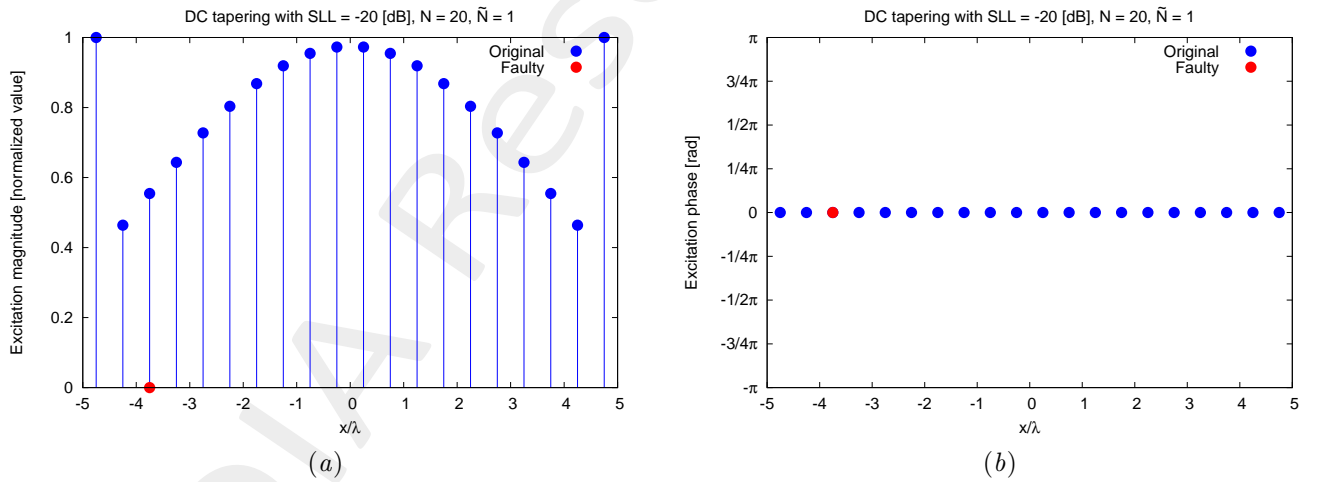


Figure 1: Original and damaged excitations for the array considered in test case 1: amplitude (a) and phase (b).

The parameters used to configure the software are the following:

- Phase 1
  - Desired SLL:  $\text{SLL}^{(1)} = -20$  [dB]
  - Mask main lobe width:  $\text{BW}^{(1)} = 12$  [deg]

– Mask  $u$  samples count:  $K^{(1)} = 200$

• Phase 2

– Desired SLL:  $SLL^{(2)} = -20$  [dB]

– Mask main lobe width:  $BW^{(2)} = 12$  [deg]

– Mask  $u$  samples count:  $K^{(2)} = 200$

• Use Hessian: Yes

### 1.1.3 Results

Figure 2 compares the original excitations with the corrected excitations obtained with the proposed method.

