
Spline-shaped SbD Synthesis of Ogive Radome

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1 [APPROACH 2] Optimization with PSO + Kriging (update negative fitness) - EXP CORRELATION

1.0.1 Motivation and goal

Problem

The main issue encountered when using the PSO+Kriging optimizer is the fact that the optimization ends with a negative fitness value, that is not physical.

Solution

In order to overcome such a problem, a different implementation of the optimization algorithm has been taken into account in this section. During the optimization the PSO uses a Kriging predictor to estimate the fitness associated to each trial solution. However, if a negative fitness is predicted for a given particle, the following additional operations are performed:

1. **the particle with negative predicted fitness is simulated** with the full-wave solver;
2. its real fitness is computed and **substituted** to the negative prediction;
3. the particle and its real fitness are **added to the pre-existing training set**, overwriting the training sample with the highest fitness. The dimension of the training set is thus kept constant during the whole optimization process;
4. the Kriging predictor is **re-trained** with the updated training set;
5. the optimization continues, using the updated predictor for successive predictions.

Goal

The main goal is to avoid the occurrence of negative fitness values during the PSO optimization, since they are not-physical.

1.0.2 Parameters

Optimization targets

- Number of variables: $K = 5$;
- Frequency range:
 - Minimum frequency: $f_{min} = 10.75$ [GHz];
 - Maximum frequency: $f_{max} = 14.5$ [GHz];
 - Number of frequency steps: $N_f = 10$ ($\Delta f \simeq 0.42$ [GHz]);
 - Central frequency: $f_0 = \frac{f_{min} + f_{max}}{2} \simeq 12.63$ [GHz];
 - Free-space wavelength at the central frequency: $\lambda_0 = \frac{c}{f_0} = 2.38 \times 10^{-2}$ [m];
- Scanning angle range:
 - Minimum scanning angle: $\theta_{min} = 0$ [deg];
 - Maximum scanning angle: $\theta_{max} = 45$ [deg];
 - Number of angular steps: $N_\theta = 4$ ($\theta_1 = 0$ [deg], $\theta_2 = 15$ [deg], $\theta_3 = 30$ [deg], $\theta_4 = 45$ [deg]);

PSO parameters

- Population dimension: $P = 10$;
- Maximum number of iterations: $I_{max} = 200$;
- Fitness threshold: $\Phi^{th} = 10^{-20}$;
- Inertial weight: $w = 0.4$;
- Constant inertial velocity;
- Exploration coefficient: $c_1 = 2$;
- Exploitation coefficient: $c_2 = 2$;
- Random seed $s = 1, 2, \dots, 10$;
- Initialization (generation of the initial swarm): use the same seed for all the optimizations.

Kriging (Gaussian Process Regressor) parameters

- Regression model: constant (Ordinary Kriging);
- Correlation models:
 - Exponential ($p = 1$);

- Initial guess for hyper-parameters θ_h : $\theta_{h,0} = 0.5$, for $h = 1, \dots, K$;
- Lower bound for hyper-parameters θ_h : $\min \{\theta_h\} = 0.1$, for $h = 1, \dots, K$;
- Upper bound for hyper-parameters θ_h : $\max \{\theta_h\} = 20.0$, for $h = 1, \dots, K$;

Not-optimized (static) radome parameter

Parameter	Description	Value
L	Length of the radome	$1.59 \times 10^{-1} [m] \simeq 6.69 \lambda_0$
D	Base diameter of the radome	$1.27 \times 10^{-1} [m] \simeq 5.35 \lambda_0$
t_0	Thickness of the base and of the top of the radome	$8.20 \times 10^{-3} [m] \simeq \frac{\lambda_r}{2}$
z_1	z -coordinate of the spline control point 1	$\frac{L-t_0}{6}$
z_2	z -coordinate of the spline control point 2	$2\frac{L-t_0}{6}$
z_3	z -coordinate of the spline control point 3	$3\frac{L-t_0}{6}$
z_4	z -coordinate of the spline control point 4	$4\frac{L-t_0}{6}$
z_5	z -coordinate of the spline control point 5	$5\frac{L-t_0}{6}$
ν	External curvature of the radome ($\nu \in [1, 2]$)	1.449 (tangent ogive)
ϵ_r	Permittivity of the radome material	2.10 (Teflon)
$\tan\delta_r$	Tangent delta of the radome material	$\tan\delta = 3.00 \times 10^{-4} @ 10.0 [GHz]$ (Teflon)
λ_r	Wavelength in the radome material	$\lambda_r \simeq \frac{c}{f_0\sqrt{\epsilon}} \simeq 1.64 \times 10^{-1}$

Table I: List of non-optimized radome parameters.

Antenna Parameters

- Linear dipole array placed over circular ground plane (PEC)
- Number of array elements: $N_e = 8$
- Dipole length: $l_e = \frac{\lambda_0}{2}$
- Array elements spacing: $d_e = \lambda/2$
- Spacing between the array and the ground plane: $h_e = \frac{\lambda_0}{4}$

Parameters boundaries

Parameter	Description	Min	Max
t_1	Radome thickness at the quota $z = z_1$	$3.28 \times 10^{-3} [m] (0.2\lambda_r)$	$13.12 \times 10^{-3} [m] (0.8\lambda_r)$
t_2	Radome thickness at the quota $z = z_2$	$3.28 \times 10^{-3} [m] (0.2\lambda_r)$	$13.12 \times 10^{-3} [m] (0.8\lambda_r)$
t_3	Radome thickness at the quota $z = z_3$	$3.28 \times 10^{-3} [m] (0.2\lambda_r)$	$13.12 \times 10^{-3} [m] (0.8\lambda_r)$
t_4	Radome thickness at the quota $z = z_4$	$3.28 \times 10^{-3} [m] (0.2\lambda_r)$	$13.12 \times 10^{-3} [m] (0.8\lambda_r)$
t_5	Radome thickness at the quota $z = z_5$	$3.28 \times 10^{-3} [m] (0.2\lambda_r)$	$13.12 \times 10^{-3} [m] (0.8\lambda_r)$

Table II: List of all considered boundaries for the optimized radome descriptors.

1.0.3 Comparison between the different random seeds ($s \in [1, 10]$)

Predicted fitness vs. real fitness

Legend

- $\tilde{\Phi}_0$: Predicted fitness for the best particle of the initial swarm;
- Φ_0 : Actual fitness for the best particle of the initial swarm;
- $\tilde{\Phi}^{opt}$: Predicted fitness for the optimal solution found at the end of the PSO;
- Φ^{opt} : Actual fitness computed for the optimal solution found at the end of the PSO;
- $\Phi_{train}^{opt} = 1.13$: Best fitness inside the training set.

