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# **Multi-Scale Synthesis of Reflectarrays Exploiting the SbD Paradigm**

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# 1 Problem Formulation

## 1.1 Geometrical Description

We define two reference systems as in Fig. 1:

- Global Reference System (GRS): defined in  $(x', y', z')$  with:

- Feed position in  $(x'_F, y'_F, z'_F)$
- Cell position in  $(x'_C, y'_C, z'_C)$

- Local Reference System (LRS): defined in  $(x, y, z)$  with:

- Feed position in  $(x_F, y_F, z_F)$
- Cell position in  $(0, 0, 0)$

- Angle of Incidence of Feed Wave:

- $\theta_{inc} = \arccos\left(\frac{z_F}{\sqrt{x_F^2 + y_F^2 + z_F^2}}\right)$
- $\phi_{inc} = \arctan\left(\frac{y_F}{x_F}\right)$

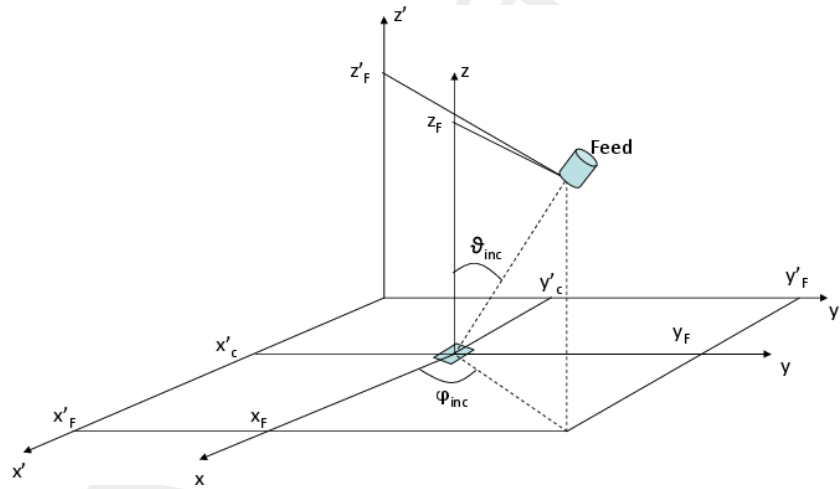


Figure 1: Geometrical description of the problem - Local and Global reference Systems.

## 1.2 Incident Field in LRS

The incident field in the LRS can be expressed as:

$$\mathbf{E}_{inc} = \left( E_{inc}^{\theta} \hat{\underline{\rho}} + E_{inc}^{\phi} \hat{\underline{\phi}} \right) \exp \left( -jk_0 \hat{\underline{\mu}}_{inc} \cdot \underline{\underline{r}} \right) \quad (1)$$

where:

- $k_0 = 2\pi/\lambda$ : is the wave number;
- $\hat{\underline{\mu}}_{inc} = -(\sin(\theta_{inc}) \cos(\phi_{inc}), \sin(\theta_{inc}) \sin(\phi_{inc}), \cos(\theta_{inc}))$ : is the propagation vector;

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### 1.3 Floquet analysis

Since we assume a Floquet analysis truncate at the first term (only 1 TE/TM mode) we can define:

$$\begin{bmatrix} E_{inc}^\theta \\ E_{inc}^\phi \end{bmatrix} = \begin{bmatrix} E_{inc}^{TM} \\ E_{inc}^{TE} \end{bmatrix} \quad (2)$$

### 1.4 Scattered Field

Given the scattering matrix ( $S$ ) in output from HFSS (Floquet first order approximation) the scattered field can be computed as:

$$\begin{bmatrix} E_{scat}^{TM} \\ E_{scat}^{TE} \end{bmatrix} = [S] \begin{bmatrix} E_{inc}^{TM} \\ E_{inc}^{TE} \end{bmatrix} \quad (3)$$

### 1.5 The Surface current computation

The surface current on the reflectarray aperture can be computed from the incident and scattered field as:

$$\begin{aligned} \mathbf{J}_s(\mathbf{r}) &= \hat{\mathbf{n}} \times \mathbf{H}_{TOT}(\mathbf{r}) \\ \mathbf{M}_s(\mathbf{r}) &= -\hat{\mathbf{n}} \times \mathbf{E}_{TOT}(\mathbf{r}) \end{aligned} \quad (4)$$

where:

- $\mathbf{H}_{TOT} = \mathbf{H}_{inc} + \mathbf{H}_{scat}$
- $\mathbf{E}_{TOT} = \mathbf{E}_{inc} + \mathbf{E}_{scat}$

But we need to have the fields expressed in cartesian components.

### 1.6 From spherical to cartesian components

This procedure is standard and is valid both for incident and scattered field:

$$\begin{bmatrix} E_{inc/scat}^x \\ E_{inc/scat}^y \\ E_{inc/scat}^z \end{bmatrix} = \begin{bmatrix} \sin(\theta_{inc}) \cos(\phi_{inc}) & \cos(\theta_{inc}) \cos(\phi_{inc}) & -\sin(\phi_{inc}) \\ \sin(\theta_{inc}) \sin(\phi_{inc}) & \cos(\theta_{inc}) \sin(\phi_{inc}) & \cos(\phi_{inc}) \\ \cos(\theta_{inc}) & -\sin(\theta_{inc}) & 0 \end{bmatrix} \begin{bmatrix} 0 \\ E_{inc/scat}^{TM} \\ E_{inc/scat}^{TE} \end{bmatrix} \quad (5)$$

### 1.7 Summary of the procedure

1. From a FEKO simulation of a feeder we obtain the Electric incident field in cartesian components;
2. We derive the spherical (and also TE/TM) components of the Electric incident field;
3. We compute the Electric scattered field;

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4. From the incident and scattered Electric field we derive the Magnetic field as:

$$\mathbf{H}_{inc/scatt} = \frac{1}{\eta} \hat{\mathbf{u}}_{inc/scat} \times \mathbf{E}_{inc/scatt} \quad (6)$$

5. Compute the Electric Currents from the total Magnetic Field.

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## 2 Preliminary Results: Square Patch Reflectarray: 25x25

### 2.1 Unit cell geometry

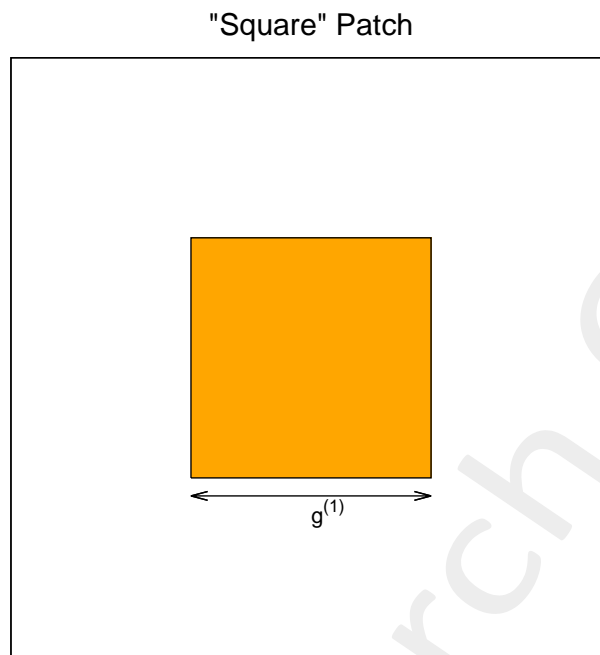


Figure 2: Square Patch unit cell.

