

Designing Phased Arrays with Subarray-only Amplitude and Phase Control based on K-means and Border Element Method

P. Rocca, L. Poli, A. Polo, and A. Massa

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Legenda

EM: Excitation Matching

BEM: Border Element Method

CPM: Contiguous Partition Method

SLL: Sidelobe Level

HPBW: Half-Power Beamwidth

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1 Comparative Assessment: K-means-based vs. BEM-based Excitation Matching

1.1 Taylor Patterns - Analysis varying Q for Different Values of Reference Sidelobe Level SLL_{ref}

OBJECTIVE: Compare the performance of the *K-means*- and *BEM*-based excitation matching methods when changing the reference pattern, and more precisely when decreasing its sidelobe level, by evaluating all the figure of merit. The performance will be analyzed also considering different values of Q , in order to confirm the outcome of the previous Section. The size of the problem is kept fixed (number of elements: $N = 32$) as well as the desired beam pointing angle.

Array Parameters

- Number of elements: $N = 32$
- Number of subarrays: $Q = 12, 16, 20, 24, 28$
- Inter-element spacing: $d = \lambda/2$
- Taylor excitation amplitudes, $SLL_{ref} = -20, -25, -30, -35, -40$ [dB]
- Pointing angle: $\theta_0 = -10$ [deg] ($u_0 = -0.174$)

CPM Clustering Parameters

- Number of iterations: $I = 50$
- Number of executions: $R = 100$

1.1.1 Taylor Pattern, $N = 32$, $SLL_{ref} = -20$ [dB], $\theta_0 = -10$ [deg]: $Q = 12, 16, 20, 24, 28$

$Q = 12$ - Excitation Matching (EM) BEM vs. K-means vs. Target

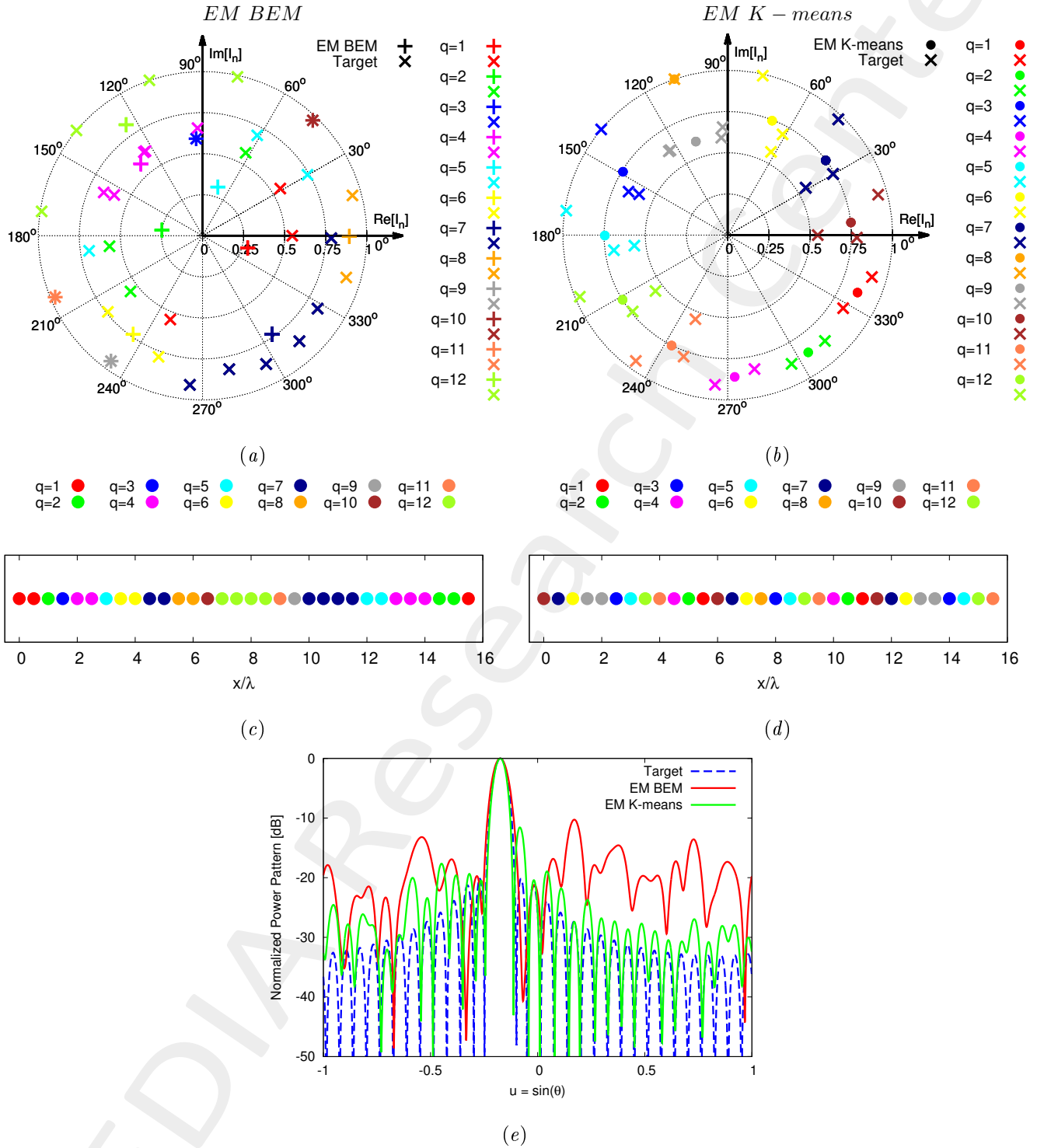


Figure 1: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 16$ - Excitation Matching (EM) BEM vs. K-means vs. Target

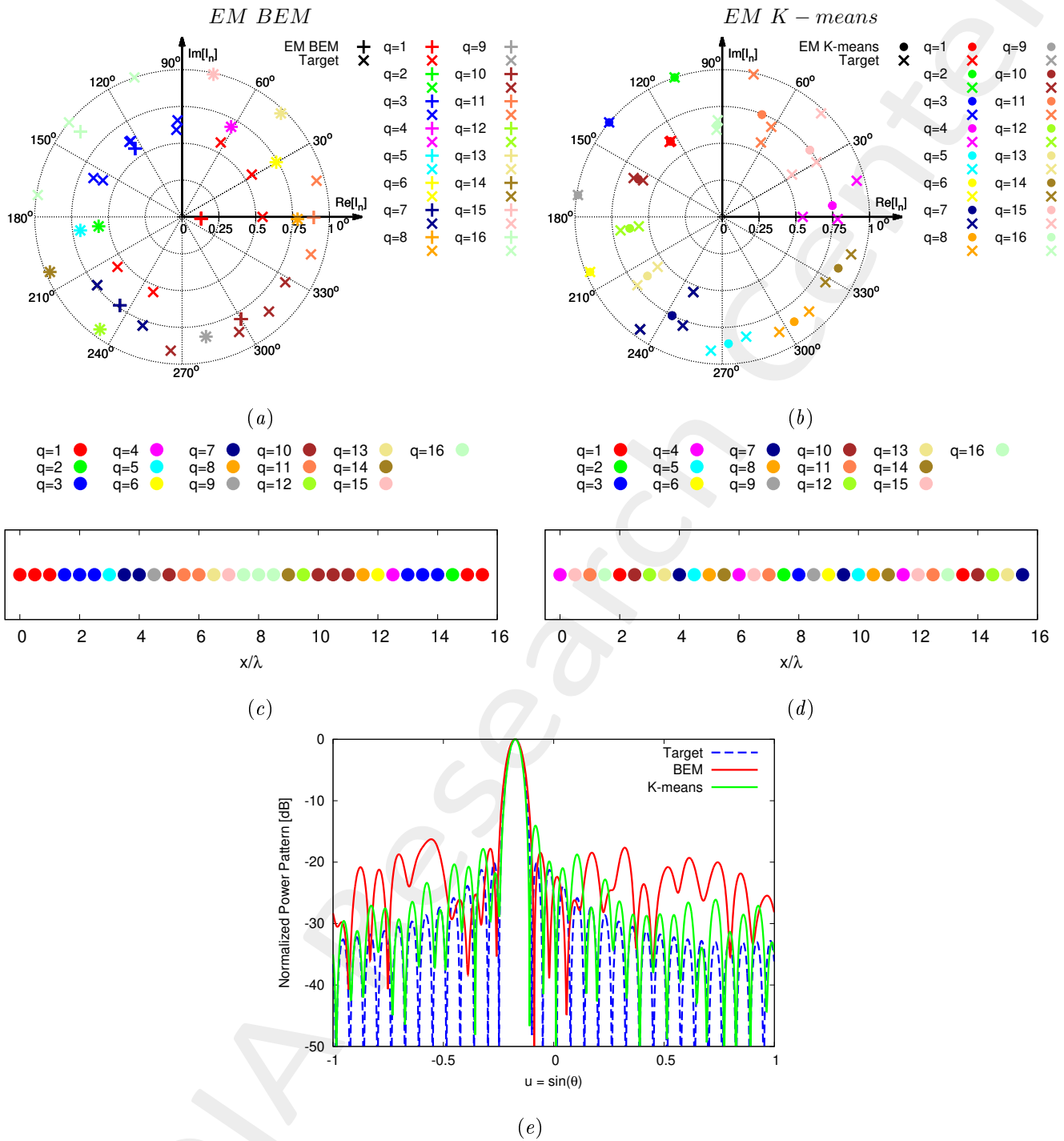


Figure 2: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 20$ - Excitation Matching (EM) BEM vs. K-means vs. Target

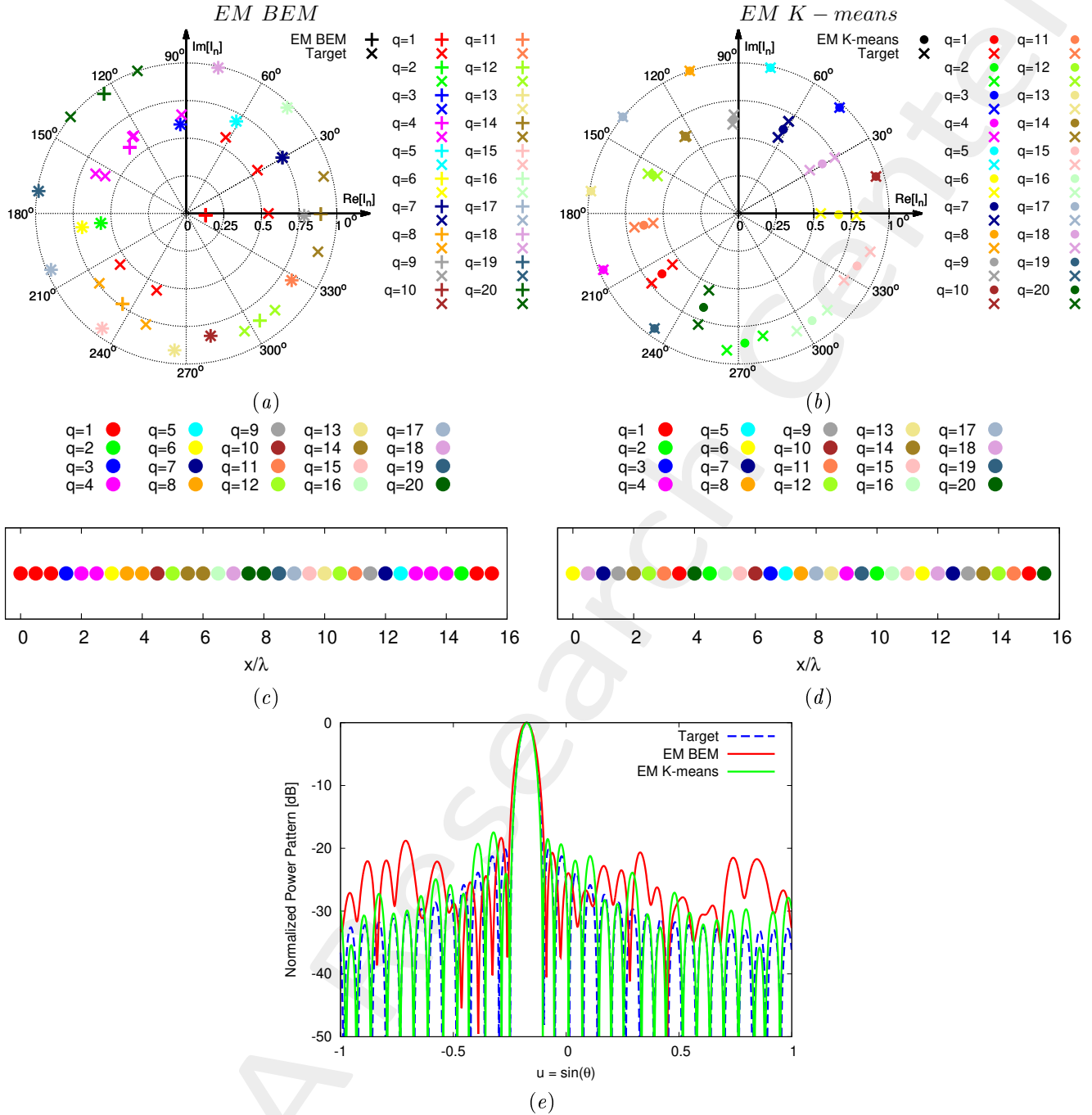


Figure 3: *Excitations Matching (EM) BEM vs. K-means vs. target* - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 24$ - Excitation Matching (EM) BEM vs. K-means vs. Target

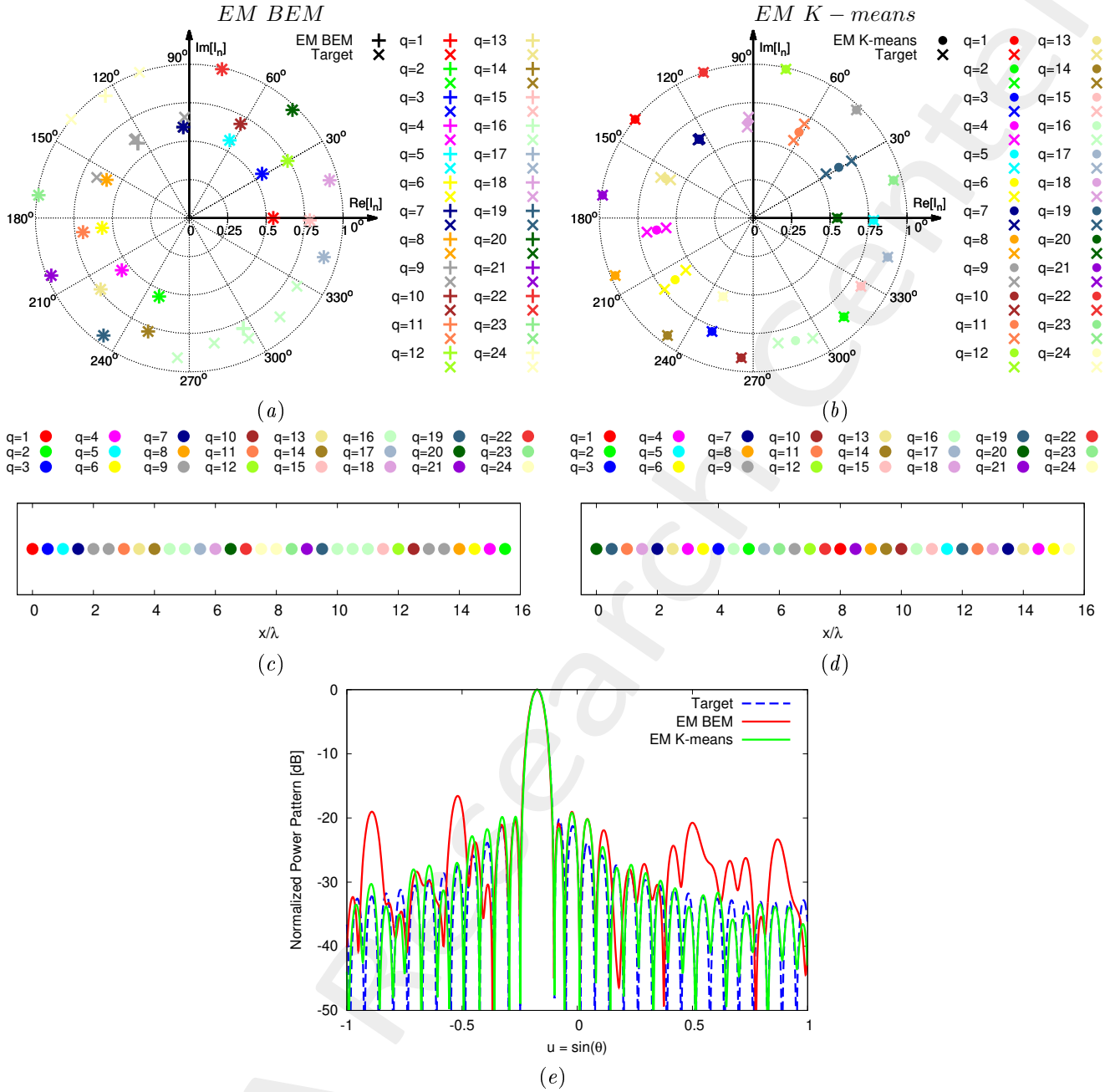


Figure 4: *Excitations Matching (EM) BEM vs. K-means vs. target* - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 28$ - Excitation Matching (EM) BEM vs. K-means vs. Target

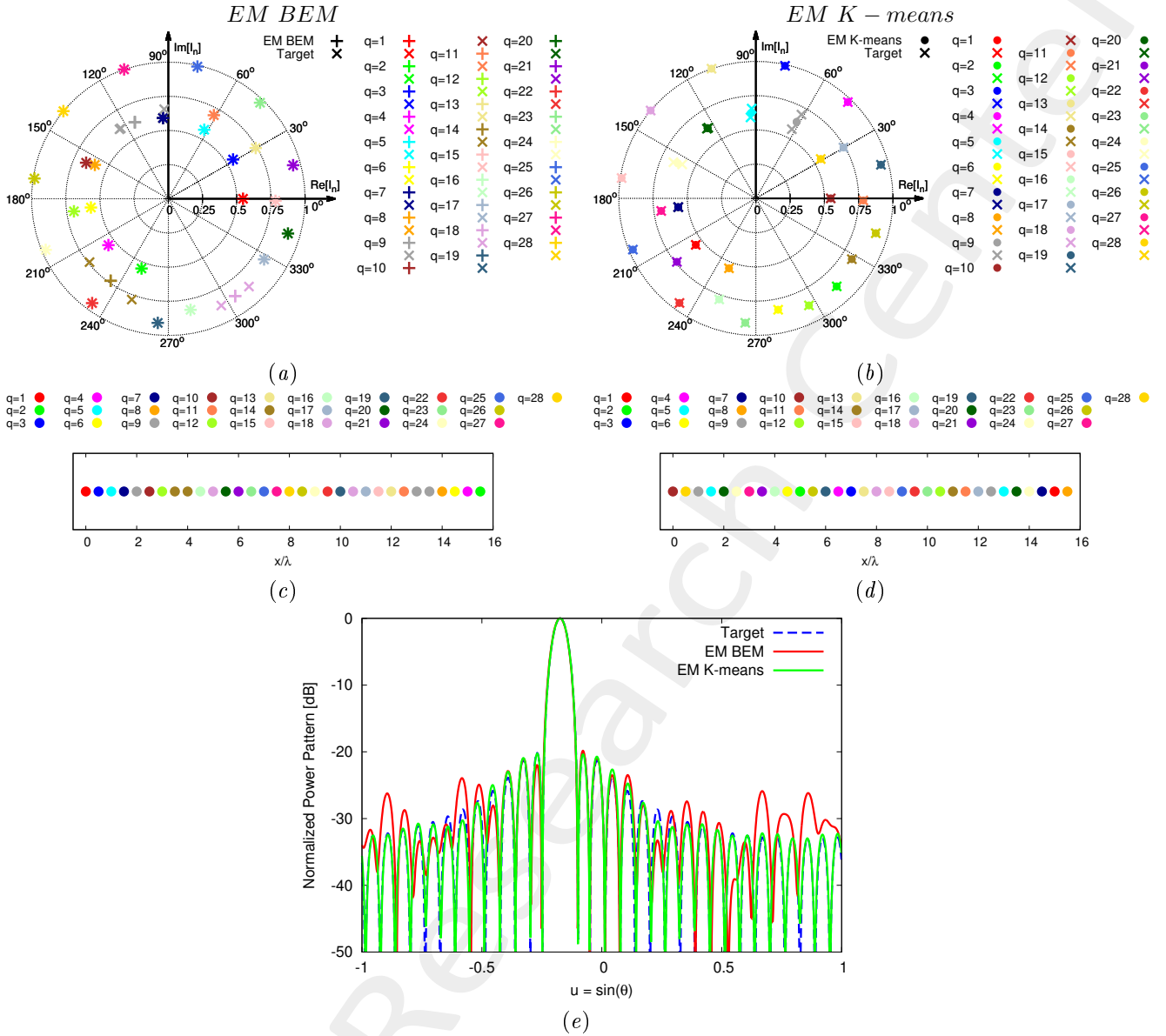


Figure 5: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

Pattern Features Resume

	<i>BEM</i>				
	<i>SLL</i> [dB]	<i>HPBW</i> [deg]	D_{\max} [dB]	Δ	Ψ
$Q = 12$	-10.27	4.11	16.09	2.82×10^{-1}	1.74×10^{-1}
$Q = 16$	-16.28	4.15	16.81	1.58×10^{-1}	9.74×10^{-2}
$Q = 20$	-18.34	4.17	17.04	1.10×10^{-1}	6.74×10^{-2}
$Q = 24$	-16.57	3.57	17.68	4.66×10^{-2}	2.87×10^{-2}
$Q = 28$	-19.84	3.59	17.82	1.01×10^{-2}	6.18×10^{-3}
$Q = 32$	-20.19	3.59	17.87	0.00	0.00
	<i>K - means</i>				
	<i>SLL</i> [dB]	<i>HPBW</i> [deg]	D_{\max} [dB]	Δ	Ψ
$Q = 12$	-11.56	3.30	17.90	5.34×10^{-2}	3.28×10^{-2}
$Q = 16$	-14.06	3.38	17.91	3.02×10^{-2}	1.86×10^{-2}
$Q = 20$	-17.46	3.49	17.88	1.06×10^{-2}	6.51×10^{-3}
$Q = 24$	-19.21	3.54	17.88	4.11×10^{-3}	2.53×10^{-3}
$Q = 28$	-20.25	3.57	17.87	6.34×10^{-4}	3.89×10^{-4}
$Q = 32$	-20.18	3.58	17.87	0.00	0.00

Table I: *Excitations Matching (EM) BEM vs. K-means solutions* - (a) Sidelobe level, *SLL*, (b) half-power beamwidth, *HPBW*, (c) directivity peak, D_{\max} , (d) pattern matching error, Δ , and (e) fitness, Ψ , values.

