A Matrix Completion Approach to Image Sparse Targets under the First-Order Born Approximation

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Abstract

In this work, a novel approach to solve the microwave inverse scattering problem under the first-order Born approximation to image sparse and weak targets is addressed. Towards this end, a single-task Bayesian compressive sensing (*ST-BCS*) solver is exploited to retrieve a preliminary guess of the unknown contrast distribution within an inaccessible domain. Then, a filtering process is used to filter out the "less reliable" contrast coefficients from the *BCS* guess. Finally, a customized matrix completion (*MC*) procedure is adopted in order to complete the retrieved images and achieve significant accuracy improvements under high levels of noise on processed data.

In order to validate the effectiveness of the proposed methodology some representative numerical benchmarks are presented considering different targets and noise levels.

1 Numerical Assessment

Goal: show the performances of BCS when dealing with a sparse scatterer

- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

1.1 L-Shaped Profile

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$
- $D = 1296$ (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2\times\frac{2\pi}{\lambda}\times\frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2\left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- N scelto in modo da essere vicino a $\#DOF$: $N = 324$ (18×18)

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$
- Full-aspect measurements
- $M \approx 2ka \rightarrow M = 27$

Sources:

- Plane waves
- $V \approx 2ka \rightarrow V = 27$
- Amplitude $A = 1$
- Frequency: 300 MHz $(\lambda = 1)$

Object:

- L made of 3 square cylinders of side $\frac{\lambda}{6} = 0.1667$ (single pixel)
- $\varepsilon_r = 2.0$
- $\sigma = 0$ [S/m]

BCS parameters:

- Initial estimate of the noise: $n_0 = 1.0 \times 10^{-3}$
- Convergenze parameter: $\tau = 1.0 \times 10^{-8}$

MC parameters:

- Threshold: $\eta = 0.2$

RESULTS: $\varepsilon_r = 2.0$

Retrieved Profiles

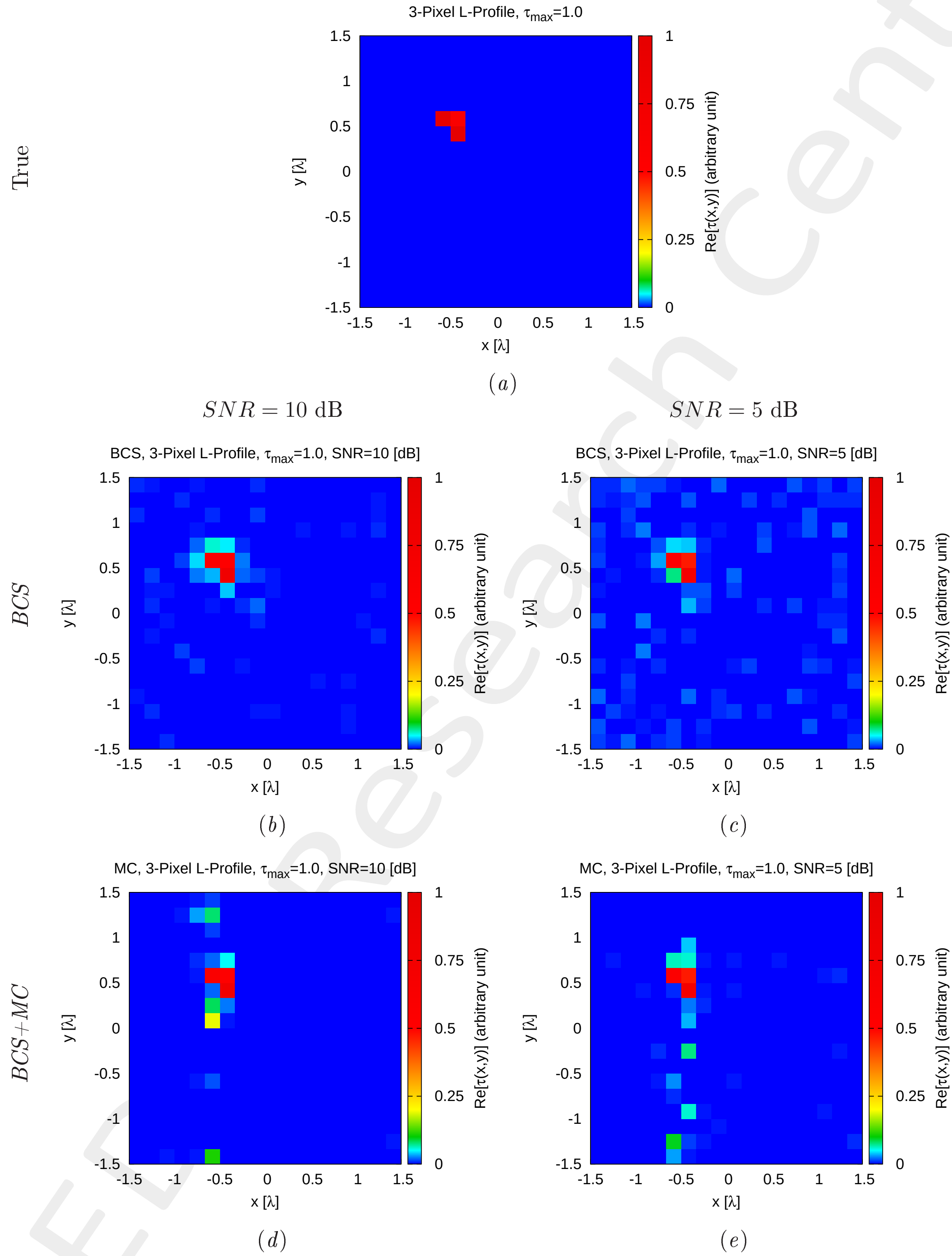

Figure 1: Actual object (a) and reconstructed object by $(b)(c)$ *BCS* and $(c)(d)$ *BCS+MC* when $(b)(d)$ $SNR = 10$ [dB], $(c)(e)$ $SNR = 5$ [dB].