Seed	# Updates	Predicted		Actual		
		$\tilde{\Phi}_0$	$\tilde{\Phi}^{opt}$	Φ_0	Φ^{opt}	$100 \frac{\Phi_{train}^{opt} - \Phi^{opt}}{\Phi_{train}^{opt}}$
1	1	1.13	3.83×10^{-2}	2.11	1.16	-2.65
2	1	1.13	9.51×10^{-3}	2.11	1.47	-30.09
3	2	1.13	1.50×10^{-2}	2.11	1.18	-4.42
4	1	1.13	2.07×10^{-1}	2.11	9.59×10^{-1}	15.13
5	2	1.13	9.84×10^{-3}	2.11	1.09	3.54
6	2	1.13	4.49×10^{-2}	2.11	9.46×10^{-1}	16.28
7	1	1.13	2.43×10^{-2}	2.11	1.52	-34.51
8	1	1.13	3.04×10^{-2}	2.11	8.48×10^{-1}	24.96
9	1	1.13	5.99×10^{-2}	2.11	1.14	-0.88
10	1	1.13	1.34×10^{-2}	2.11	1.12	0.88

Table III: Number of updates during the optimization, initial and final predicted fitness and associated real fitness, for each considered random seed $s \in [1, 10]$.

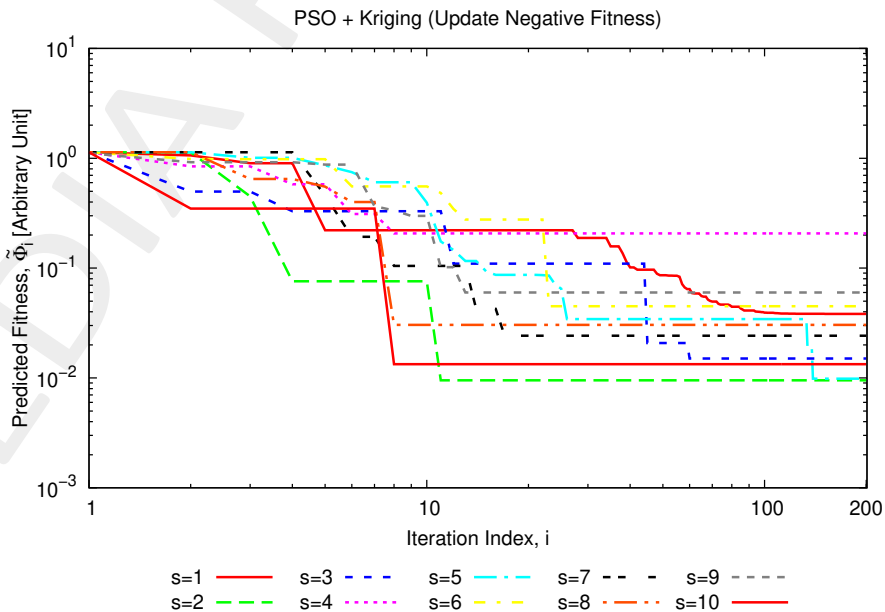


Figure 1: Predicted fitness vs. iteration index, for different random seeds ($s \in [1, 10]$).

Statistics (over $S = 10$ seeds)

Predicted				Actual			
$\min \{\tilde{\Phi}^{opt}\}$	$\max \{\tilde{\Phi}^{opt}\}$	$\text{avg} \{\tilde{\Phi}^{opt}\}$	$\text{std} \{\tilde{\Phi}^{opt}\}$	$\min \{\Phi^{opt}\}$	$\max \{\Phi^{opt}\}$	$\text{avg} \{\Phi^{opt}\}$	$\text{std} \{\Phi^{opt}\}$
9.51×10^{-3}	2.07×10^{-1}	4.52×10^{-2}	5.61×10^{-2}	8.48×10^{-1}	1.52	1.14	2.04×10^{-1}

Lowest Training Sample

Table IV: Statistics (min, max, average and standard deviation) of the predicted and actual fitness obtained over $S = 10$ seeds.

1.0.4 Difference between predicted and actual final fitness over $S = 10$ seeds

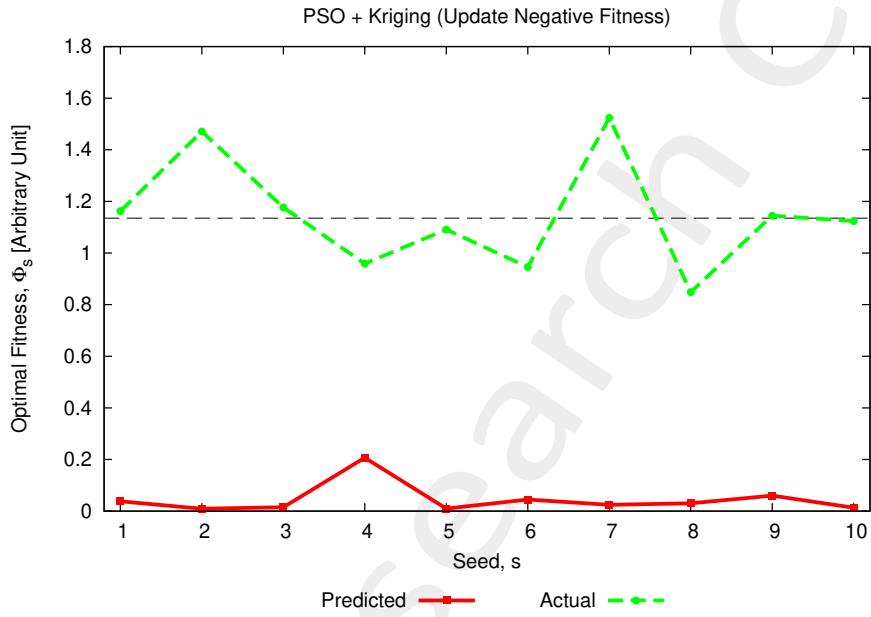


Figure 2: Predicted and actual final fitness for different random seeds ($s \in [1, 10]$).