1.1.2 Taylor Pattern, $N = 32$, $SLL_{ref} = -25$ [dB], $\theta_0 = -10$ [deg]: $Q = 12, 16, 20, 24, 28$

$Q = 12$ - Excitation Matching (EM) BEM vs. K-means vs. Target

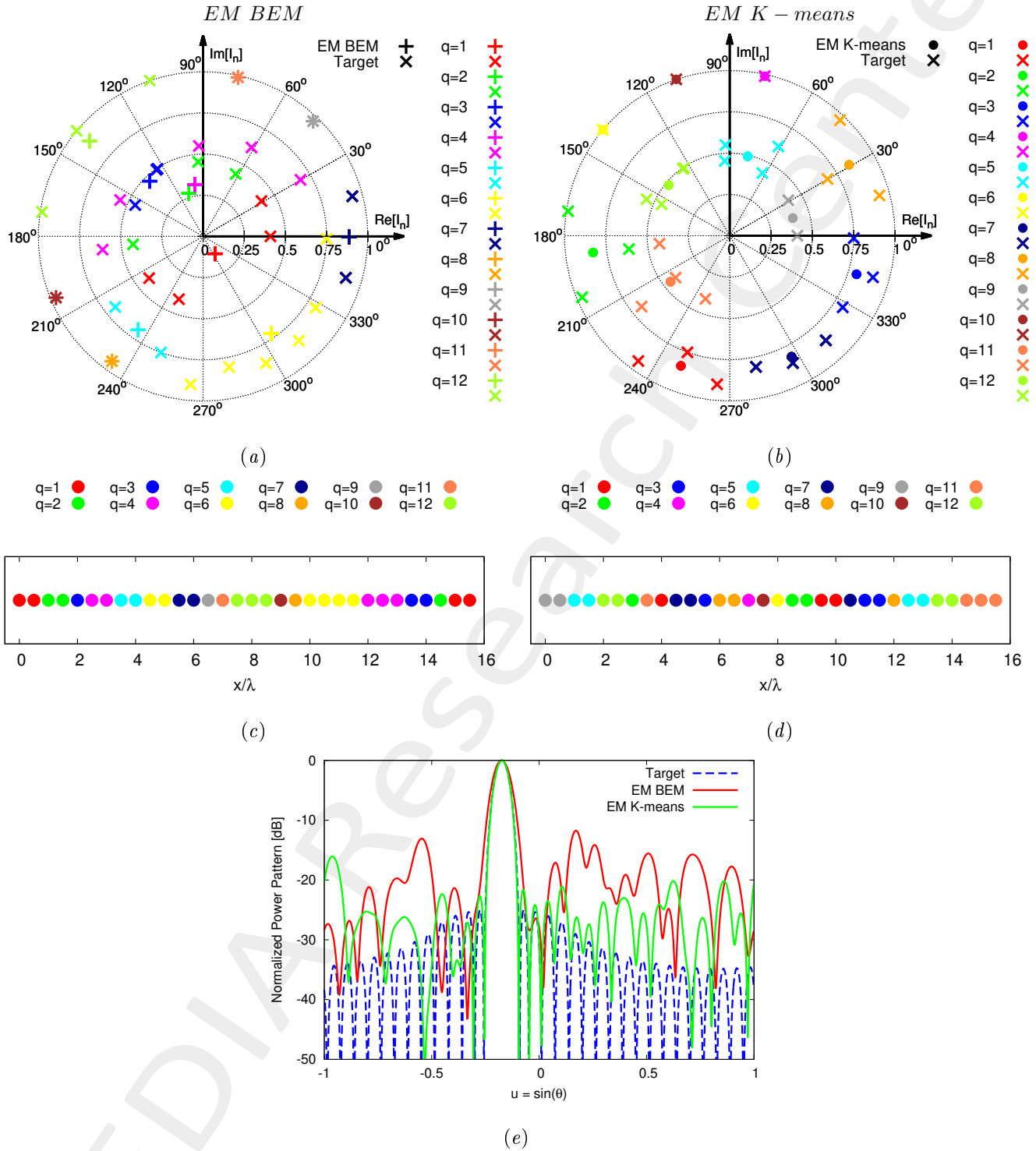


Figure 6: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 16$ - Excitation Matching (EM) BEM vs. K-means vs. Target

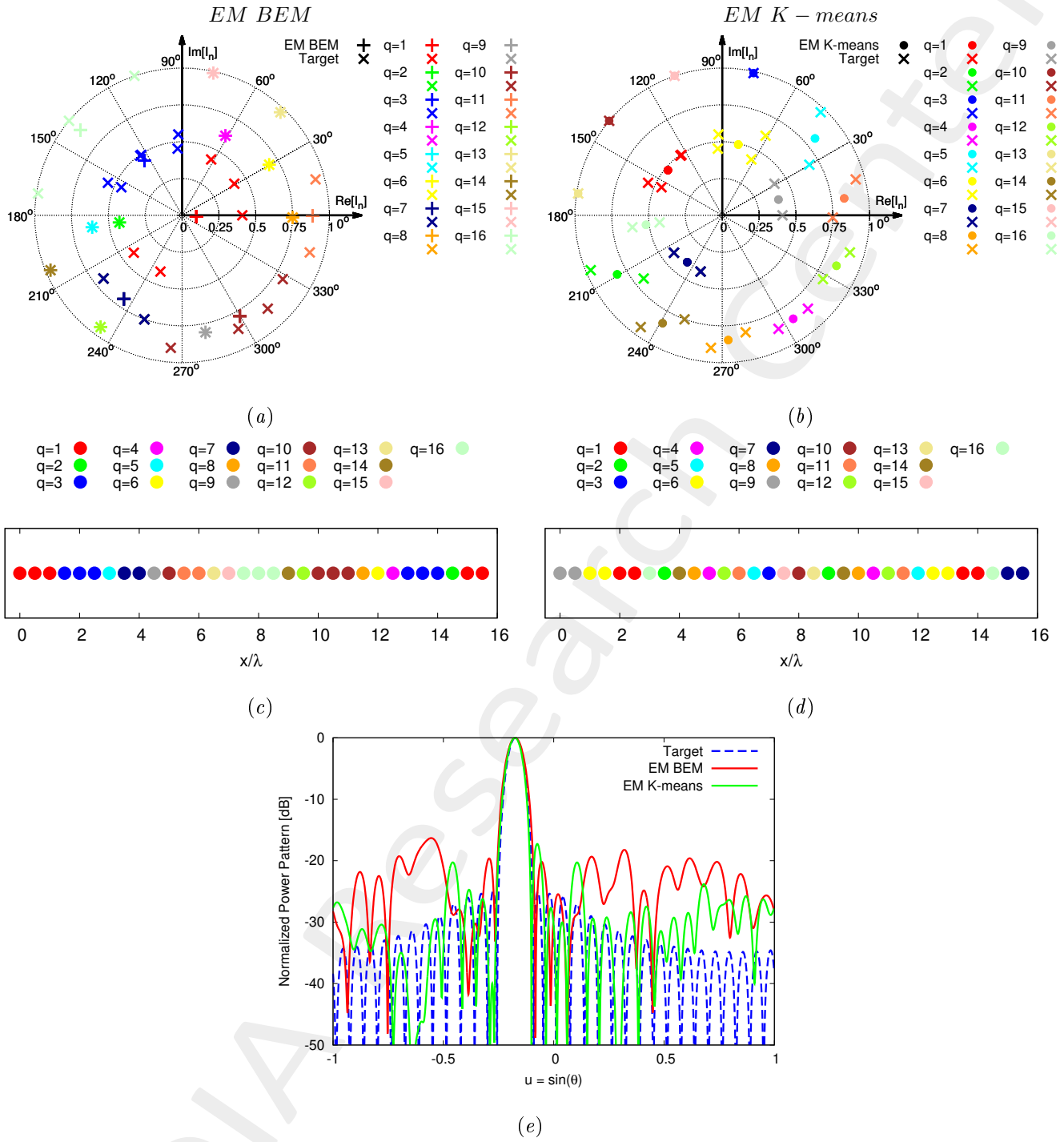


Figure 7: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 20$ - Excitation Matching (EM) BEM vs. K-means vs. Target

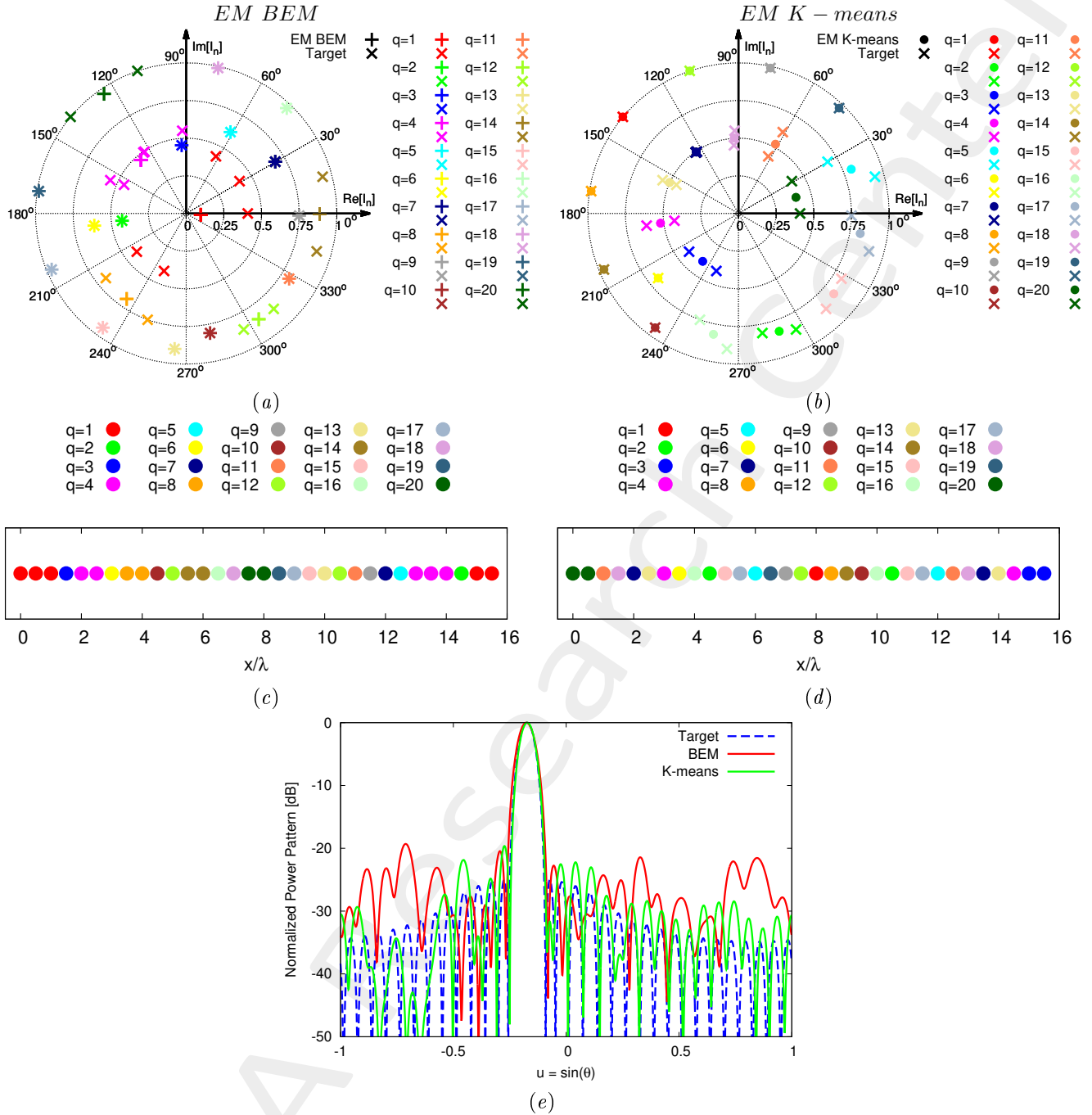


Figure 8: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 24$ - Excitation Matching (EM) BEM vs. K-means vs. Target

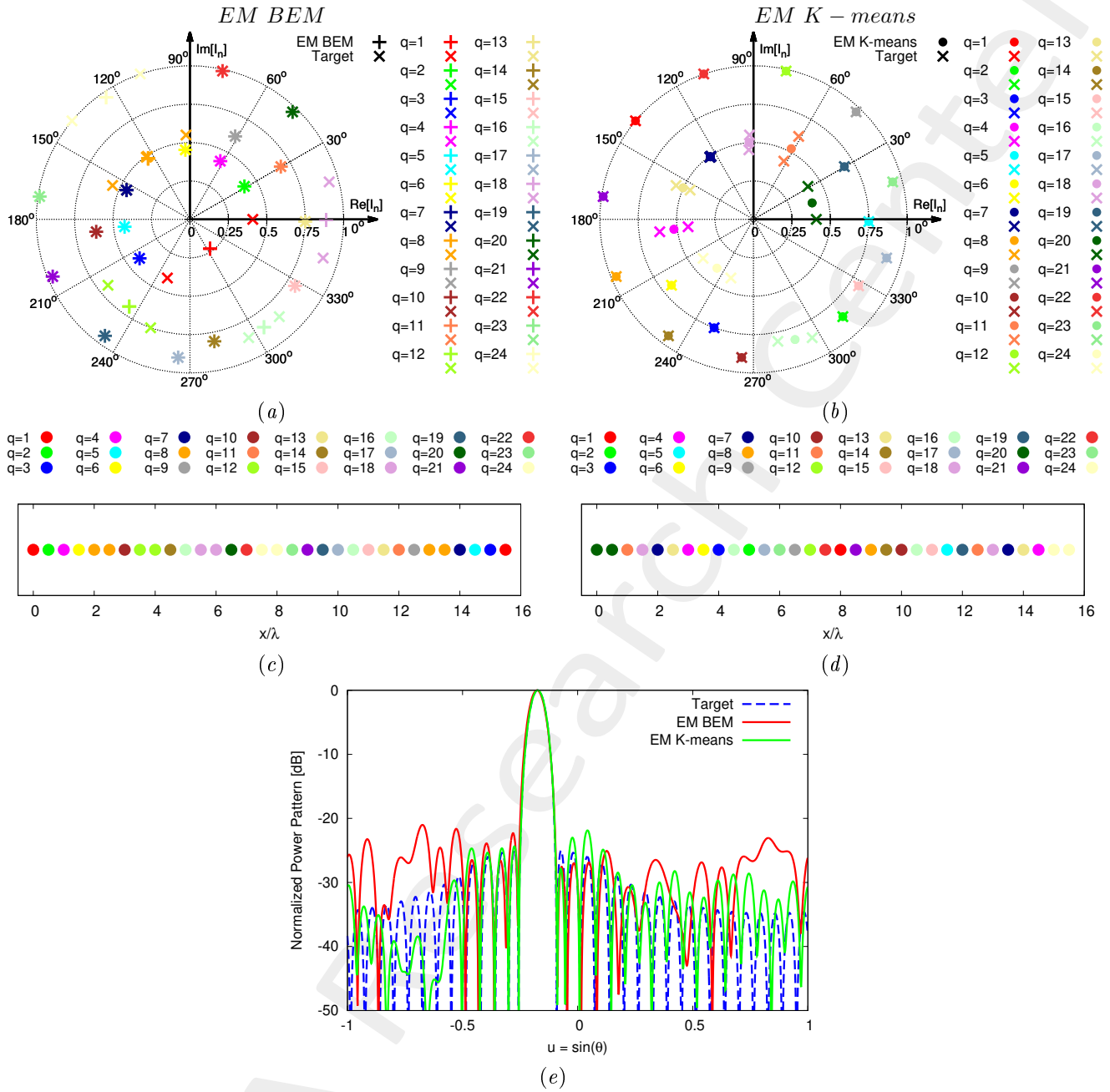


Figure 9: *Excitations Matching (EM) BEM vs. K-means vs. target* - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 28$ - Excitation Matching (EM) BEM vs. K-means vs. Target

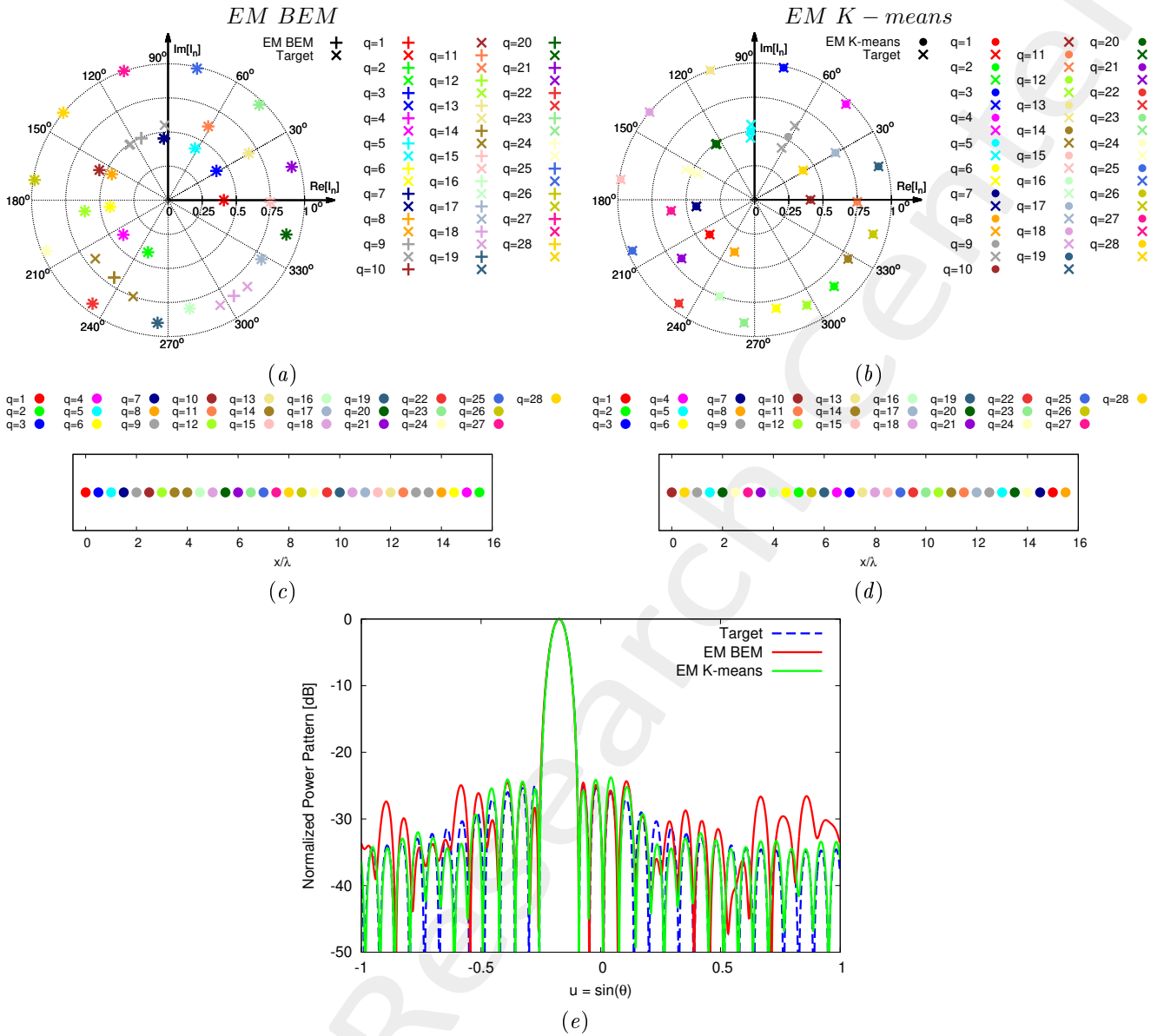


Figure 10: *Excitations Matching (EM) BEM vs. K-means vs. target* - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

Pattern Features Resume

<i>BEM</i>					
	<i>SLL</i> [dB]	<i>HPBW</i> [deg]	D_{\max} [dB]	Δ	Ψ
$Q = 12$	-11.76	4.67	15.85	2.37×10^{-1}	1.29×10^{-1}
$Q = 16$	-16.32	4.34	16.67	1.33×10^{-1}	7.21×10^{-2}
$Q = 20$	-19.31	4.37	16.90	8.09×10^{-2}	4.39×10^{-2}
$Q = 24$	-21.02	3.94	17.38	4.44×10^{-2}	2.41×10^{-2}
$Q = 28$	-24.35	3.81	17.61	9.35×10^{-3}	5.08×10^{-3}
$Q = 32$	-25.02	3.80	17.67	0.00	0.00
<i>K - means</i>					
	<i>SLL</i> [dB]	<i>HPBW</i> [deg]	D_{\max} [dB]	Δ	Ψ
$Q = 12$	-16.06	3.74	17.45	6.29×10^{-2}	3.41×10^{-2}
$Q = 16$	-17.67	3.78	17.56	3.23×10^{-2}	1.75×10^{-2}
$Q = 20$	-19.64	3.77	17.63	1.54×10^{-2}	8.38×10^{-3}
$Q = 24$	-21.87	3.80	17.63	6.85×10^{-3}	3.72×10^{-3}
$Q = 28$	-23.76	3.78	17.67	1.55×10^{-3}	8.42×10^{-4}
$Q = 32$	-25.01	3.80	17.66	0.00	0.00

Table II: *Excitations Matching (EM) BEM vs. K-means solutions* - Sidelobe level, *SLL*, half-power beamwidth, *HPBW*, directivity peak, D_{\max} , pattern matching error, Δ , and fitness, Ψ , values.

1.1.3 Taylor Pattern, $N = 32$, $SLL_{ref} = -30$ [dB], $\theta_0 = -10$ [deg]: $Q = 12, 16, 20, 24, 28$

$Q = 12$ - Excitation Matching (EM) BEM vs. K-means vs. Target

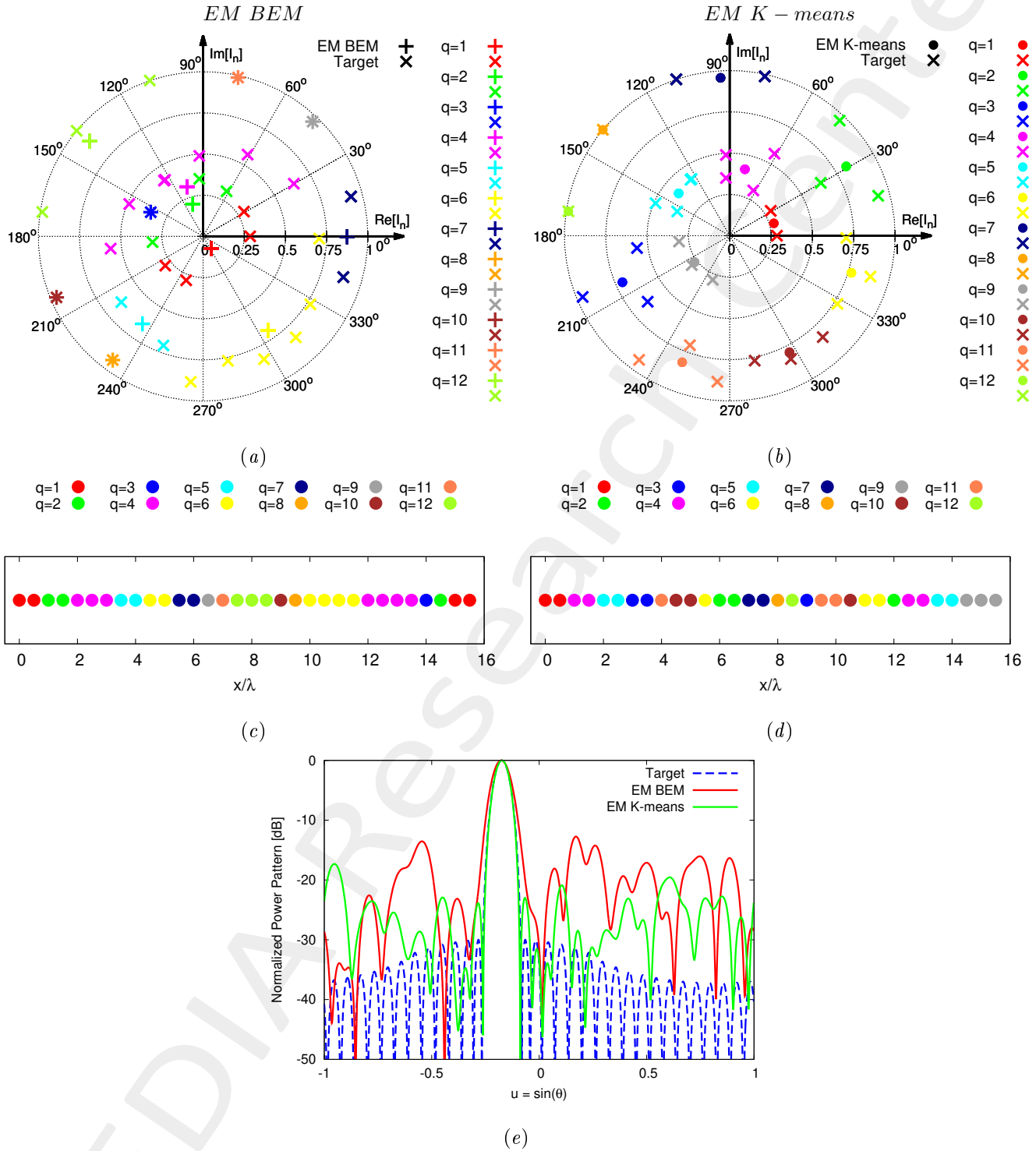


Figure 11: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 16$ - Excitation Matching (EM) BEM vs. K-means vs. Target

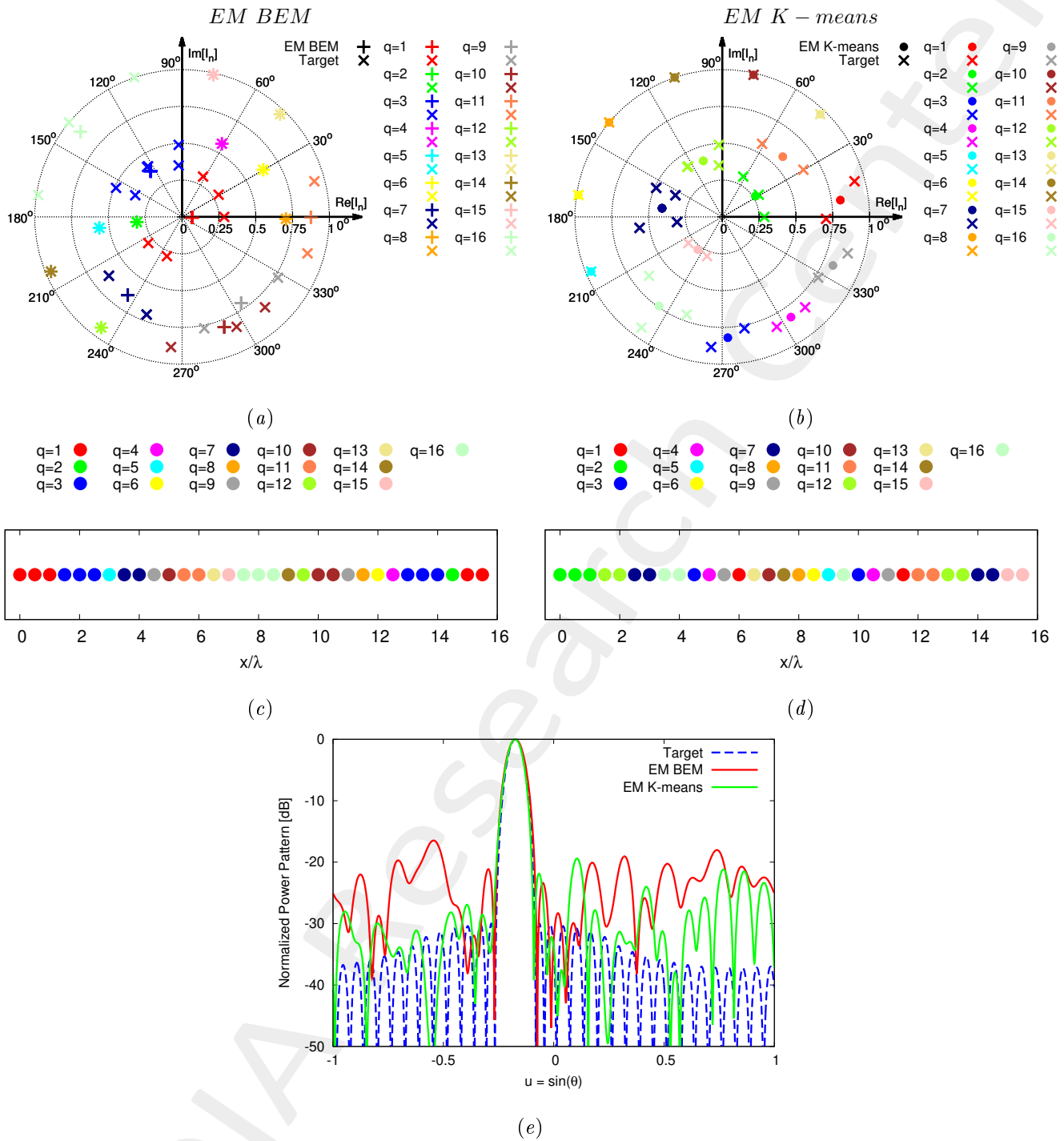


Figure 12: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 20$ - Excitation Matching (EM) BEM vs. K-means vs. Target

