

# Novel Strategies for Designing Phased Arrays with Subarray-only Amplitude and Phase Control based on K-means and Border Element Method

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## Legenda

*EM*: Excitation Matching

*BEM*: Border Element Method

*CPM*: Contiguous Partition Method

*SLL*: Sidelobe Level

*HPBW*: Half-Power Beamwidth

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# 1 Mathematical Background

Let us consider a linear array of  $N$  elements equally-spaced ( $d$  being the inter-element distance) along the  $x$ -axis. The array elements are assumed to be grouped into  $Q$  ( $Q < N$ ) non-overlapping clusters as depicted in Fig. 1, each  $q$ -th ( $q = 1, \dots, Q$ ) one composed by  $N_q$  elements (non necessarily physically contiguous) so that  $\sum_{q=1}^Q N_q = N$ . Moreover, each  $q$ -th ( $q = 1, \dots, Q$ ) subarray is equipped with an amplifier and a phase shifter providing an amplitude weight and a phase delay equal to  $\alpha_q$  and  $\varphi_q$ , respectively. The array factor term corresponding to such an array is given by

$$AF(\theta) = \sum_{q=1}^Q I_q \left\{ \sum_{n=1}^N \delta_{c_n q} e^{j[k(n-1)d \sin \theta]} \right\}, \quad (1)$$

in which  $I_q = \alpha_q e^{j\varphi_q}$  is the complex excitation, of the  $q$ -th subarray,  $c_n \in \mathbf{c} = \{c_n \in \mathbb{N}_0 | 1 \leq c_n \leq Q, n = 1, \dots, N\}$ , is an integer number identifying the membership of the  $n$ -th ( $n = 1, \dots, N$ ) array element to the  $q$ -th ( $q = 1, \dots, Q$ ) cluster,  $\delta_{c_n q}$  is the Kronecker delta function equal to  $\delta_{c_n q} = 1$  if  $c_n = q$  and  $\delta_{c_n q} = 0$  otherwise, and  $k = \frac{2\pi}{\lambda}$  is the wavenumber, being  $\lambda$  the wavelength.

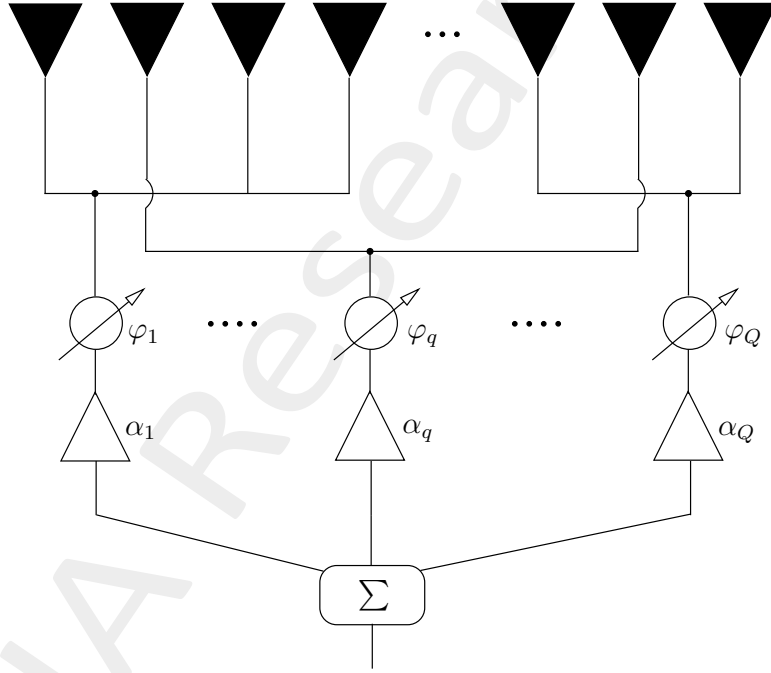


Figure 1: Sketch of the array architecture.

## 2 Excitation Matching Strategies

**Sub-Array Synthesis Problem** - Determine the optimal clustering of the array elements into  $Q$  sub-arrays,  $\mathbf{c}^{opt}$ , exploiting an excitation matching strategy, so that the following metric turns out to be minimized

$$\Psi(\mathbf{c}) = \frac{1}{N} \sum_{n=1}^N |v_n - \chi_n(\mathbf{c})|^2 \quad (2)$$

in which  $v_n$  is the reference complex excitation of the  $n$ -th element and

$$\chi_n(\mathbf{c}) = \frac{\sum_{q=1}^Q \sum_{n=1}^N \delta_{c_n q} v_n}{\sum_{q=1}^Q \sum_{n=1}^N \delta_{c_n q}}, \quad n = 1, \dots, N. \quad (3)$$

Two different approaches will be proposed in the following to identify the best sub-array configuration  $\mathbf{c}^{opt}$ : (i) the *K-means* clustering method and (ii) a customized version of the *Border Element Method (BEM)* proposed that suitably adapted for amplitude and phase clustering.