- Mean Absolute Error (MAE): $MAE = \frac{1}{S} \sum_{s=1}^S |\tilde{\Phi}_s^{opt} - \Phi_s^{opt}|$
- Normalized Mean Error (NME): $NME = \frac{1}{S} \sum_{s=1}^S \frac{|\tilde{\Phi}_s^{opt} - \Phi_s^{opt}|}{|\Phi_s^{opt}|}$
- Matching Error (ME): $ME = \frac{1}{S} \frac{\sum_{s=1}^S |\tilde{\Phi}_s^{opt} - \Phi_s^{opt}|^2}{\sum_{s=1}^S |\Phi_s^{opt}|^2}$

MAE	NME	ME
1.94	1.39	1.90

Table V: Difference between predicted and actual fitness for all considered random seeds.

1.0.5 Comparison with the PSO that doesn't update the training set (seed $s = 8$)

The following figure directly compares the evolution of the predicted fitness for the same optimal seed ($s = 8$) for

1. the PSO+Kriging updating the negative fitness during the evolution;
2. the PSO+Kriging that does not update the training set during the evolution.

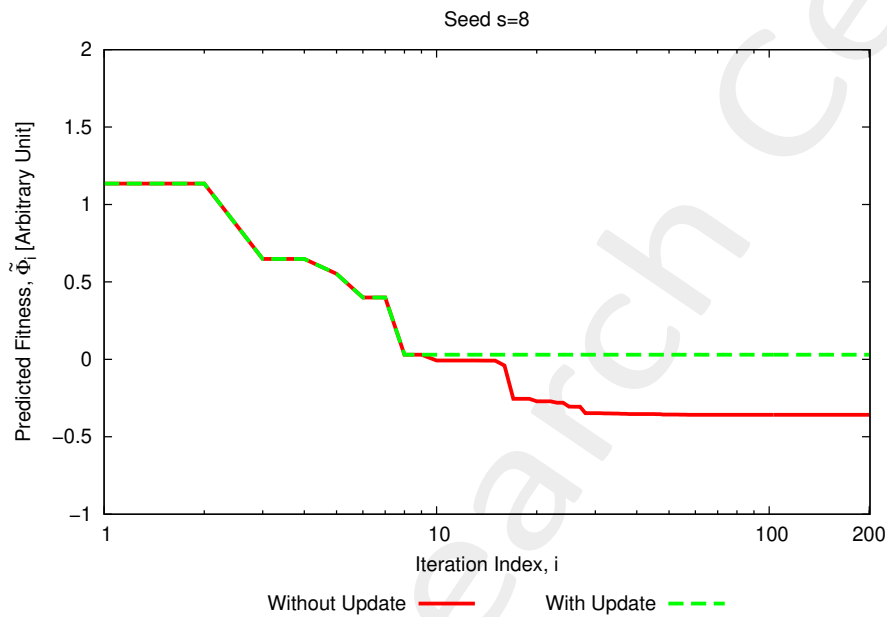


Figure 3: Predicted fitness for seed $s = 8$, when using the two optimization methods (with and without (iteration 1) update of the negative fitness).

As it can be seen, given the same seed and initial swarm, the two fitness curves perfectly overlap up to iteration $i = 8$. This is because at iteration $i = 9$ a negative fitness is predicted for one of the particles in the swarm, and one of the two optimizations continues with an updated training set, while the other one continues with negative fitness values.

1.0.6 Optimized parameters ($t_1^{opt}, \dots, t_5^{opt}$) vs. seed

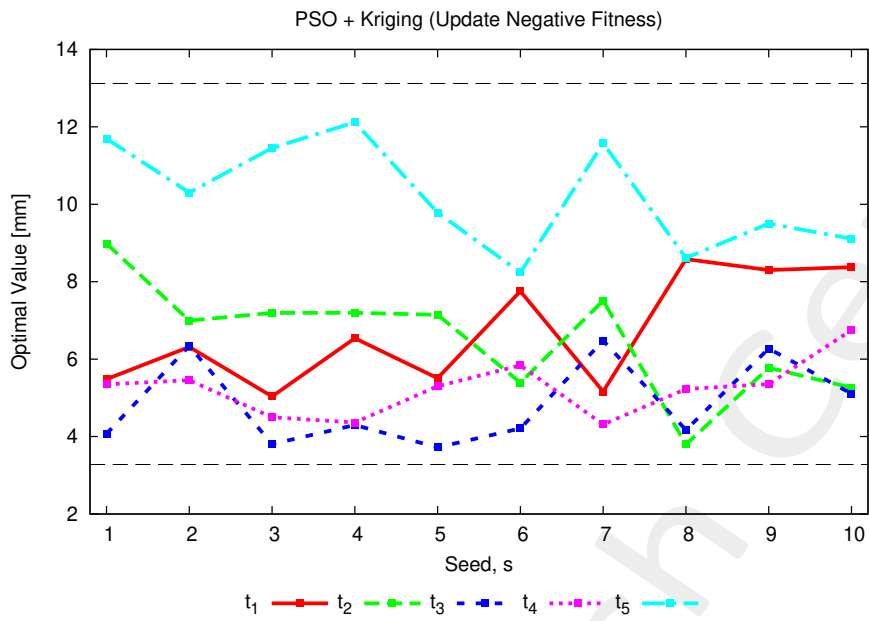


Figure 4: Predicted and actual final fitness for different random seeds ($s \in [1, 10]$).

1.0.7 Best solution found (min. actual fitness)

- Seed: $s = 8$;
- True fitness: $\Phi^{opt} = 8.48 \times 10^{-1}$;
- Average BSE error: $BSE_{avg} = \sqrt{\Phi^{opt}} = 0.92$ [deg].

1.0.8 Analysis of the best solution (seed $s = 8$)

Fitness evolution

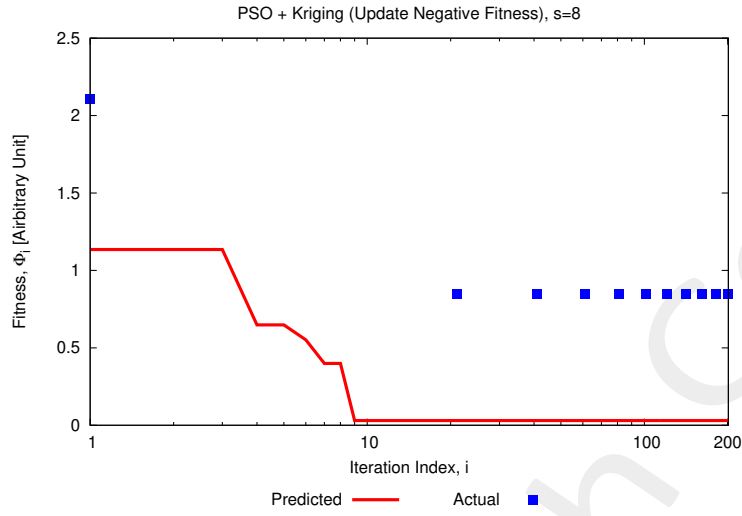


Figure 5: Evolution of the (predicted) fitness and actual fitness computed for the intermediate optimal particles each 20 iterations (“control points”).