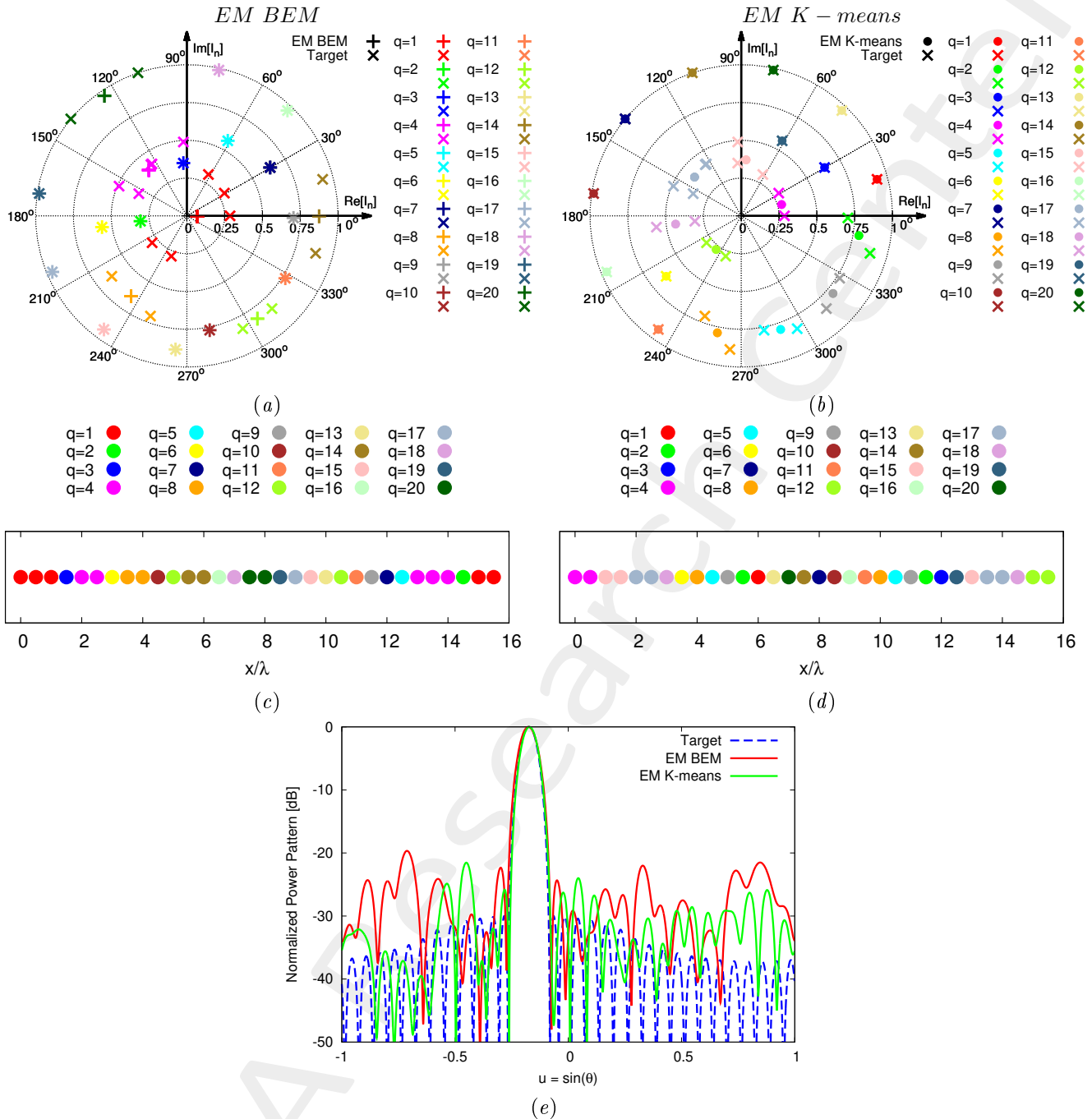


Figure 13: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 24$ - Excitation Matching (EM) BEM vs. K-means vs. Target

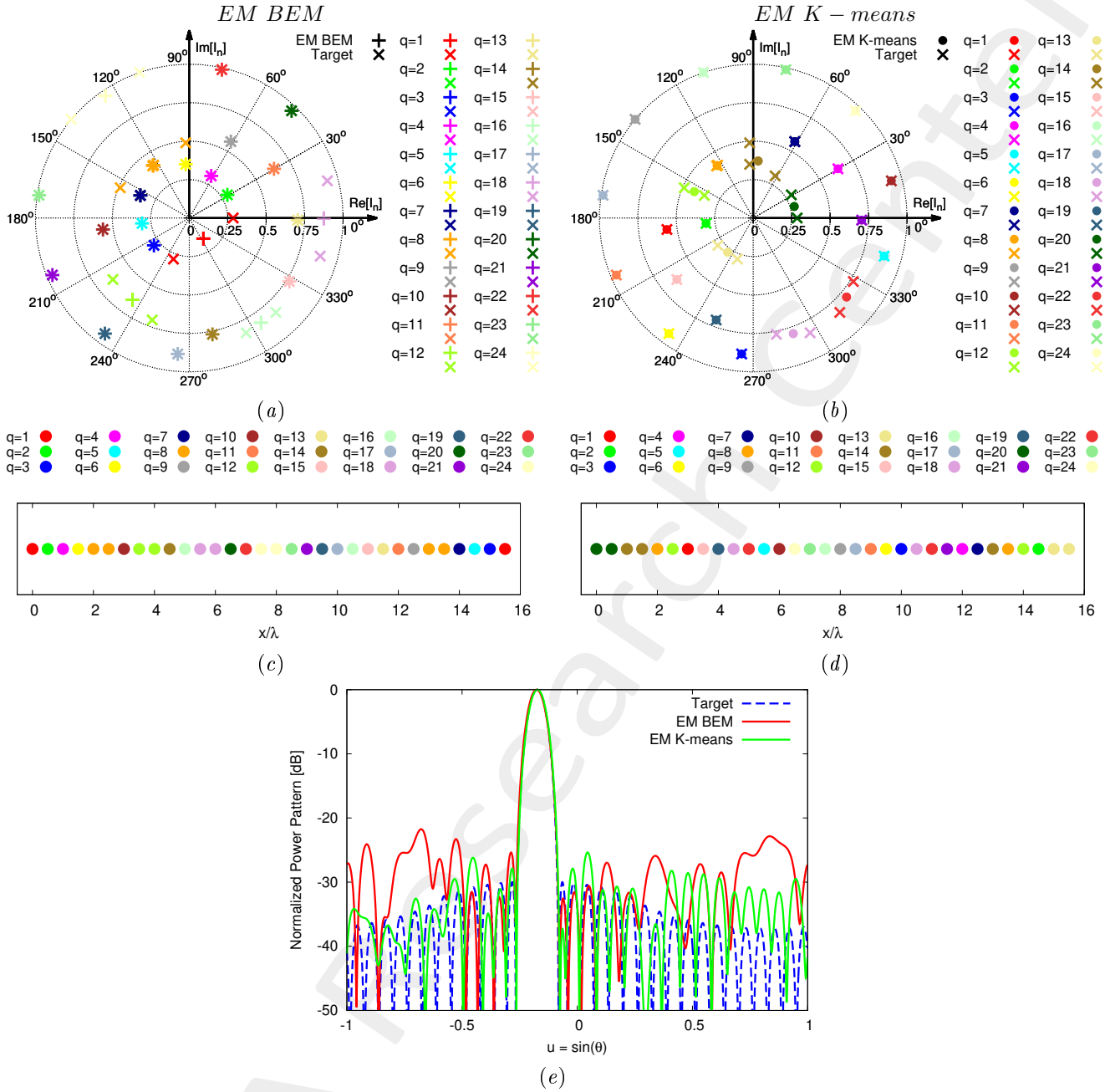


Figure 14: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 28$ - Excitation Matching (EM) BEM vs. K-means vs. Target

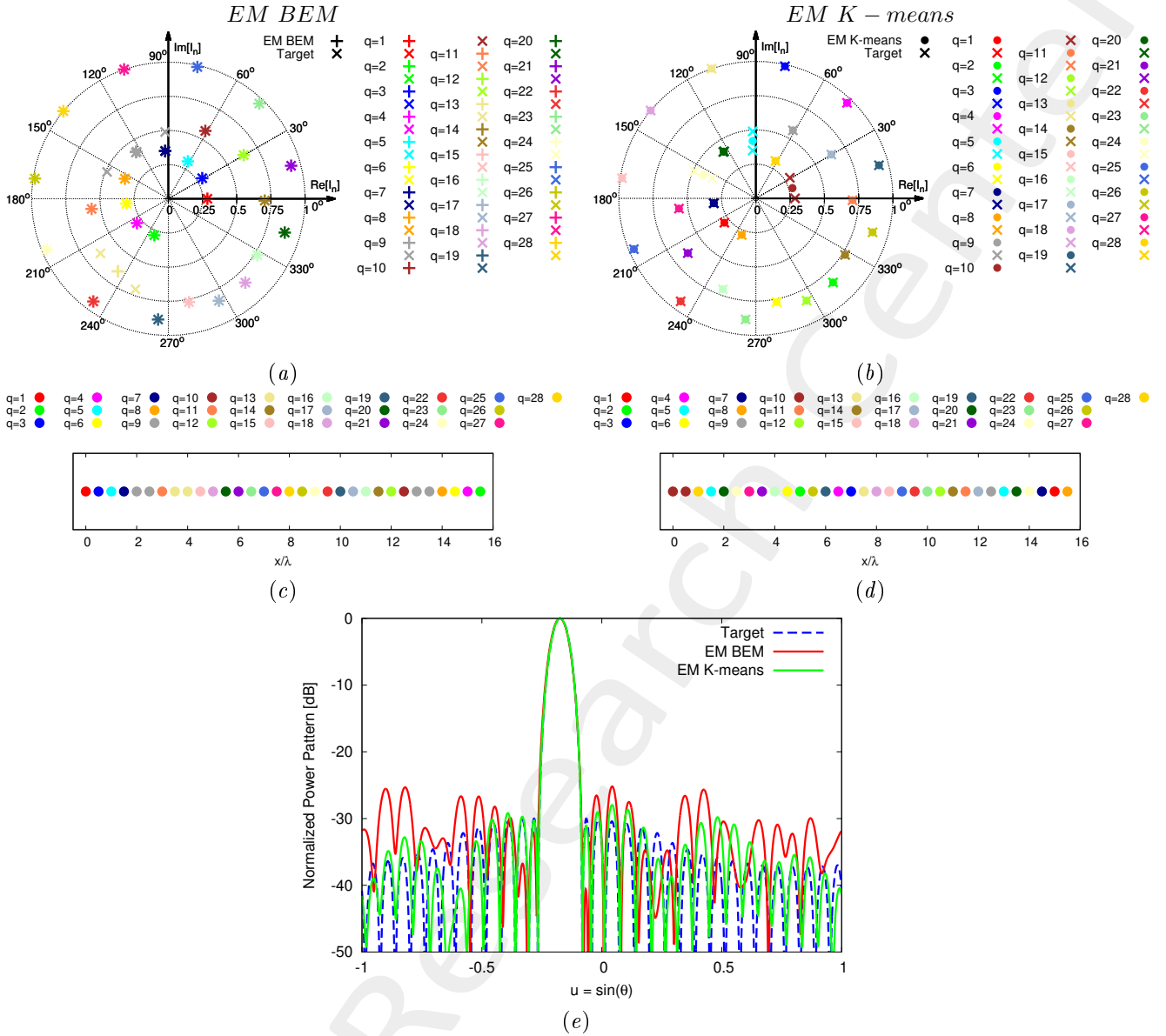


Figure 15: *Excitations Matching (EM) BEM vs. K-means vs. target* - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

Pattern Features Resume

	<i>BEM</i>				
	<i>SLL</i> [dB]	<i>HPBW</i> [deg]	D_{\max} [dB]	Δ	Ψ
$Q = 12$	-12.75	4.93	15.69	2.14×10^{-1}	1.05×10^{-1}
$Q = 16$	-16.48	4.52	16.52	1.14×10^{-1}	5.58×10^{-2}
$Q = 20$	-19.67	4.55	16.76	5.94×10^{-2}	2.92×10^{-2}
$Q = 24$	-21.76	4.17	17.15	3.85×10^{-2}	1.89×10^{-2}
$Q = 28$	-25.19	4.07	17.33	1.28×10^{-2}	6.27×10^{-3}
$Q = 32$	-30.00	4.04	17.42	0.00	0.0
	<i>K - means</i>				
	<i>SLL</i> [dB]	<i>HPBW</i> [deg]	D_{\max} [dB]	Δ	Ψ
$Q = 12$	-17.31	3.98	17.21	6.59×10^{-2}	3.23×10^{-2}
$Q = 16$	-19.41	4.07	17.24	3.44×10^{-2}	1.69×10^{-2}
$Q = 20$	-21.53	4.04	17.34	1.81×10^{-2}	8.86×10^{-3}
$Q = 24$	-25.36	4.05	17.37	7.89×10^{-3}	3.87×10^{-3}
$Q = 28$	-27.96	4.04	17.41	2.01×10^{-3}	9.87×10^{-4}
$Q = 32$	-30.00	4.04	17.42	0.00	0.00

Table III: *Excitations Matching (EM) BEM vs. K-means solutions* - Sidelobe level, *SLL*, half-power beamwidth, *HPBW*, directivity peak, D_{\max} , pattern matching error, Δ , and fitness, Ψ , values.

1.1.4 Taylor Pattern, $N = 32$, $SLL_{ref} = -35$ [dB], $\theta_0 = -10$ [deg]: $Q = 12, 16, 20, 24, 28$

$Q = 12$ - Excitation Matching (EM) BEM vs. K-means vs. Target

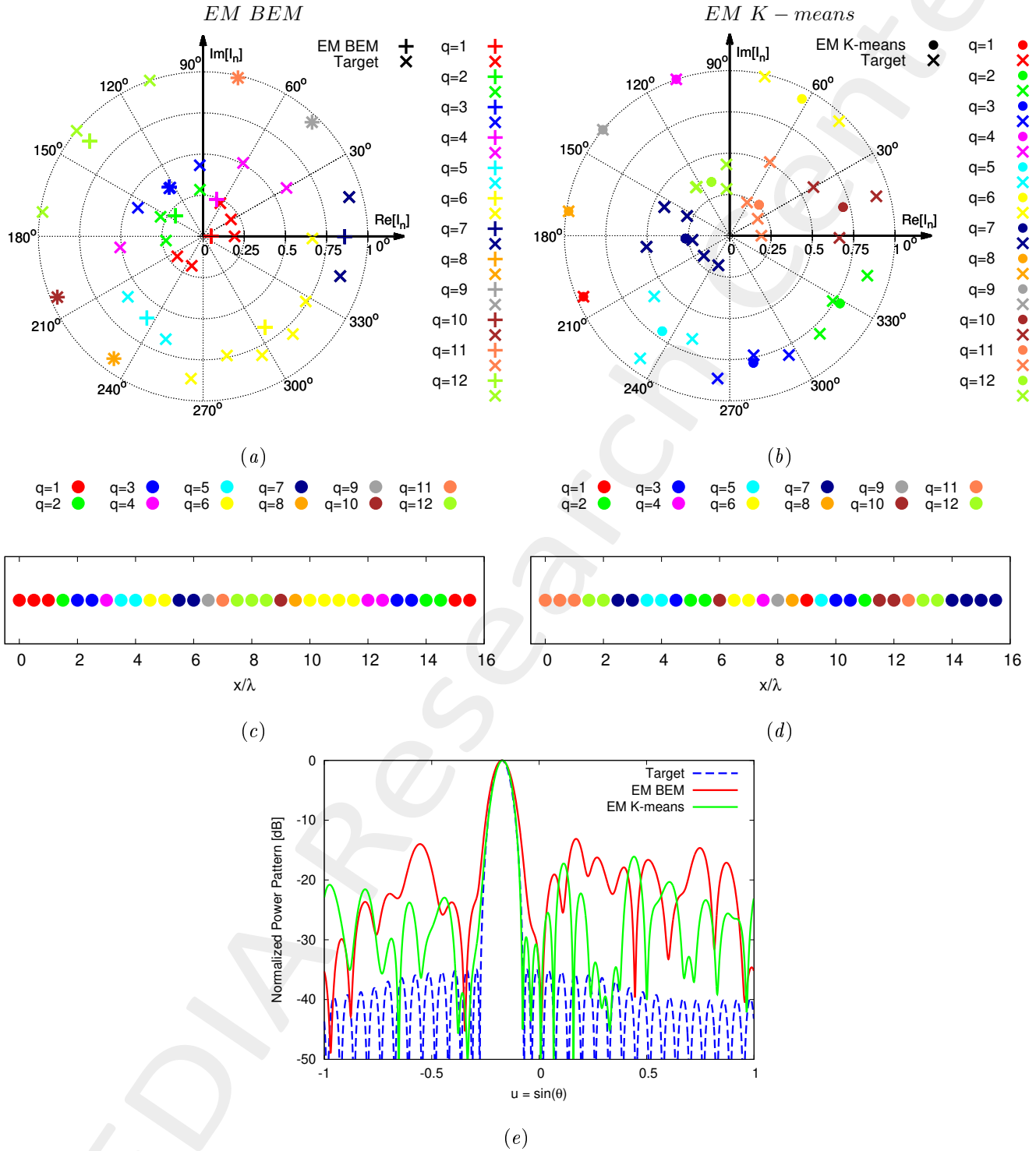


Figure 16: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 16$ - Excitation Matching (EM) BEM vs. K-means vs. Target

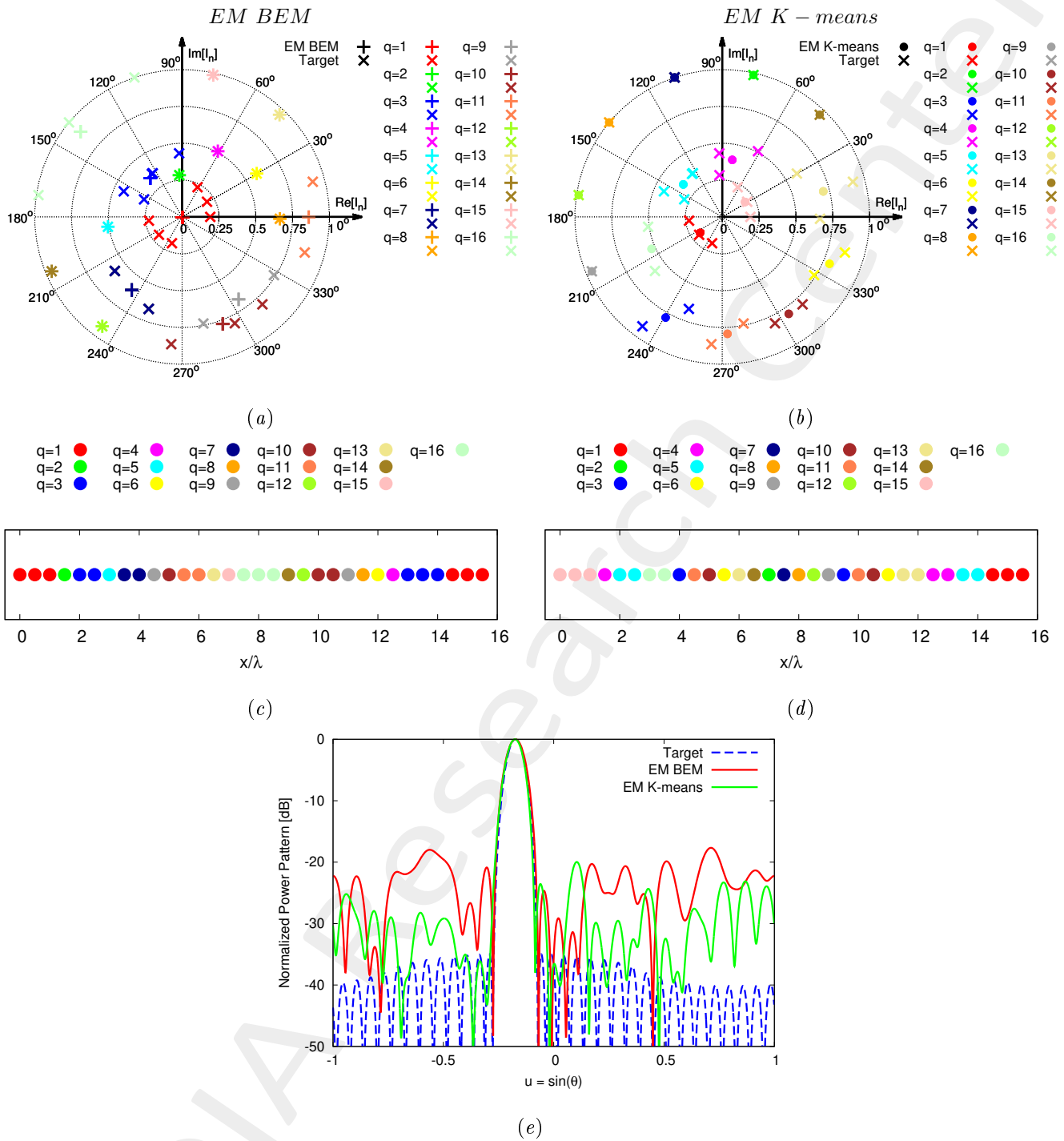


Figure 17: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 20$ - Excitation Matching (EM) BEM vs. K-means vs. Target

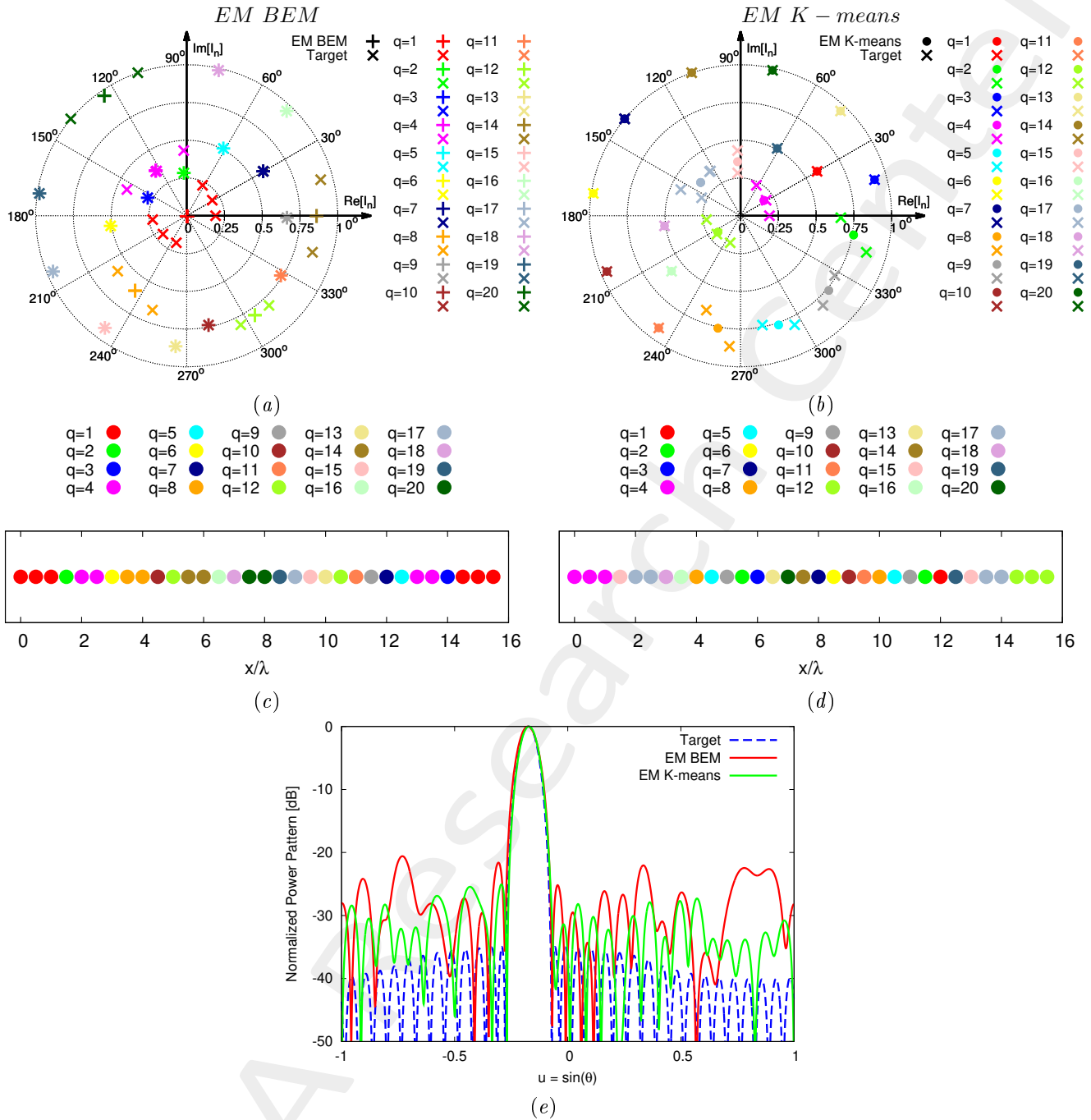


Figure 18: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 24$ - Excitation Matching (EM) BEM vs. K-means vs. Target