### 2.2 Accuracy vs. Training Samples

#### 2.2.1 Matrix Norm and Phase mean squared errors

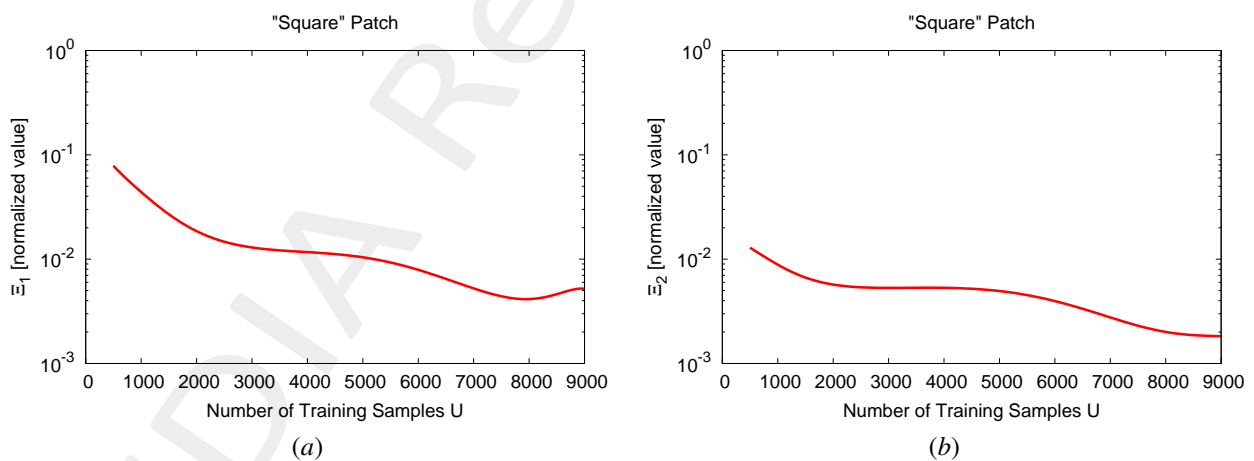


Figure 3: Square Patch Reflectarray  $25 \times 25$  - Accuracy vs. Training Samples: Matrix norm error (a) and Phase mean squared error (b).

### 2.2.2 True vs. Predicted

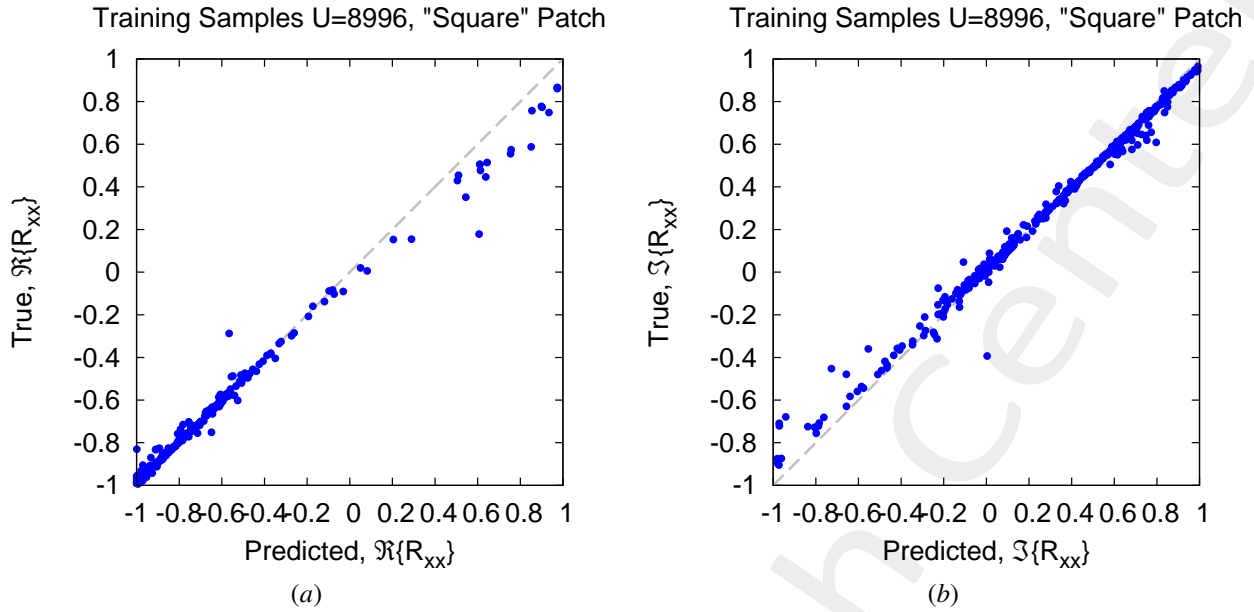


Figure 4: Square Patch Reflectarray  $25 \times 25$  - Accuracy vs. Training Samples -  $R_{xx}$ : Real (a) and Imaginary (b) part of true vs. predicted.

### 2.3 Optimization target

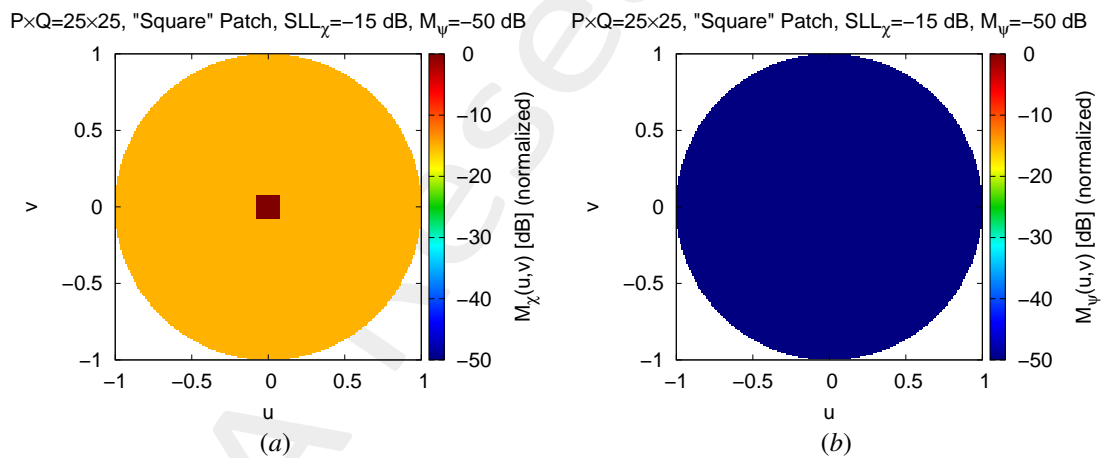


Figure 5: Square Patch Reflectarray  $25 \times 25$  - Optimization target: SLL on the wanted polarization(a), mask on the unwanted polarization (b).