Retrieved Profiles

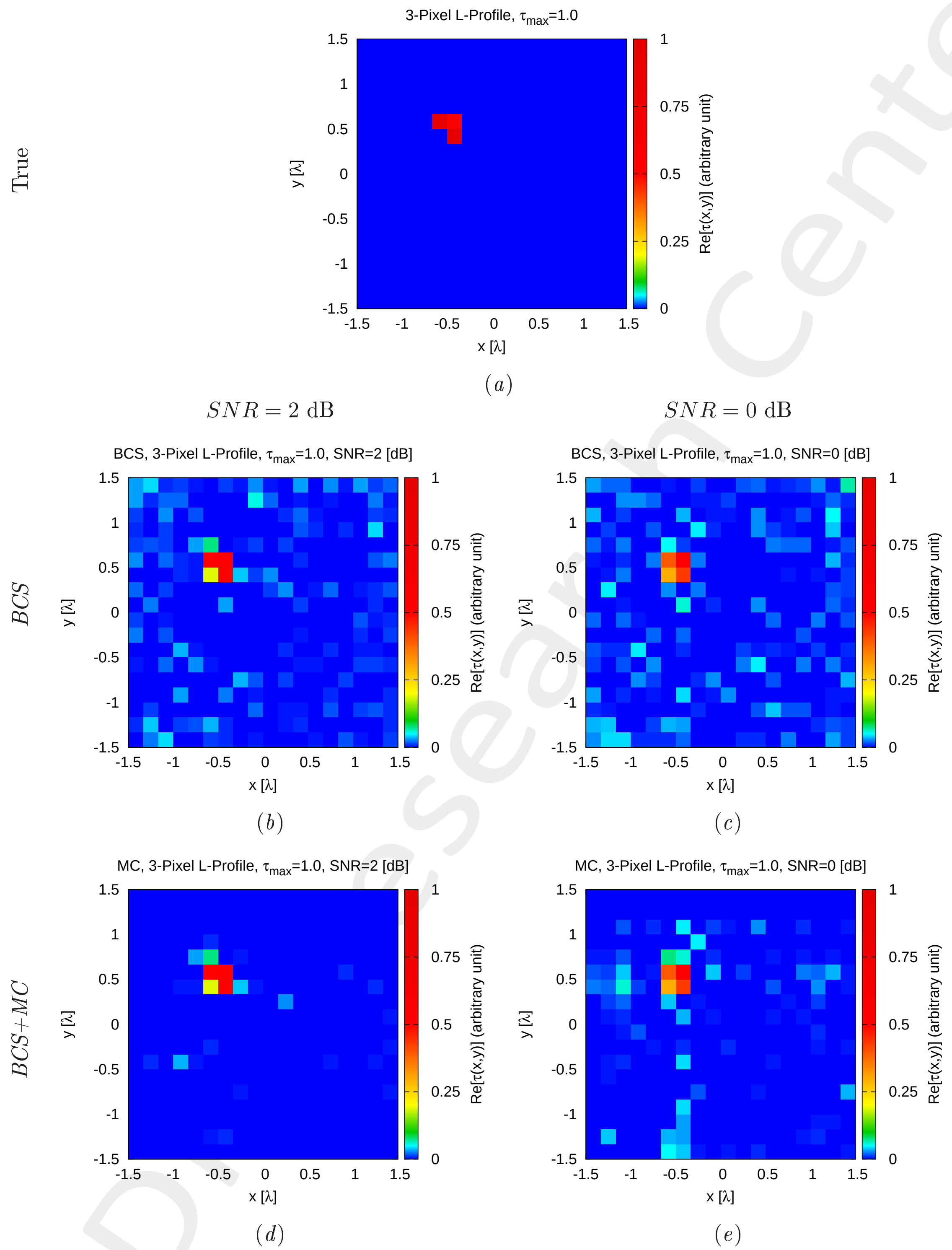

Figure 2: Actual object (a) and reconstructed object by $(b)(c)$ BCS and $(c)(d)$ $BCS{+}MC$ when $(b)(d)$ $SNR = 2$ [dB], $(c)(e)$ $SNR = 0$ [dB].

