Failure Correction in Linear Arrays Through Compressive Sensing

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1 Introduction

The performance of phased array antennas depends on the excitations of the single radiating elements. When one of these elements fails, the overall radiation pattern is impacted negatively.

Failure correction aims at restoring the original pattern of a phased array with a number of damaged elements as closely as possible by correcting the excitations of the working elements. Minimum-complexity failure correction does so by changing as few excitations as possible.

The problem of failure correction has been thoroughly studied in the literature, and a wide range of techniques have been applied: numerical techniques, vector space projection, Genetic Algorithms, Particle Swarm Optimization, Differential Evolution, Simulated Annealing, and other nature-inspired optimization algorithms. The proposed methods also differ in the attribute of the array being modified: the magnitudes, the phases or the positions of the singular elements, or a combination of the three.

2 Mathematical Formulation

2.1 Embedded Element Factor

The far-field embedded element factor $\psi_n(\theta, \phi)$ of the n-th element of an array is defined as

$$\psi_n(\theta, \phi) = e^{jk\mathbf{p}_n \cdot \widehat{\mathbf{r}}} \tag{1}$$

where

- $\mathbf{p}_n$ is the position of the n-th element

$$\mathbf{p}_n = x_n \widehat{\mathbf{x}} + y_n \widehat{\mathbf{y}} + z_n \widehat{\mathbf{z}}, \tag{2}$$

- $\widehat{\mathbf{r}}$ is the direction considered

$$\widehat{\mathbf{r}} = \sin(\theta)\cos(\phi)\widehat{\mathbf{x}} + \sin(\theta)\sin(\phi)\widehat{\mathbf{y}} + \cos(\theta)\widehat{\mathbf{z}}. \tag{3}$$

In this document, we are only going to consider Uniform Linear Arrays positioned along the $[0, d(N-1)]$ interval of the x axis, where d is the inter-element distance and is equal to

$$d = \lambda/2. \tag{4}$$

Thus, the position of the n-th element of an array is given by

$$\mathbf{p}_n = x_n \widehat{\mathbf{x}}, \tag{5}$$

with

$$x_n = d(n-1). \tag{6}$$

Moreover, due to the circular symmetry of the system around the x axis, we are only interested in computing the embedded element factors for directions parallel to the xy plane. More specifically, it is possible to set $\phi = 0$, and thus the beam direction $\widehat{\mathbf{r}}$ of interest is given by

$$\widehat{\mathbf{r}} = \sin(\theta)\widehat{\mathbf{x}} + \cos(\theta)\widehat{\mathbf{z}}. \tag{7}$$

Inserting Equation (5) and (7) in (1), a simplified expression for the embedded element factor is obtained

$$\psi_n(\theta) = e^{jkx_n \sin(\theta)}, \tag{8}$$

where $k = 2\pi/\lambda$ is the wavenumber.e

The above expression can be rewritten as a function of $u = \sin(\theta)$ to obtain

$$\psi_n(u) = e^{jkx_n u}. \tag{9}$$

For the rest of the document, the last expression will be used to compute the embedded element factor.

2.2 Minimum-complexity Failure correction problem definition

Consider a one-dimensional Uniform Linear Array comprising N isotropic radiating elements, distributed along the x axis at a uniform distance $d = \lambda/2$ between $x = 0$ and $x = d(N-1)$. Suppose that the excitations of each element are reconfigurable, and let the column vector $\mathbf{w}_{\text{orig}} = [w_{\text{orig},1}, \ldots, w_{\text{orig},N}]^T \in \mathbb{C}^N$ represent the original excitations (or weights) of the radiating elements. The field $E_{\text{orig}}(u)$ radiated by the array with the original excitations is given by

$$E_{\text{orig}}(u) \triangleq \sum_{n=1}^{N} w_{\text{orig},n}\psi_n(u). \tag{10}$$

Suppose that a number D of radiating elements is damaged, and as a result their excitations change to a new value, which is immutable. Let the column vector $\mathbf{w}_{\text{faulty}} = [w_{\text{faulty},1}, \ldots, w_{\text{faulty},N}]^T \in \mathbb{C}^N$ represent the excitations of all the elements of the array in the damaged state, and let Ω be the set of the D indexes of the damaged elements.

The field $E_{\text{faulty}}(u)$ radiated by the array with the damaged excitations is given by

$$E_{\text{faulty}}(u) \triangleq \sum_{n=1}^{N} w_{\text{faulty},n}\psi_n(u). \tag{11}$$

Depending on the damaged excitations, the damaged pattern might differ significantly from the original pattern. Even though the excitations of the damaged elements are no longer reconfigurable, the excitations of the non-damaged elements can still be reconfigured to a different value. In this way, it is possible to partially restore the properties of the original pattern.