## 2.4 Optimization results

### 2.4.1 Cost Function

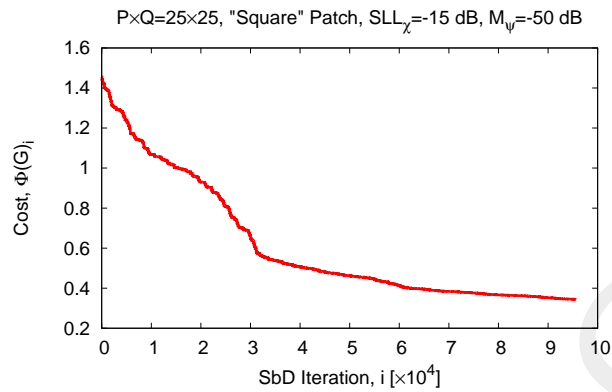


Figure 6: Square Patch Reflectarray  $25 \times 25$  - Optimization: Cost function behavior.

### 2.4.2 Geometrical Design

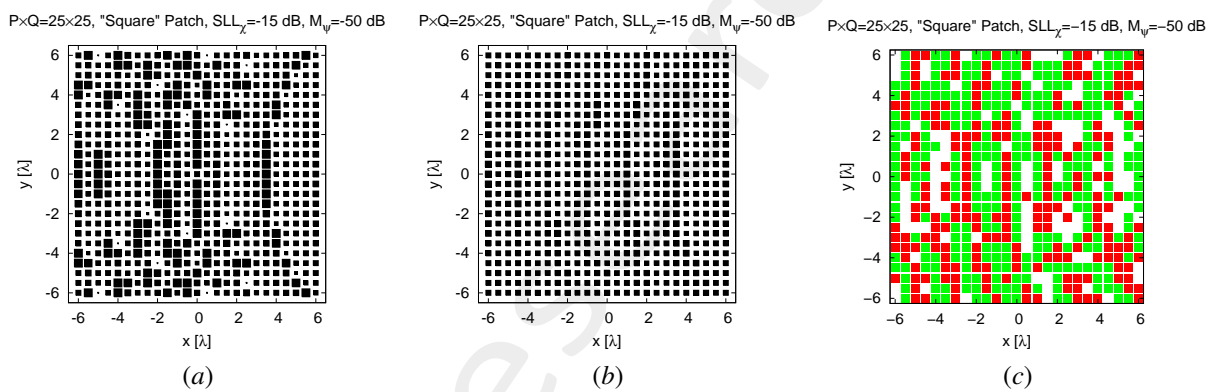


Figure 7: Square Patch Reflectarray  $25 \times 25$  - Optimization: Starting reflectarray configuration(a), optimized reflectarray configuration (b) and the differential (c) between starting and optimal design.



### 2.4.3 Reflection Coefficient

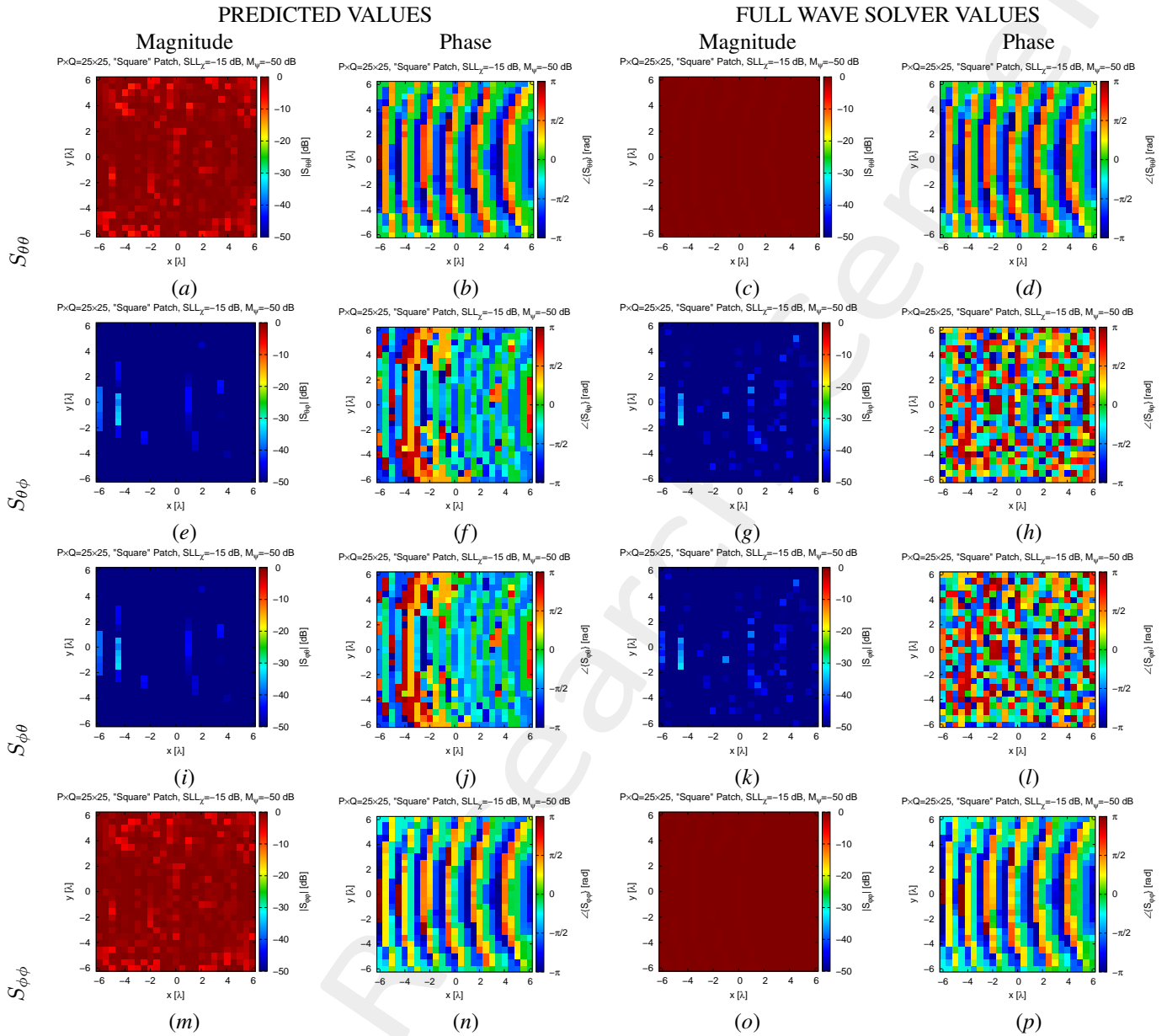


Figure 8: Square Patch Reflectarray  $25 \times 25$  - Optimization - Reflection Coefficients: predicted(a)(b)(e)(f)(i)(j)(m)(n) vs. full-wave simulation (c)(d)(g)(h)(k)(l)(o)(p) of the magnitude(a)(c)(e)(g)(i)(k)(m)(o) and phase (b)(d)(f)(h)(j)(l)(n)(p) of  $S_{\theta\theta}$ (a)(b)(c)(d),  $S_{\theta\phi}$ (e)(f)(g)(h),  $S_{\phi\theta}$ (i)(j)(k)(l) and  $S_{\phi\phi}$ (m)(n)(o)(p).