Optimized parameters ($t_1^{opt}, \dots, t_5^{opt}$)

Parameter	Description	Optimized Value [m]	Min [m]	Max [m]
t_1	Radome thickness at the quota $z = z_1$	8.58×10^{-3}	3.28×10^{-3}	13.12×10^{-3}
t_2	Radome thickness at the quota $z = z_2$	3.79×10^{-3}	3.28×10^{-3}	13.12×10^{-3}
t_3	Radome thickness at the quota $z = z_3$	4.17×10^{-3}	3.28×10^{-3}	13.12×10^{-3}
t_4	Radome thickness at the quota $z = z_4$	5.23×10^{-3}	3.28×10^{-3}	13.12×10^{-3}
t_5	Radome thickness at the quota $z = z_5$	8.61×10^{-3}	3.28×10^{-3}	13.12×10^{-3}

Table VI: Optimized parameters for the best seed ($s = 8$).

Pointing error (BSE) vs. frequency

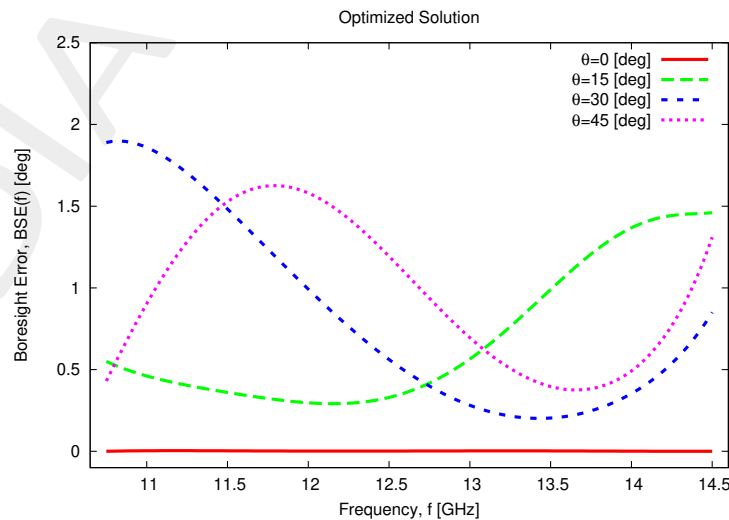


Figure 6: Pointing error (BSE) vs. frequency.

Directivity patterns

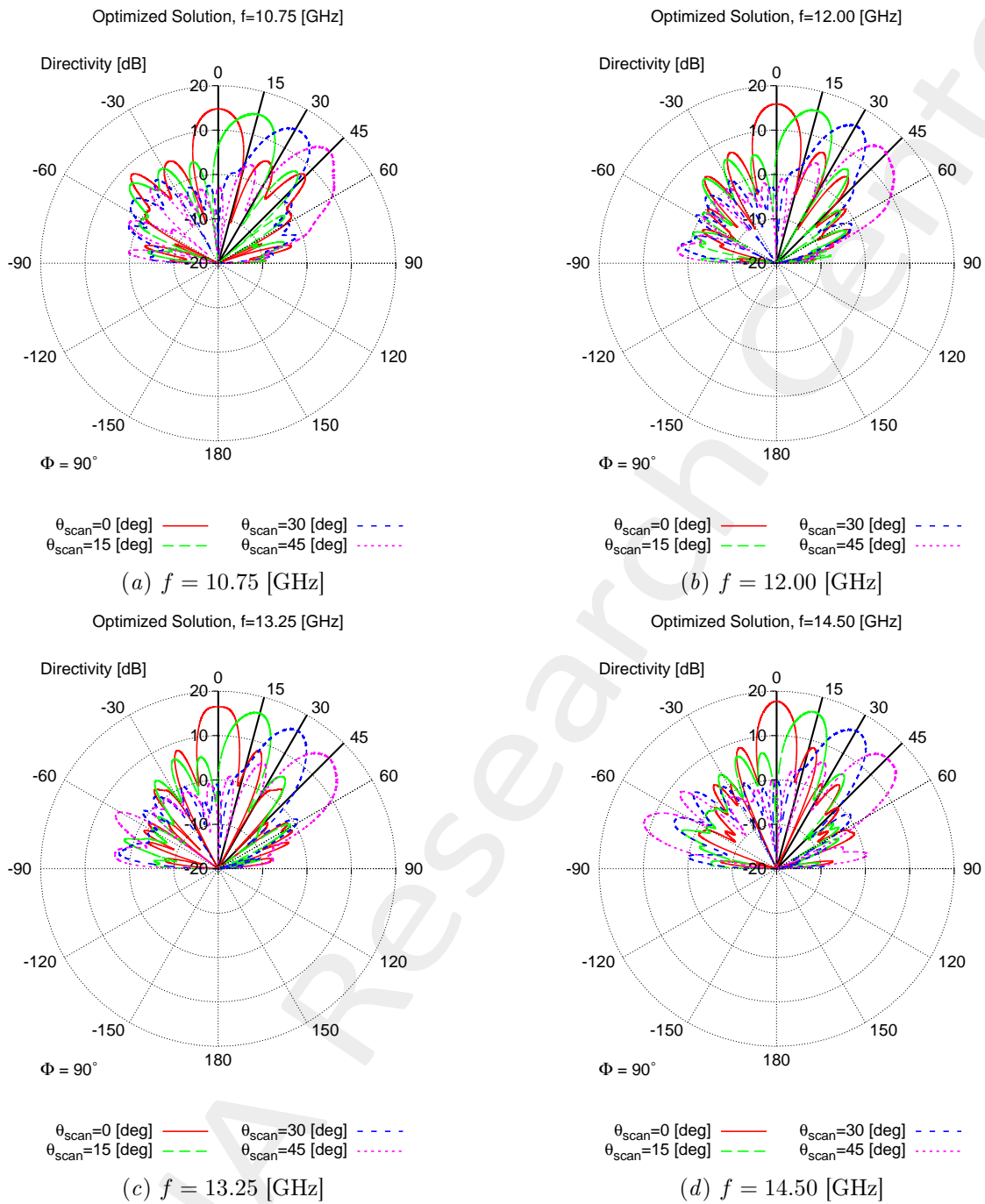


Figure 7: Radiated pattern for the optimized solution at (a) 10.75 [GHz], (b) 12.00 [GHz], (c) 13.25 [GHz] and (d) 14.50 [GHz].