The minimum-complexity failure correction problem is that of finding a set of corrected weights $\mathbf{w}_{\text{corr}} = [w_{\text{corr},1}, \ldots, w_{\text{corr},N}]^T \in \mathbb{C}^N$, that radiate a field $E_{\text{corr}}(u)$ given by

$$E_{\text{corr}}(u) \triangleq \sum_{n=1}^{N} w_{\text{corr},n}\psi_n(u). \tag{12}$$

The corrected weights should be so that (a) the excitations of the damaged elements are fixed to their damaged value,

$w_{\text{corr},n} = w_{\text{faulty},n}, \forall n \in \Omega$, (b) the radiated field $E_{\text{corr}}(u)$ matches the original field $E_{\text{orig}}(u)$ is the main lobe direction u_{ML}, (c) the radiated field $E_{\text{corr}}(u)$ has a given SLL requirement, and (d) the minimum number of elements is reconfigured.

The minimum-complexity failure correction problem can be formulated as follows:

- **Minimum-complexity Failure Correction problem (initial formulation)** —Given the set of the damaged elements' indices Ω, the excitations of the array in the original and damaged state, $\mathbf{w}_{\text{orig}}$ and $\mathbf{w}_{\text{faulty}}$ respectively, the desired side lobe region, and the desired SLL, find the set of corrected excitations

$\mathbf{w}_{\mathrm{corr}}$, that

$$\text{minimizes} \qquad \left\|\Delta \mathbf{w}_{\mathrm{orig}} - \mathbf{w}_{\mathrm{corr}}\right\|_0 \tag{13}$$

$$\text{subject to} \qquad \begin{cases} w_{\mathrm{corr},n} = w_{\mathrm{faulty},n} \quad \forall n \in \Omega \\ \left|E_{\mathrm{corr}}(u_{\mathrm{ML}})\right|^2 = \left|E_{\mathrm{orig}}(u_{\mathrm{ML}})\right|^2 \\ \left|E_{\mathrm{corr}}(u)\right|^2 \leq \mathrm{SLL}_{\mathrm{desired}} \left|E_{\mathrm{corr}}(u_{\mathrm{ML}})\right|^2 \quad \forall u \in \mathrm{Sidelobe} \end{cases} \tag{14}$$

where $\|\cdot\|_1$ is the l_1-norm, u_{ML} is the main lobe direction of the original pattern and Sidelobe represents the observation angles outside the assigned main lobe region.

2.3 Independent variables reduction

Let us distinguish between the corrected excitations $\mathbf{w}_{\text{corr,mut}}$ that are working and thus can be mutated, from the corrected excitations $\mathbf{w}_{\text{corr,immut}}$ of the faulty elements that can not be mutated

$$\mathbf{w}_{\text{corr,mut}} \triangleq [w_{\text{corr},n}, n \notin \Omega]^T \in \mathbb{C}^{N-D}, \tag{15}$$

$$\mathbf{w}_{\text{corr,immut}} \triangleq [w_{\text{corr},n}, n \in \Omega]^T \in \mathbb{C}^{D}. \tag{16}$$

Similarly, let us distinguish the original excitations $\mathbf{w}_{\text{orig,mut}}$ for the working elements, and the original excitations $\mathbf{w}_{\text{orig,immut}}$ for the faulty elements,

$$\mathbf{w}_{\text{orig,mut}} \triangleq [w_{\text{orig},n}, n \notin \Omega]^T \in \mathbb{C}^{N-D}, \tag{17}$$

$$\mathbf{w}_{\text{orig,immut}} \triangleq [w_{\text{orig},n}, n \in \Omega]^T \in \mathbb{C}^{D}. \tag{18}$$

Given the condition (a) mentioned above or, equivalently, the first constraint of (14), a valid solution to the MFC problem needs to satisfy

$$\mathbf{w}_{\text{corr,immut}} = \mathbf{w}_{\text{orig,immut}}. \tag{19}$$

Therefore, we can rephrase the original problem only in terms of the mutable corrected weights, thus dropping the first constraint.