## 2.4.4 Superficial Currents

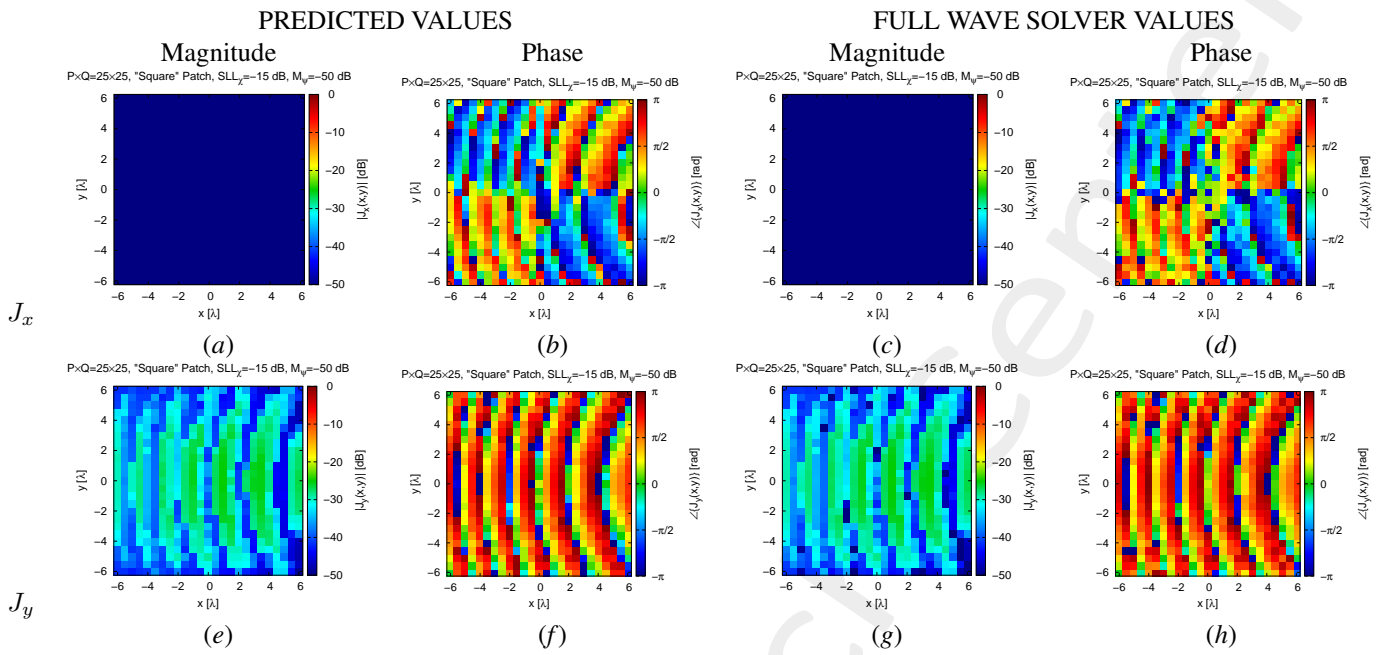


Figure 9: Square Patch Reflectarray  $25 \times 25$  - Optimization - Superficial Currents: predicted(a)(b)(e)(f) vs. full-wave simulation (c)(d)(g)(h) of the magnitude(a)(c)(e)(g) and phase (b)(d)(f)(h) of  $J_x$ (a)(b)(c)(d) and  $J_y$ (e)(f)(g)(h).

## 2.4.5 Fields

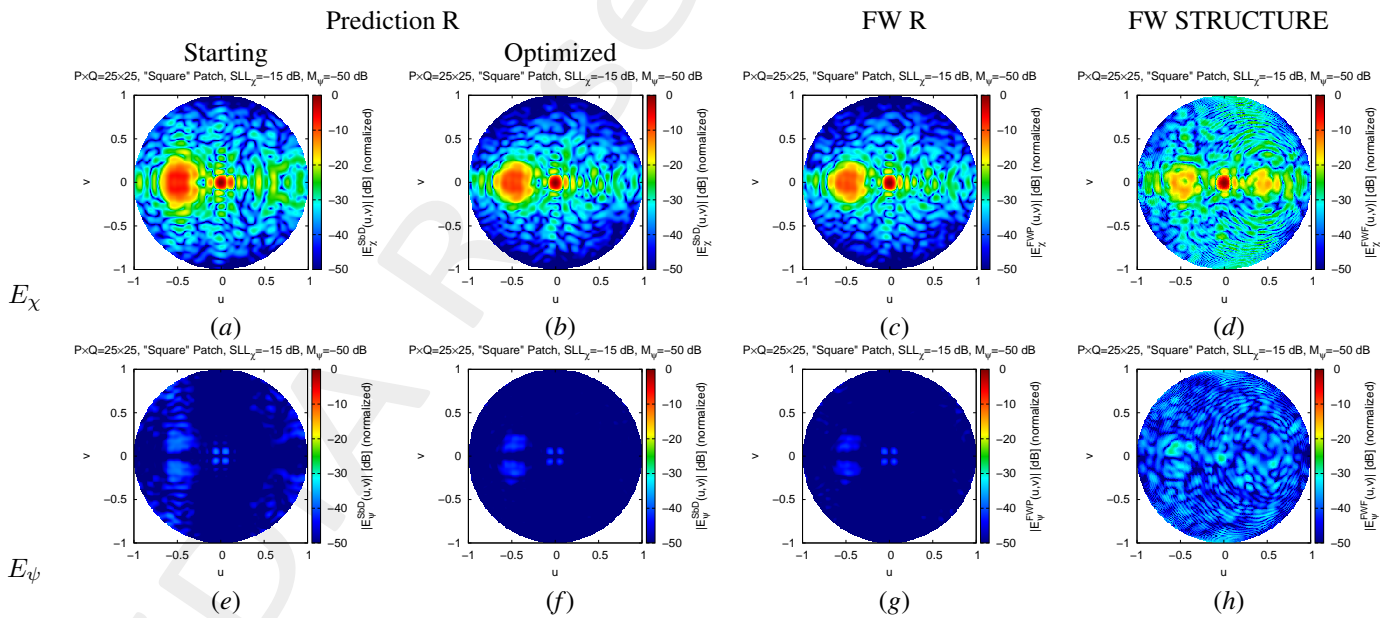


Figure 10: Square Patch Reflectarray  $25 \times 25$  - Optimization - Radiated Fields: predicted(a)(b)(e)(f) vs. full-wave simulation of R (c)(g) vs. full-wave simulation of the entire structure (d)(h) of the magnitude of  $E_\chi$ (a)(b)(c)(d) and  $E_\psi$ (e)(f)(g)(h).

## 2.4.6 Fields Cut

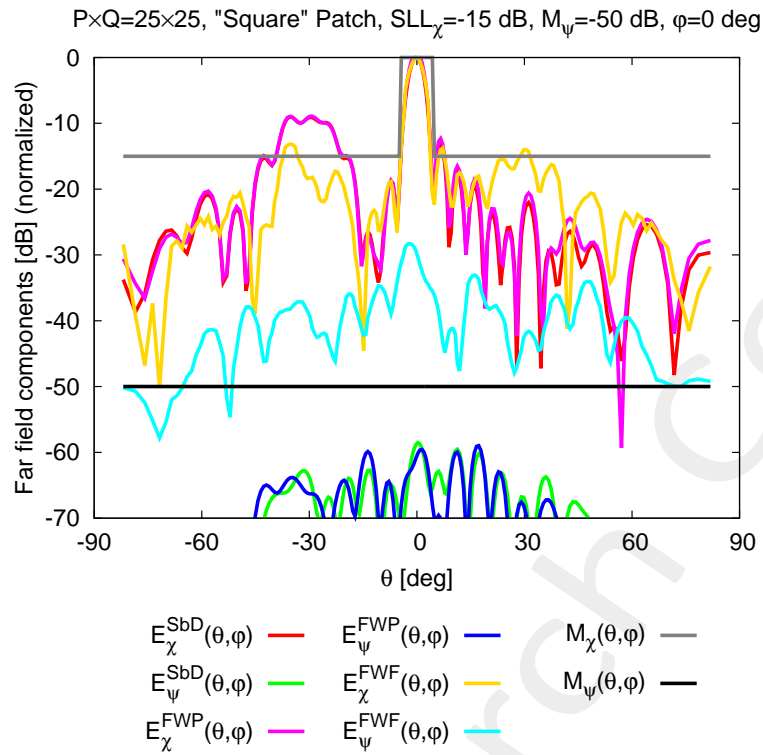


Figure 11: Square Patch Reflectarray  $25 \times 25$  - Optimization - Radiated Field Cut with the comparison.