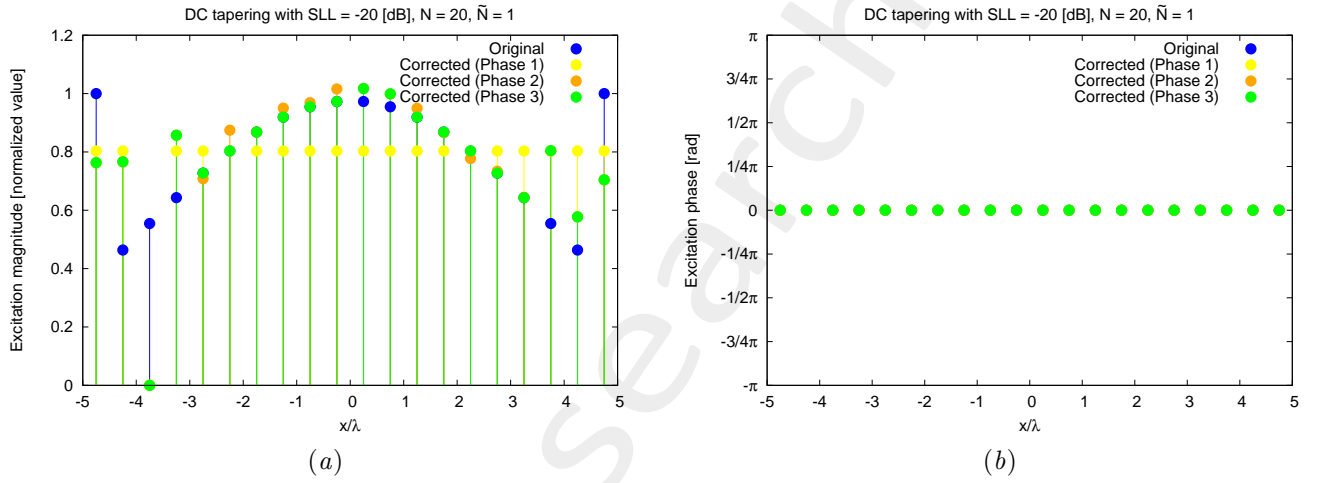


Figure 2: Original and corrected excitations for the array considered in test case 1: amplitude (a) and phase (b).

Figure 3 compares the original, faulty and corrected radiation patterns.

