## Domino-Tiling of Orthogonal Polygon Phased Arrays: An Exhaustive Search Approach

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### 1 Introduction

This work presents an innovative tiling optimization strategy for arbitrary orthogonal-polygon shaped apertures. An exhaustive search approach, together with a multi-objective strategy, has been used in order to obtain optimal tiling configurations, jointly optimizing two different pattern features of interest. A simple example validating the proposed method has been finally reported.

#### 2 Numerical Results

#### 2.1 Orthogonal Polygon #7 (Hexagon Small)

#### **Array Analysis Parameters:**

- Total Number of Elements: P = 48
- Spacing:  $d = \lambda/2$
- Number of Samples along u: 256
- Number of Samples along v: 256
- Steering  $\theta$  Direction:  $\theta_s = 0$
- Steering  $\phi$  Direction:  $\phi_s = 0$

#### **Tiling Parameters:**

- Number of Inner Lattice Points:  $N_{inn} = 33$
- Number of Outer Lattice Points:  $N_{out} = 33$
- Number of boundary points N = 33
- Number of cells  $N_{cel} = 48$
- Number of tiling Configurations: T = 39028

# 2.1.1 ETM-MOP - CP Reference Excitations, Symmetric Mask - Mask Matching vs {SLL, D, HPBW} , SLL vs {D, HPBW}

#### **Reference Fully-Populated Array:**

- Number of Samples along u: 512 2 < u < 2
- Number of Samples along v: 512 2 < v < 2
- Steering  $\theta$  Direction:  $\theta_s = 0$
- Steering  $\phi$  Direction:  $\phi_s = 0$
- Tapering: CP Symmetric Mask
- Side lobe level: SLL = -20.6[dB]
- Main Lobe Window Width along u:  $MW_u = 0.28$  [u]
- Main Lobe Window Width along v:  $MW_v = 0.28$  [v]

#### Synthesis Mask Representation:



#### **Cost Function:**

- $OBJ^{(1)} = D$
- $OBJ^{(2)} = HPBW_{AZ}$
- $OBJ^{(3)} = HPBW_{EL}$
- $OBJ^{(4)} = SLL$
- $OBJ^{(5)} = \int_{-1}^{1} \int_{-1}^{1} \left[ M\left(u,v\right) P\left(u,v;\underline{C}\right) \right] \mathcal{H}\left[ P\left(u,v;\underline{C}\right) M\left(u,v\right) \right] dudv$

Results





	Solution ID	SLL [dB]	D [dBi]	$HPBW_{az}$ [deg]	$HPBW_{el}$ [deg]	Mask Matching
min - Mask - D	28240	-20.03	21.27	14.54	17.64	$9.99 \times 10^{-7}$
max - D - max - Mask	21292	-17.12	21.38	14.33	17.47	$7.73  imes 10^{-4}$
MMD - mask - D	14026	-19.16	21.22	14.53	17.20	$1.60 \times 10^{-4}$
Compr - mask - D	15108	-19.60	21.31	14.55	17.67	$2.30 \times 10^{-5}$
min - mask - max - AZ	33723	-20.01	21.24	14.53	17.48	$9.99 \times 10^{-7}$
max - mask - min - AZ	27997	-17.42	21.32	14.25	17.70	$5.74 \times 10^{-4}$
Compr - mask - AZ	18059	-19.66	21.29	14.47	17.70	$2.10 \times 10^{-5}$
MMD - mask - AZ	27997	-17.42	21.32	14.25	17.70	$5.74 \times 10^{-4}$
min - mask - max - EL	33723	-20.01	21.24	14.53	17.48	$9.99 \times 10^{-7}$
max - mask - min - EL	13627	-18.13	21.28	14.53	17.00	$3.32 \times 10^{-4}$
MMD - Mask - EL	13627	-18.13	21.28	14.53	17.00	$3.32 \times 10^{-4}$
Compr - Mask - EL	14027	-19.83	21.23	14.53	17.34	$1.60 \times 10^{-5}$
min - Mask - SLL	39027	-20.10	21.26	14.53	17.74	0
MMD - Mask - SLL	39027	-20.10	21.26	14.53	17.74	0
min - D - min - SLL	39027	-20.10	21.26	14.53	17.74	0
max - SLL - max - D	21292	-17.12	21.38	14.33	17.47	$7.73 \times 10^{-4}$
Compr - SLL - D	15009	-19.11	21.35	14.54	17.62	$1.24 \times 10^{-4}$
MMD - SLL - D	39027	-20.10	21.26	14.53	17.74	0
max - SLL - min - AZ	27997	-17.42	21.32	14.25	17.70	$5.74 \times 10^{-4}$
min - SLL - max - AZ	39027	-20.10	21.26	14.53	17.74	0
Compr - SLL - AZ	18106	-19.07	21.30	14.39	17.67	$1.33 \times 10^{-4}$
MMD - SLL - AZ	39027	-20.10	21.26	14.53	17.74	0
max - SLL - min - EL	13627	-18.13	21.28	14.53	17.00	$3.32 \times 10^{-4}$
min - SLL - max - EL	39027	-20.10	21.26	14.53	17.74	0
$\overline{M}\overline{M}\overline{D} - SLL - EL$	39027	-20.10	21.26	14.53	17.74	0
Compr - SLL - EL	14027	-19.83	21.23	14.53	17.34	$1.60 \times 10^{-5}$