### 3 Square Patch Reflectarray: $25 \times 25$ - $R_f = 10$

#### 3.1 Optimization target

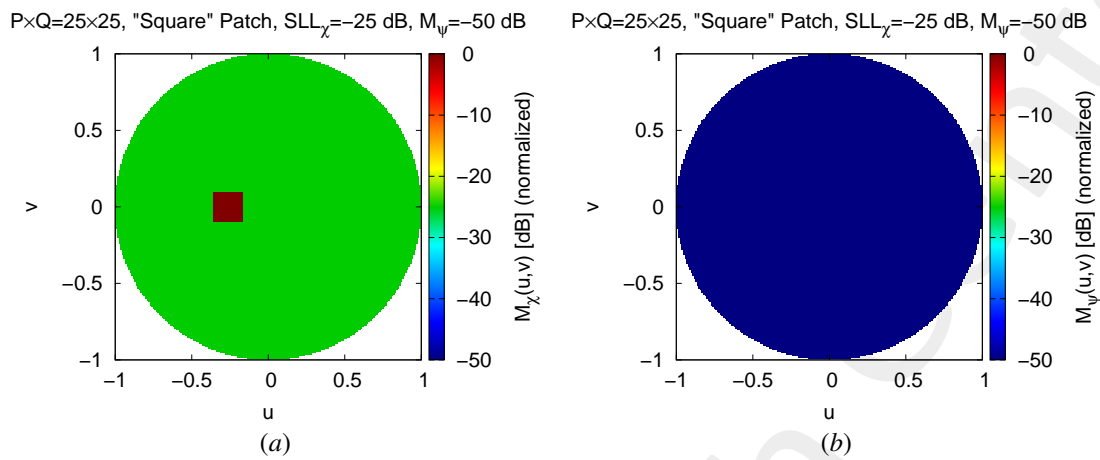


Figure 12: Square Patch Reflectarray  $25 \times 25$   $R_f = 10$  - Optimization target: SLL on the wanted polarization(a), mask on the unwanted polarization (b).

#### 3.2 Optimization results

##### 3.2.1 Cost Function

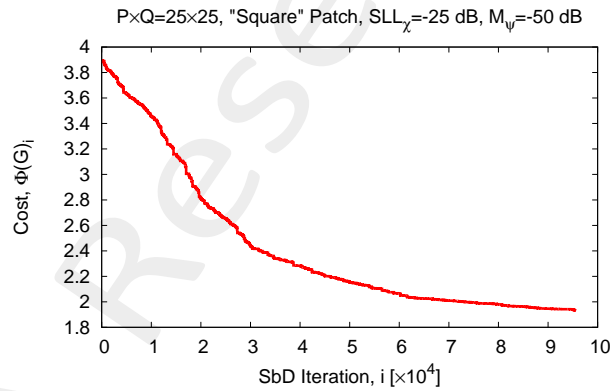


Figure 13: Square Patch Reflectarray  $25 \times 25$   $R_f = 10$  - Optimization: Cost function behavior.

### 3.2.2 Geometrical Design

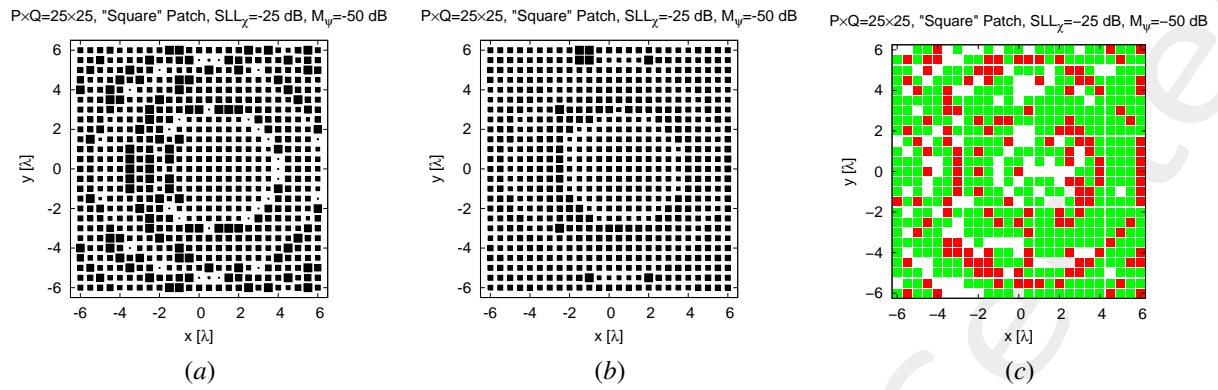


Figure 14: Square Patch Reflectarray  $25 \times 25 R_f = 10$  - Optimization: Starting reflectarray configuration (a), optimized reflectarray configuration (b) and the differential (c) between starting and optimal design.