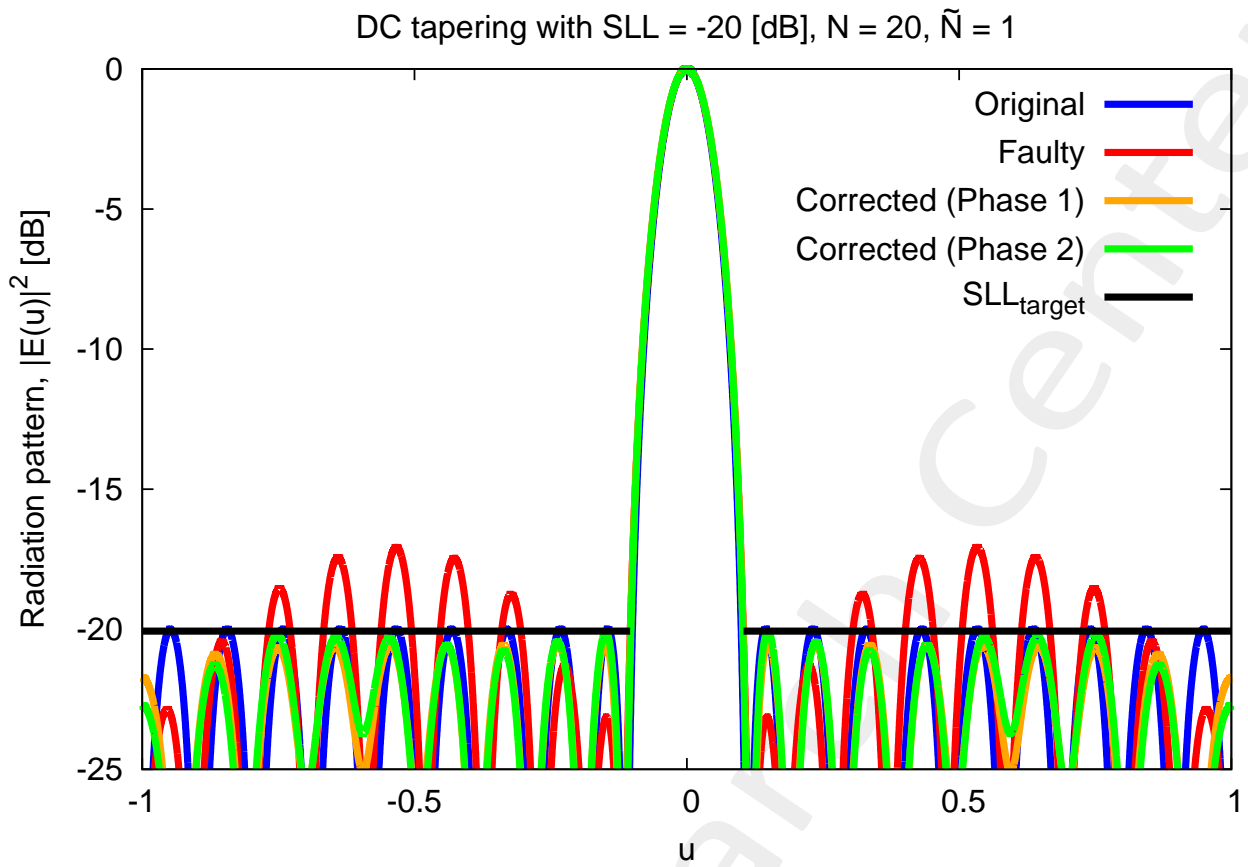


Figure 3: The radiation pattern for the original, faulty and corrected excitations.

Figure 4 shows the value of the L1-norm cost function for each iteration of the algorithm.