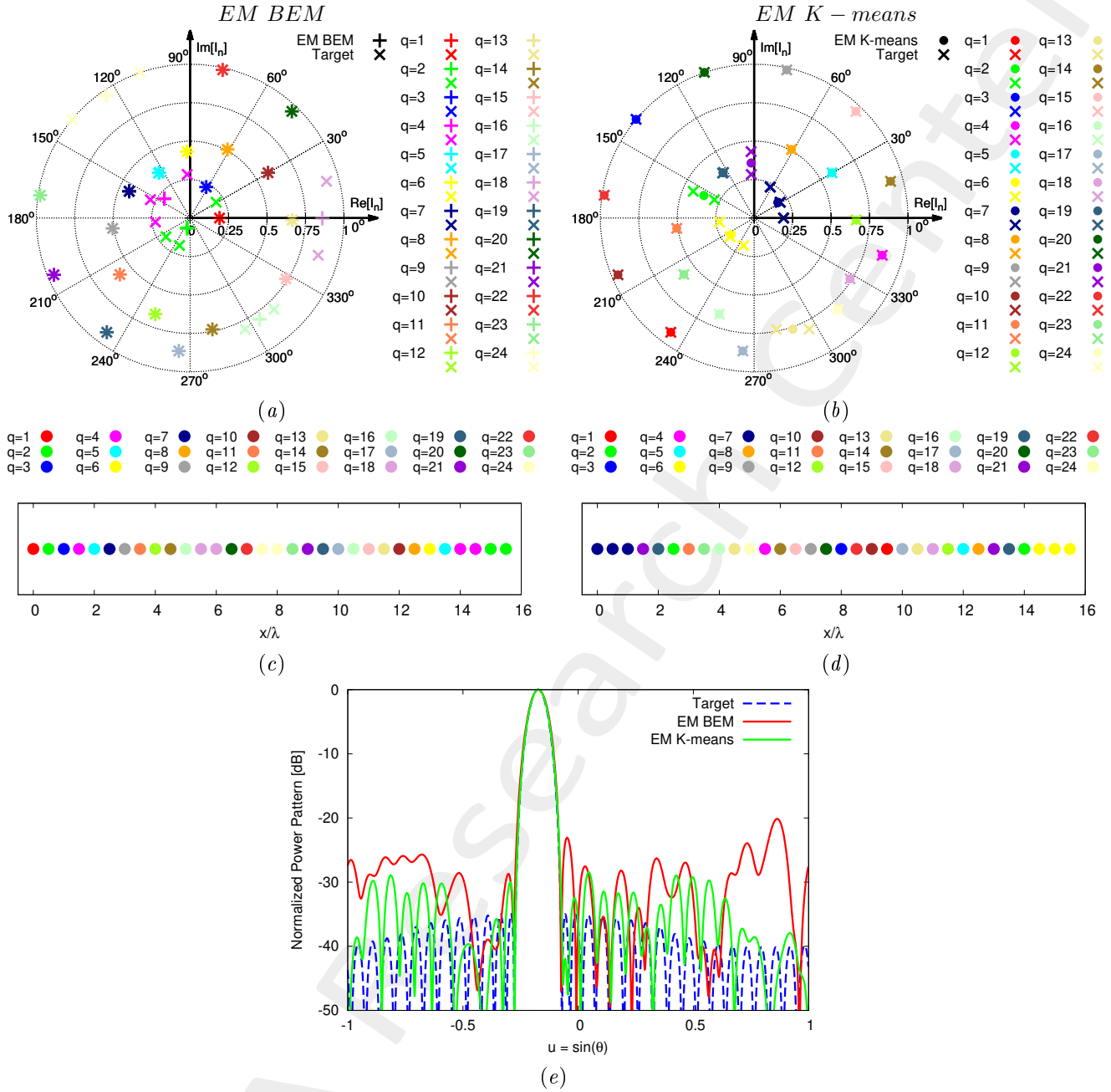


Figure 19: *Excitations Matching (EM) BEM vs. K-means vs. target* - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 28$ - Excitation Matching (EM) BEM vs. K-means vs. Target

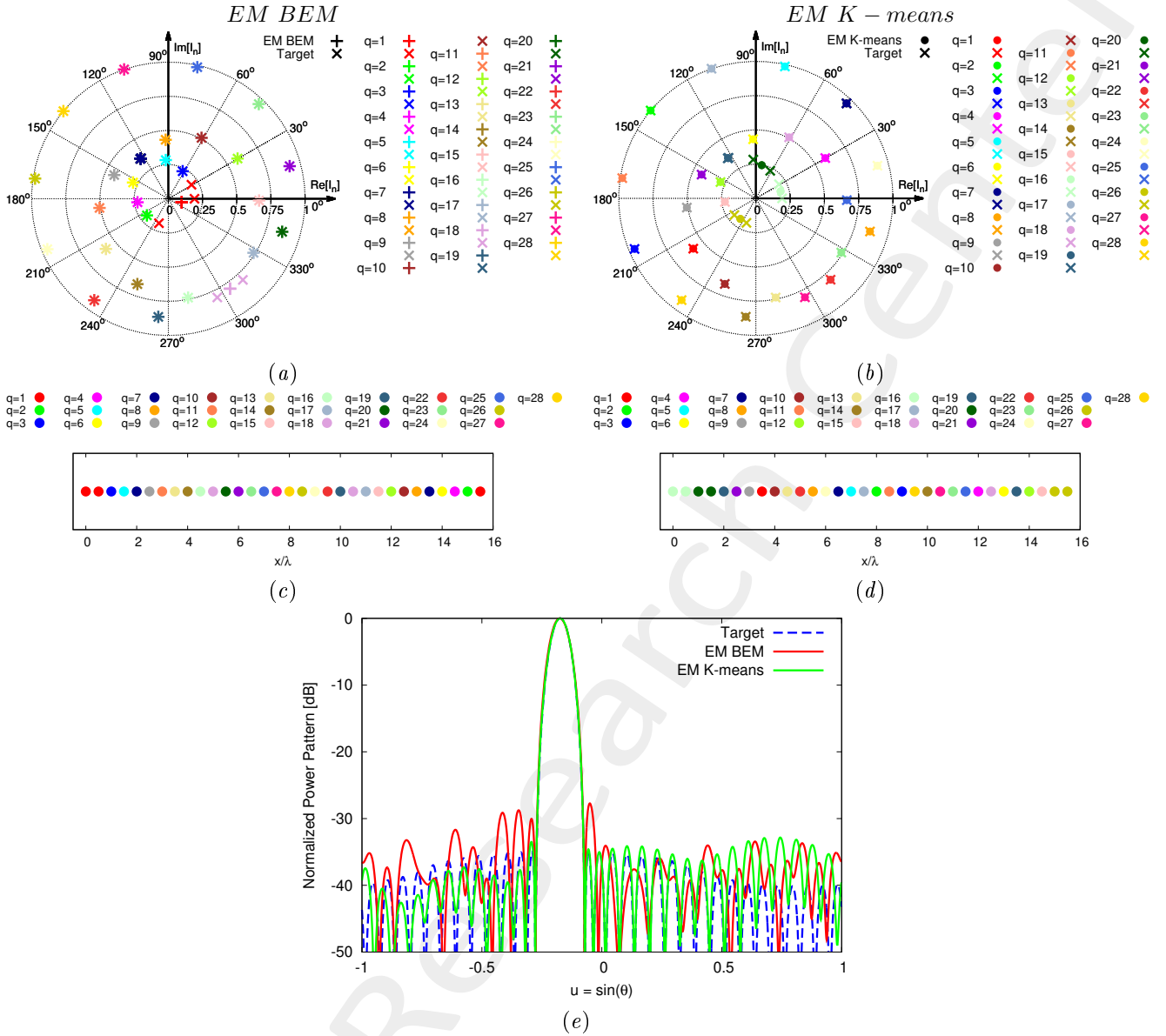


Figure 20: *Excitations Matching (EM) BEM vs. K-means vs. target* - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

Pattern Features Resume

	<i>BEM</i>				
	<i>SLL</i> [dB]	<i>HPBW</i> [deg]	D_{\max} [dB]	Δ	Ψ
$Q = 12$	-13.12	5.05	15.66	1.86×10^{-1}	8.40×10^{-2}
$Q = 16$	-17.67	4.78	16.30	1.05×10^{-1}	4.73×10^{-2}
$Q = 20$	-20.59	4.83	16.52	4.87×10^{-2}	2.21×10^{-2}
$Q = 24$	-20.14	4.55	16.80	3.25×10^{-2}	1.47×10^{-2}
$Q = 28$	-27.72	4.45	17.01	7.44×10^{-3}	3.37×10^{-3}
$Q = 32$	-35.01	4.28	17.17	0.00	0.00
	<i>K - means</i>				
	<i>SLL</i> [dB]	<i>HPBW</i> [deg]	D_{\max} [dB]	Δ	Ψ
$Q = 12$	-16.09	4.26	16.87	6.84×10^{-2}	3.10×10^{-2}
$Q = 16$	-19.97	4.33	17.00	3.33×10^{-2}	1.50×10^{-2}
$Q = 20$	-25.06	4.34	17.05	1.62×10^{-2}	7.32×10^{-3}
$Q = 24$	-28.45	4.34	17.09	6.45×10^{-3}	2.92×10^{-3}
$Q = 28$	-32.83	4.30	17.14	1.52×10^{-3}	6.89×10^{-4}
$Q = 32$	-35.01	4.28	17.17	0.00	0.00

Table IV: *Excitations Matching (EM) BEM vs. K-means solutions* - Sidelobe level, *SLL*, half-power beamwidth, *HPBW*, directivity peak, D_{\max} , pattern matching error, Δ , and fitness, Ψ , values.

1.1.5 Taylor Pattern, $N = 32$, $SLL_{ref} = -40$ [dB], $\theta_0 = -10$ [deg]: $Q = 12, 16, 20, 24, 28$

$Q = 12$ - Excitation Matching (EM) BEM vs. K-means vs. Target

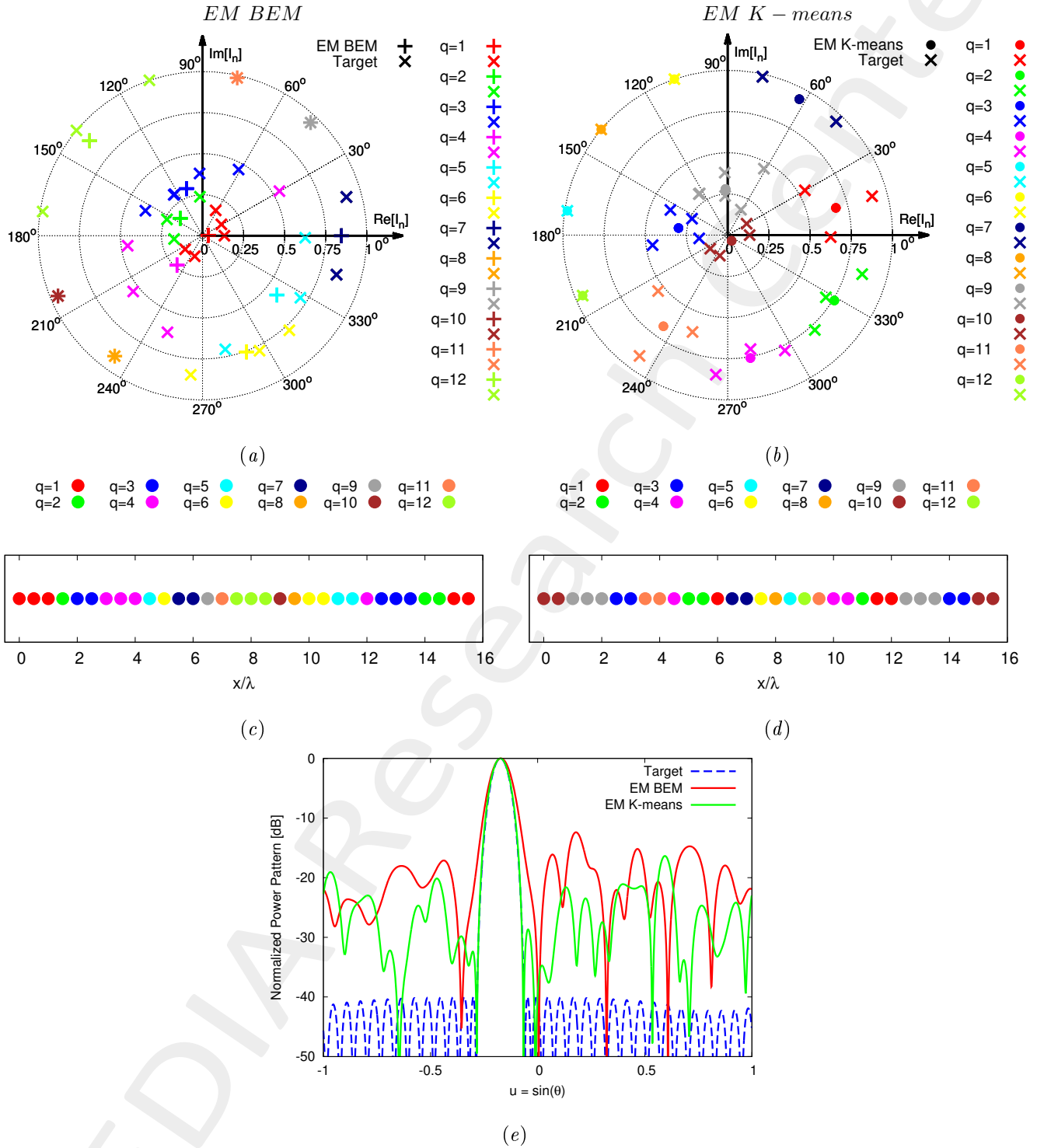


Figure 21: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 16$ - Excitation Matching (EM) BEM vs. K-means vs. Target

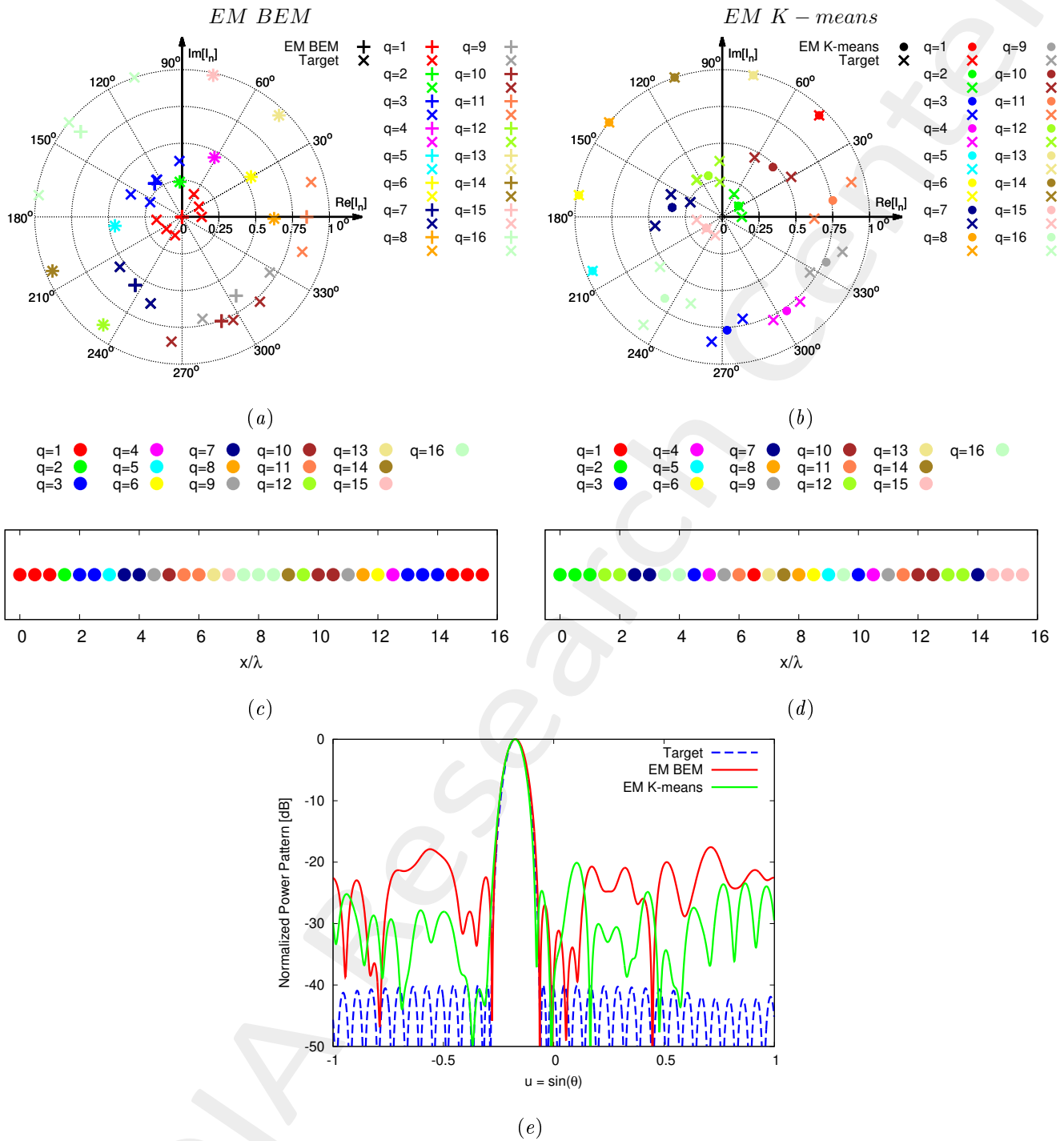


Figure 22: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 20$ - Excitation Matching (EM) BEM vs. K-means vs. Target

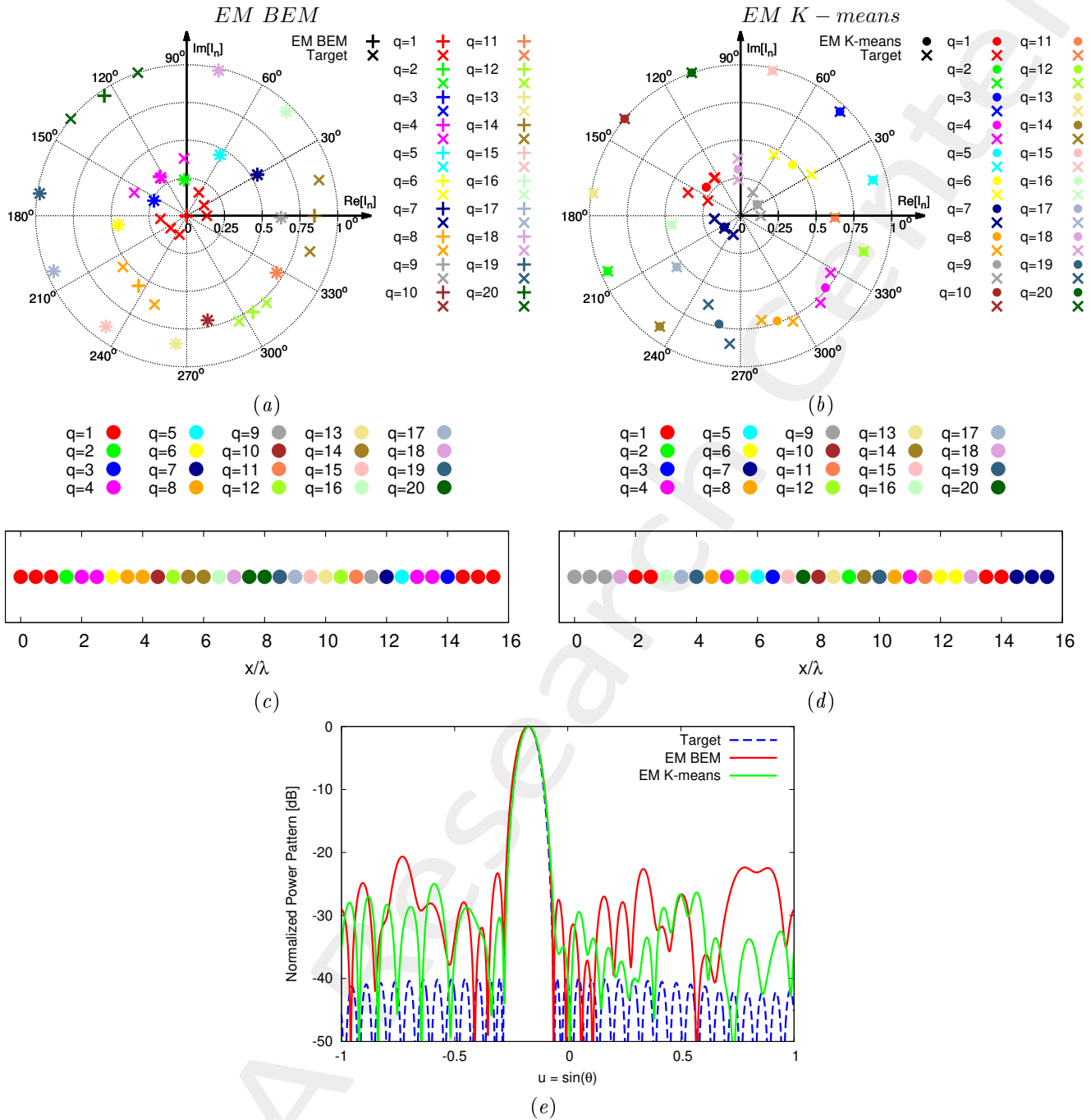


Figure 23: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 24$ - Excitation Matching (EM) BEM vs. K-means vs. Target

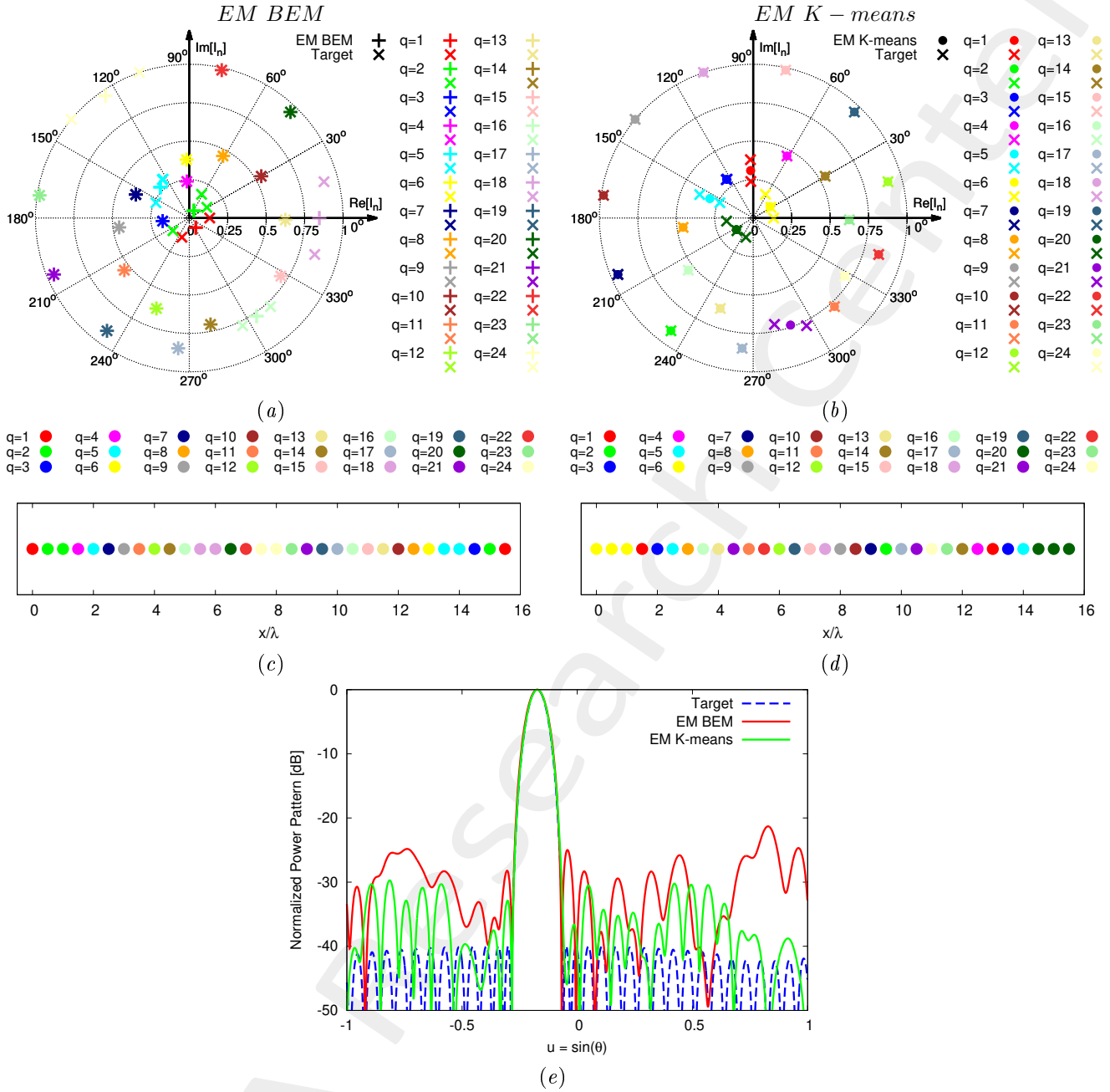


Figure 24: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 28$ - Excitation Matching (EM) BEM vs. K-means vs. Target