### 3.2.3 Reflection Coefficient

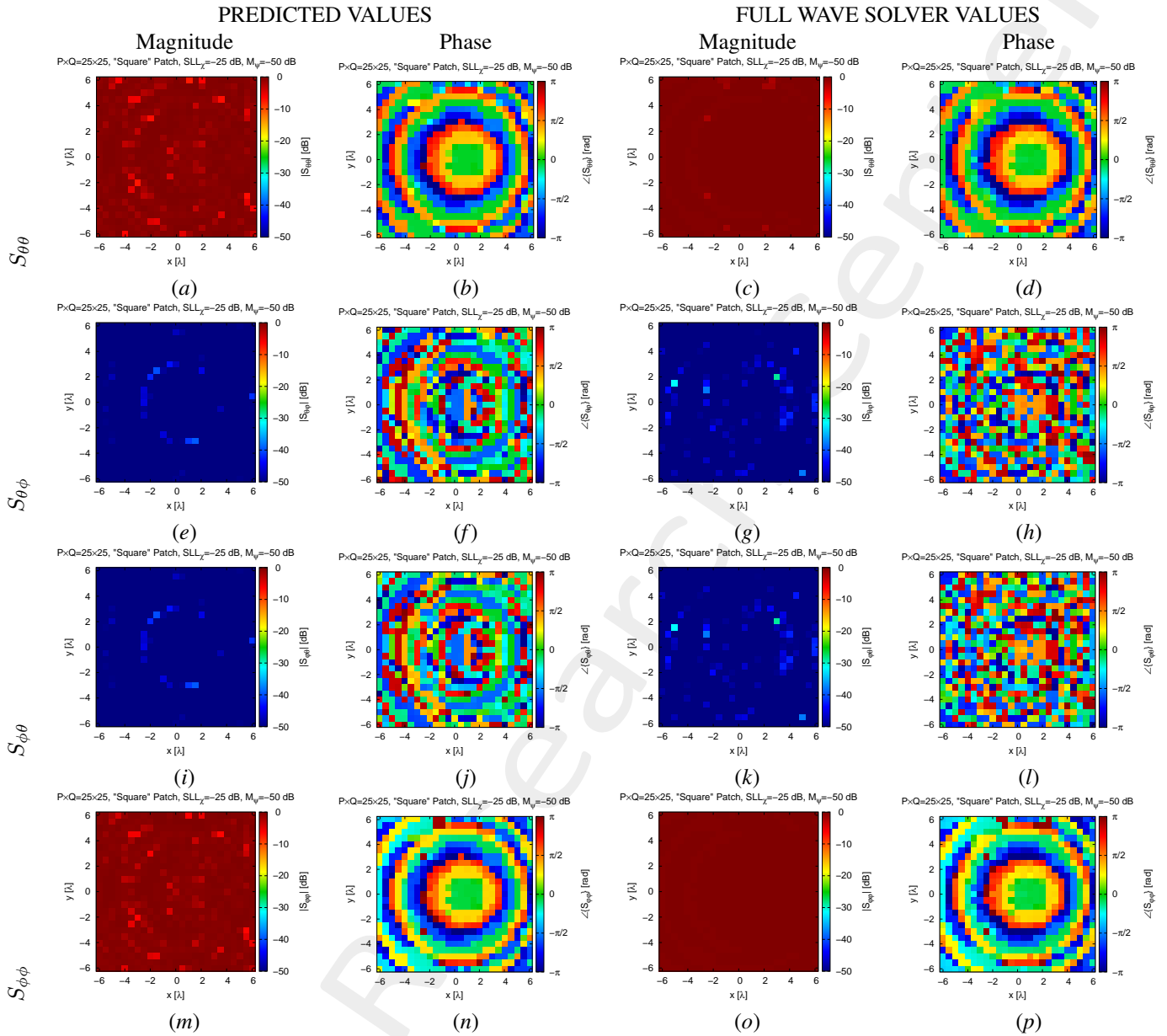


Figure 15: Square Patch Reflectarray  $25 \times 25$   $R_f = 10$  - Optimization - Reflection Coefficients: predicted(a)(b)(e)(f)(i)(j)(m)(n) vs. full-wave simulation (c)(d)(g)(h)(k)(l)(o)(p) of the magnitude(a)(c)(e)(g)(i)(k)(m)(o) and phase (b)(d)(f)(h)(j)(l)(n)(p) of  $S_{\theta\theta}$ (a)(b)(c)(d),  $S_{\theta\phi}$ (e)(f)(g)(h),  $S_{\phi\theta}$ (i)(j)(k)(l) and  $S_{\phi\phi}$ (m)(n)(o)(p).

### 3.2.4 Superficial Currents

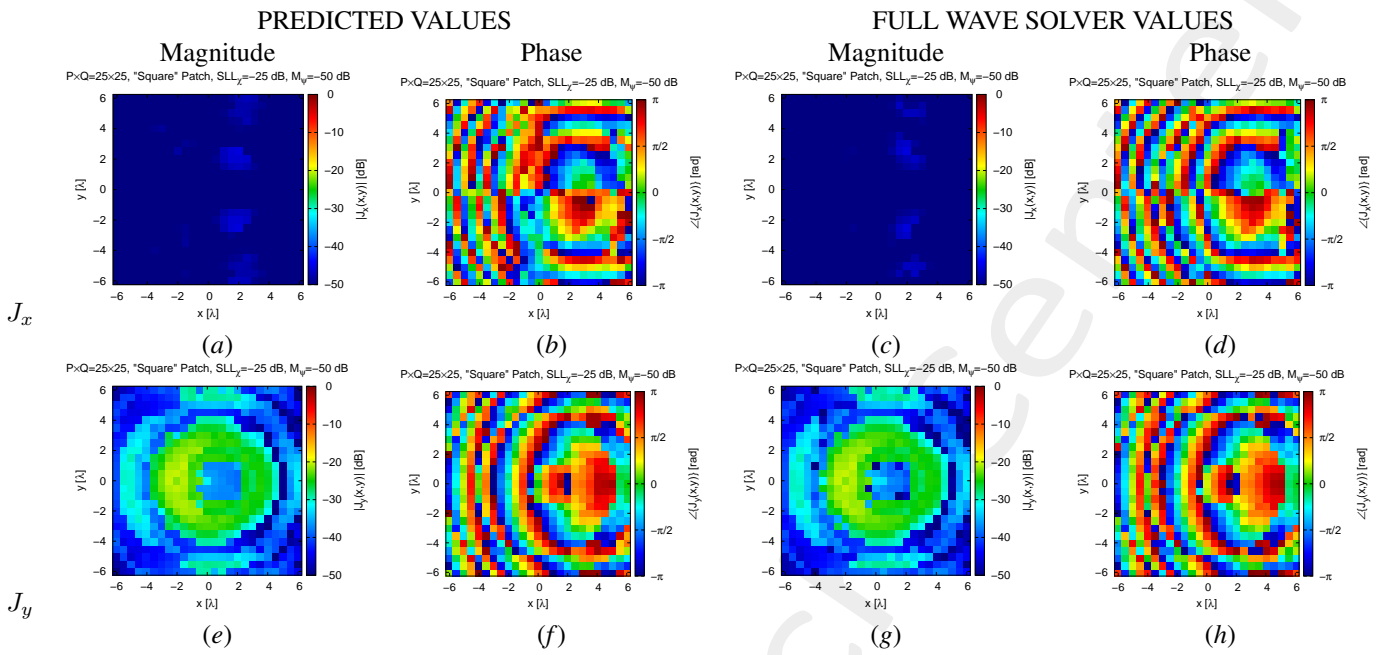


Figure 16: Square Patch Reflectarray  $25 \times 25$   $R_f = 10$  - Optimization - Superficial Currents: predicted(a)(b)(e)(f) vs. full-wave simulation (c)(d)(g)(h)of the magnitude(a)(c)(e)(g) and phase (b)(d)(f)(h) of  $J_x$ (a)(b)(c)(d) and  $J_y$ (e)(f)(g)(h).