2 [APPROACH 2] Optimization with PSO + Kriging (update negative fitness) - GAUSS CORRELATION

2.0.1 Parameters

Optimization targets

- Number of variables: $K = 5$;
- Frequency range:
 - Minimum frequency: $f_{min} = 10.75$ [GHz];
 - Maximum frequency: $f_{max} = 14.5$ [GHz];
 - Number of frequency steps: $N_f = 10$ ($\Delta f \simeq 0.42$ [GHz]);
 - Central frequency: $f_0 = \frac{f_{min} + f_{max}}{2} \simeq 12.63$ [GHz];
 - Free-space wavelength at the central frequency: $\lambda_0 = \frac{c}{f_0} = 2.38 \times 10^{-2}$ [m];
- Scanning angle range:
 - Minimum scanning angle: $\theta_{min} = 0$ [deg];
 - Maximum scanning angle: $\theta_{max} = 45$ [deg];
 - Number of angular steps: $N_\theta = 4$ ($\theta_1 = 0$ [deg], $\theta_2 = 15$ [deg], $\theta_3 = 30$ [deg], $\theta_4 = 45$ [deg]);

PSO parameters

- Population dimension: $P = 10$;
- Maximum number of iterations: $I_{max} = 200$;
- Fitness threshold: $\Phi^{th} = 10^{-20}$;
- Inertial weight: $w = 0.4$;
- Constant inertial velocity;
- Exploration coefficient: $c_1 = 2$;
- Exploitation coefficient: $c_2 = 2$;
- Random seed $s = 1, 2, \dots, 10$;
- Initialization (generation of the initial swarm): use the same seed for all the optimizations.

Kriging (Gaussian Process Regressor) parameters

- Regression model: constant (Ordinary Kriging);

- Correlation models:
 - **Gaussian** ($p = 2$):
- Initial guess for hyper-parameters θ_h : $\theta_{h,0} = 0.5$, for $h = 1, \dots, K$;
- Lower bound for hyper-parameters θ_h : $\min \{\theta_h\} = 0.1$, for $h = 1, \dots, K$;
- Upper bound for hyper-parameters θ_h : $\max \{\theta_h\} = 20.0$, for $h = 1, \dots, K$;

Not-optimized (static) radome parameter

Parameter	Description	Value
L	Length of the radome	$1.59 \times 10^{-1} [m] \simeq 6.69 \lambda_0$
D	Base diameter of the radome	$1.27 \times 10^{-1} [m] \simeq 5.35 \lambda_0$
t_0	Thickness of the base and of the top of the radome	$8.20 \times 10^{-3} [m] \simeq \frac{\lambda_r}{2}$
z_1	z -coordinate of the spline control point 1	$\frac{L-t_0}{6}$
z_2	z -coordinate of the spline control point 2	$2 \frac{L-t_0}{6}$
z_3	z -coordinate of the spline control point 3	$3 \frac{L-t_0}{6}$
z_4	z -coordinate of the spline control point 4	$4 \frac{L-t_0}{6}$
z_5	z -coordinate of the spline control point 5	$5 \frac{L-t_0}{6}$
ν	External curvature of the radome ($\nu \in [1, 2]$)	1.449 (tangent ogive)
ε_r	Permittivity of the radome material	2.10 (Teflon)
$\tan \delta_r$	Tangent delta of the radome material	$\tan \delta = 3.00 \times 10^{-4} @ 10.0 [GHz]$ (Teflon)
λ_r	Wavelength in the radome material	$\lambda_r \simeq \frac{c}{f_0 \sqrt{\varepsilon}} \simeq 1.64 \times 10^{-1}$

Table VII: List of non-optimized radome parameters.

Antenna Parameters

- Linear dipole array placed over circular ground plane (PEC)
- Number of array elements: $N_e = 8$
- Dipole length: $l_e = \frac{\lambda_0}{2}$
- Array elements spacing: $d_e = \lambda/2$
- Spacing between the array and the ground plane: $h_e = \frac{\lambda_0}{4}$

Parameters boundaries

Parameter	Description	Min	Max
t_1	Radome thickness at the quota $z = z_1$	$3.28 \times 10^{-3} [m] (0.2\lambda_r)$	$13.12 \times 10^{-3} [m] (0.8\lambda_r)$
t_2	Radome thickness at the quota $z = z_2$	$3.28 \times 10^{-3} [m] (0.2\lambda_r)$	$13.12 \times 10^{-3} [m] (0.8\lambda_r)$
t_3	Radome thickness at the quota $z = z_3$	$3.28 \times 10^{-3} [m] (0.2\lambda_r)$	$13.12 \times 10^{-3} [m] (0.8\lambda_r)$
t_4	Radome thickness at the quota $z = z_4$	$3.28 \times 10^{-3} [m] (0.2\lambda_r)$	$13.12 \times 10^{-3} [m] (0.8\lambda_r)$
t_5	Radome thickness at the quota $z = z_5$	$3.28 \times 10^{-3} [m] (0.2\lambda_r)$	$13.12 \times 10^{-3} [m] (0.8\lambda_r)$

Table VIII: List of all considered boundaries for the optimized radome descriptors.

2.0.2 Comparison between the different random seeds ($s \in [1, 10]$)

Predicted fitness vs. real fitness

Legend

- $\tilde{\Phi}_0$: Predicted fitness for the best particle of the initial swarm;
- Φ_0 : Actual fitness for the best particle of the initial swarm;
- $\tilde{\Phi}^{opt}$: Predicted fitness for the optimal solution found at the end of the PSO;
- Φ^{opt} : Actual fitness computed for the optimal solution found at the end of the PSO;
- $\Phi_{train}^{opt} = 1.13$: Best fitness inside the training set.