### 2.1 K-means Method

For the standard K-means implementation the centroid, therefore is legitimate to refer to the centroid as the cluster mean.

- **Step 0 - Initialization:** Definition of the initial clustering configuration. In the proposed implementation, the initial barycenters (or centroids) of the  $Q$  clusters are randomly initialized:  $I_q = \alpha e^{j\beta}$  ( $q = 1, \dots, Q$  and  $i = 0$ ,  $i$  being the iteration index) where  $\alpha$  and  $\beta$  are randomly chosen within the ranges  $\alpha \in [0 : 1]$  and  $\beta \in [0, 2\pi]$ , respectively;
- **Step 1 - Assignment Step:** In this step each element excitation is assigned to a cluster, the one whose barycenter (or centroid) has the minimum Euclidean distance from the reference excitation value. More in detail, two sub-steps can be identified:
  - *Step 1.a - Distance Computation:* The Euclidean distance is computed between excitation values and the cluster barycenters:

$$d_{nq} = \|v_n - I_q\| \quad n = 1, \dots, N; \quad q = 1, \dots, Q \quad (4)$$

- *Step 1.b - Assignment:* The element excitations are assigned to the cluster whose barycenter has the least Euclidean distance computed in (4). The entries  $c_n$  ( $n = 1, \dots, N$ ) of the sub-array configuration vector,  $\mathbf{c}^i$ , at  $i$ -th iteration are thus defined as follows:

$$c_n = q \Leftrightarrow \|v_n - I_q\| \leq \|v_n - I_j\|, \quad \forall j, 1 \leq j \leq Q \quad (5)$$

- **Step 2 - Update Step:** In this step the cluster means, which correspond to the clusters barycenters, are updated considering the elements belonging to each cluster:

$$I_q = \frac{\sum_{n=1}^N \delta_{c_n q} v_n}{\sum_{n=1}^N \delta_{c_n q}}, \quad (6)$$

- **Step 3 - Stopping Criteria:** Once initialized the algorithm will iterate through the assignment step and the update step until it reaches convergence or exceeded the maximum number of iterations. The convergence is reached when the cluster members no longer changes, i.e. the case in which the assignment step does not provide any movement of the data points from one cluster to another.

## 2.2 Border Element Method

while the optimal values of the sub-array coefficients  $\mathbf{I}^{opt} = \{I_q^{opt} | q = 1, \dots, Q\}$  are computed analytically. More in detail,

The algorithm works as follows:

- **Step 1 - Initialization** - Define the list  $\mathbf{L} = \{\ell_n; n = 1, \dots, N\}$  of the  $N$  complex-valued entries  $\ell_n = \xi_n e^{j\gamma_n}$  ordered in increasing order considering the magnitude (namely,  $\xi_1 = \min_n \{\xi_n\}$  and  $\xi_N = \max_n \{\xi_n\}$ ). Moreover, for elements having the same magnitude, the entries are ordered considering an increasing order also in terms of phase (namely,  $\gamma_1 = \min_{m, \xi_m = \xi_1} \{\arg \langle \ell_m \rangle\}$  and  $\gamma_N = \max_{m, \xi_m = \xi_N} \{\arg \langle \ell_m \rangle\}$ ). Set the initial ( $i = 0$ ,  $i$  being the iteration index) sub-array configuration,  $\mathbf{c}^i$ , by randomly selecting  $Q - 1$  cut points of the list  $\mathbf{L}$  among the  $N - 1$  admissible ones ( $Q - 1 < N - 1$ ) and go to Step 2;
- **Step 2 - Sub-Array Coefficients Synthesis** - For the  $i$ -th trial sub-array configuration,  $\mathbf{c}^i$ , compute the optimal sub-array excitations ( $q = 1, \dots, Q$ )

$$I_q = \frac{\sum_{n=1}^N \delta_{c_n q} v_n}{\sum_{n=1}^N \delta_{c_n q}}, \quad (7)$$

which represent the arithmetic mean of the complex excitations of the array elements belonging to the same  $q$ -th cluster;