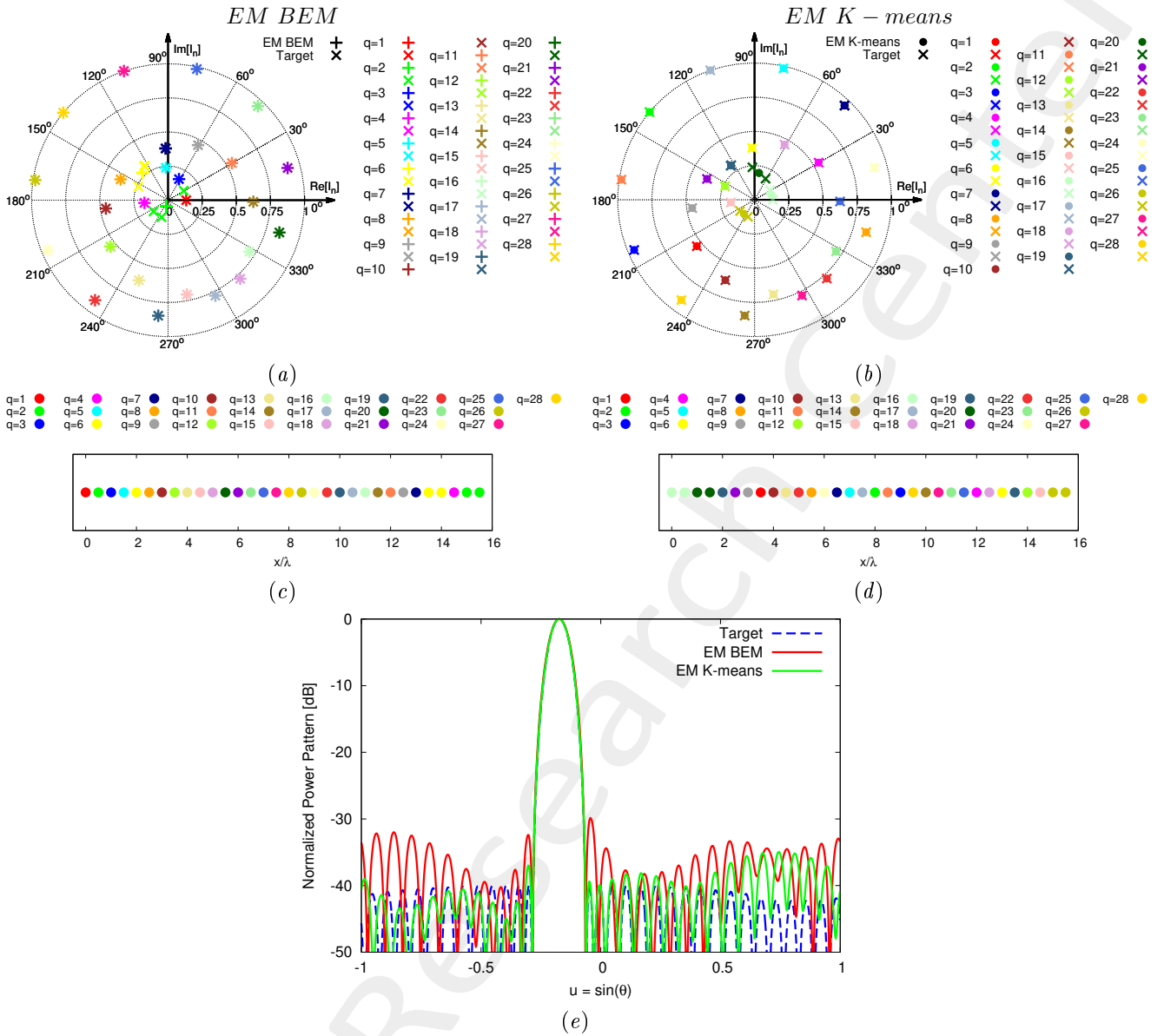


Figure 25: *Excitations Matching (EM) BEM vs. K-means vs. target* - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

Pattern Features Resume

	<i>BEM</i>				
	<i>SLL</i> [dB]	<i>HPBW</i> [deg]	D_{\max} [dB]	Δ	Ψ
$Q = 12$	-12.39	5.42	15.30	1.92×10^{-1}	8.13×10^{-2}
$Q = 16$	-17.56	4.92	16.18	9.77×10^{-2}	4.14×10^{-2}
$Q = 20$	-20.65	4.97	16.41	4.04×10^{-2}	1.71×10^{-2}
$Q = 24$	-21.31	4.75	16.63	2.80×10^{-2}	1.19×10^{-2}
$Q = 28$	-29.87	4.68	16.78	4.67×10^{-3}	1.98×10^{-3}
$Q = 32$	-39.98	4.50	16.95	0.00	0.00
	<i>K - means</i>				
	<i>SLL</i> [dB]	<i>HPBW</i> [deg]	D_{\max} [dB]	Δ	Ψ
$Q = 12$	-16.36	4.66	16.52	6.70×10^{-2}	2.84×10^{-2}
$Q = 16$	-20.10	4.52	16.80	3.27×10^{-2}	1.39×10^{-2}
$Q = 20$	-24.95	4.56	16.83	1.48×10^{-2}	6.28×10^{-3}
$Q = 24$	-29.72	4.54	16.88	5.06×10^{-3}	2.14×10^{-3}
$Q = 28$	-34.96	4.52	16.92	9.80×10^{-4}	4.15×10^{-4}
$Q = 32$	-39.97	4.49	16.95	0.00	0.00

Table V: *Excitations Matching (EM) BEM vs. K-means solutions* - Sidelobe level, *SLL*, half-power beamwidth, *HPBW*, directivity peak, D_{\max} , pattern matching error, Δ , and fitness, Ψ , values.

1.1.6 Comparative Resume

Analysis vs. Q , $\theta_0 = -5$ [deg]

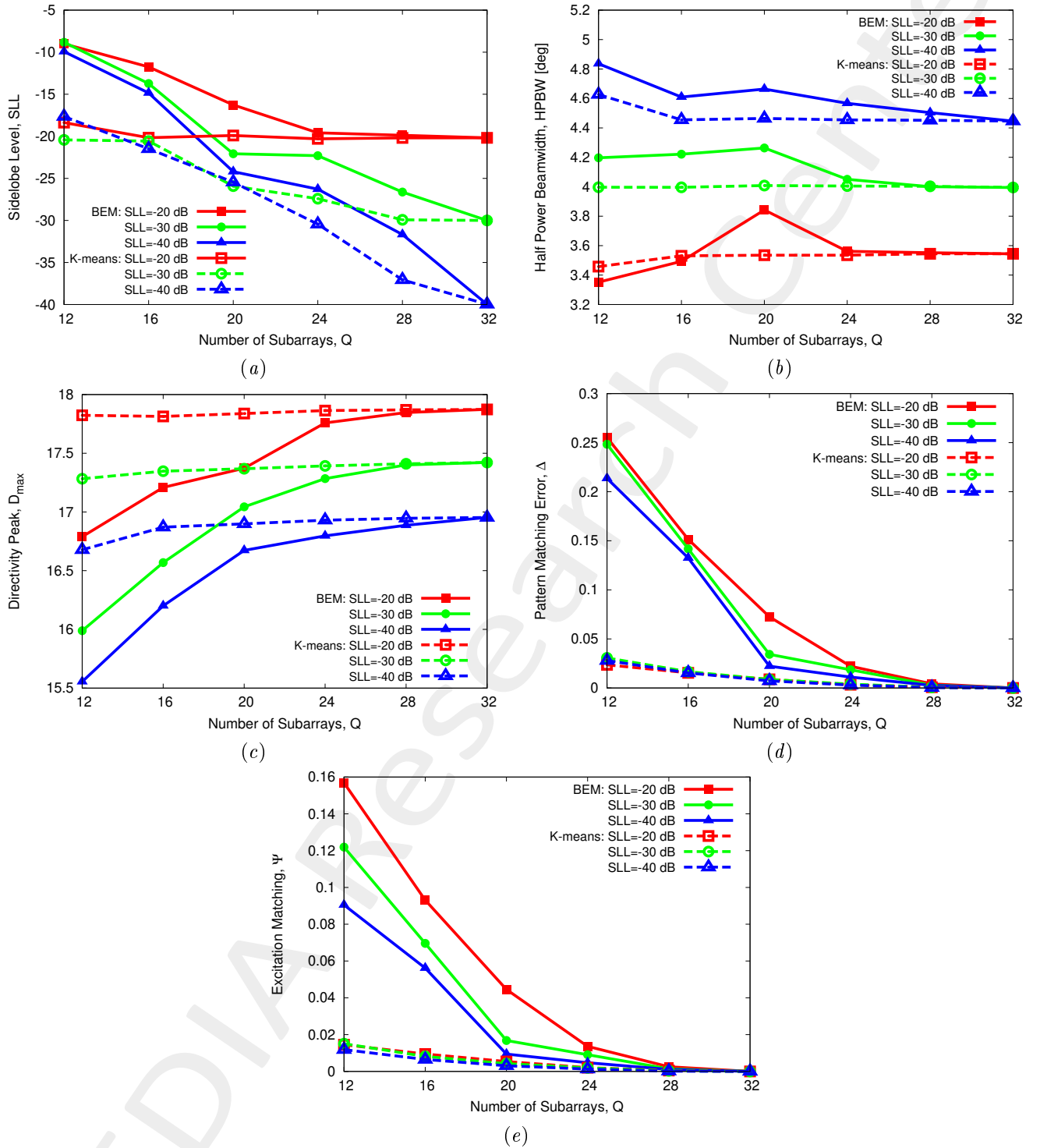


Figure 26: *Excitations Matching (EM) BEM vs. K-means* - Sidelobe level, SLL , half-power beamwidth, $HPBW$, directivity peak, D_{max} , pattern matching error, Δ , and fitness, Ψ , values as a function of the number of subarrays, Q .

Analysis vs. Q , $\theta_0 = -10$ [deg]

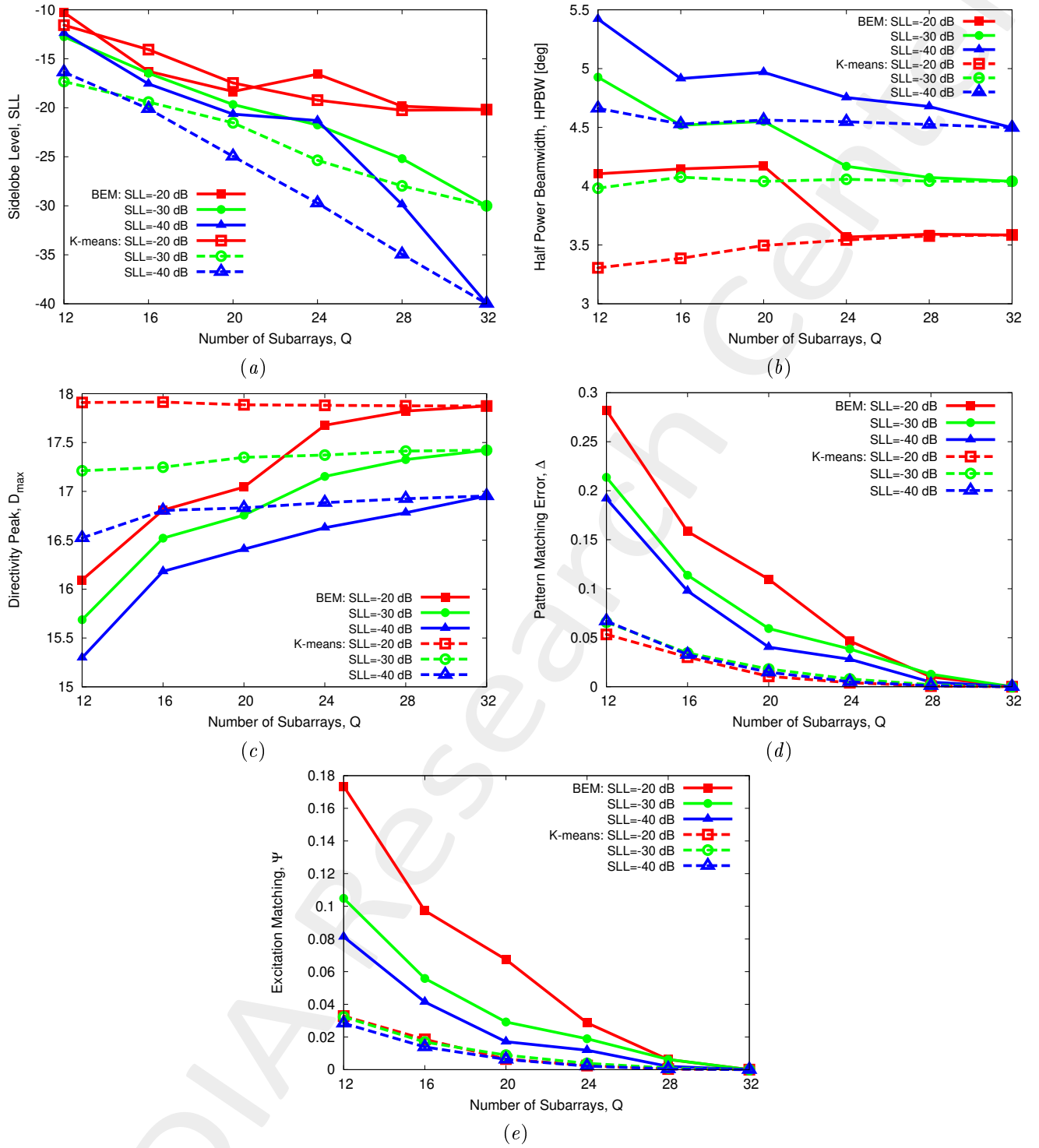


Figure 27: *Excitations Matching (EM) BEM vs. K-means* - Sidelobe level, SLL , half-power beamwidth, $HPBW$, directivity peak, D_{max} , pattern matching error, Δ , and fitness, Ψ , values as a function of the number of subarrays, Q .

Analysis vs. Q , $\theta_0 = -15$ [deg]

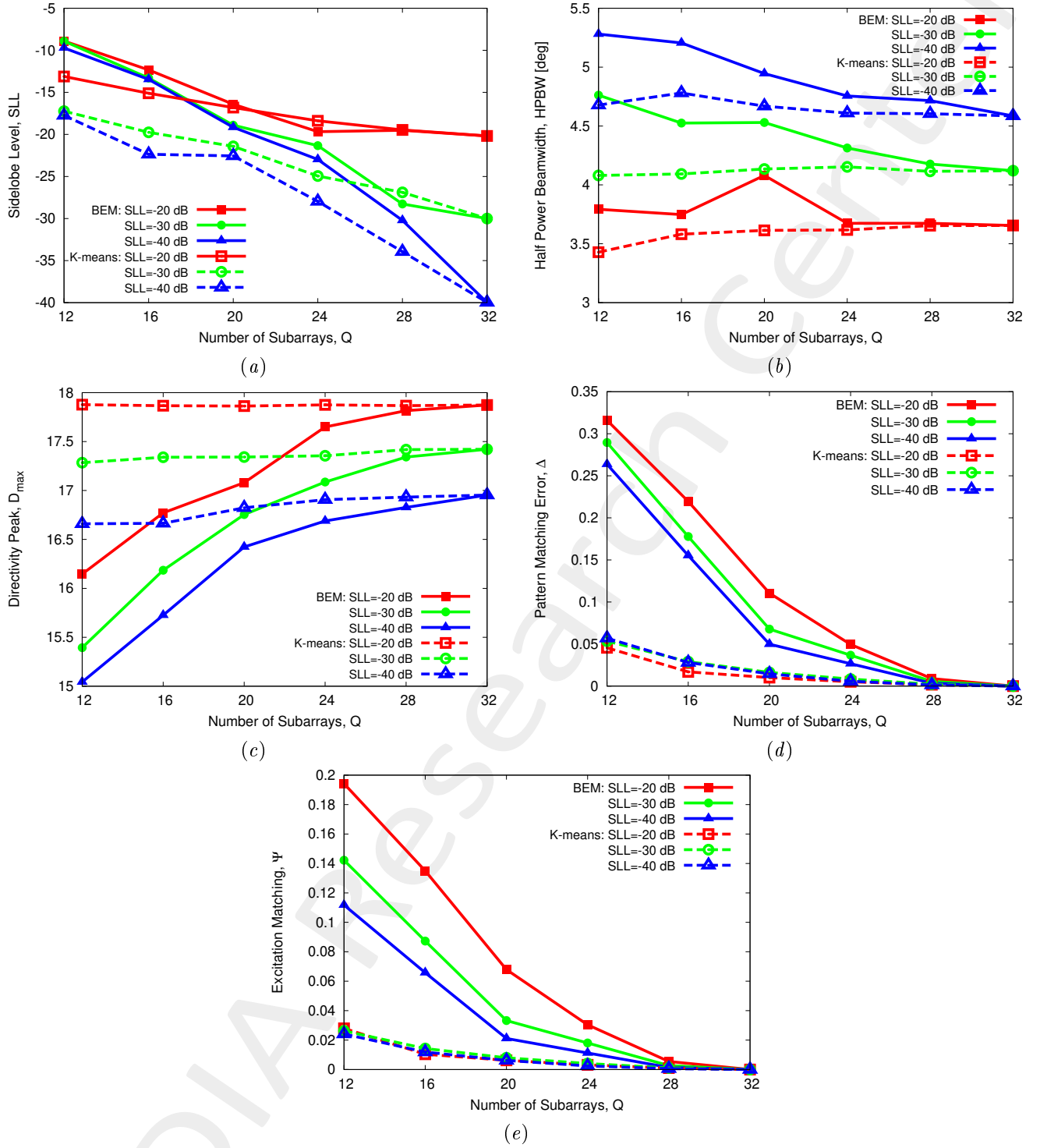


Figure 28: *Excitations Matching (EM) BEM vs. K-means* - Sidelobe level, SLL , half-power beamwidth, $HPBW$, directivity peak, D_{max} , pattern matching error, Δ , and fitness, Ψ , values as a function of the number of subarrays, Q .

Analysis vs. $SLL_{ref}, \theta_0 = -5$ [deg]

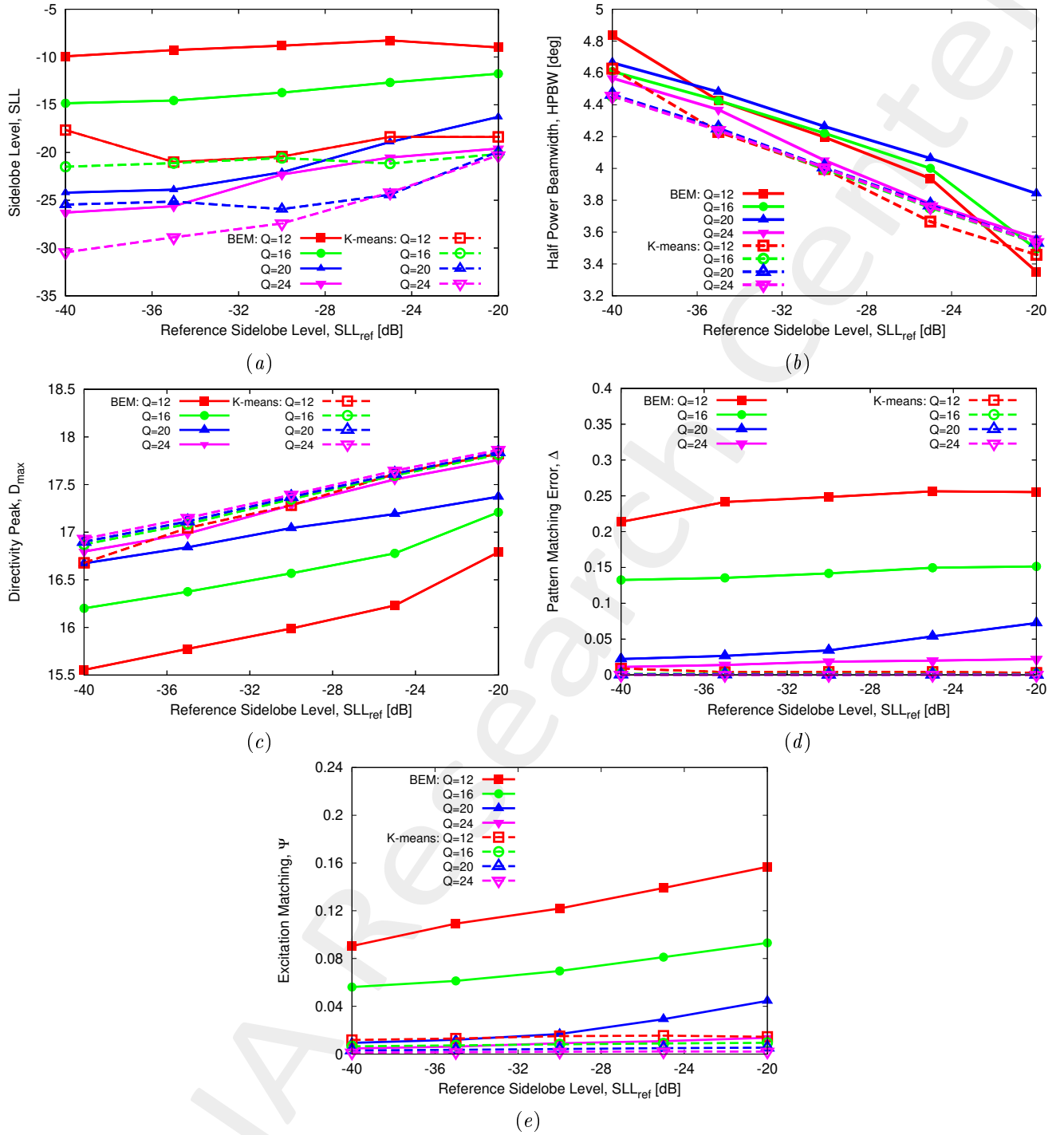


Figure 29: Excitation Matching (EM) BEM vs. K-means - Sidelobe level, SLL , half-power beamwidth, $HPBW$, directivity peak, D_{max} , pattern matching error, Δ , and fitness, Ψ , values as a function of the target sidelobe level, SLL_{ref} .

Analysis vs. $SLL_{ref}, \theta_0 = -10$ [deg]

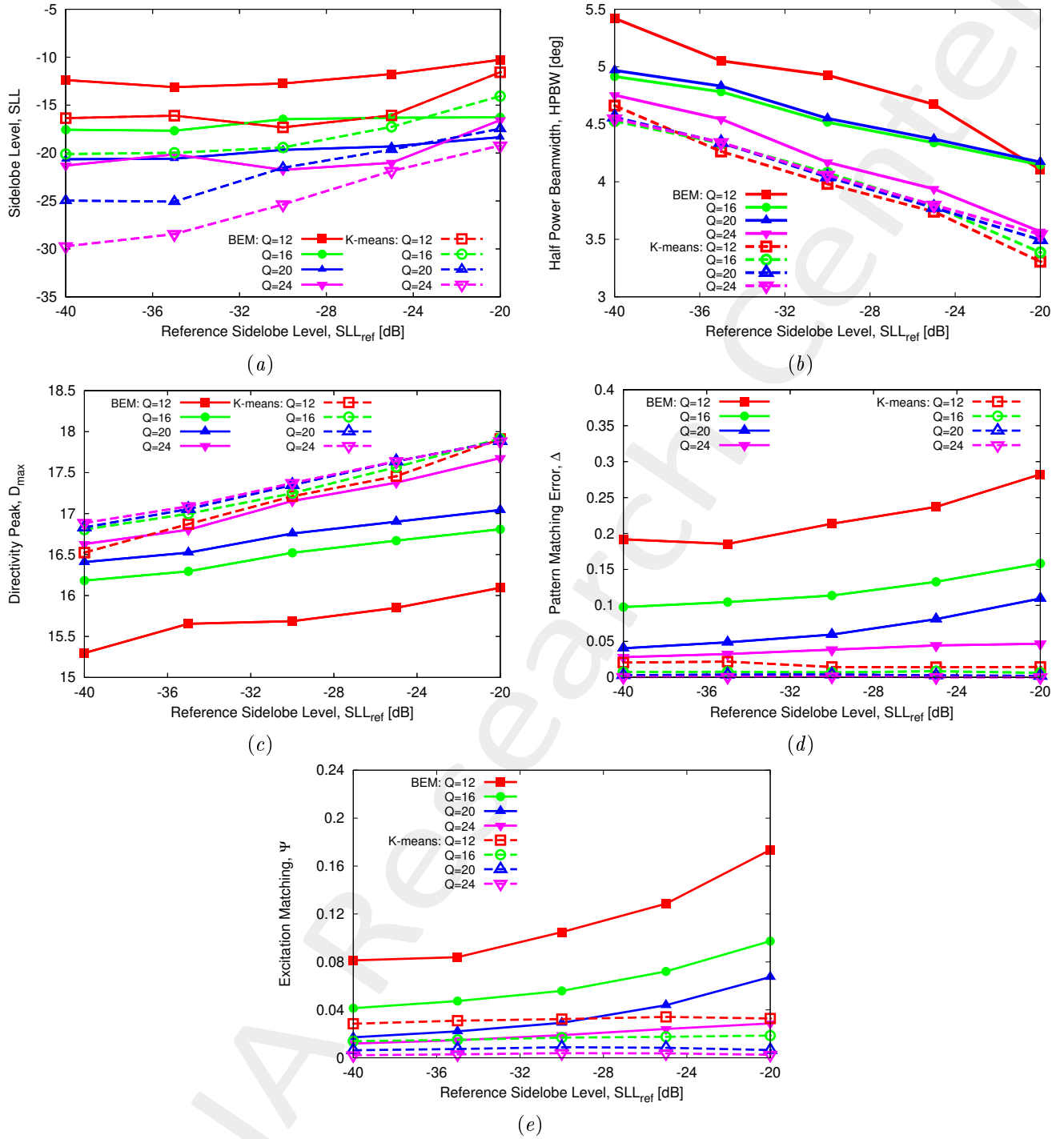


Figure 30: Excitation Matching (EM) BEM vs. K-means - Sidelobe level, SLL , half-power beamwidth, $HPBW$, directivity peak, D_{max} , pattern matching error, Δ , and fitness, Ψ , values as a function of the target sidelobe level, SLL_{ref} .