Retrieved Profiles

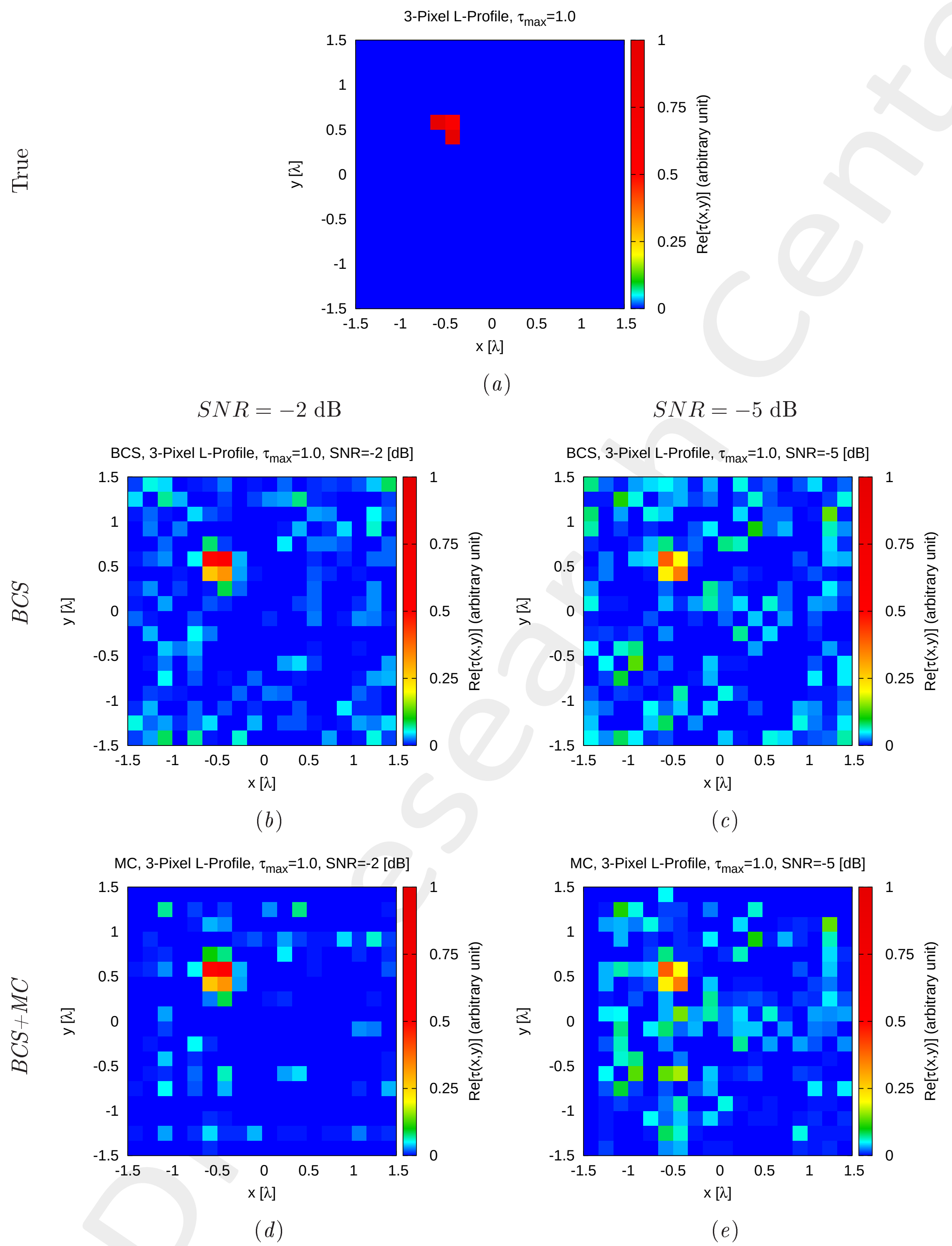

Figure 3: Actual object (a) and reconstructed object by $(b)(c)$ *BCS* and $(c)(d)$ *BCS+MC* when $(b)(d)$ $SNR = -2$ [dB], $(c)(e)$ $SNR = -5$ [dB].