### 3.2.5 Fields

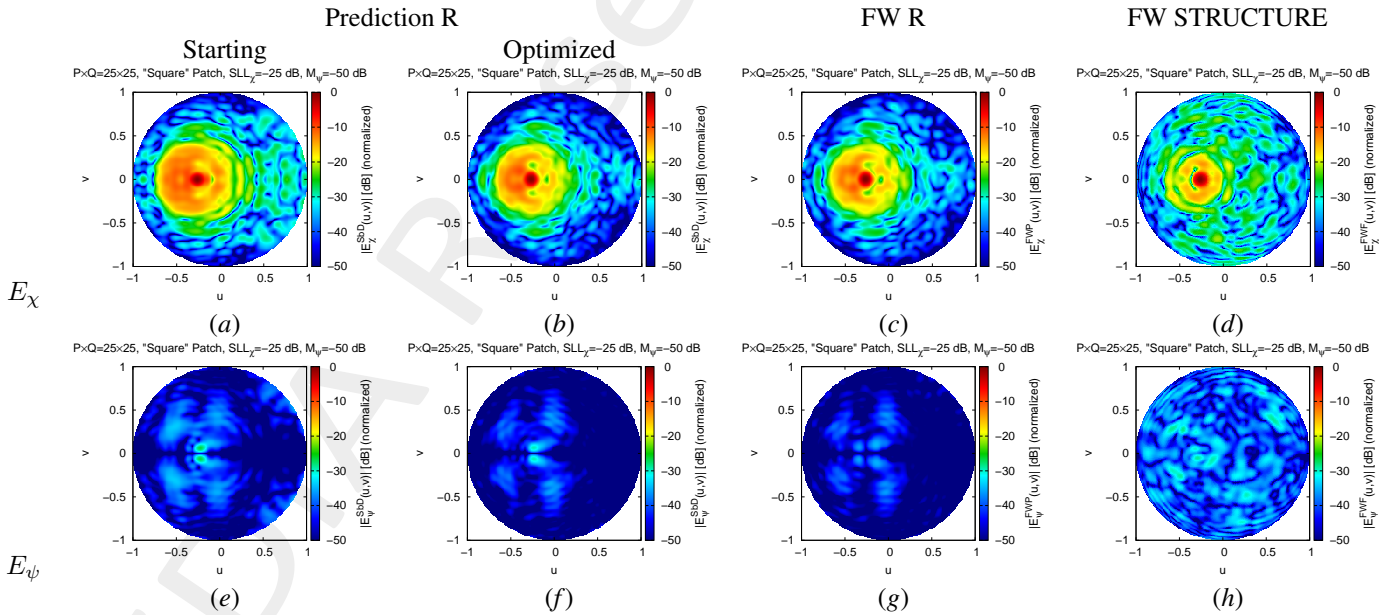


Figure 17: Square Patch Reflectarray  $25 \times 25$   $R_f = 10$  - Optimization - Radiated Fields: predicted(a)(b)(e)(f) vs. full-wave simulation of R (c)(g) vs. full-wave simulation of the entire structure (d)(h) of the magnitude of  $E_\chi$ (a)(b)(c)(d) and  $E_\psi$ (e)(f)(g)(h).

### 3.2.6 Fields Cut

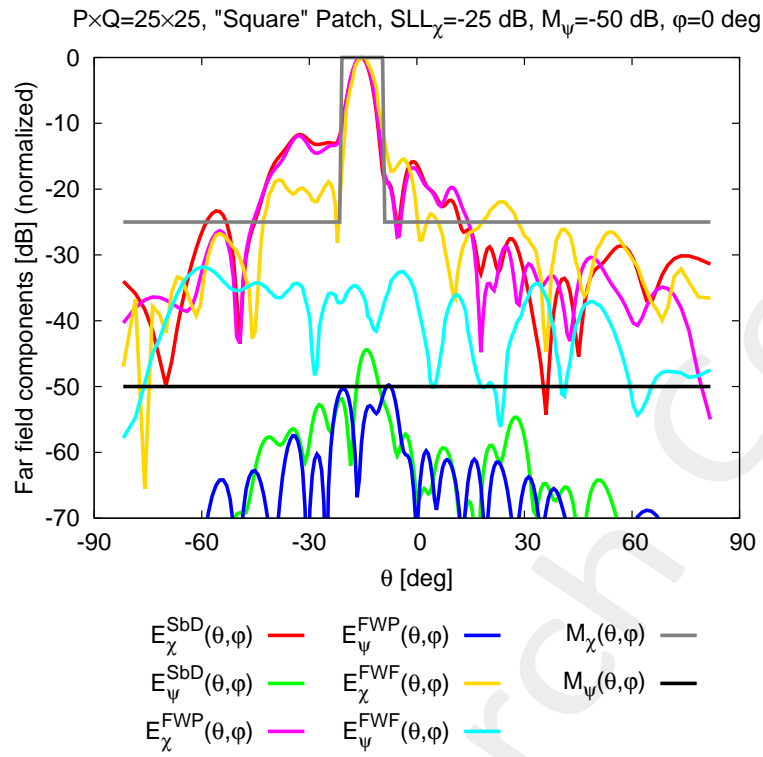


Figure 18: Square Patch Reflectarray  $25 \times 25$   $R_f = 10$  - Optimization - Radiated Field Cut with the comparison.



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More information on the topics of this document can be found in the following list of references.

## References

- [1] P. Rocca, M. Benedetti, M. Donelli, D. Franceschini, and A. Massa, "Evolutionary optimization as applied to inverse problems," *Inverse Problems - 25 th Year Special Issue of Inverse Problems, Invited Topical Review*, vol. 25, pp. 1-41, Dec. 2009.
- [2] P. Rocca, G. Oliveri, and A. Massa, "Differential Evolution as applied to electromagnetics," *IEEE Antennas Propag. Mag.*, vol. 53, no. 1, pp. 38-49, Feb. 2011.
- [3] P. Rocca, N. Anselmi, A. Polo, and A. Massa, "An irregular two-sizes square tiling method for the design of isophoric phased arrays," *IEEE Trans. Antennas Propag.*, vol. 68, no. 6, pp. 4437-4449, Jun. 2020.
- [4] P. Rocca, N. Anselmi, A. Polo, and A. Massa, "Modular design of hexagonal phased arrays through diamond tiles," *IEEE Trans. Antennas Propag.*, vol.68, no. 5, pp. 3598-3612, May 2020.
- [5] N. Anselmi, L. Poli, P. Rocca, and A. Massa, "Design of simplified array layouts for preliminary experimental testing and validation of large AESAs," *IEEE Trans. Antennas Propag.*, vol. 66, no. 12, pp. 6906-6920, Dec. 2018.
- [6] N. Anselmi, P. Rocca, M. Salucci, and A. Massa, "Contiguous phase-clustering in multibeam-on-receive scanning arrays," *IEEE Trans. Antennas Propag.*, vol. 66, no. 11, pp. 5879-5891, Nov. 2018.
- [7] G. Oliveri, G. Gottardi, F. Robol, A. Polo, L. Poli, M. Salucci, M. Chuan, C. Massagrande, P. Vinetti, M. Mattivi, R. Lombardi, and A. Massa, "Co-design of unconventional array architectures and antenna elements for 5G base station," *IEEE Trans. Antennas Propag.*, vol. 65, no. 12, pp. 6752-6767, Dec. 2017.
- [8] N. Anselmi, P. Rocca, M. Salucci, and A. Massa, "Irregular phased array tiling by means of analytic schemata-driven optimization," *IEEE Trans. Antennas Propag.*, vol. 65, no. 9, pp. 4495-4510, September 2017.
- [9] N. Anselmi, P. Rocca, M. Salucci, and A. Massa, "Optimization of excitation tolerances for robust beamforming in linear arrays," *IET Microwaves, Antennas & Propagation*, vol. 10, no. 2, pp. 208-214, 2016.
- [10] P. Rocca, R. J. Mailloux, and G. Toso, "GA-Based optimization of irregular sub-array layouts for wideband phased arrays design," *IEEE Antennas and Wireless Propag. Lett.*, vol. 14, pp. 131-134, 2015.
- [11] P. Rocca, M. Donelli, G. Oliveri, F. Viani, and A. Massa, "Reconfigurable sum-difference pattern by means of parasitic elements for forward-looking monopulse radar," *IET Radar, Sonar & Navigation*, vol 7, no. 7, pp. 747-754, 2013.
- [12] P. Rocca, L. Manica, and A. Massa, "Ant colony based hybrid approach for optimal compromise sum-difference patterns synthesis," *Microwave Opt. Technol. Lett.*, vol. 52, no. 1, pp. 128-132, Jan. 2010.
- [13] P. Rocca, L. Manica, and A. Massa, "An improved excitation matching method based on an ant colony optimization for suboptimal-free clustering in sum-difference compromise synthesis," *IEEE Trans. Antennas Propag.*, vol. 57, no. 8, pp. 2297-2306, Aug. 2009.