- **Step 3 - Complex Excitation-Matching Evaluation** - Compute the excitation-matching value (2) in correspondence with  $\mathbf{c}^i$  and  $\mathbf{I}^i$ :  $\Psi^i = \Psi(\mathbf{c}^i, \mathbf{I}^i)$ . Compare  $\Psi^i$  with the best value found so far,  $\Psi_{opt}^{i-1} = \min_{h=1, \dots, i-1} \{\Psi^h\}$ . If  $\Psi^i < \Psi_{opt}^{i-1}$ , then set  $\Psi_{opt} = \Psi^i$  and update the current (i.e.,  $i$ -th) best sub-array configuration and phase vector:  $\mathbf{c}_{opt} \leftarrow \mathbf{c}^i$  and  $\mathbf{I}_{opt} \leftarrow \mathbf{I}^i$ ;

- Step 4 - **Convergence Check** - Stop the *External Iterative* loop if the maximum number of iterations  $T_{max}$  has been reached ( $i \geq T_{max}$ ) or the stationary condition

$$\frac{|T_{stat} \Psi_{opt}^{i-1} - \sum_{h=2}^{T_{stat}+1} \Psi_{opt}^{i-h}|}{\Psi_{opt}^i} \leq \eta \quad (8)$$

holds true,  $T_{stat}$  and  $\eta$  being a user-defined number of iterations and a numerical threshold, respectively;

- Step 5 - **Sub-Array Configuration Update** - Update the iteration index ( $i \leftarrow i + 1$ ) and define a new sub-array aggregation  $\mathbf{c}_i$  by changing the position of - at least - one of the  $Q - 1$  cut points of the list  $\mathbf{L}$  that define the previous clustering configuration,  $\mathbf{c}_{i-1}$ , according to the *BEM* algorithm, then go to Step 2.

### 3 Figure of Merit

Starting from the definition of the power pattern expressed as a function of the direction cosine angular variable  $u = \sin \theta$ ,

$$PP(u) = \left| \sum_{q=1}^Q I_q \left\{ \sum_{n=1}^N \delta_{c_n q} e^{j[k(n-1)u]} \right\} \right|^2 \quad (9)$$

the following figure of merit have been selected to suitably evaluate the performance of the array synthesis method:

- **Sidelobe Level,  $SLL$ :** expressed in [dB], the difference between the main beam peak value and the maximum sidelobe peak value  $\left[ \frac{(\max_{u \notin \Omega} PP(u))}{PP(u_0)} \right]$ , being  $\Omega$  the angular region of the main beam and  $u_0 = \sin \theta_0$  the direction of maximum radiation];
- **Maximum directivity:** defined as the following ratio

$$D_{\max} = \frac{4\pi |PP(u_0)|^2}{\mathcal{P}}, \quad (10)$$

being  $\mathcal{P}$  the total power radiated by the array  $\mathcal{P} = \int_0^{2\pi} \int_0^\pi |AF(\theta)|^2 d\theta d\phi$ . Such parameter is usually expressed in [dB] as  $D_{\max}]_{dB} = 10 \log_{10} D_{\max}$ .

- **Half Power Beamwidth,  $HPBW$ :** expressed in degrees [deg], angle between the two directions in which the radiation intensity is one-half of the main beam, measured in a plane containing the direction of the main.
- **Excitation Matching,** expressed as

$$\Psi(\mathbf{c}) = \frac{1}{N} \sum_{n=1}^N |v_n - \chi_n(\mathbf{c})|^2 \quad (11)$$

in which  $v_n$  is the reference complex excitation of the  $n$ -th element and  $\chi_n(\mathbf{c})$  is defined in (3) and  $\mathbf{c}$  is the vector of the subarray aggregations;

- **Pattern Matching Error,** expressed as

$$\Delta = \frac{\sum_{k=1}^K \{PP(u_k) - PP_{ref}(u_k)\}}{\sum_{k=1}^K \{PP_{ref}(u_k)\}} \quad (12)$$

where  $PP_{ref}(u)$  is the reference power pattern,  $k = 1, \dots, K$  being  $K$  the number of samples.



More information on the topics of this document can be found in the following list of references.