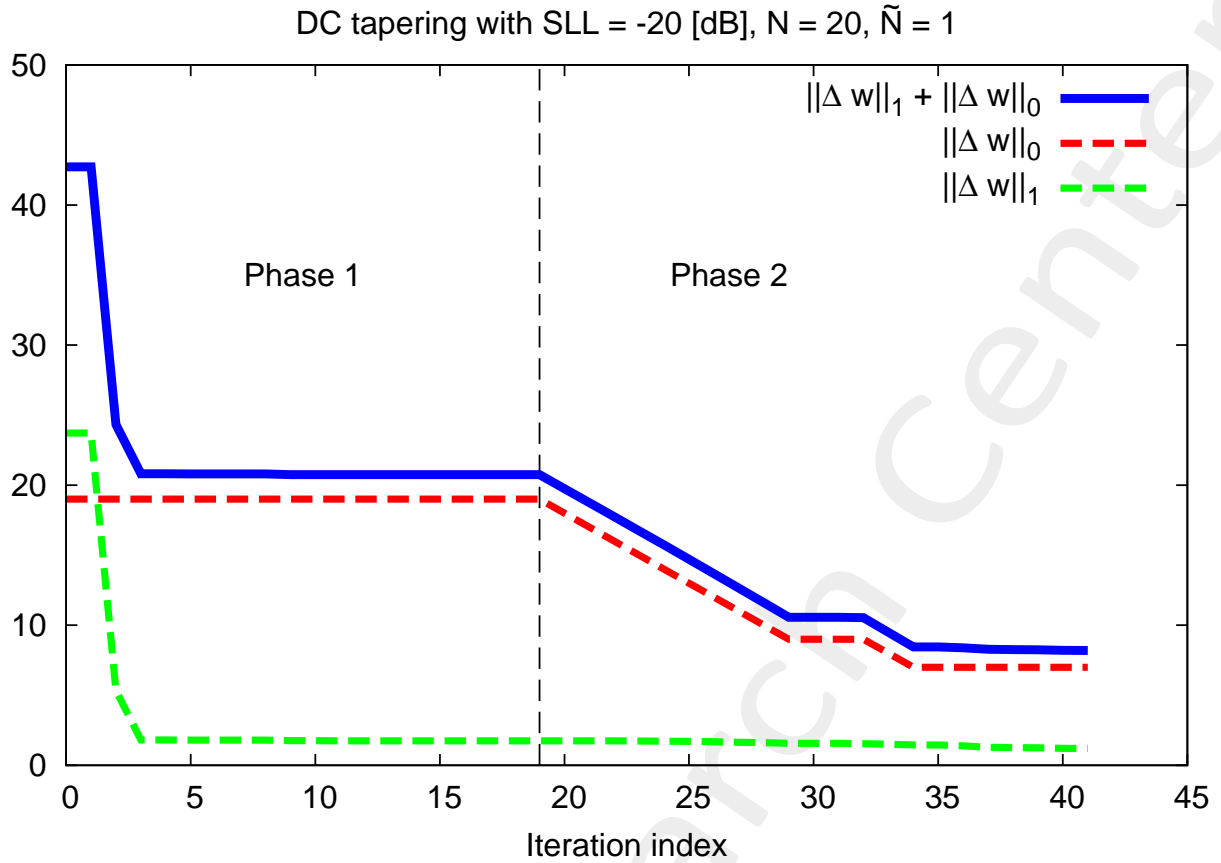


Figure 4: The value of the L1-norm cost function for each iteration of the algorithm.

Table I reports the SLL of the radiation patterns for the original, faulty and corrected excitations.

	Pattern SLL [dB]	HPBW [deg]	DRR	$\ \mathbf{w}_{\text{corr,mut}} - \mathbf{w}_{\text{orig,mut}}\ _1$	$\ \mathbf{w}_{\text{corr,mut}} - \mathbf{w}_{\text{orig,mut}}\ _0$
Original excitations	-20.00	5.36	0.464		
Faulty excitations	-20.00	5.36	0.464		
Corrected excitations (init.)	-13.19	5.07	1.0	3.1	20
Corrected excitations (Phase 1)	-20.46	5.37	0.505	0.647	20
Corrected excitations (Phase 2)	-20.00	5.36	0.464	0.00	0

Table I: Comparison of the original, faulty and corrected excitations.

#### 1.1.4 Observations

The proposed method succeeded in providing a set a corrected excitations. Moreover, more than 40% of the excitations are kept unchanged.

## 1.2 Test case 2—Dolph-Chebyshev, $N=20$ , $SLL=-20$ [dB], No faulty element

### 1.2.1 Goal of the analysis

The goal of test case 2 is that of **demonstrating the functionality** of the MFC method developed. We consider the case where no element is damaged, and we expect to find the original excitation set as a solution.

### 1.2.2 Parameters

The array considered in test case 2 has the following properties

- Number of array elements:  $N = 20$
- Tapering: Dolph-Chebyshev,  $SLL=-20$  [dB]
- Damaged element indexes set:  $\Omega = \{\}$
- Number of faulty elements:  $D = 0$
- Damaged element excitation:  $\mathbf{w}_{\text{corr,immut}} = \mathbf{\emptyset}$

Figure 5 shows the original excitations and the damaged ones.