Analysis vs. $SLL_{ref}, \theta_0 = -15$ [deg]

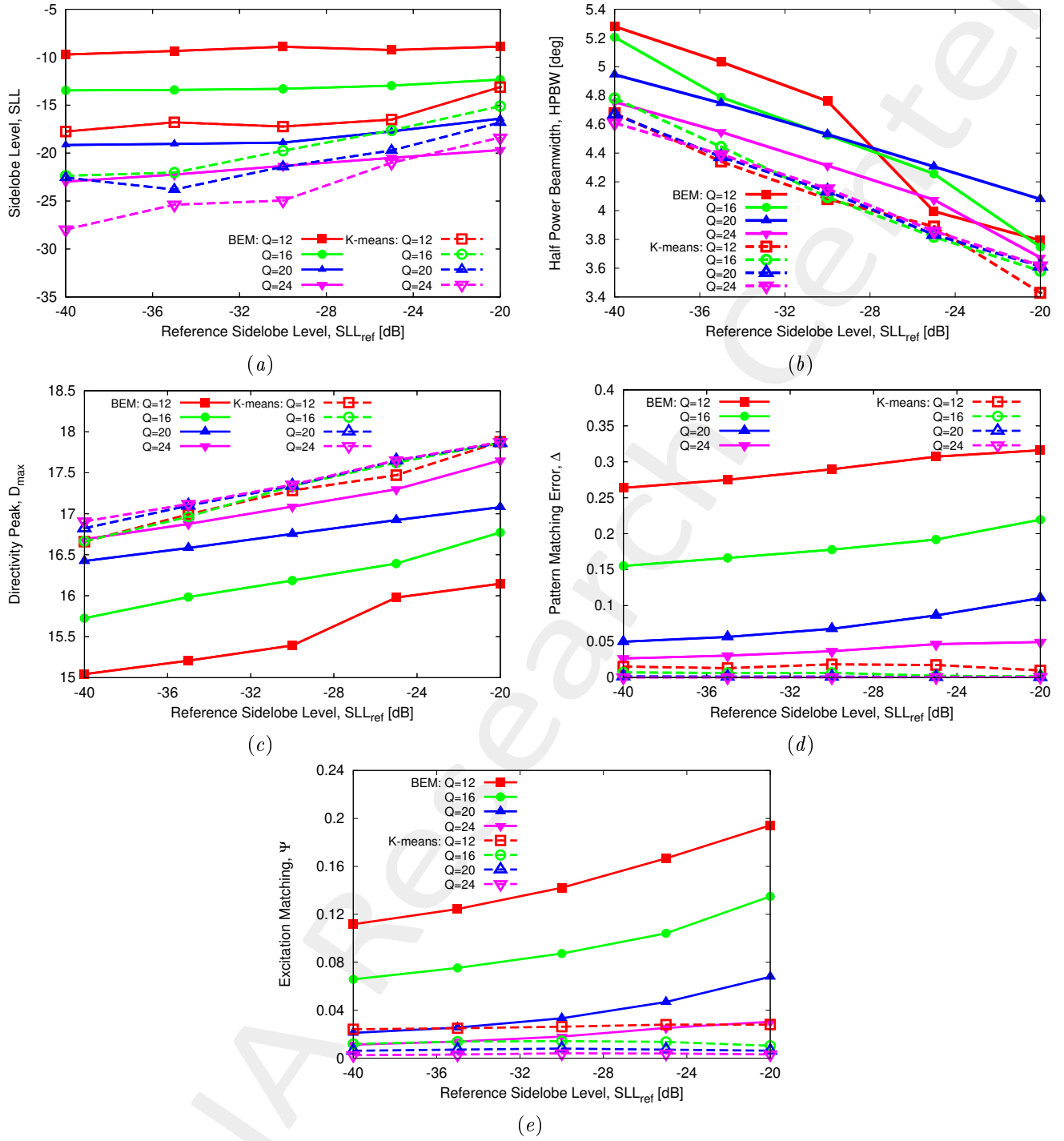


Figure 31: Excitation Matching (EM) BEM vs. K-means - Sidelobe level, SLL , half-power beamwidth, $HPBW$, directivity peak, D_{max} , pattern matching error, Δ , and fitness, Ψ , values as a function of the target sidelobe level, SLL_{ref} .

OUTCOMES:

- Generally, the *EM K – means* outperforms the *EM BEM* method in terms of sidelobe level, half-power beamwidth, directivity peak, pattern matching error and excitation matching error whatever the sidelobe level of the reference pattern SLL_{ref} and whatever the number of subarrays Q .

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1.2 Taylor Patterns - Analysis varying Pointing Angle θ_0

OBJECTIVE: Compare the performance of the *K-means*- and *BEM*-based excitation matching methods when changing the beam pointing angle of the patterns and thus the phases of the reference excitations [the angle of the points in the complex plane]. The size of the problem is kept fixed (number of elements: $N = 32$) as well as the tapering on the reference excitations (and accordingly the sidelobe level of the reference pattern).

Array Parameters

- Number of elements: $N = 32$
- Number of subarrays: $Q = 16$
- Inter-element spacing: $d = \lambda/2$
- Taylor excitation amplitudes, $SLL_{ref} = -30$ [dB]
- Pointing angle: $\theta_0 = -2, -4, -6, -8, -10, -15, -20$ [deg]

K-means Clustering Method Parameters

- Number of iterations: $I = 50$
- Number of executions: $R = 100$

1.2.1 Taylor Pattern, $N = 32$, $Q = 16$, $SLL_{ref} = -30$ [dB]: $\theta_0 = -2, -4, -6, -8, -10, -15, -20$

$\theta_0 = -2$ [deg] - Excitation Matching (EM) BEM vs. K-means vs. Target

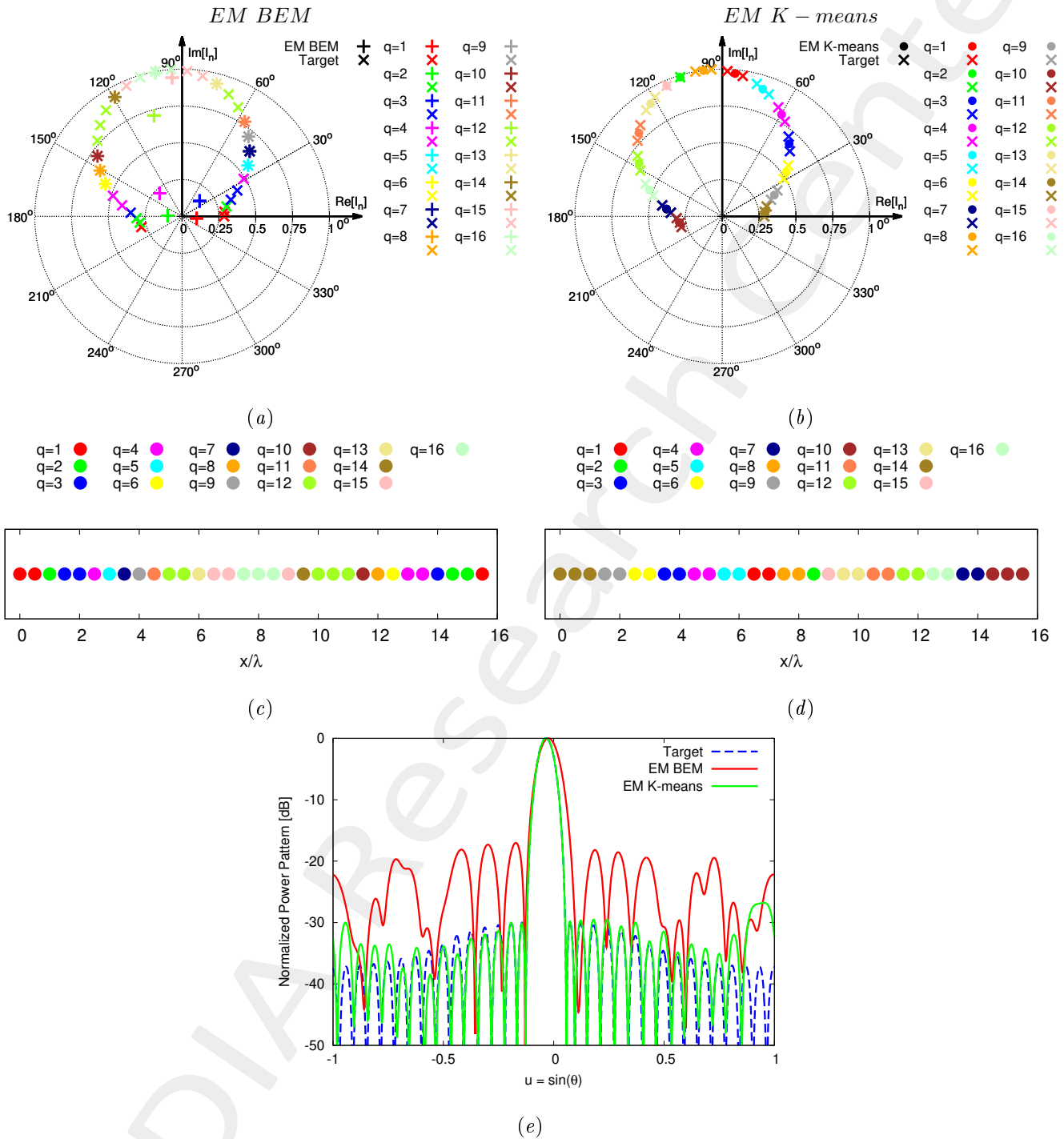


Figure 32: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$\theta_0 = -4$ [deg] - Excitation Matching (EM) BEM vs. K-means vs. Target

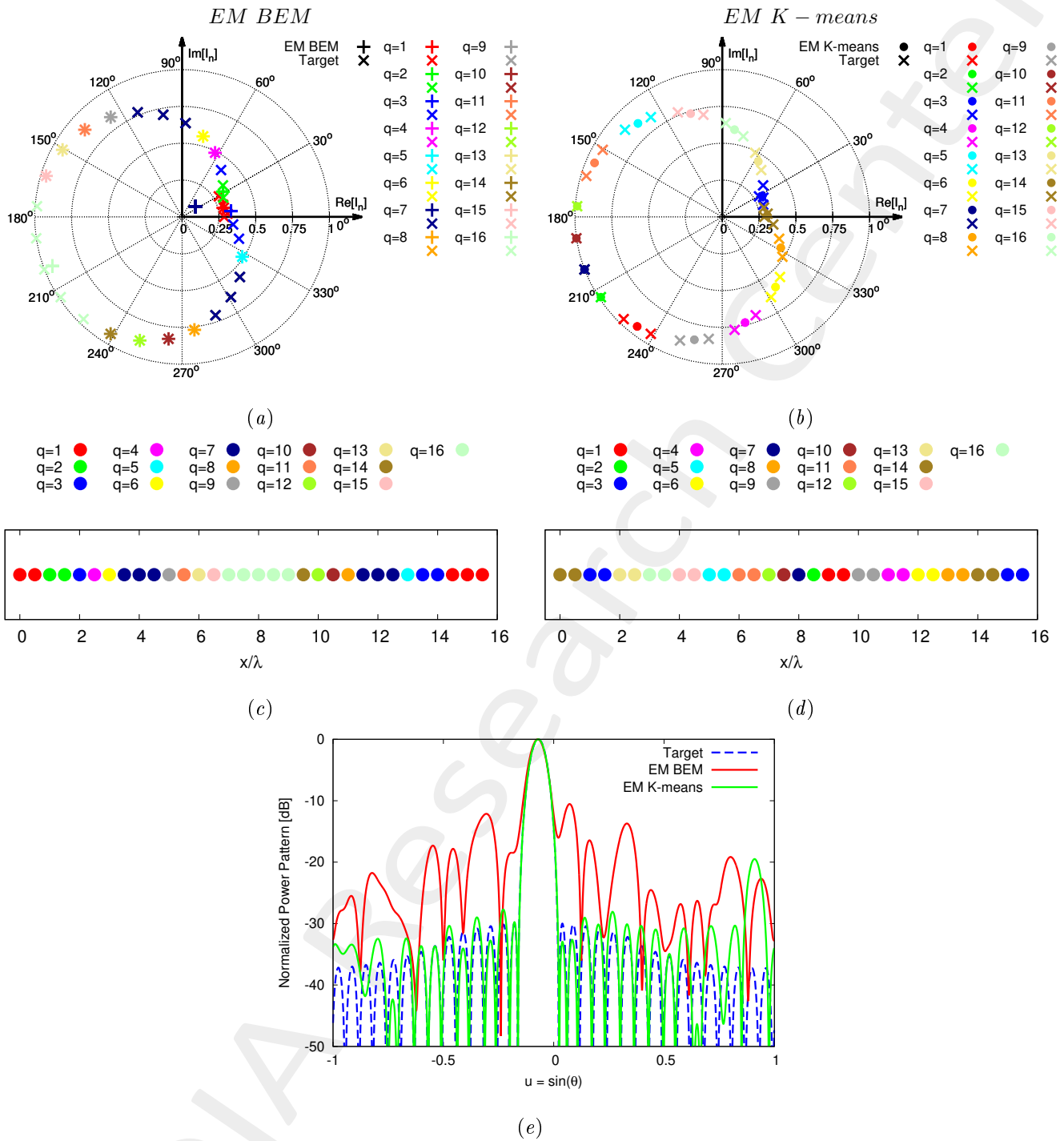


Figure 33: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$\theta_0 = -6$ [deg] - Excitation Matching (EM) BEM vs. K-means vs. Target

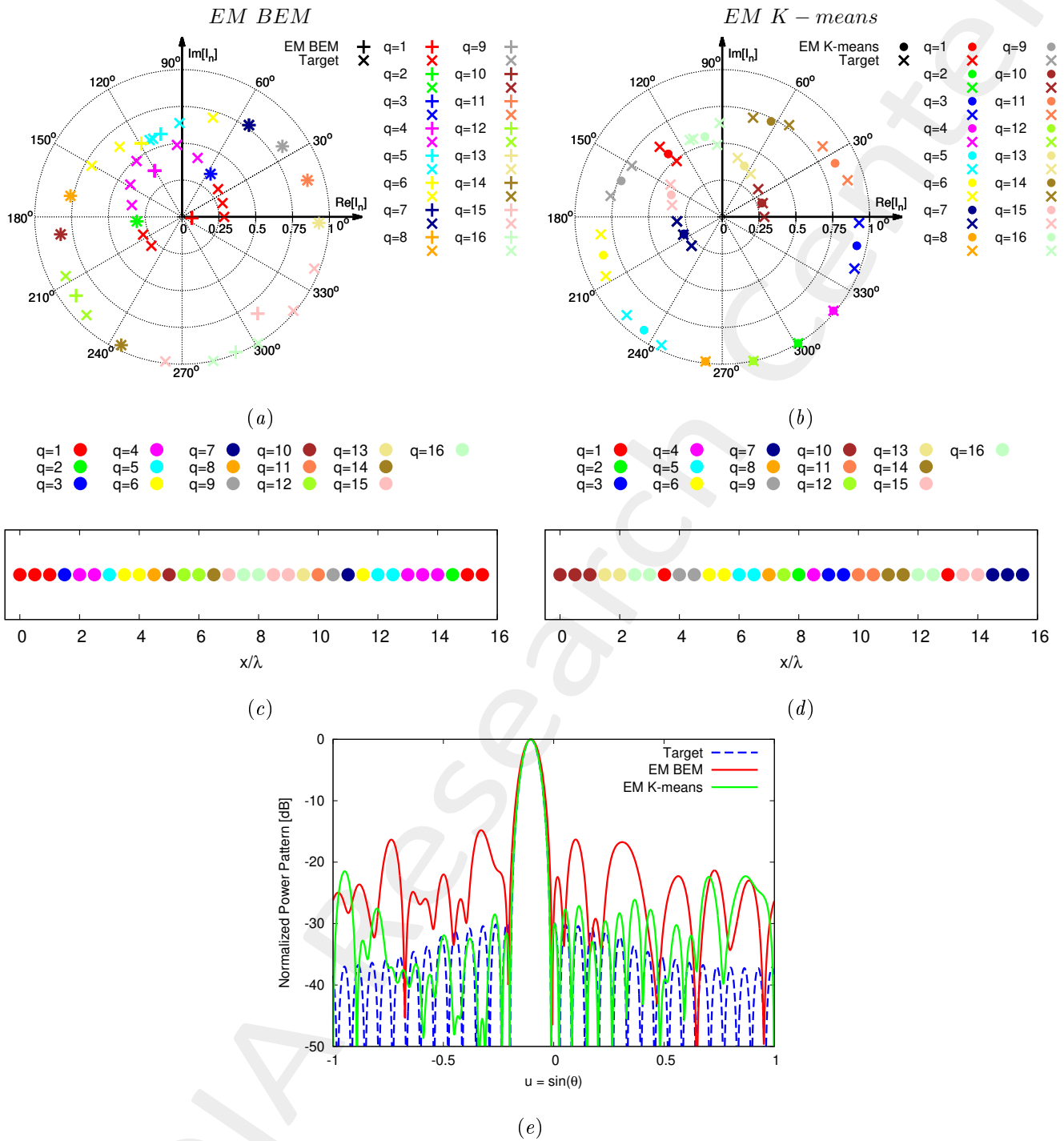


Figure 34: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$\theta_0 = -8$ [deg] - Excitation Matching (EM) BEM vs. K-means vs. Target

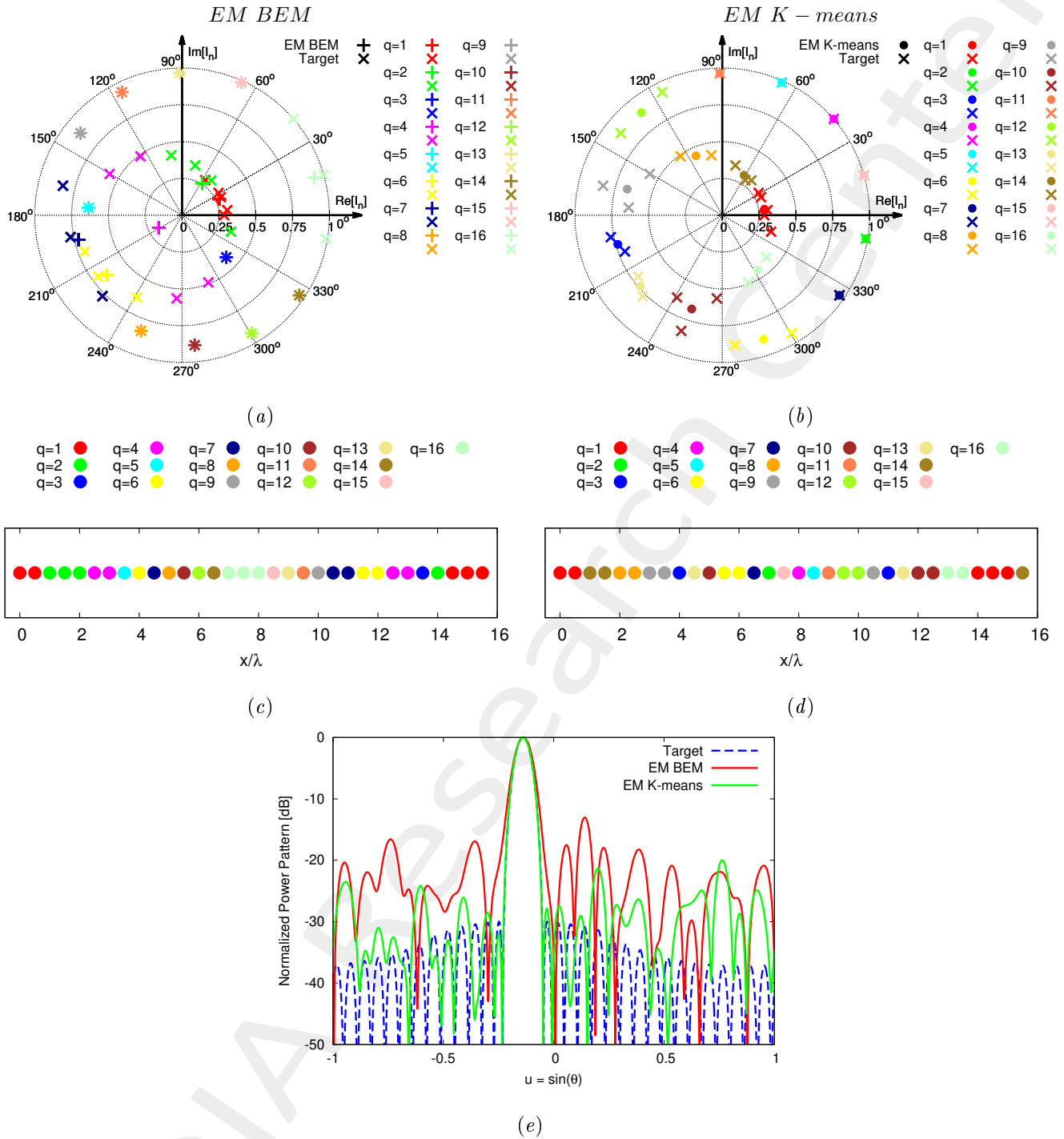


Figure 35: *Excitations Matching (EM) BEM vs. K-means vs. target* - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$\theta_0 = -15$ [deg] - Excitation Matching (EM) BEM vs. K-means vs. Target

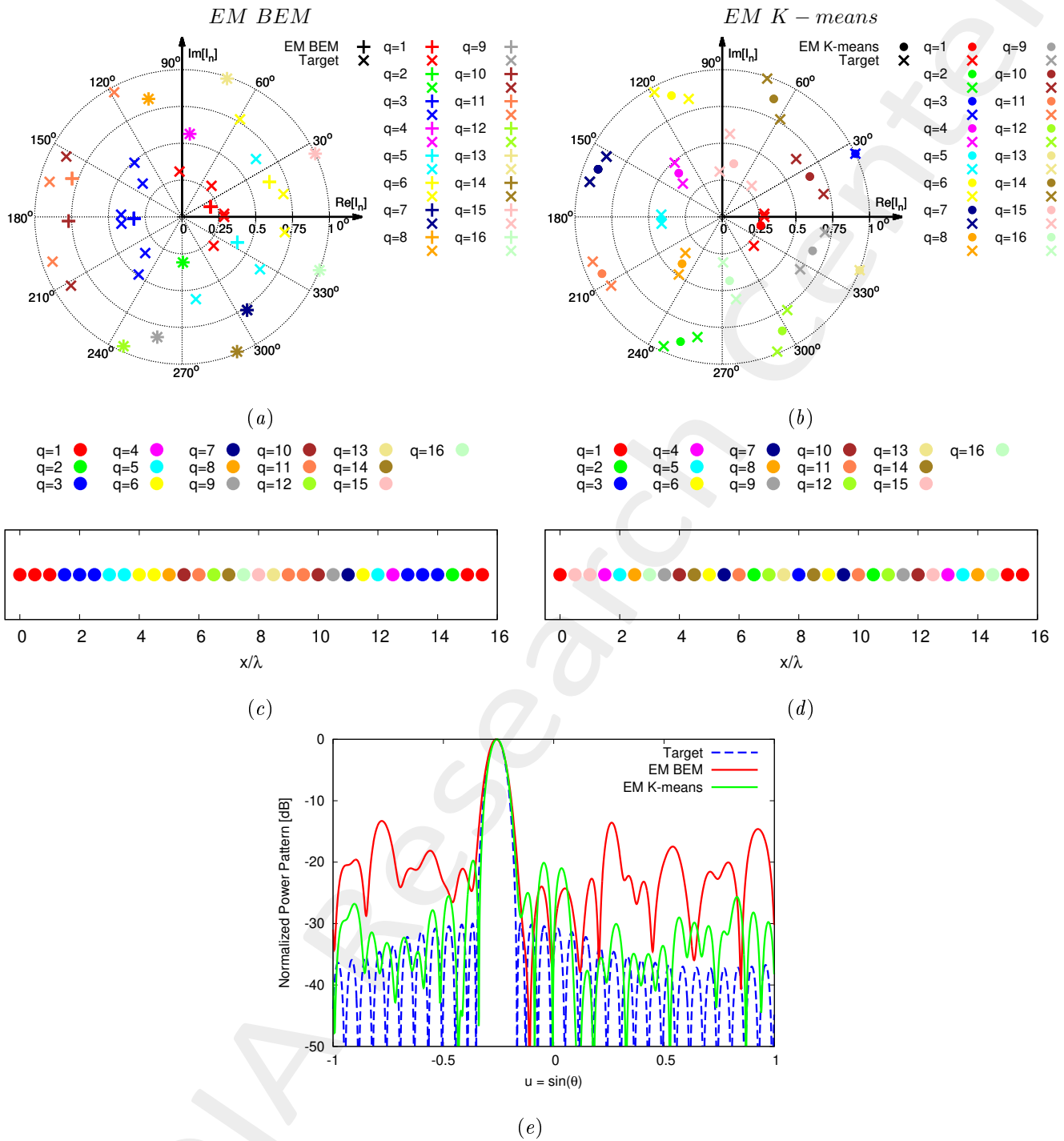


Figure 36: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$\theta_0 = -20$ [deg] - Excitation Matching (EM) BEM vs. K-means vs. Target