Confidence Levels

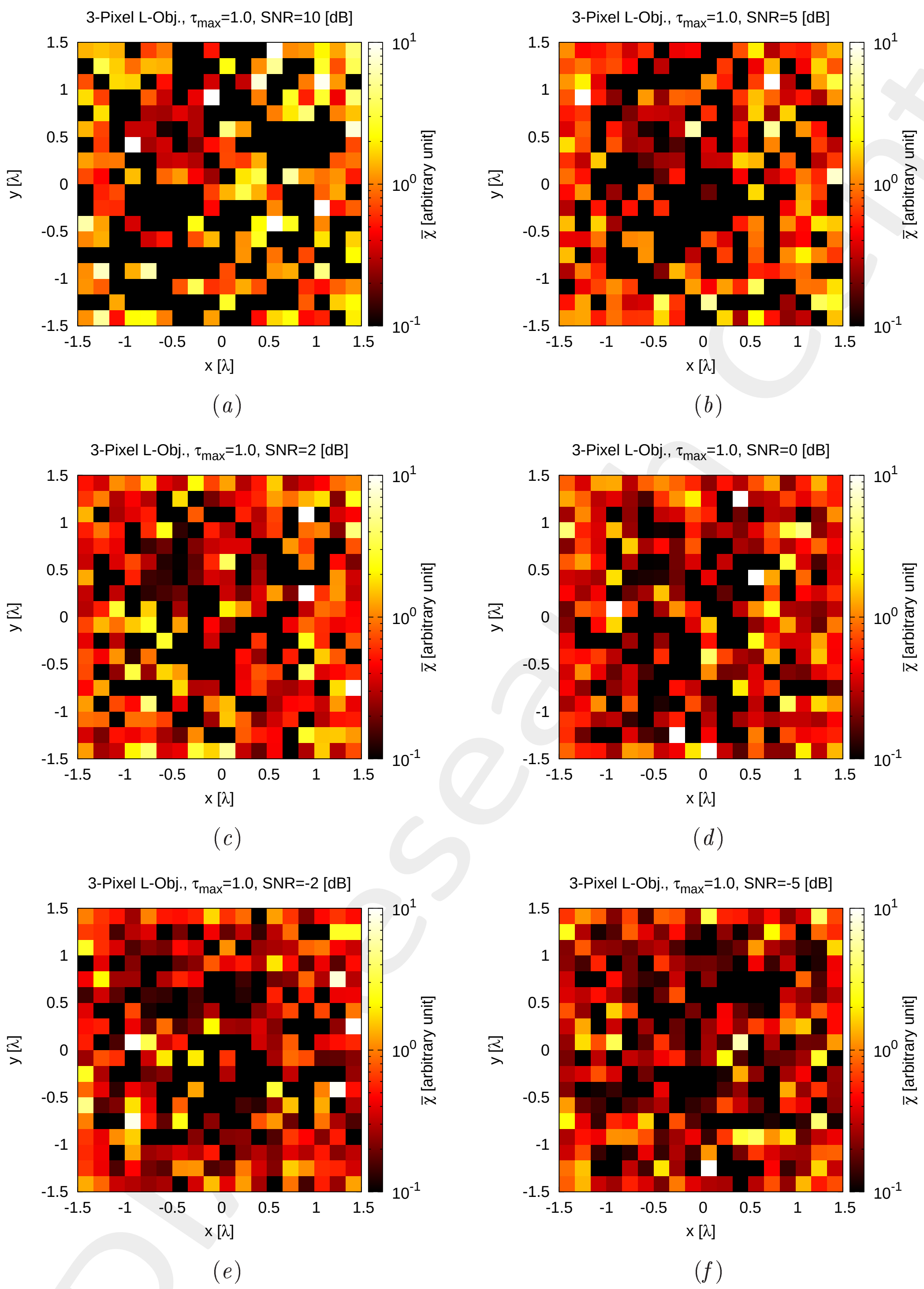

Figure 4: Confidence Levels when (a) $SNR = 10$ [dB], (b) $SNR = 5$ [dB], (c) $SNR = 2$ [dB], (d) $SNR = 0$ [dB], (e) $SNR = -2$ [dB], (f) $SNR = -5$ [dB].

1.2 Large Square Profile

Goal: show the performances of BCS when dealing with a sparse scatterer

- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$
- $D = 1296$ (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2\times\frac{2\pi}{\lambda}\times\frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2\left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- N scelto in modo da essere vicino a $\#DOF$: $N = 324$ (18×18)

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$
- Full-aspect measurements
- $M \approx 2ka \rightarrow M = 27$

Sources:

- Plane waves
- $V \approx 2ka \rightarrow V = 27$
- Amplitude $A = 1$
- Frequency: 300 MHz ($\lambda = 1$)

Object:

- Square cylinder of side $\frac{\lambda}{3} = 0.333$ (4 pixels)
- $\varepsilon_r = 1.5$
- $\sigma = 0$ [S/m]

BCS parameters:

- Initial estimate of the noise: $n_0 = 1.0 \times 10^{-3}$
- Convergenze parameter: $\tau = 1.0 \times 10^{-8}$

MC parameters:

- Threshold: $\eta = 0.2$

RESULTS: $\varepsilon_r = 1.5$

Retrieved Profiles

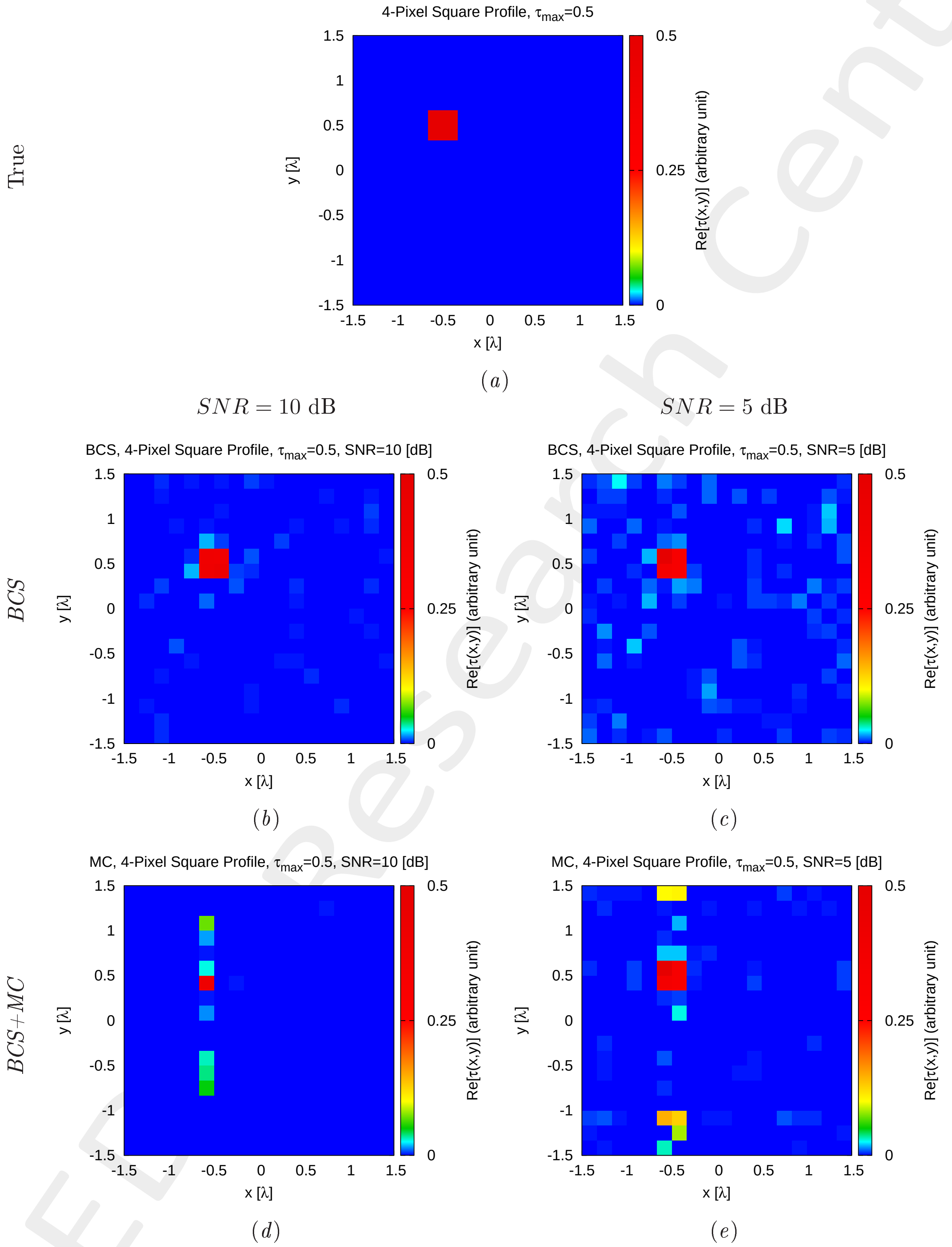

Figure 5: Actual object (a) and reconstructed object by $(b)(c)$ BCS and $(c)(d)$ $BCS{+}MC$ when $(b)(d)$ $SNR = 10$ [dB], $(c)(e)$ $SNR = 5$ [dB].