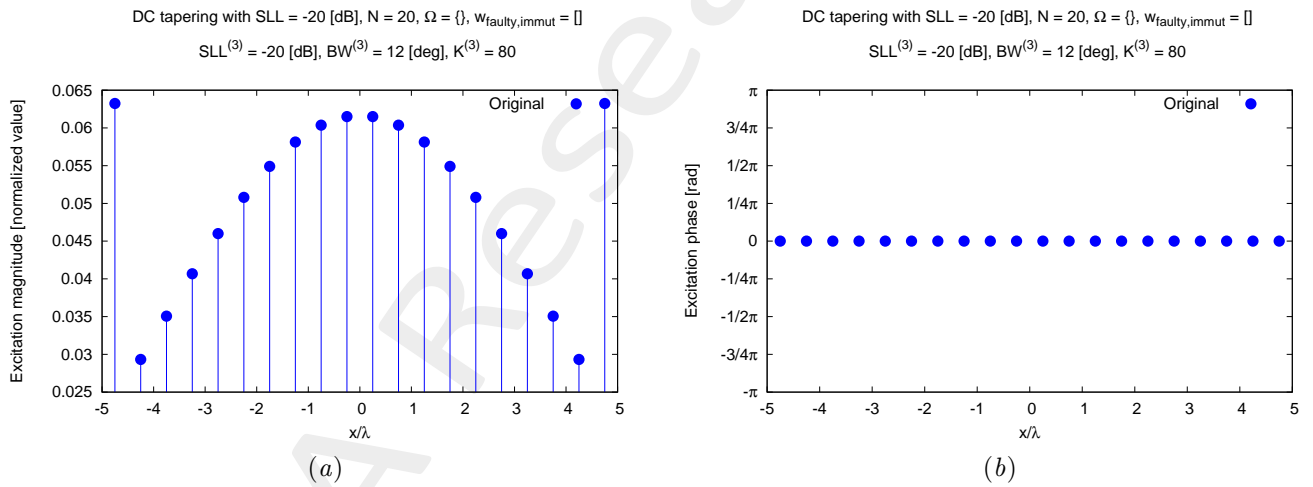


Figure 5: Original and damaged excitations for the array considered in test case 2: amplitude (a) and phase (b).

The parameters used to configure the software are the following:

- Phase 2
  - Desired SLL:  $SLL^{(1)} = -20.5$  [dB]
  - Mask main lobe width:  $BW^{(1)} = 12$  [deg]
  - Mask  $u$  samples count:  $K^{(1)} = 200$
- Phase 3

- Desired SLL:  $SLL^{(2)} = -20$  [dB]
- Mask main lobe width:  $BW^{(2)} = 12$  [deg]
- Mask  $u$  samples count:  $K^{(2)} = 200$

- Use Hessian: Yes

### 1.2.3 Results

Figure 6 compares the original excitations with the corrected excitations obtained with the proposed method.

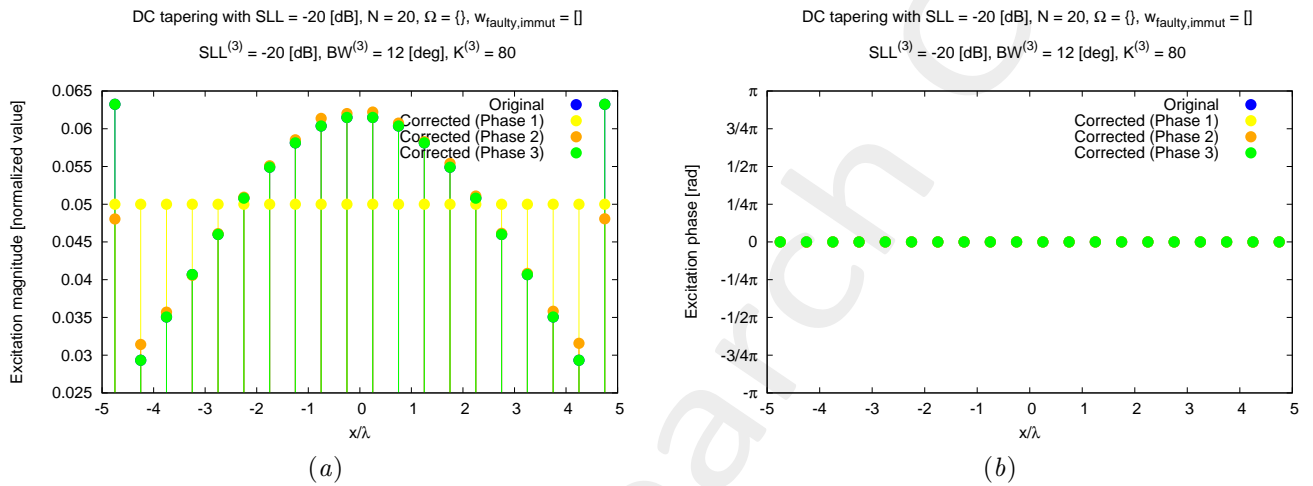


Figure 6: Original and corrected excitations for the array considered in test case 2: amplitude (a) and phase (b).

Figure 7 compares the original, faulty and corrected radiation patterns.