Table I: Pattern descriptors for the presented solutions.

#### **Observations:**

In plot (b), due to the mask matching value of the solution ID = 39027, this value are not representable in the plot. However, in other plots that don't consider mask matching values (e.g. (e)(f)(g)), the solution ID = 39027 is correctly represented.

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ELEDIA Research Center - Student Report Figure 5: Tiling solutions 10-12.

#### **Observations:**

- The twelve solutions are chosen among all pareto solutions, considering different criteria: the solution that minimise the first objective, the solution that minimise the second objective, the solution given by Minimum Manhattan distance and a solution that represent a good compromise between objectives.
- In the plot (a) the MMD represent a poor result due to low levels of mask matching and directivity.
- In the plot (b) all the solutions are overlapped each others.
- In plots (c)(d) the MMD is overlapped with the solution that minimise the HPBW.
- In plots (e)(f) the MMD is overlapped with the solution that maximise the HPBW.
- In the plot (g) the MMD is overlapped with the solution that minimise SLL and D.

When a black circle is present there will be an overlap between different solutions, so one is hided from the other. Some solutions are present in more plots, so in the table, every ID is associated to the value that minimise or represent.

## 2.1.2 ETM-MOP - CP Reference Excitations, Asymmetric Mask - Mask Matching vs {SLL, D, HPBW} , SLL vs {D, HPBW}

#### **Reference Fully-Populated Array:**

- Number of Samples along u: 512 2 < u < 2
- Number of Samples along v:  $512 \ -2 < v < 2$
- Steering  $\theta$  Direction:  $\theta_s = 0$
- Steering  $\phi$  Direction:  $\phi_s = 0$
- Tapering: CP Asymmetric Mask
- Side lobe level: SLL = -20.6[dB]
- Main Lobe Window Width along u:  $MW_u = 0.28$  [u]
- Main Lobe Window Width along v:  $MW_v = 0.28$  [v]

#### Synthesis Mask Representation:



#### **Cost Function:**

- $OBJ^{(1)} = D$
- $OBJ^{(2)} = HPBW_{AZ}$
- $OBJ^{(3)} = HPBW_{EL}$
- $OBJ^{(4)} = SLL$

• 
$$OBJ^{(5)} = \int_{-1}^{1} \int_{-1}^{1} \left[ M\left(u,v\right) - P\left(u,v;\underline{C}\right) \right] \mathcal{H}\left[ P\left(u,v;\underline{C}\right) - M\left(u,v\right) \right] dudv$$

Results



Figure 6: Pareto front of the ETM solutions.