Retrieved Profiles

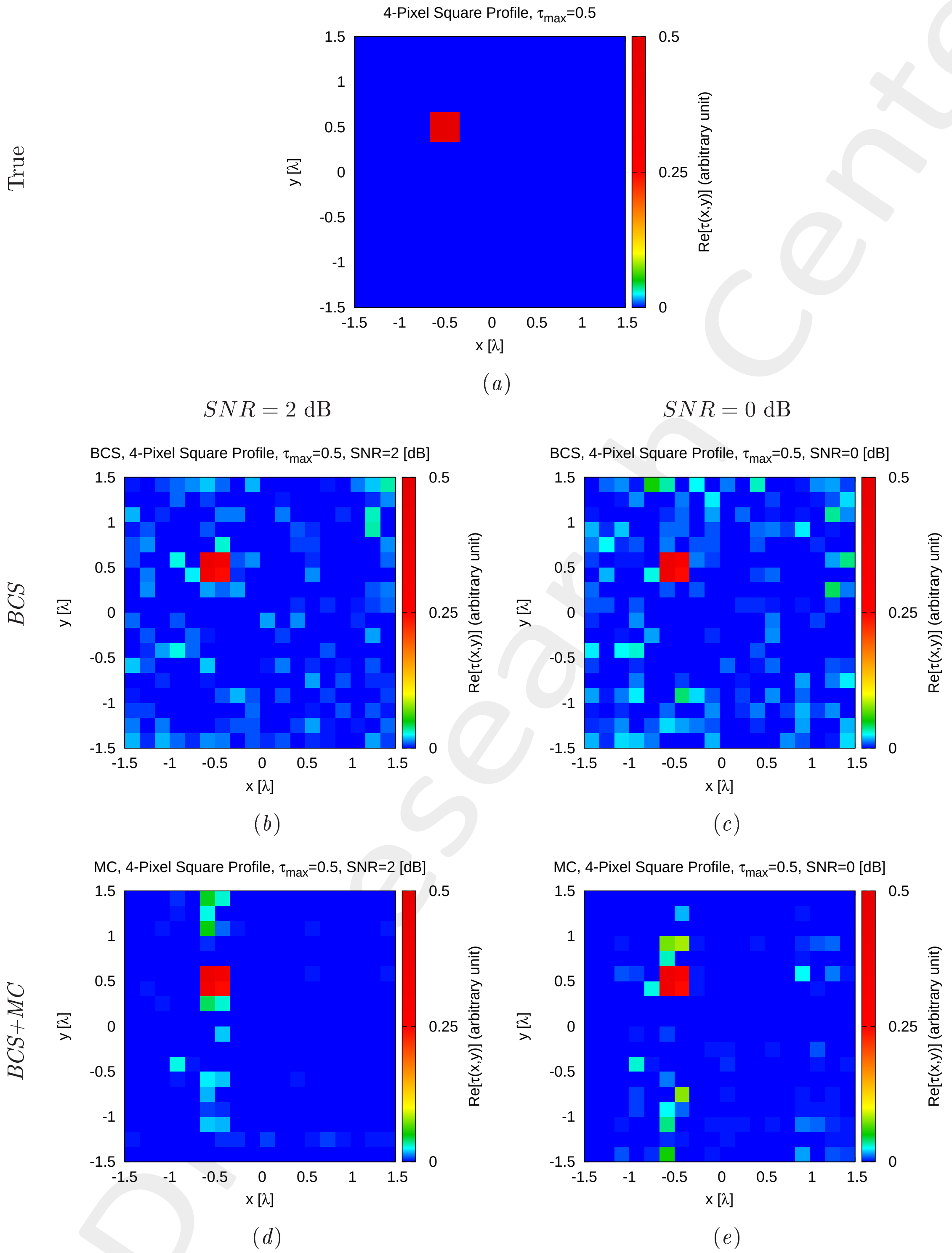

Figure 6: Actual object (a) and reconstructed object by $(b)(c)$ *BCS* and $(c)(d)$ *BCS+MC* when $(b)(d)$ $SNR = 2$ [dB], $(c)(e)$ $SNR = 0$ [dB].