DC tapering with SLL = -20 [dB],  $N = 20$ ,  $\Omega = \{\}$ ,  $w_{\text{faulty,immut}} = []$

$\text{SLL}^{(3)} = -20$  [dB],  $\text{BW}^{(3)} = 12$  [deg],  $K^{(3)} = 80$

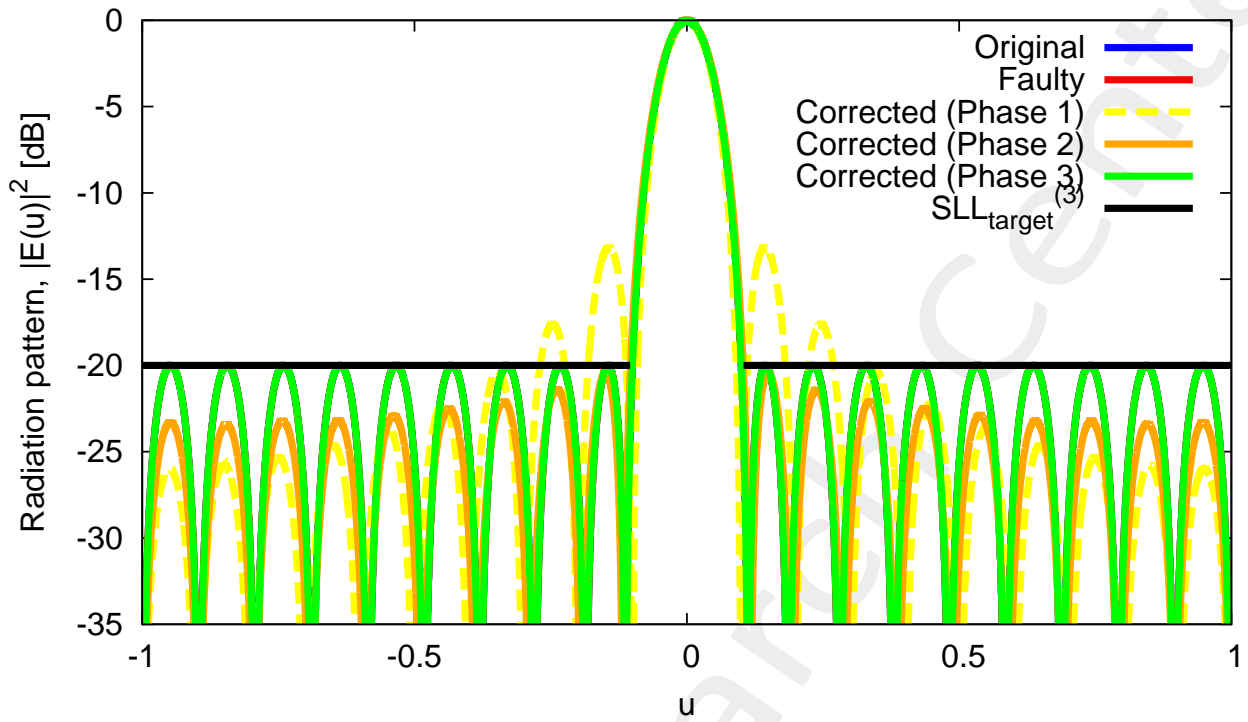


Figure 7: The radiation pattern for the original, faulty and corrected excitations.

Figure 8 shows the value of the L1-norm cost function for each iteration of the algorithm.