Seed	# Updates	Predicted		Actual		
		$\tilde{\Phi}_0$	$\tilde{\Phi}^{opt}$	Φ_0	Φ^{opt}	$100 \frac{\Phi_{train}^{opt} - \Phi^{opt}}{\Phi_{train}^{opt}}$
1	1	8.16×10^{-1}	1.08×10^{-2}	1.32	1.37	-21.24
2	1	8.16×10^{-1}	6.18×10^{-1}	1.32	9.79×10^{-1}	13.36
3	1	8.16×10^{-1}	4.21×10^{-2}	1.32	1.08	4.42
4	1	8.16×10^{-1}	1.22×10^{-3}	1.32	1.54	-36.28
5	1	8.16×10^{-1}	1.01×10^{-1}	1.32	8.85×10^{-1}	21.68
6	1	8.16×10^{-1}	1.63×10^{-1}	1.32	1.09	3.54
7	1	8.16×10^{-1}	6.61×10^{-1}	1.32	1.65	-46.02
8	2	8.16×10^{-1}	1.06×10^{-3}	1.32	9.28×10^{-1}	17.88
9	1	8.16×10^{-1}	6.06×10^{-2}	1.32	9.16×10^{-1}	18.94
10	2	8.16×10^{-1}	3.01×10^{-3}	1.32	1.33	-17.70

Table IX: Number of updates during the optimization, initial and final predicted fitness and associated real fitness, for each considered random seed $s \in [1, 10]$.

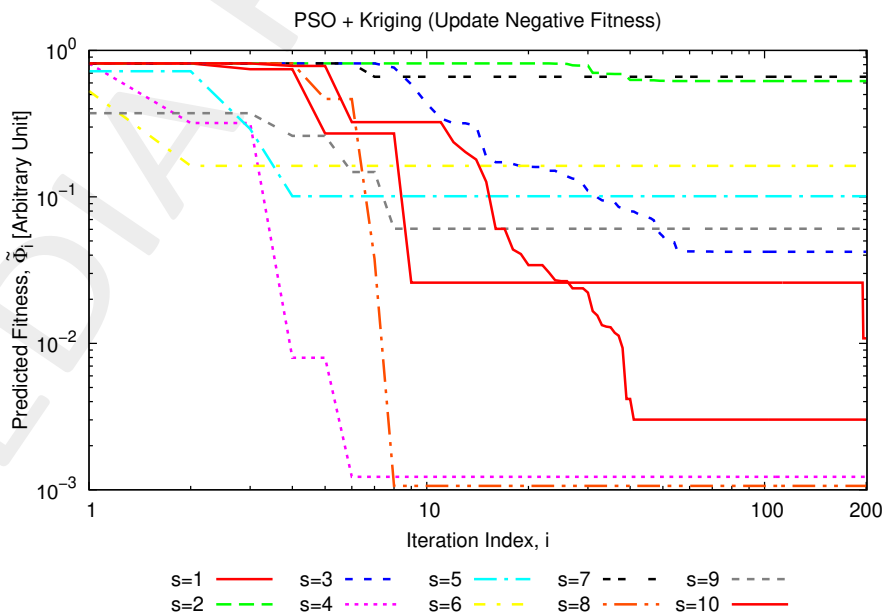


Figure 8: Predicted fitness vs. iteration index, for different random seeds ($s \in [1, 10]$).

Statistics (over $S = 10$ seeds)

Predicted				Actual			
$\min \{\tilde{\Phi}^{opt}\}$	$\max \{\tilde{\Phi}^{opt}\}$	$\text{avg} \{\tilde{\Phi}^{opt}\}$	$\text{std} \{\tilde{\Phi}^{opt}\}$	$\min \{\Phi^{opt}\}$	$\max \{\Phi^{opt}\}$	$\text{avg} \{\Phi^{opt}\}$	$\text{std} \{\Phi^{opt}\}$
1.06×10^{-3}	6.61×10^{-1}	1.66×10^{-1}	2.42×10^{-1}	8.85×10^{-1}	1.65	1.18	2.61×10^{-1}

Lowest Training Sample

Table X: Statistics (min, max, average and standard deviation) of the predicted and actual fitness obtained over $S = 10$ seeds.

2.0.3 Difference between predicted and actual final fitness over $S = 10$ seeds

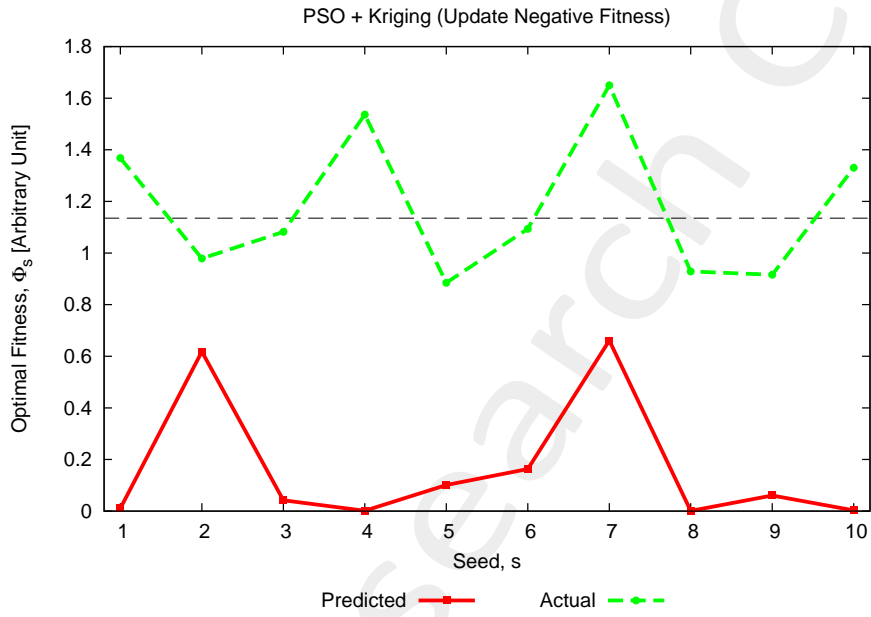


Figure 9: Predicted and actual final fitness for different random seeds ($s \in [1, 10]$).

- Mean Absolute Error (MAE): $MAE = \frac{1}{S} \sum_{s=1}^S |\tilde{\Phi}_s^{opt} - \Phi_s^{opt}|$
- Normalized Mean Error (NME): $NME = \frac{1}{S} \sum_{s=1}^S \frac{|\tilde{\Phi}_s^{opt} - \Phi_s^{opt}|}{|\Phi_s^{opt}|}$
- Matching Error (ME): $ME = \frac{1}{S} \frac{\sum_{s=1}^S |\tilde{\Phi}_s^{opt} - \Phi_s^{opt}|^2}{\sum_{s=1}^S |\Phi_s^{opt}|^2}$

MAE	NME	ME
1.01	8.59×10^{-1}	7.72×10^{-1}

Table XI: Difference between predicted and actual fitness for all considered random seeds.