- 
- [14] P. Rocca, L. Manica, and A. Massa, "Hybrid approach for sub-arrayed monopulse antenna synthesis," *Electronics Letters*, vol. 44, no. 2, pp. 75-76, Jan. 2008.
- [15] P. Rocca, L. Manica, F. Stringari, and A. Massa, "Ant colony optimization for tree-searching based synthesis of monopulse array antenna," *Electronics Letters*, vol. 44, no. 13, pp. 783-785, Jun. 19, 2008.
- [16] G. Oliveri, A. Gelmini, A. Polo, N. Anselmi, and A. Massa, "System-by-design multi-scale synthesis of task-oriented reflectarrays," *IEEE Trans. Antennas Propag.*, vol. 68, no. 4, pp. 2867-2882, Apr. 2020.
- [17] M. Salucci, F. Robol, N. Anselmi, M. A. Hannan, P. Rocca, G. Oliveri, M. Donelli, and A. Massa, "S-Band spline-shaped aperture-stacked patch antenna for air traffic control applications," *IEEE Tran. Antennas Propag.*, vol. 66, no. 8, pp. 4292-4297, Aug. 2018.
- [18] M. Salucci, L. Poli, A. F. Morabito, and P. Rocca, "Adaptive nulling through subarray switching in planar antenna arrays," *Journal of Electromagnetic Waves and Applications*, vol. 30, no. 3, pp. 404-414, February 2016
- [19] T. Moriyama, L. Poli, and P. Rocca, "Adaptive nulling in thinned planar arrays through genetic algorithms," *IEICE Electronics Express*, vol. 11, no. 21, pp. 1-9, Sep. 2014.
- [20] L. Poli, P. Rocca, M. Salucci, and A. Massa, "Reconfigurable thinning for the adaptive control of linear arrays," *IEEE Trans. Antennas Propag.*, vol. 61, no. 10, pp. 5068-5077, Oct. 2013.
- [21] P. Rocca, L. Poli, G. Oliveri, and A. Massa, "Adaptive nulling in time-varying scenarios through time-modulated linear arrays," *IEEE Antennas Wireless Propag. Lett.*, vol. 11, pp. 101-104, 2012.
- [22] M. Benedetti, G. Oliveri, P. Rocca, and A. Massa, "A fully-adaptive smart antenna prototype: ideal model and experimental validation in complex interference scenarios," *Progress in Electromagnetic Research*, PIER 96, pp. 173-191, 2009.
- [23] M. Benedetti, R. Azaro, and A. Massa, "Memory enhanced PSO-based optimization approach for smart antennas control in complex interference scenarios," *IEEE Trans. Antennas Propag.*, vol. 56, no. 7, pp. 1939-1947, Jul. 2008.
- [24] M. Benedetti, R. Azaro, and A. Massa, "Experimental validation of a fully-adaptive smart antenna prototype," *Electronics Letters*, vol. 44, no. 11, pp. 661-662, May 2008.
- [25] R. Azaro, L. Ioriatti, M. Martinelli, M. Benedetti, and A. Massa, "An experimental realization of a fully-adaptive smart antenna," *Microwave Opt. Technol. Lett.*, vol. 50, no. 6, pp. 1715-1716, Jun. 2008.
- [26] M. Donelli, R. Azaro, L. Fimognari, and A. Massa, "A planar electronically reconfigurable Wi-Fi band antenna based on a parasitic microstrip structure," *IEEE Antennas Wireless Propag. Lett.*, vol. 6, pp. 623-626, 2007.
- [27] M. Benedetti, R. Azaro, D. Franceschini, and A. Massa, "PSO-based real-time control of planar uniform circular arrays," *IEEE Antennas Wireless Propag. Lett.*, vol. 5, pp. 545-548, 2006.
- [28] G. Oliveri, P. Rocca, M. Salucci, and A. Massa, "Holographic smart EM skins for advanced beam power shaping in next generation wireless environments," *IEEE J. Multiscale Multiphysics Comput. Tech.*, vol. 6, pp. 171-182, Oct. 2021.

- 
- [29] M. Salucci, L. Tenuti, G. Gottardi, A. Hannan, and A. Massa, "System-by-design method for efficient linear array miniaturisation through low-complexity isotropic lenses," *Electronic Letters*, vol. 55, no. 8, pp. 433-434, May 2019.
- [30] M. Salucci, N. Anselmi, S. Goudos, and A. Massa, "Fast design of multiband fractal antennas through a system-by-design approach for NB-IoT applications," *EURASIP J. Wirel. Commun. Netw.*, vol. 2019, no. 1, pp. 68-83, Mar. 2019.
- [31] M. Salucci, G. Oliveri, N. Anselmi, and A. Massa, "Material-by-design synthesis of conformal miniaturized linear phased arrays," *IEEE Access*, vol. 6, pp. 26367-26382, 2018.
- [32] M. Salucci, G. Oliveri, N. Anselmi, G. Gottardi, and A. Massa, "Performance enhancement of linear active electronically-scanned arrays by means of MbD-synthesized metalenses," *Journal of Electromagnetic Waves and Applications*, vol. 32, no. 8, pp. 927-955, 2018.
- [33] G. Oliveri, M. Salucci, N. Anselmi and A. Massa, "Multiscale System-by-Design synthesis of printed WAIMs for waveguide array enhancement," *IEEE J. Multiscale Multiphysics Computat. Techn.*, vol. 2, pp. 84-96, 2017.
- [34] A. Massa and G. Oliveri, "Metamaterial-by-Design: Theory, methods, and applications to communications and sensing - Editorial," *EPJ Applied Metamaterials*, vol. 3, no. E1, pp. 1-3, 2016.
- [35] G. Oliveri, F. Viani, N. Anselmi, and A. Massa, "Synthesis of multi-layer WAIM coatings for planar phased arrays within the system-by-design framework," *IEEE Trans. Antennas Propag.*, vol. 63, no. 6, pp. 2482-2496, June 2015.
- [36] G. Oliveri, L. Tenuti, E. Bekele, M. Carlin, and A. Massa, "An SbD-QCTO approach to the synthesis of isotropic metamaterial lenses," *IEEE Antennas Wireless Propag. Lett.*, vol. 13, pp. 1783-1786, 2014.
- [37] A. Massa, G. Oliveri, P. Rocca, and F. Viani, "System-by-Design: a new paradigm for handling design complexity," *8th European Conference on Antennas Propag. (EuCAP 2014)*, The Hague, The Netherlands, pp. 1180-1183, Apr. 6-11, 2014.
- [38] P. Rocca, G. Oliveri, R. J. Mailloux, and A. Massa, "Unconventional phased array architectures and design Methodologies - A review," *Proceedings of the IEEE*, vol. 104, no. 3, pp. 544-560, March 2016.