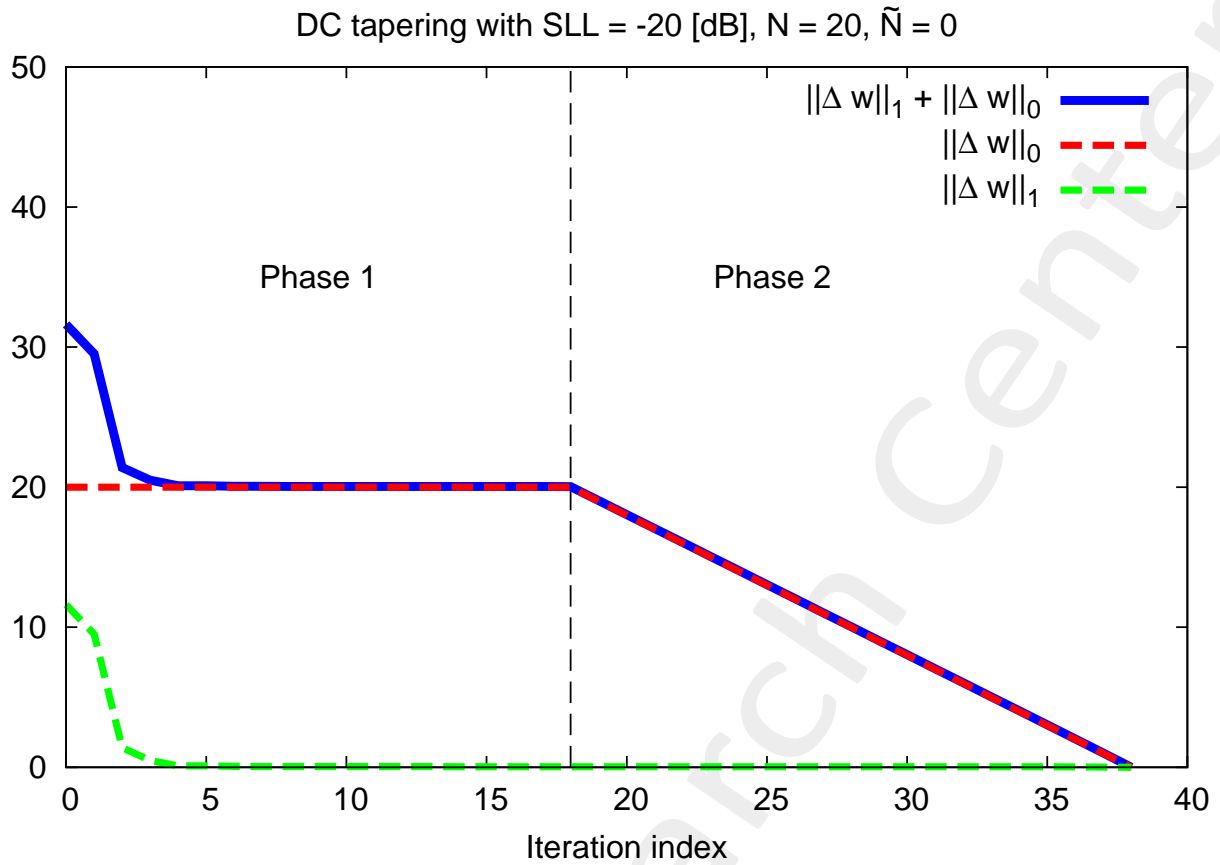


Figure 8: The value of the L1-norm cost function for each iteration of the algorithm.

Table II reports the SLL of the radiation patterns for the original, faulty and corrected excitations.

	Pattern SLL [dB]	HPBW [deg]	DRR	$\ w_{\text{corr,mut}} - w_{\text{orig,mut}}\ _1$	$\ w_{\text{corr,mut}} - w_{\text{orig,mut}}\ _0$
Original excitations	-20.00	5.36	0.464		
Faulty excitations	-17.08	5.46	0.464		
Corrected excitations (init.)	-14.89	5.20	1.0	2.80	19
Corrected excitations (Phase 1)	-20.52	5.51	0.568	1.74	19
Corrected excitations (Phase 2)	-20.11	5.49	0.568	1.50	8

Table II: Comparison of the original, faulty and corrected excitations.

#### 1.2.4 Observations

The array in Test case 2 has no damaged element, and we expected to find the original excitation set as a solution. The algorithm produces a set of excitations which is exactly equal to the original one.

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More information on the topics of this document can be found in the following list of references.

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