2.0.4 Comparison with the PSO that doesn't update the training set (seed $s = 5$)

The following figure directly compares the evolution of the predicted fitness for the same optimal seed ($s = 5$) for

1. the PSO+Kriging updating the negative fitness during the evolution;
2. the PSO+Kriging that does not update the training set during the evolution.

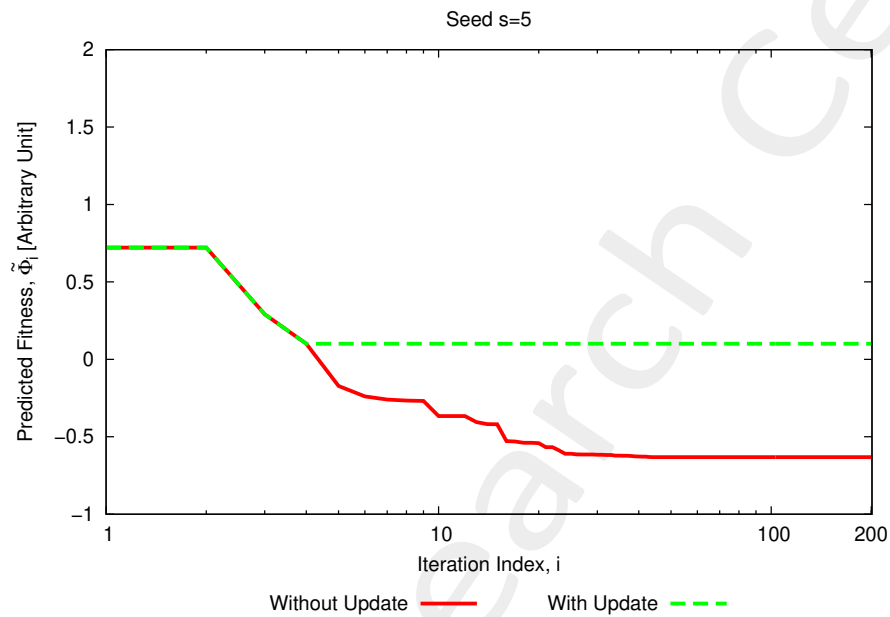


Figure 10: Predicted fitness for seed $s = 5$, when using the two optimization methods (with and without (iteration 1) update of the negative fitness).

As it can be seen, given the same seed and initial swarm, the two fitness curves perfectly overlap up to iteration $i = 5$.

2.0.5 Optimized parameters ($t_1^{opt}, \dots, t_5^{opt}$) vs. seed

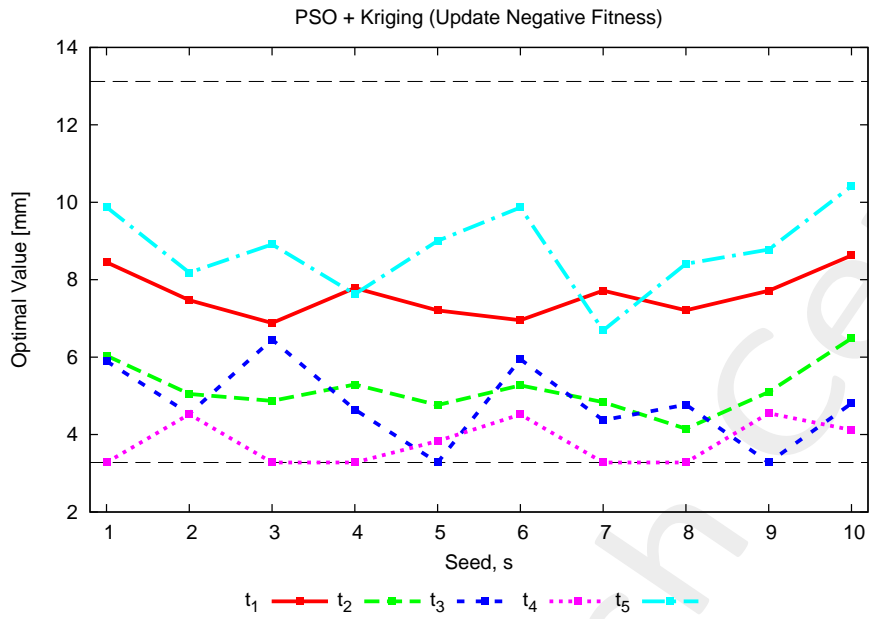


Figure 11: Predicted and actual final fitness for different random seeds ($s \in [1, 10]$).

2.0.6 Best solution found (min. actual fitness)

- Seed: $s = 5$;
- True fitness: $\Phi^{opt} = 8.85 \times 10^{-1}$;
- Average BSE error: $BSE_{avg} = \sqrt{\Phi^{opt}} = 0.94$ [deg].

2.0.7 Analysis of the best solution (seed $s = 5$)

Fitness evolution

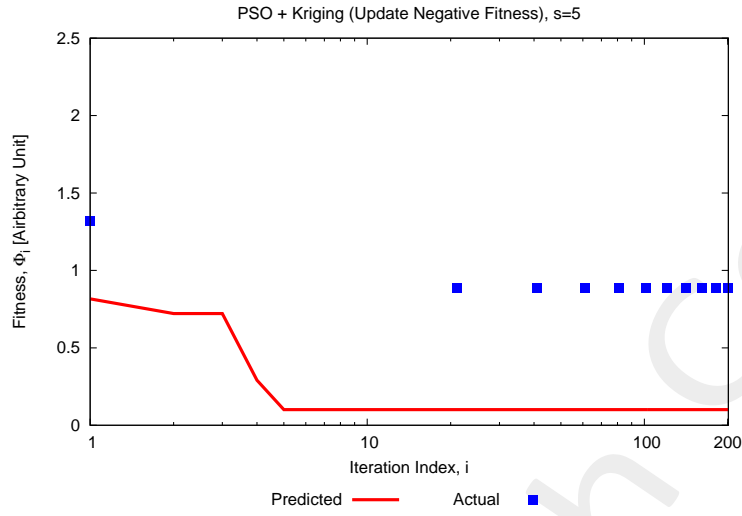


Figure 12: Evolution of the (predicted) fitness and actual fitness computed for the intermediate optimal particles each 20 iterations (“control points”).