## References

- [1] P. Rocca, G. Oliveri, R. J. Mailloux, and A. Massa, "Unconventional phased array architectures and design Methodologies - A review," *Proceedings of the IEEE*, vol. 104, no. 3, pp. 544-560, March 2016.
- [2] P. Rocca, L. Poli, A. Polo, and A. Massa, "Optimal excitation matching strategy for sub-arrayed phased linear arrays generating arbitrary shaped beams," *IEEE Trans. Antennas Propag.*, vol. 68, no. 6, pp. 4638-4647, Jun. 2020.
- [3] G. Oliveri, G. Gottardi and A. Massa, "A new meta-paradigm for the synthesis of antenna arrays for future wireless communications," *IEEE Trans. Antennas Propag.*, vol. 67, no. 6, pp. 3774-3788, Jun. 2019.
- [4] P. Rocca, M. H. Hannan, L. Poli, N. Anselmi, and A. Massa, "Optimal phase-matching strategy for beam scanning of sub-arrayed phased arrays," *IEEE Trans. Antennas and Propag.*, vol. 67, no. 2, pp. 951-959, Feb. 2019.
- [5] N. Anselmi, P. Rocca, M. Salucci, and A. Massa, "Contiguous phase-clustering in multibeam-on-receive scanning arrays," *IEEE Trans. Antennas Propag.*, vol. 66, no. 11, pp. 5879-5891, Nov. 2018.
- [6] L. Poli, G. Oliveri, P. Rocca, M. Salucci, and A. Massa, "Long-Distance WPT Unconventional Arrays Synthesis," *Journal of Electromagnetic Waves and Applications*, vol. 31, no. 14, pp. 1399-1420, Jul. 2017.
- [7] G. Gottardi, L. Poli, P. Rocca, A. Montanari, A. Aprile, and A. Massa, "Optimal Monopulse Beamforming for Side-Looking Airborne Radars," *IEEE Antennas Wireless Propag. Lett.*, vol. 16, pp. 1221-1224, 2017.
- [8] G. Oliveri, M. Salucci, and A. Massa, "Synthesis of modular contiguously clustered linear arrays through a sparseness-regularized solver," *IEEE Trans. Antennas Propag.*, vol. 64, no. 10, pp. 4277-4287, Oct. 2016.
- [9] P. Rocca, G. Oliveri, R. J. Mailloux, and A. Massa, "Unconventional phased array architectures and design Methodologies - A review," *Proceedings of the IEEE = Special Issue on 'Phased Array Technologies', Invited Paper*, vol. 104, no. 3, pp. 544-560, March 2016.
- [10] P. Rocca, M. D'Urso, and L. Poli, "Advanced strategy for large antenna array design with subarray-only amplitude and phase contr," *IEEE Antennas and Wireless Propag. Lett.*, vol. 13, pp. 91-94, 2014.
- [11] L. Manica, P. Rocca, G. Oliveri, and A. Massa, "Synthesis of multi-beam sub-arrayed antennas through an excitation matching strategy," *IEEE Trans. Antennas Propag.*, vol. 59, no. 2, pp. 482-492, Feb. 2011.
- [12] G. Oliveri, "Multi-beam antenna arrays with common sub-array layouts," *IEEE Antennas Wireless Propag. Lett.*, vol. 9, pp. 1190-1193, 2010.
- [13] P. Rocca, R. Haupt, and A. Massa, "Sidelobe reduction through element phase control in sub-arrayed array antennas," *IEEE Antennas Wireless Propag. Lett.*, vol. 8, pp. 437-440, 2009.