For the sake of brevity, let us denote with $\Delta\mathbf{w}$ the excitation corrections,

$$\Delta\mathbf{w} = \mathbf{w}_{\text{orig,mut}} - \mathbf{w}_{\text{corr,mut}}. \tag{20}$$

- **Minimum-complexity Failure Correction problem (reduced variables formulation)**—Given the set of the damaged elements' indices Ω, the excitations of the array in the original and damaged state, $\mathbf{w}_{\text{orig}}$ and $\mathbf{w}_{\text{faulty}}$ respectively, the desired side lobe region, and the desired SLL, find the set of mutable corrected excitations $\mathbf{w}_{\text{corr,mut}}$, that

$$\text{minimizes} \quad \|\Delta\mathbf{w}\|_0 \tag{21}$$

$$\text{subject to} \quad \begin{cases} |E_{\text{corr}}(u_{\text{ML}})|^2 = |E_{\text{orig}}(u_{\text{ML}})|^2 \\ |E_{\text{corr}}(u)|^2 \leq \text{SLL}_{\text{desired}} \, |E_{\text{corr}}(u_{\text{ML}})|^2 \quad \forall u \in \text{Sidelobe} \end{cases}. \tag{22}$$

2.4 Problem discretization

We can discretize the problem by considering a finite set of K points $U = \{u_1, \ldots, u_K\} \subset$ Sidelobe where the SLL constraint is enforced. This operation results in K single-point inequality constraints, where the k-th inequality is given by

$$|E_{\mathrm{corr}}(u_k)|^2 \leq \mathrm{SLL}_{\mathrm{desired}} |E_{\mathrm{orig}}(u_{\mathrm{ML}})|^2 \tag{23}$$

In this document, the set U of samples is chosen in the following way,

$$U(K, \mathrm{BW}) = \left\{-1 + k\frac{1 - \mathrm{BW}/2}{\frac{K}{2} - 1}, k \in \left[0, \frac{K}{2} - 1\right]\right\} \cup \left\{1 - k\frac{1 - \mathrm{BW}/2}{\frac{K}{2} - 1}, k \in \left[0, \frac{K}{2} - 1\right]\right\} \tag{24}$$

where K is the number of u samples to consider and is assumed to be even, and BW is the desired main lobe beamwidth.

- **Minimum-complexity Failure Correction problem** (**discretized formulation**)—Given the set of the damaged elements' indices Ω, the excitations of the array in the original and damaged state, $\mathbf{w}_{\mathrm{orig}}$ and $\mathbf{w}_{\mathrm{faulty}}$ respectively, the desired SLL and beamwidth, and the number K of angular samples to consider, find the set of mutable corrected excitations $\mathbf{w}_{\mathrm{corr,mut}}$, that

$$\text{minimizes} \quad \|\Delta\mathbf{w}\|_0 \tag{25}$$

$$\text{subject to} \quad \begin{cases} |E_{\mathrm{corr}}(u_{\mathrm{ML}})|^2 = |E_{\mathrm{orig}}(u_{\mathrm{ML}})|^2 \\ |E_{\mathrm{corr}}(u)|^2 \leq \mathrm{SLL}_{\mathrm{desired}} |E_{\mathrm{corr}}(u_{\mathrm{ML}})|^2 \quad \forall u \in U(K, \mathrm{BW}) \end{cases}. \tag{26}$$

2.5 Proposed algorithm

The three phases of the proposed algorithm are the following:

- **Phase 1**—First, an initial, coarse solution to the failure correction problem is computed, as described in Section 2.5.1.
- **Phase 2**—The optimization problem defined in (25), is solved using the interior-point algorithm provided by MATLAB's `fmincon()`, as described in Section 2.5.2.
- **Phase 3**—The solution from the procedure in Phase 2 is further processed in an iterative fashion in order to remove the corrections that have little or no impact on the radiated field. This procedure is described in more detail in Section 2.5.3.

2.5.1 Phase 1: the initialization procedure

For the initialization phase, all the reconfigurable excitations are set to a fixed value. The choice of this value is not critical to the end results. Here, the average of the original mutable excitations is chosen as the fixed value. The set of corrected excitations $\mathbf{w}^{(1)}_{\text{corr,mut}}$ for Phase 1 is then given by

$$w^{(1)}_{\text{corr,mut},n} = \frac{1}{N - N_f} \sum_{p \notin \Omega} w_{\text{orig,mut},p} \tag{27}$$

2.5.2 Phase 2: l_1-norm minimization

Using the corrections $\mathbf{w}^{(1)}_{\text{corr,mut}}$ obtained in the previous phase as starting point, the following minimization problem is solved using an interior-point algorithm:

$$\text{find } \mathbf{w}^{(2)}_{\text{corr,mut}} \text{ that minimizes } \|\Delta\mathbf{w}\|_1 \tag{28}$$

$$\text{subject to } |E_{\text{corr}}(u_k)|^2 \leq \text{SLL}^{(2)}_{\text{target}} \, |E_{\text{corr}}(0)|^2 \quad \forall u_k \in U(K^{(2)}, \text{BW}^{(2)}_{\text{target}}), \tag{29}$$

where $\text{SLL}^{(2)}_{\text{target}}, K^{(2)}$, and $\text{BW}^{(2)}_{\text{target}}$ are user-defined parameters for Phase 2 of the algorithm.