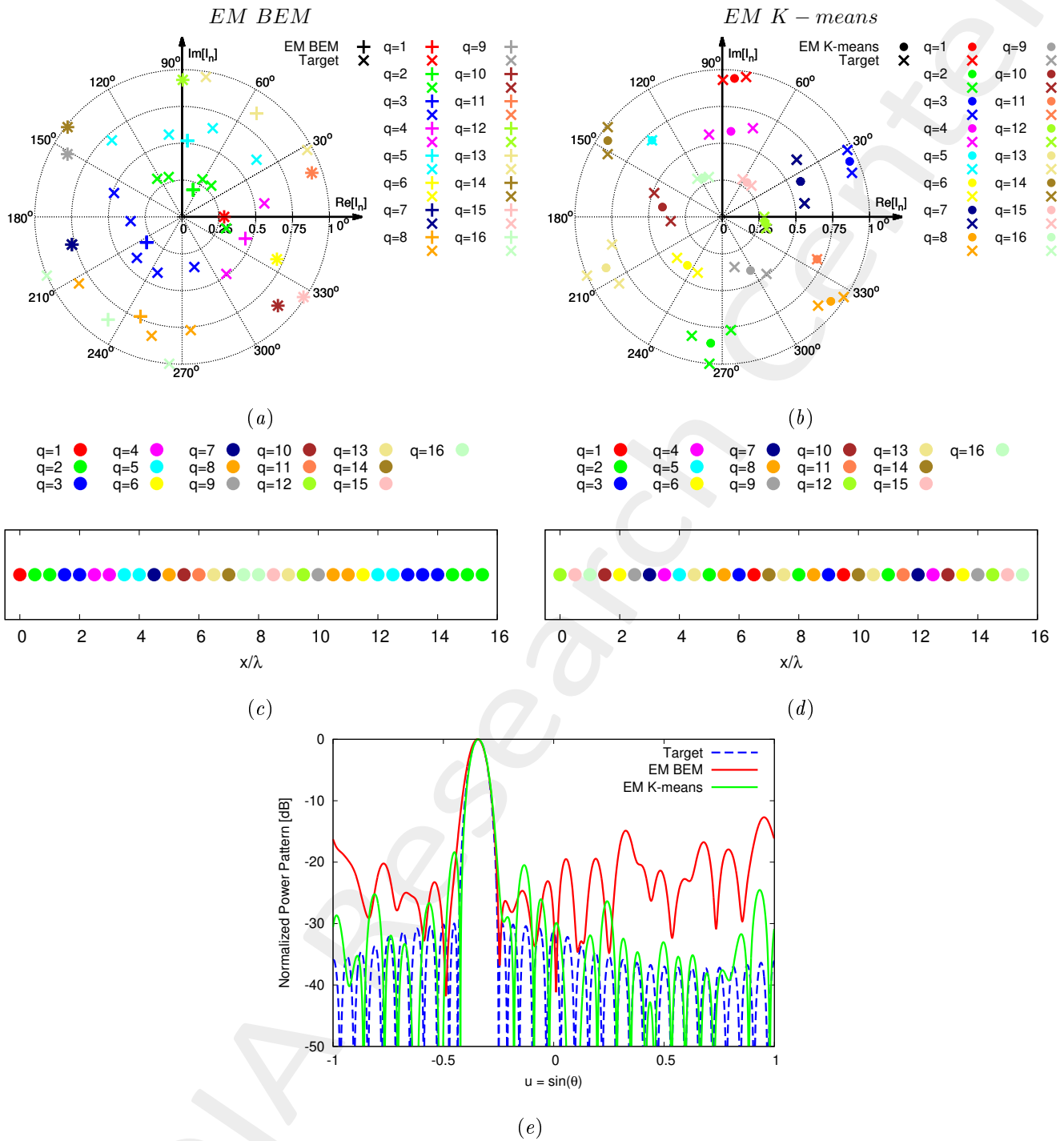


Figure 37: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

Pattern Features Resume

	<i>BEM</i>				
	<i>SLL</i> [DB]	<i>HPBW</i> [DEG]	D_{\max} [DB]	Δ	Ψ
$\theta_0 = 0$ [DEG]	-30.00	3.98	17.42	0.00	0.00
$\theta_0 = -2$ [DEG]	-17.03	5.09	15.99	1.60×10^{-1}	7.87×10^{-2}
$\theta_0 = -4$ [DEG]	-10.54	4.13	16.20	2.06×10^{-1}	1.01×10^{-1}
$\theta_0 = -6$ [DEG]	-14.81	4.55	16.38	1.30×10^{-1}	6.37×10^{-2}
$\theta_0 = -8$ [DEG]	-13.04	4.42	16.37	1.33×10^{-1}	6.52×10^{-2}
$\theta_0 = -10$ [DEG]	-16.48	4.52	16.52	1.14×10^{-1}	5.58×10^{-2}
$\theta_0 = -15$ [DEG]	-13.30	4.53	16.19	1.78×10^{-1}	8.73×10^{-2}
$\theta_0 = -20$ [DEG]	-12.70	4.69	16.26	1.65×10^{-1}	8.09×10^{-2}
	<i>K - means</i>				
	<i>SLL</i> [DB]	<i>HPBW</i> [DEG]	D_{\max} [DB]	Δ	Ψ
$\theta_0 = 0$ [DEG]	-30.20	3.98	17.42	0.00	0.00
$\theta_0 = -2$ [DEG]	-28.29	3.96	17.40	4.14×10^{-3}	2.03×10^{-3}
$\theta_0 = -4$ [DEG]	-25.01	3.97	17.38	1.08×10^{-2}	5.28×10^{-3}
$\theta_0 = -6$ [DEG]	-23.52	3.98	17.32	2.26×10^{-2}	1.11×10^{-2}
$\theta_0 = -8$ [DEG]	-22.98	3.96	17.33	2.77×10^{-2}	1.36×10^{-2}
$\theta_0 = -10$ [DEG]	-21.42	3.95	17.31	3.44×10^{-2}	1.69×10^{-2}
$\theta_0 = -15$ [DEG]	-22.58	4.01	17.38	2.91×10^{-2}	1.43×10^{-2}
$\theta_0 = -20$ [DEG]	-20.53	4.07	17.43	2.24×10^{-2}	1.10×10^{-2}

Table VI: *Excitations Matching (EM) BEM vs. K-means solutions* - Sidelobe level, *SLL*, half-power beamwidth, *HPBW*, directivity peak, D_{\max} , pattern matching error, Δ , and fitness, Ψ , values.

1.2.2 Comparative Resume

Analysis vs. Q

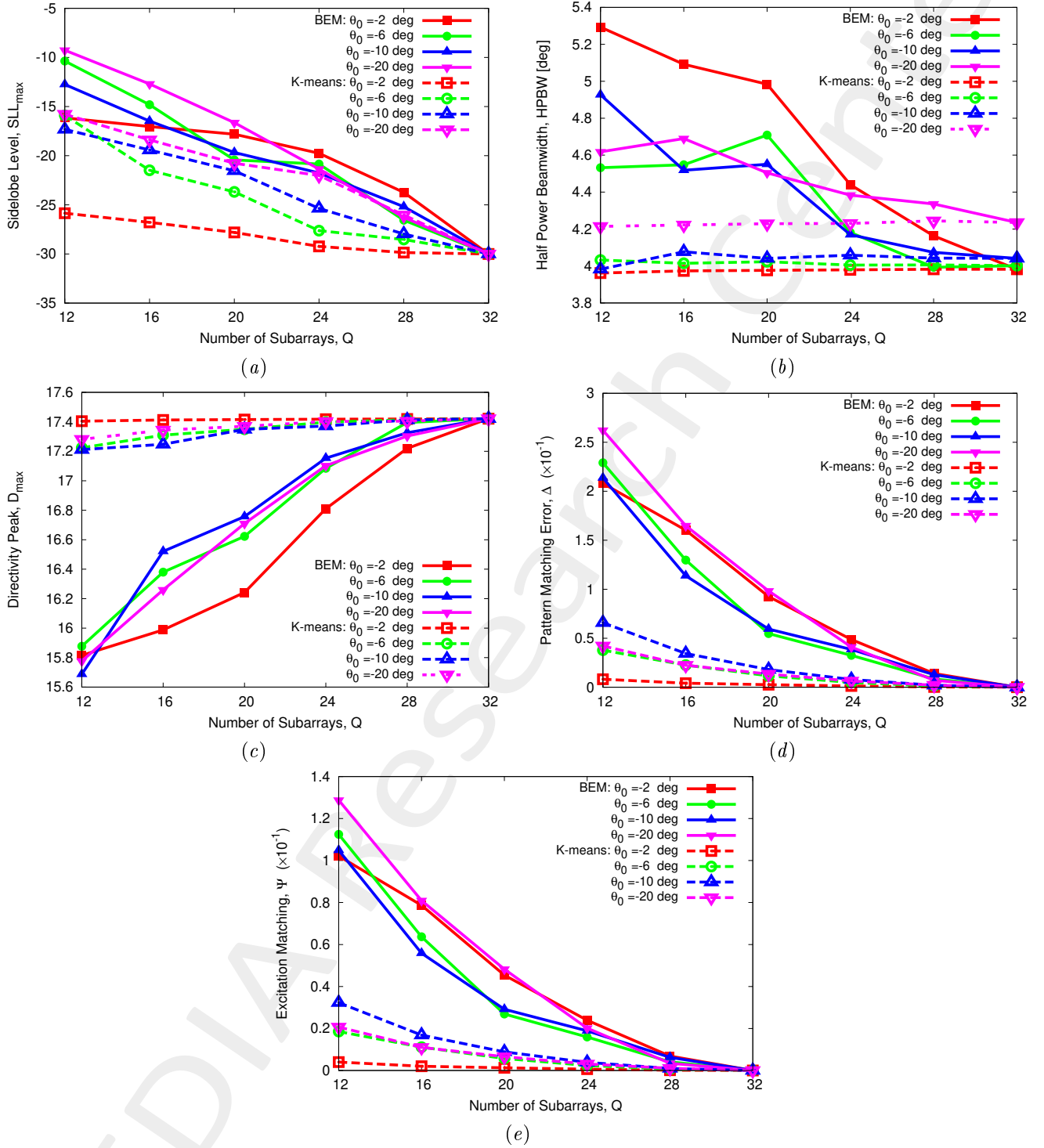


Figure 38: *Excitation Matching (EM) BEM vs. K-means* - Sidelobe level, SLL , half-power beamwidth, $HPBW$, directivity peak, D_{max} , pattern matching error, Δ , and fitness, Ψ , values as a function of the number of subarrays, Q .

OUTCOMES:

- Generally, the *EM K – means* outperforms the *EM BEM* method in terms of sidelobe level, half-power beamwidth, directivity peak, pattern matching error and excitation matching error whatever the pointing angle θ_0 .

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1.3 Cosecant-Squared Patterns - Analysis varying Q

OBJECTIVE: Compare the performance of the $K - means$ - and BEM -based excitation matching methods considering a kind of pattern alternative to the pencil beam, the cosecant-squared pattern. An analysis varying the number of subarrays will be carried out, keeping a fixed beam pointing angle as well as the array size.

Array Parameters

- Array Parameters
- Number of elements: $N = 17$
- Number of subarrays: $Q = 6, 8, 9, 11, 15$
- Inter-element spacing: $d = \lambda/2$
- Cosecant-square pattern, $SLL_{ref} \simeq -30$ [dB]
- Pointing angle: $\theta_0 = 0$ [deg]

K-means Clustering Method Parameters

- Number of iterations: $I = 50$
- Number of executions: $R = 100$

1.3.1 Cosecant-Squared Pattern, $N = 32$, $SLL_{ref} \simeq -30$ [dB], $\theta_0 = 0$ [deg]: $Q = 6, 8, 9, 11, 15$

$Q = 6$ - Excitation Matching (EM) BEM vs. K-means vs. Target

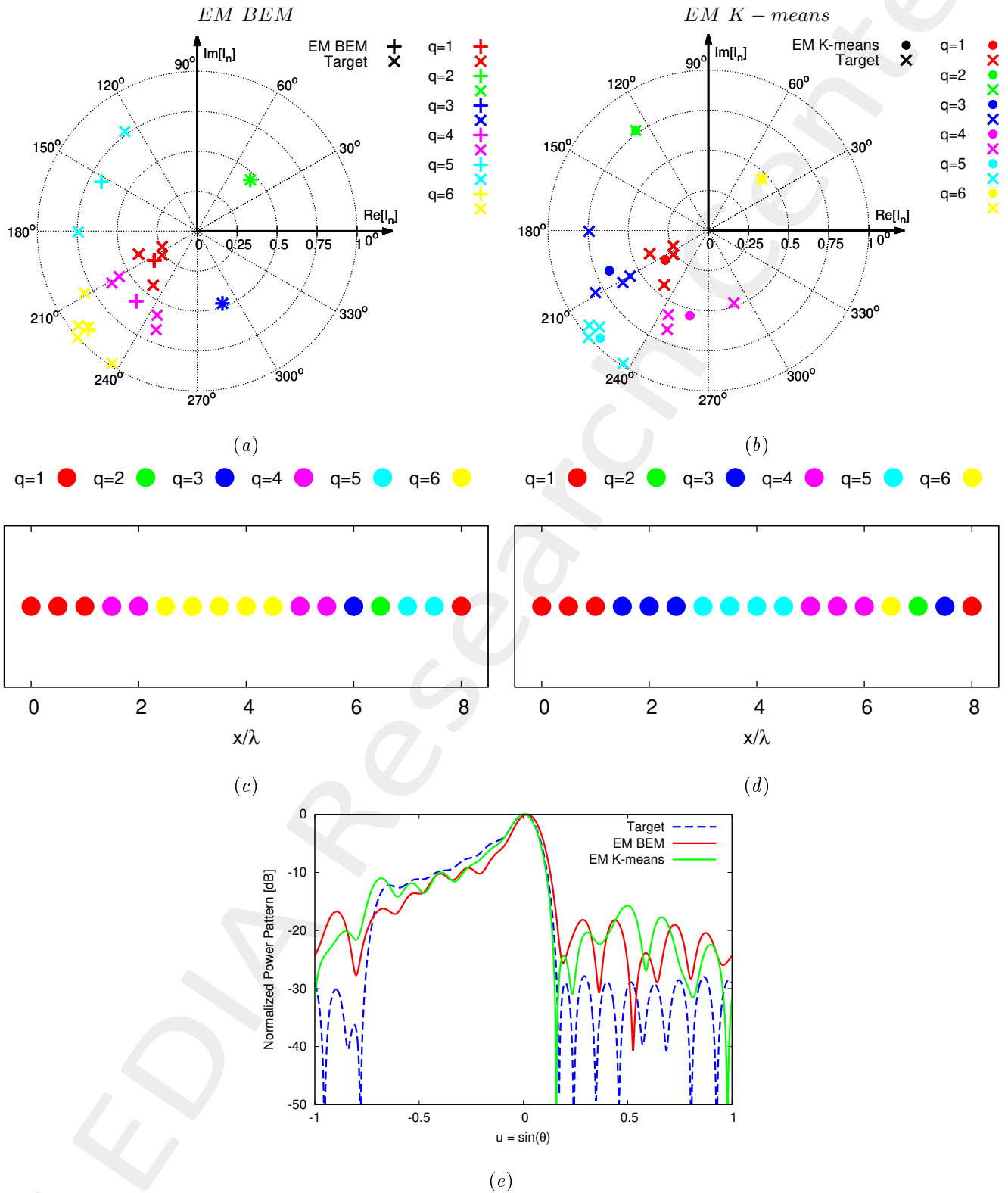


Figure 39: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 8$ - Excitation Matching (EM) BEM vs. K-means vs. Target

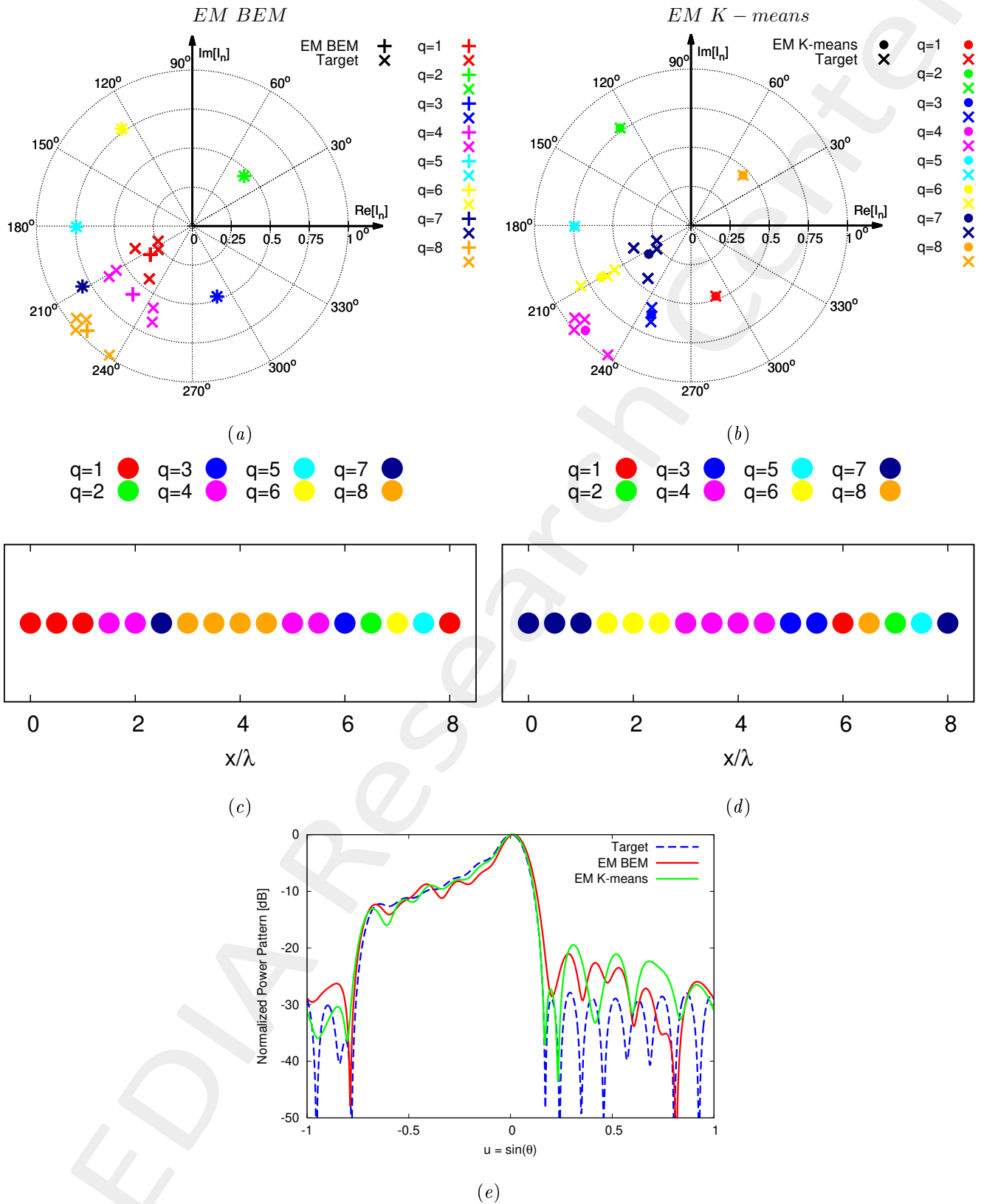


Figure 40: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 9$ - Excitation Matching (EM) BEM vs. K-means vs. Target

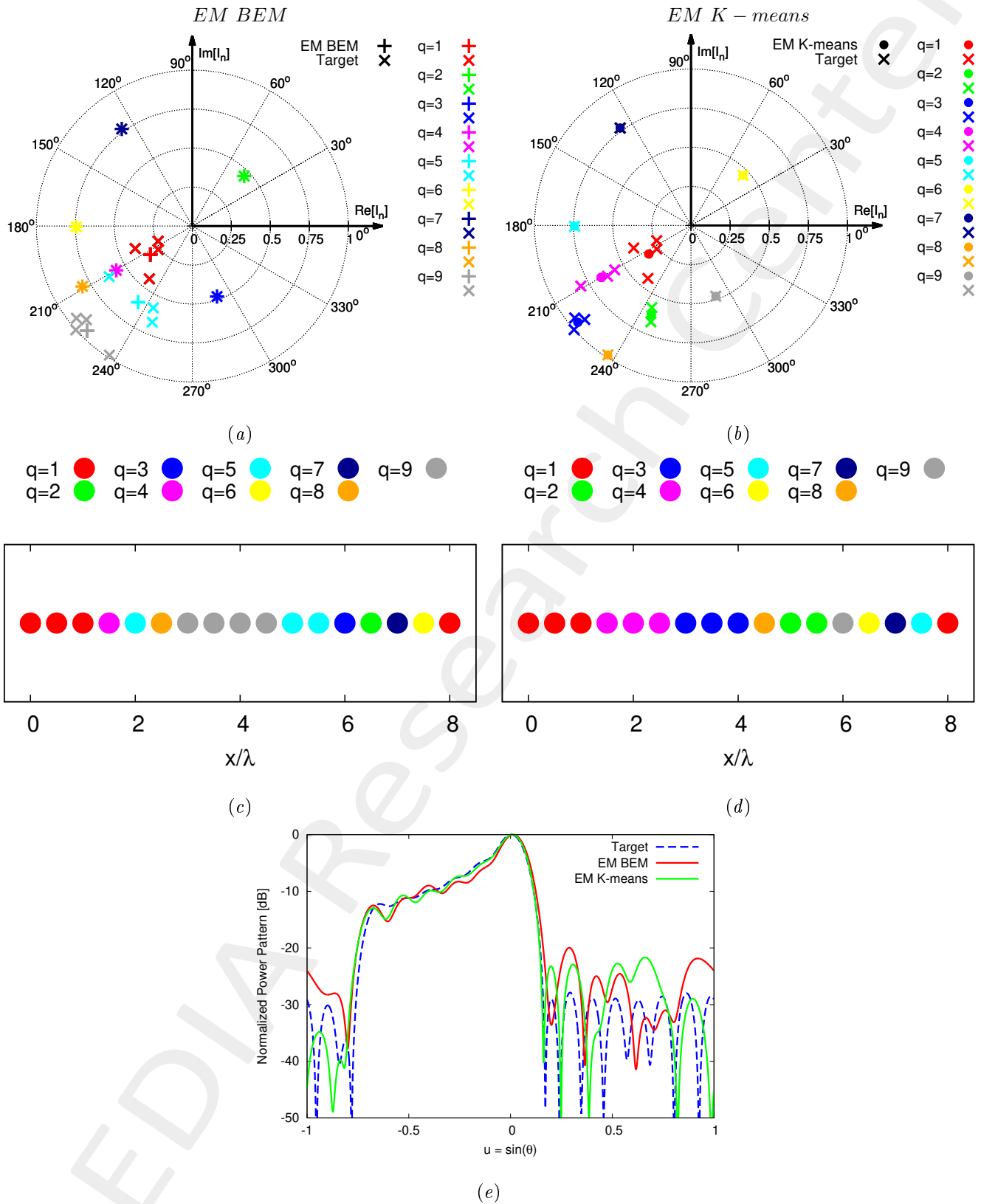


Figure 41: *Excitations Matching (EM) BEM vs. K-means vs. target* - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 11$ - Excitation Matching (EM) BEM vs. K-means vs. Target