- [14] P. Rocca, L. Manica, R. Azaro, and A. Massa, "A hybrid approach for the synthesis of sub-arrayed monopulse linear arrays," *IEEE Trans. Antennas Propag.*, vol. 57, no. 1, pp. 280-283, Jan. 2009.
- [15] L. Manica, P. Rocca, M. Benedetti, and A. Massa, "A fast graph-searching algorithm enabling the efficient synthesis of sub-arrayed planar monopulse antennas," *IEEE Trans. Antennas Propag.*, vol. 57, no. 3, pp. 652-664, Mar. 2009.
- [16] P. Rocca, L. Manica, A. Martini, and A. Massa, "Compromise sum-difference optimization through the iterative contiguous partition method," *IET Microwaves, Antennas & Propagation*, vol. 3, no. 2, pp. 348-361, 2009.
- [17] L. Manica, P. Rocca, and A. Massa, "An excitation matching procedure for sub-arrayed monopulse arrays with maximum directivity," *IET Radar, Sonar & Navigation*, vol. 3, no. 1, pp. 42-48, Feb. 2009.
- [18] L. Manica, P. Rocca, and A. Massa, "Design of subarrayed linear and planar array antennas with SLL control based on an excitation matching approach," *IEEE Trans. Antennas Propag.*, vol. 57, no. 6, pp. 1684-1691, Jun. 2009.
- [19] L. Manica, P. Rocca, A. Martini, and A. Massa, "An innovative approach based on a tree-searching algorithm for the optimal matching of independently optimum sum and difference excitations," *IEEE Trans. Antennas Propag.*, vol. 56, no. 1, pp. 58-66, Jan. 2008.
- [20] P. Rocca, L. Manica, and A. Massa, "An effective excitation matching method for the synthesis of optimal compromises between sum and difference patterns in planar arrays," *Progress in Electromagnetic Research B*, vol. 3, pp. 115-130, 2008.
- [21] P. Rocca, L. Manica, and A. Massa, "Directivity optimization in planar sub-arrayed monopulse antenna," *Progress in Electromagnetic Research L*, vol. 4, pp. 1-7, 2008.
- [22] P. Rocca, L. Manica, M. Pastorino, and A. Massa, "Boresight slope optimization of sub-arrayed linear arrays through the contiguous partition method," *IEEE Antennas Wireless Propag. Lett.*, vol. 8, pp. 253-257, 2008.
- [23] P. Rocca, L. Manica, and A. Massa, "Synthesis of monopulse antennas through the iterative contiguous partition method," *Electronics Letters*, vol. 43, no. 16, pp. 854-856, Aug. 2007.
- [24] P. Rocca, L. Manica, A. Martini, and A. Massa, "Synthesis of large monopulse linear arrays through a tree-based optimal excitations matching," *IEEE Antennas Wireless Propag. Lett.*, vol. 7, pp. 436-439, 2007.
- [25] P. Rocca, N. Anselmi, A. Polo, and A. Massa, "An irregular two-sizes square tiling method for the design of isophoric phased arrays," *IEEE Trans. Antennas Propag.*, vol. 68, no. 6, pp. 4437-4449, Jun. 2020.
- [26] P. Rocca, N. Anselmi, A. Polo, and A. Massa, "Modular design of hexagonal phased arrays through diamond tiles," *IEEE Trans. Antennas Propag.*, vol. 68, no. 5, pp. 3598-3612, May 2020.

- [27] N. Anselmi, L. Poli, P. Rocca, and A. Massa, "Design of simplified array layouts for preliminary experimental testing and validation of large AESAs," *IEEE Trans. Antennas Propag.*, vol. 66, no. 12, pp. 6906-6920, Dec. 2018.
- [28] G. Oliveri, G. Gottardi, F. Robol, A. Polo, L. Poli, M. Salucci, M. Chuan, C. Massagrande, P. Vinetti, M. Mattivi, R. Lombardi, and A. Massa, "Co-design of unconventional array architectures and antenna elements for 5G base station," *IEEE Trans. Antennas Propag.*, vol. 65, no. 12, pp. 6752-6767, Dec. 2017.
- [29] N. Anselmi, P. Rocca, M. Salucci, and A. Massa, "Irregular phased array tiling by means of analytic schemata-driven optimization," *IEEE Trans. Antennas Propag.*, vol. 65, no. 9, pp. 4495-4510, September 2017.
- [30] N. Anselmi, P. Rocca, M. Salucci, and A. Massa, "Optimization of excitation tolerances for robust beamforming in linear arrays," *IET Microwaves, Antennas & Propagation*, vol. 10, no. 2, pp. 208-214, 2016.
- [31] P. Rocca, R. J. Mailloux, and G. Toso, "GA-Based optimization of irregular sub-array layouts for wideband phased arrays design," *IEEE Antennas and Wireless Propag. Lett.*, vol. 14, pp. 131-134, 2015.
- [32] P. Rocca, M. Donelli, G. Oliveri, F. Viani, and A. Massa, "Reconfigurable sum-difference pattern by means of parasitic elements for forward-looking monopulse radar," *IET Radar, Sonar & Navigation*, vol. 7, no. 7, pp. 747-754, 2013.
- [33] P. Rocca, L. Manica, and A. Massa, "Ant colony based hybrid approach for optimal compromise sum-difference patterns synthesis," *Microwave Opt. Technol. Lett.*, vol. 52, no. 1, pp. 128-132, Jan. 2010.
- [34] P. Rocca, L. Manica, and A. Massa, "An improved excitation matching method based on an ant colony optimization for suboptimal-free clustering in sum-difference compromise synthesis," *IEEE Trans. Antennas Propag.*, vol. 57, no. 8, pp. 2297-2306, Aug. 2009.
- [35] P. Rocca, L. Manica, and A. Massa, "Hybrid approach for sub-arrayed monopulse antenna synthesis," *Electronics Letters*, vol. 44, no. 2, pp. 75-76, Jan. 2008.
- [36] P. Rocca, L. Manica, F. Stringari, and A. Massa, "Ant colony optimization for tree-searching based synthesis of monopulse array antenna," *Electronics Letters*, vol. 44, no. 13, pp. 783-785, Jun. 19, 2008.