2.5.3 Phase 3: l_0-norm minimization

The solution $\mathbf{w}^{(2)}_{\text{corr}}$ obtained in Phase 2 of the proposed algorithm is further processed in an iterative fashion ($i = 1, 2, \dots$) to remove the corrections that have little impact on the radiated field, as long as the pattern still satisfies a mask defined by three parameters ($\text{SLL}^{(3)}$, $K^{(3)}$and $\text{BW}^{(3)}$).

The used algorithm is the following

- **Step 3-A**: Initialize the set of required corrections Γ to the empty set

$$\Gamma \leftarrow \{\} \tag{30}$$

  As corrections are marked as required, their index will be added to the set Γ.

- **Step 3-B**: For the i-th iteration of the algorithm and starting from $i = 1$, find the position $p^{(i)}$ of the correction with minimal non-zero magnitude from the set of corrections of the previous iteration, $\mathbf{w}^{(3,i-1)}_{\text{corr,mut}}$, that has not been marked as required

$$\begin{aligned} \text{find } p^{(i)} \text{ that minimizes} \quad & \min\left\{\left|w^{(3,i-1)}_{\text{corr,mut},p^{(i)}} - w_{\text{orig,mut},p^{(i)}}\right|\right\} \\ \text{subject to} \quad & \begin{cases} \left|w^{(3,i-1)}_{\text{corr,mut},p^{(i)}} - w_{\text{orig,mut},p^{(i)}}\right| \neq 0 \\ p^{(i)} \in \Gamma \end{cases} \end{aligned}$$

  For the first iteration, substitute $\mathbf{w}^{(3,i-1)}_{\text{corr,mut}}$ with the solution $\mathbf{w}^{(2)}_{\text{corr,mut}}$ from the previous step. If no correction can be found that satisfies all constraints, the iteration stops.

- **Step 3-C**: Now, consider a new set of excitations $\mathbf{w}^{(3,i)}_{\text{corr,mut}}$ obtained by removing the $p^{(i)}$-th correction from $\mathbf{w}_{\text{corr,mut}}^{(3,i-1)}$

$$w_{\text{corr,mut},n}^{(3,i)} \leftarrow \begin{cases} w_{\text{corr,mut},n}^{(3,i-1)} & \text{if } n \neq p^{(i)} \\ w_{\text{orig,mut},n} & \text{if } n = p^{(i)} \end{cases} \tag{31}$$

The new set of excitations $\mathbf{w}_{\text{corr,mut}}^{(3,i)}$ will then have a L0 cost which is 1 lower than that of the previous set of excitations.

- **Step 3-D**: Check if the new pattern $E_{\text{corr}}^{(3,i)}(u) = \sum_{n=1}^{N} w_{\text{corr},n}^{(3,i)} \psi_n(u)$ for the new set of excitations satisfies

$$\begin{cases} \left| E_{\text{corr}}^{(3,i)}(u_{\text{ML}}) \right|^2 = \left| E_{\text{corr}}^{(3,i)}(u_{\text{ML}}) \right|^2 \\ \left| E_{\text{corr}}^{(3,i)}(u_k) \right|^2 \leq \text{SLL}^{(3)} \left| E_{\text{orig}}(u_{\text{ML}}) \right|^2 \quad \forall u_k \in U(K^{(3)}, \text{BW}^{(3)}) \end{cases} \tag{32}$$

where $\text{SLL}^{(3)}$ and $\text{BW}^{(3)}$, $K^{(3)}$are user-defined parameters. If the pattern $E_{\text{corr}}^{(3,i)}$ satisfies the mask, the set of required excitations is reset to the empty set

$$\Gamma \leftarrow \{\} \tag{33}$$

Otherwise, the $p^{(i)}$-th correction is restored and added to the set of required corrections

$$\begin{aligned} E_{\text{corr}}^{(3,i)} &\leftarrow E_{\text{corr}}^{(3,i-1)} \\ \Gamma &\leftarrow \Gamma \cup \{p^{(i)}\} \end{aligned}$$

In both cases, the iteration then continues from Phase 3-B with incremented value of i.

$$i \leftarrow i + 1 \tag{34}$$

More information on the topics of this document can be found in the following list of references.

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