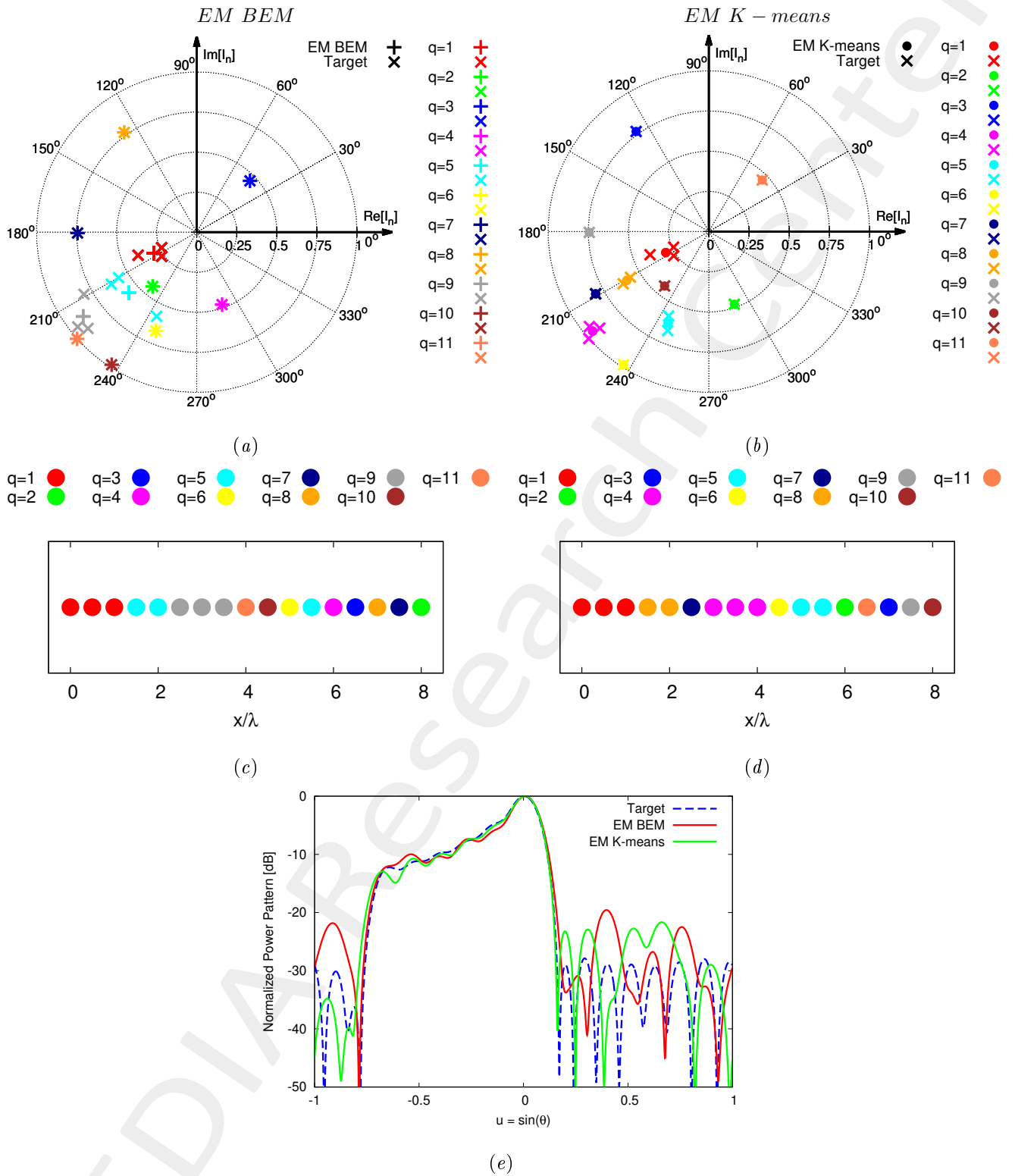


Figure 42: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 15$ - Excitation Matching (EM) BEM vs. K-means vs. Target

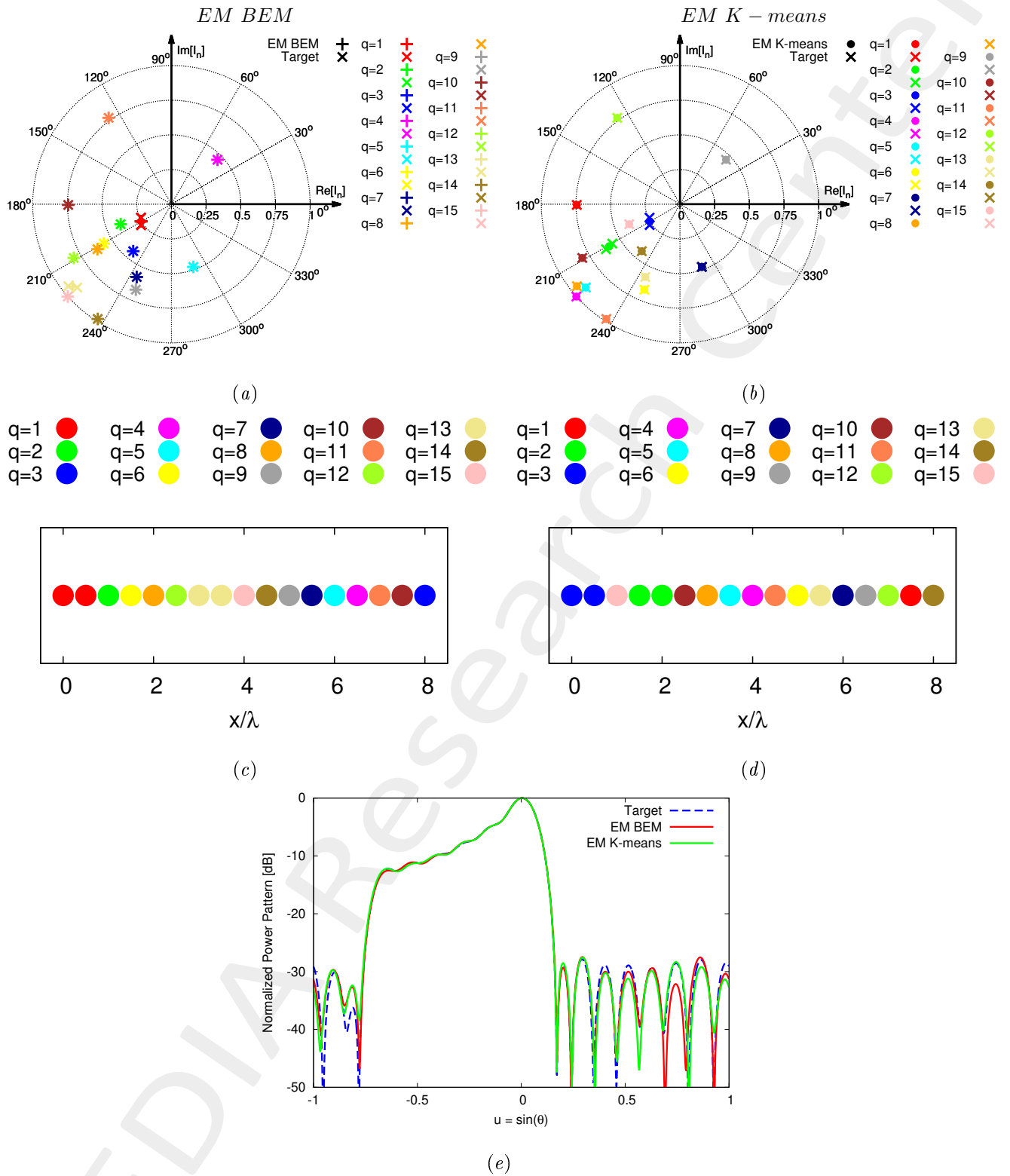


Figure 43: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

Pattern Features Resume

	<i>BEM</i>		<i>K – means</i>	
	Δ	Ψ	Δ	Ψ
$Q = 6$	7.13×10^{-2}	3.31×10^{-2}	4.75×10^{-2}	2.20×10^{-2}
$Q = 8$	3.27×10^{-2}	1.52×10^{-2}	1.91×10^{-2}	8.85×10^{-3}
$Q = 9$	2.68×10^{-2}	1.24×10^{-2}	1.15×10^{-2}	5.32×10^{-3}
$Q = 11$	1.61×10^{-2}	7.45×10^{-3}	3.60×10^{-3}	1.67×10^{-3}
$Q = 15$	4.44×10^{-4}	2.06×10^{-4}	4.14×10^{-4}	1.92×10^{-4}

Table VII: *Excitations Matching (EM) BEM vs. K-means solutions* - Pattern matching error, Δ , and fitness, Ψ , values.

OUTCOMES:

- Generally, the *EM K – means* outperforms the *EM BEM* method in terms of pattern matching error and excitation matching error whatever the number of subarrays Q .

1.4 Flat-Top Patterns - Analysis varying Q

OBJECTIVE: Compare the performance of the $K - means$ - and BEM -based excitation matching methods considering a kind of pattern alternative to the pencil beam, the flat-top pattern. An analysis varying the number of subarrays will be carried out, keeping a fixed beam pointing angle as well as the array size.

Array Parameters

- Number of elements: $N = 32$
- Number of subarrays: $Q = 12, 16, 20, 28$
- Inter-element spacing: $d = \lambda/2$
- Flat-top pattern, $SLL_{ref} = 25$ [dB]
- Pointing angle: $\theta_0 = 0$ [deg]

K-means Clustering Method Parameters

- Number of iterations: $I = 50$
- Number of executions: $R = 100$

1.4.1 Flat-Top Pattern, $N = 32$, $SLL_{ref} = -20$ [dB], $\theta_0 = 0$ [deg]: $Q = 12, 16, 20, 28$

$Q = 12$ - Excitation Matching (EM) BEM vs. K-means vs. Target

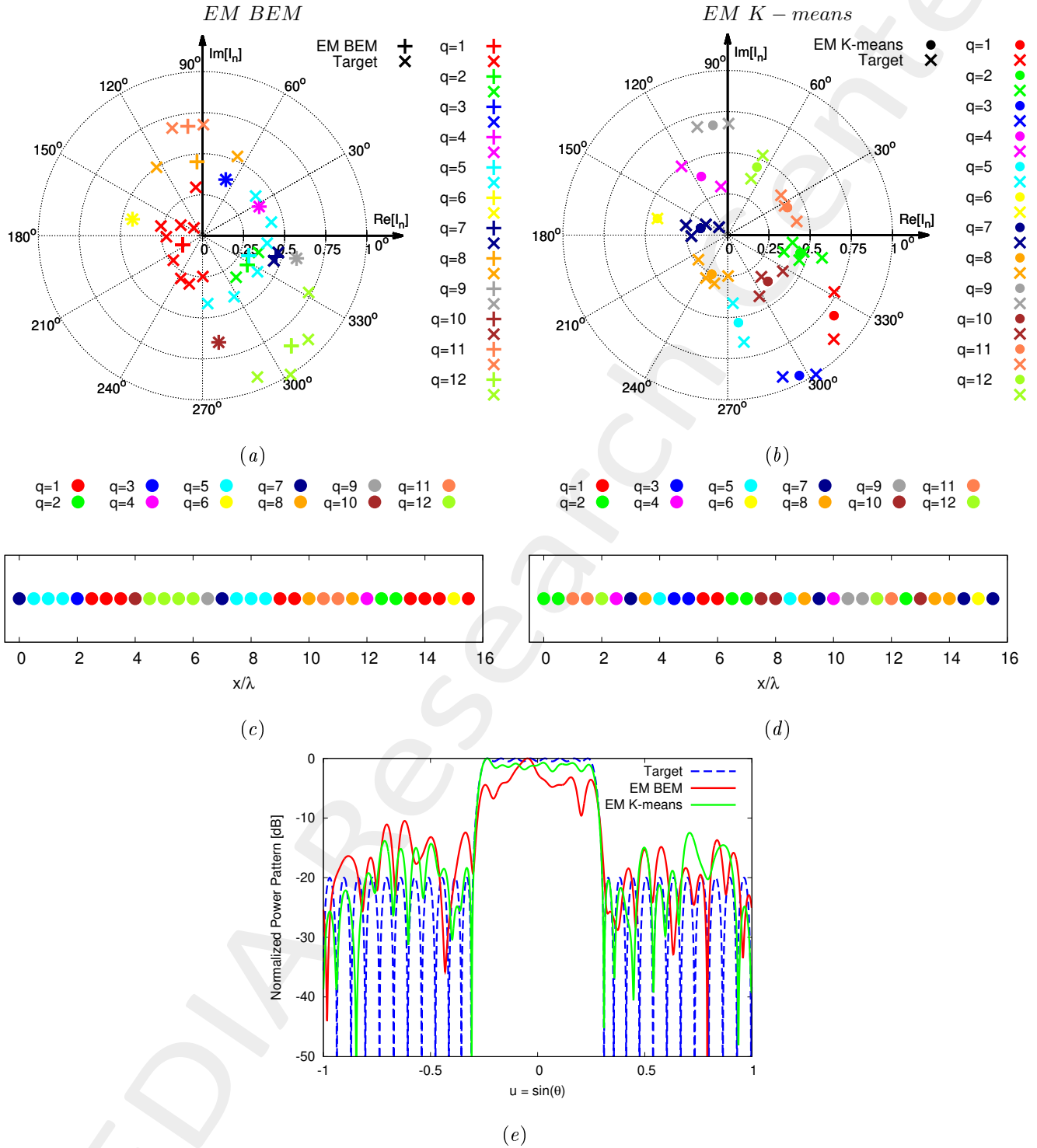


Figure 44: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 16$ - Excitation Matching (EM) BEM vs. K-means vs. Target

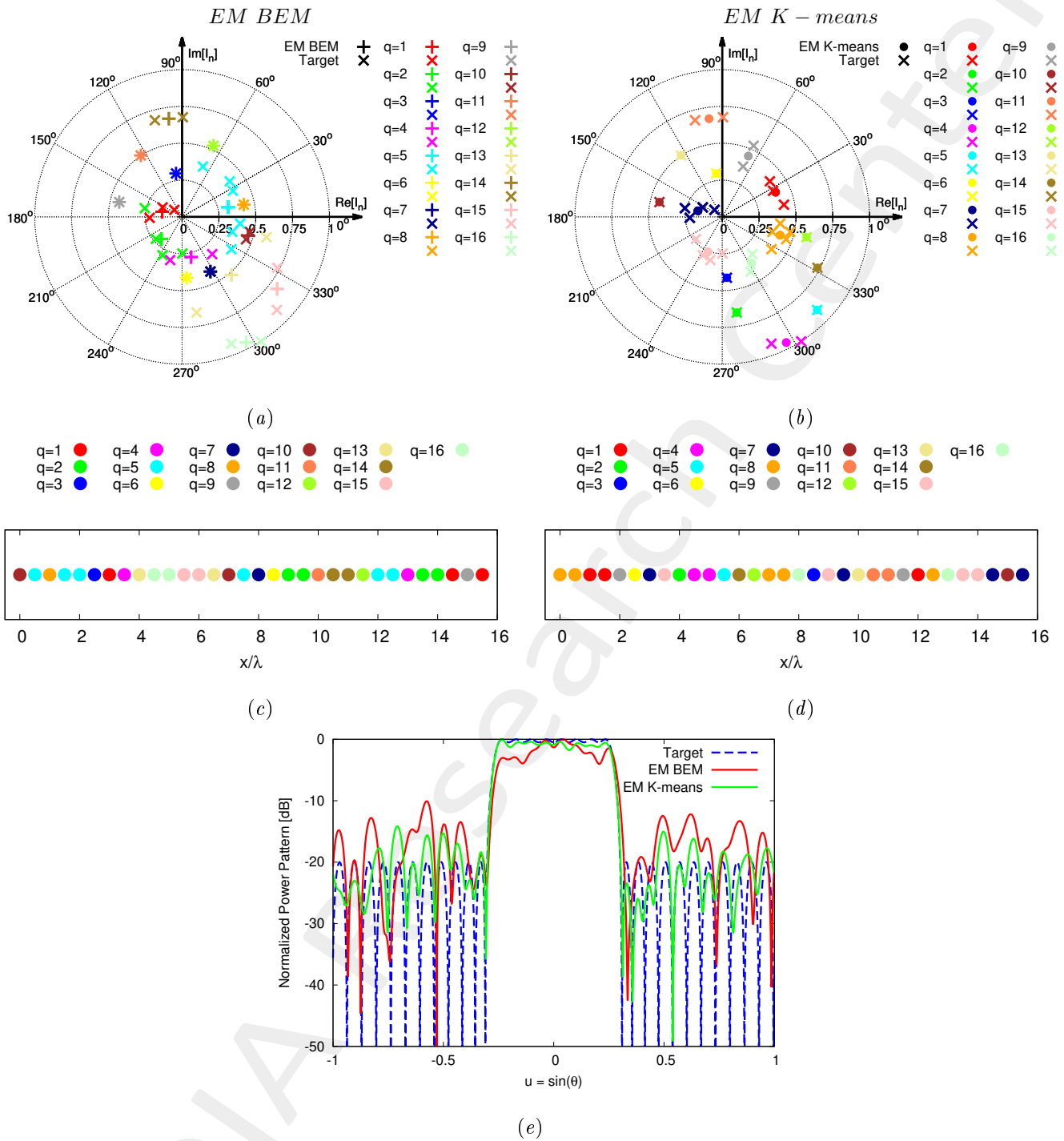


Figure 45: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 20$ - Excitation Matching (EM) BEM vs. K-means vs. Target

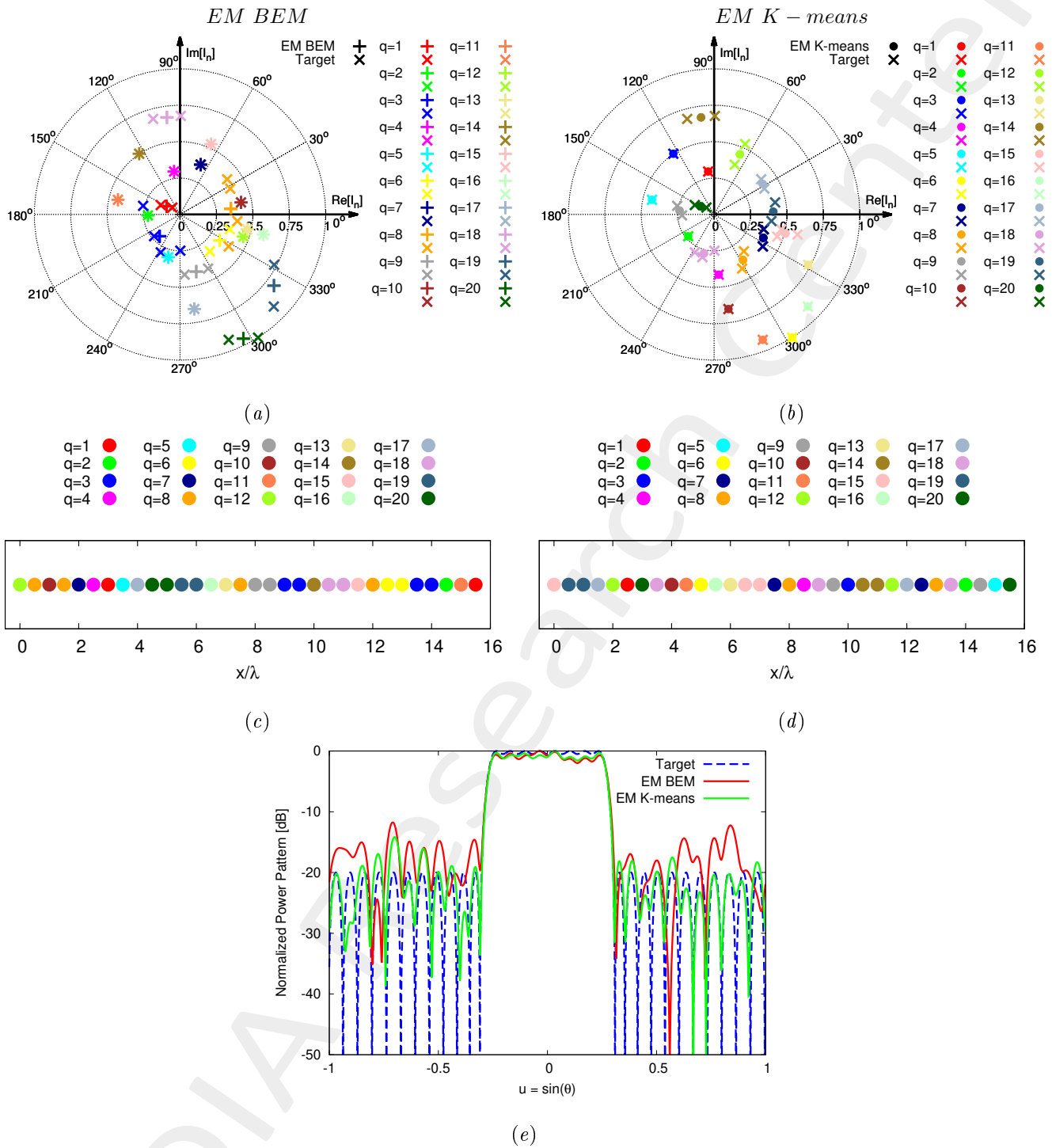


Figure 46: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

$Q = 28$ - Excitation Matching (EM) BEM vs. K-means vs. Target

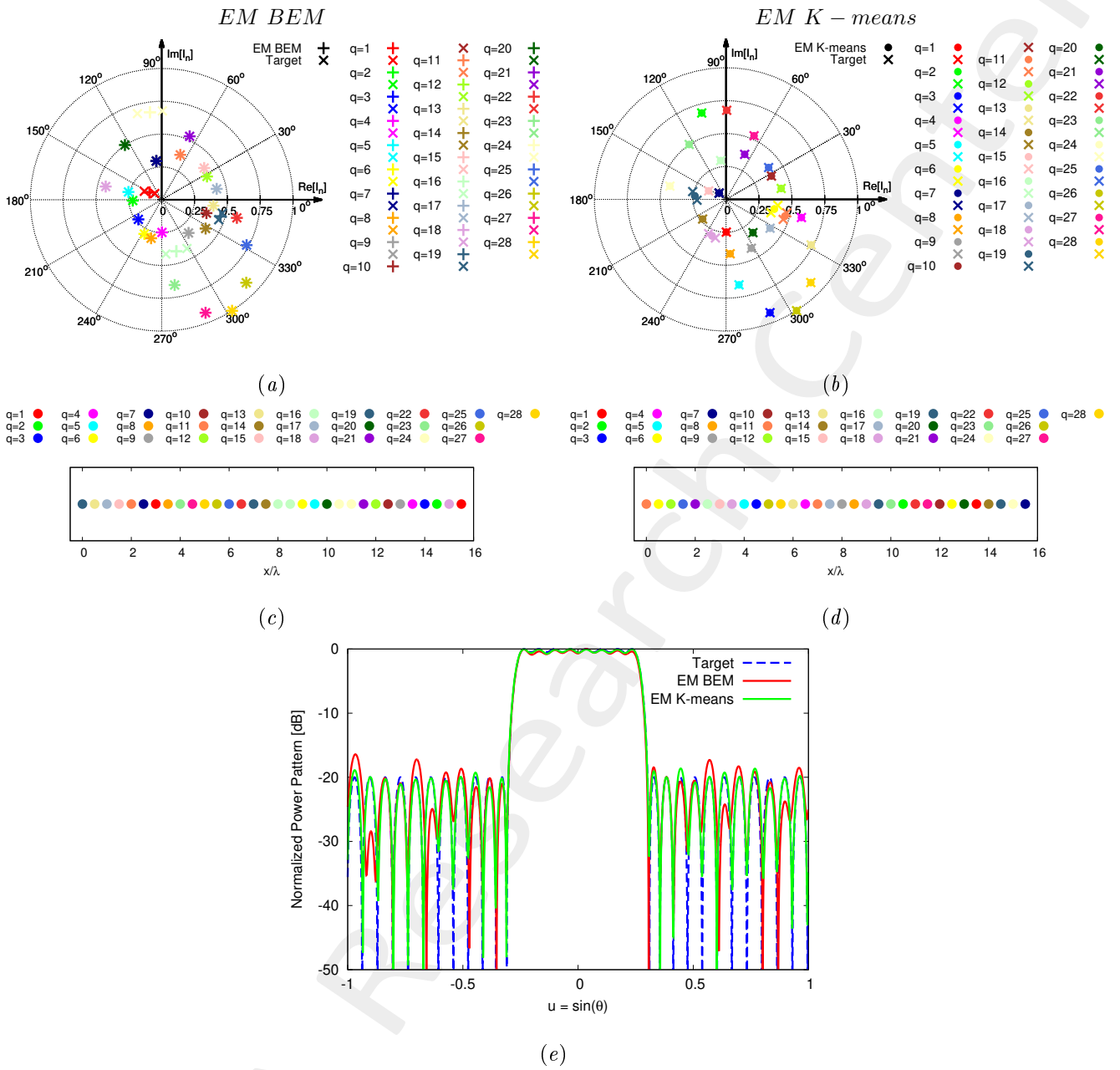


Figure 47: Excitations Matching (EM) BEM vs. K-means vs. target - (a)(b) Clustered excitations and (c)(d) subarray configurations for (a)(c) BEM and (b)(d) K-means methods and along with (e) the power patterns comparison.

Pattern Features Resume

	<i>BEM</i>		<i>K – means</i>	
	Δ	Ψ	Δ	Ψ
$Q = 12$	1.49×10^{-1}	3.78×10^{-2}	3.64×10^{-2}	9.23×10^{-3}
$Q = 16$	9.40×10^{-2}	2.38×10^{-2}	2.01×10^{-2}	5.09×10^{-3}
$Q = 20$	4.37×10^{-2}	1.11×10^{-2}	1.03×10^{-2}	2.62×10^{-3}
$Q = 28$	4.57×10^{-3}	1.16×10^{-3}	1.14×10^{-3}	2.88×10^{-4}

Table VIII: *Excitations Matching (EM) BEM vs. K-means solutions* - Pattern matching error, Δ , and fitness, Ψ , values.

OUTCOMES:

- Generally, the *EM K – means* outperforms the *EM BEM* method in terms of pattern matching error and excitation matching error whatever the number of subarrays Q .

More information on the topics of this document can be found in the following list of references.

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