	Solution ID	SLL [dB]	D [dBi]	$HPBW_{az}$ [deg]	$HPBW_{el}$ [deg]	MaskMatching
min - Mask - D	39028	-18.53	21.31	15.01	17.39	$2.00 \times 10^{-6}$
max - D - max - Mask	21441	-17.40	21.41	14.83	17.22	$4.95 \times 10^{-4}$
MMD - mask - D	13879	-18.29	21.27	15.05	16.82	$2.74 \times 10^{-4}$
Compr - mask - D	29646	-18.26	21.35	14.98	17.37	$2.00 \times 10^{-5}$
min - mask - max - AZ	39028	-18.53	21.31	15.01	17.39	$2.00 \times 10^{-6}$
max - mask - min - AZ	8491	-16.00	21.37	14.74	17.09	$7.00 \times 10^{-4}$
Compr - mask - AZ	28375	-18.47	21.32	14.89	17.35	$2.70 \times 10^{-5}$
MMD - mask - AZ	8491	-16.00	21.37	14.74	17.09	$7.00 \times 10^{-4}$
min - mask - max - EL	39028	-18.53	21.31	15.01	17.39	$2.00 \times 10^{-6}$
max - mask - min - EL	13627	-17.40	21.31	15.04	16.66	$3.35 \times 10^{-4}$
MMD - Mask - EL	13627	-17.40	21.31	15.04	16.66	$3.35 \times 10^{-4}$
Compr - Mask - EL	33576	-19.26	21.30	15.05	17.10	$1.20 \times 10^{-5}$
max - Mask - min - SLL	35236	-19.68	21.30	15.05	17.17	$1.10 \times 10^{-5}$
max - SLL - min - Mask	39028	-18.53	21.31	15.01	17.39	$2.00 \times 10^{-6}$
MMD - Mask - SLL	35236	-19.68	21.30	15.05	17.17	$1.10 \times 10^{-5}$
Compr - Mask - SLL	35178	-19.25	21.32	15.02	17.32	$7.00 \times 10^{-6}$
min - D - min - SLL	35236	-19.68	21.30	15.05	17.17	$1.10 \times 10^{-5}$
max - SLL - max - D	21441	-17.40	21.41	14.83	17.22	$4.95 \times 10^{-4}$
Compr - SLL - D	15140	-18.57	21.37	15.00	17.31	$1.40 \times 10^{-4}$
MMD - SLL - D	35236	-19.68	21.30	15.05	17.17	$1.10 \times 10^{-5}$
max - SLL - min - AZ	8491	-16.00	21.37	14.74	17.09	$7.00 \times 10^{-4}$
min - SLL - max - AZ	35236	-19.68	21.30	15.05	17.17	$1.10 \times 10^{-5}$
Compr - SLL - AZ	25258	-19.04	21.31	14.83	17.53	$1.28 \times 10^{-4}$
MMD - SLL - AZ	35236	-19.68	21.30	15.05	17.17	$1.10 \times 10^{-5}$
max - SLL - min - EL	13627	-17.40	21.31	15.04	16.66	$3.35 \times 10^{-4}$
min - SLL - max - EL	35236	-19.68	21.30	15.05	17.17	$1.10 \times 10^{-5}$
$MMD - SLL - \overline{EL}$	35236	-19.68	21.30	15.05	17.17	$1.10 \times 10^{-5}$
Compr - SLL - EL	35233	-19.07	21.29	15.05	17.00	$8.00 \times 10^{-5}$

Table II: Pattern descriptors for the presented solutions.









ELEDIA Research Center - Student Report Figure 10: Tiling solutions 10-12.



Figure 11: Tiling solutions 13.

#### **Observations:**

- The twelve solutions are chosen among all pareto solutions, considering different criteria: the solution that minimise the first objective, the solution that minimise the second objective, the solution given by Minimum Manhattan distance and a solution that represent a good compromise between objectives.
- In the plot (a) the MMD represent a poor result due to low levels of mask matching and directivity.
- In the plot (b) the MMD is overlapped with the solution that minimise the *SLL*.
- In the plot (c) the MMD is overlapped with the solution that maximise the HPBW.
- In the plot (d) the MMD is overlapped with the solution that minimise the HPBW.
- In plots (e)(f) the MMD is overlapped with the solution that maximise the HPBW.
- In the plot (g) the MMD is overlapped with the solution that minimise SLL and D.

When a black circle is present there will be an overlap between different solutions, so one is hided from the other. Some solutions are present in more plots, so in the table, every ID is associated to the value that minimise or represent.

More information on the topics of this document can be found in the following list of references.

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