Optimized parameters ($t_1^{opt}, \dots, t_5^{opt}$)

Parameter	Description	Optimized Value [m]	Min [m]	Max [m]
t_1	Radome thickness at the quota $z = z_1$	7.21	3.28×10^{-3}	13.12×10^{-3}
t_2	Radome thickness at the quota $z = z_2$	4.76	3.28×10^{-3}	13.12×10^{-3}
t_3	Radome thickness at the quota $z = z_3$	3.28	3.28×10^{-3}	13.12×10^{-3}
t_4	Radome thickness at the quota $z = z_4$	3.83	3.28×10^{-3}	13.12×10^{-3}
t_5	Radome thickness at the quota $z = z_5$	9.00	3.28×10^{-3}	13.12×10^{-3}

Table XII: Optimized parameters for the best seed ($s = 8$).

Pointing error (BSE) vs. frequency

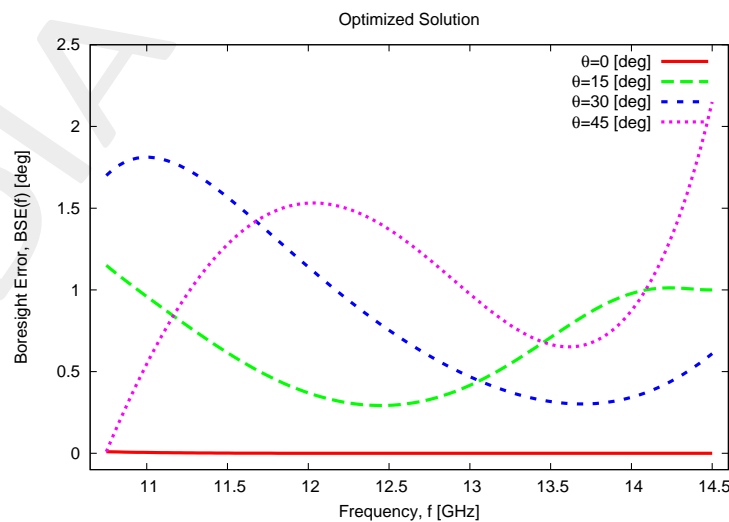


Figure 13: Pointing error (BSE) vs. frequency.

Directivity patterns

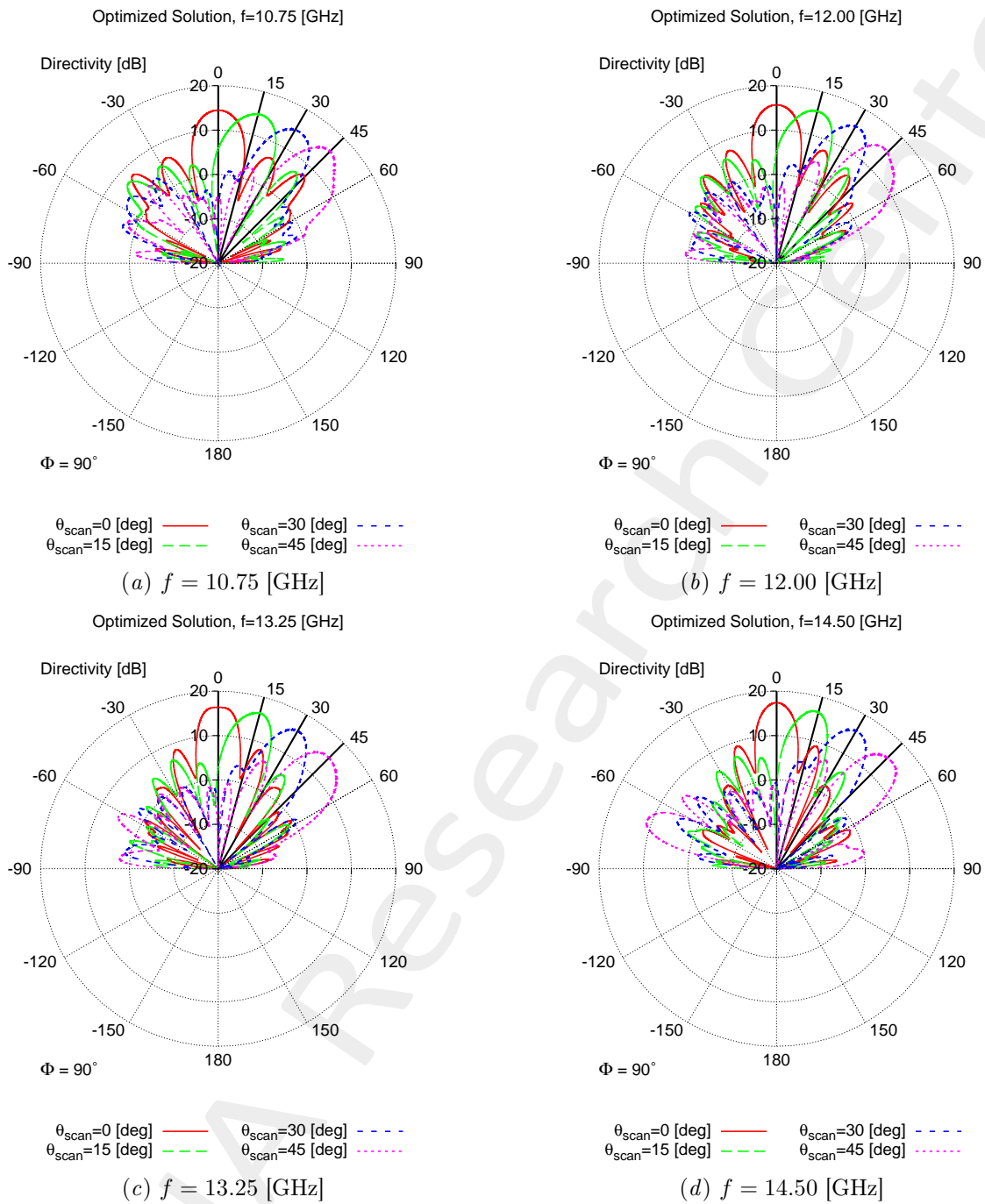


Figure 14: Radiated pattern for the optimized solution at (a) 10.75 [GHz], (b) 12.00 [GHz], (c) 13.25 [GHz] and (d) 14.50 [GHz].

More information on the topics of this document can be found in the following list of references.

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