Retrieved Profiles

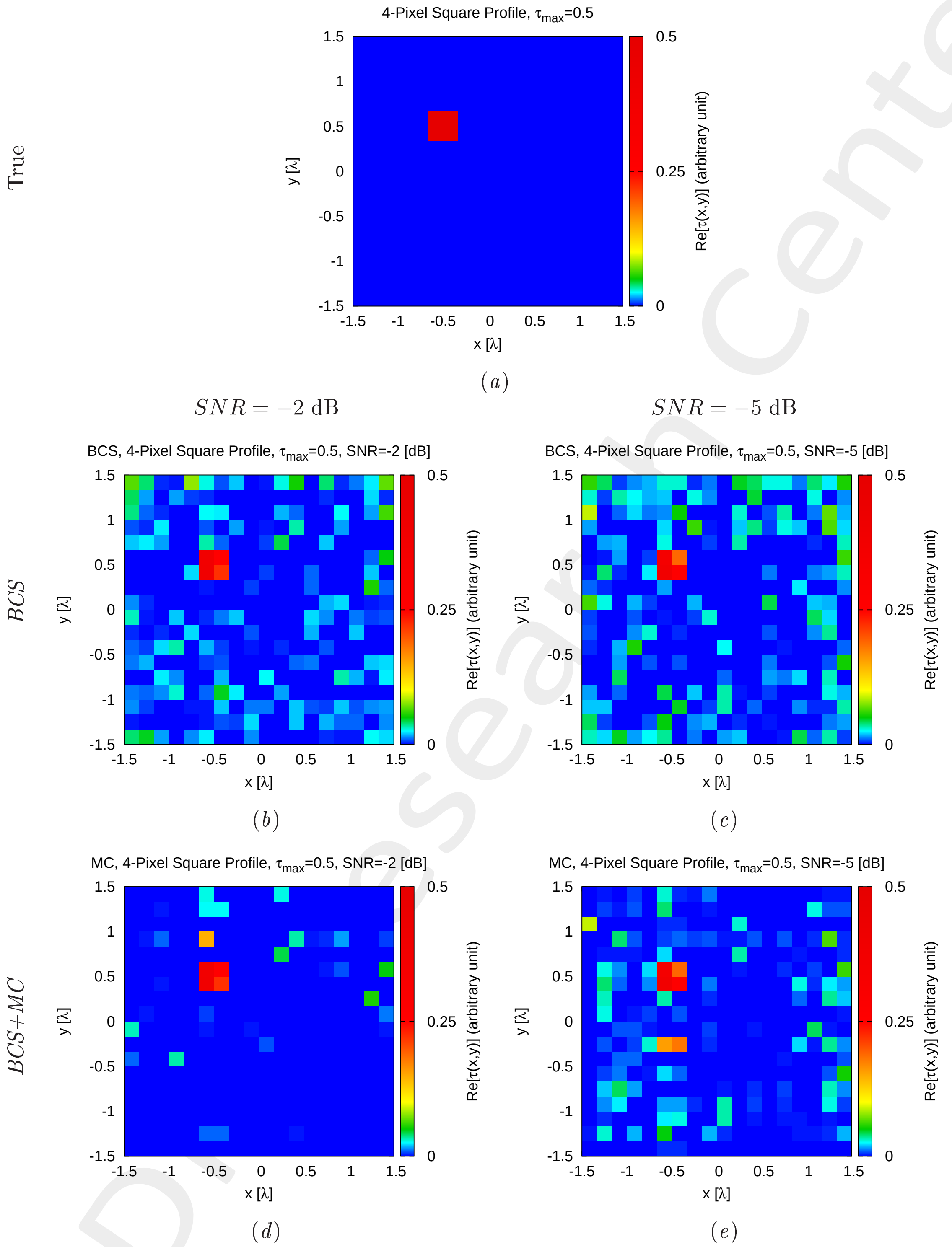

Figure 7: Actual object (a) and reconstructed object by $(b)(c)$ *BCS* and $(c)(d)$ *BCS+MC* when $(b)(d)$ $SNR = -2$ [dB], $(c)(e)$ $SNR = -5$ [dB].

Confidence Levels

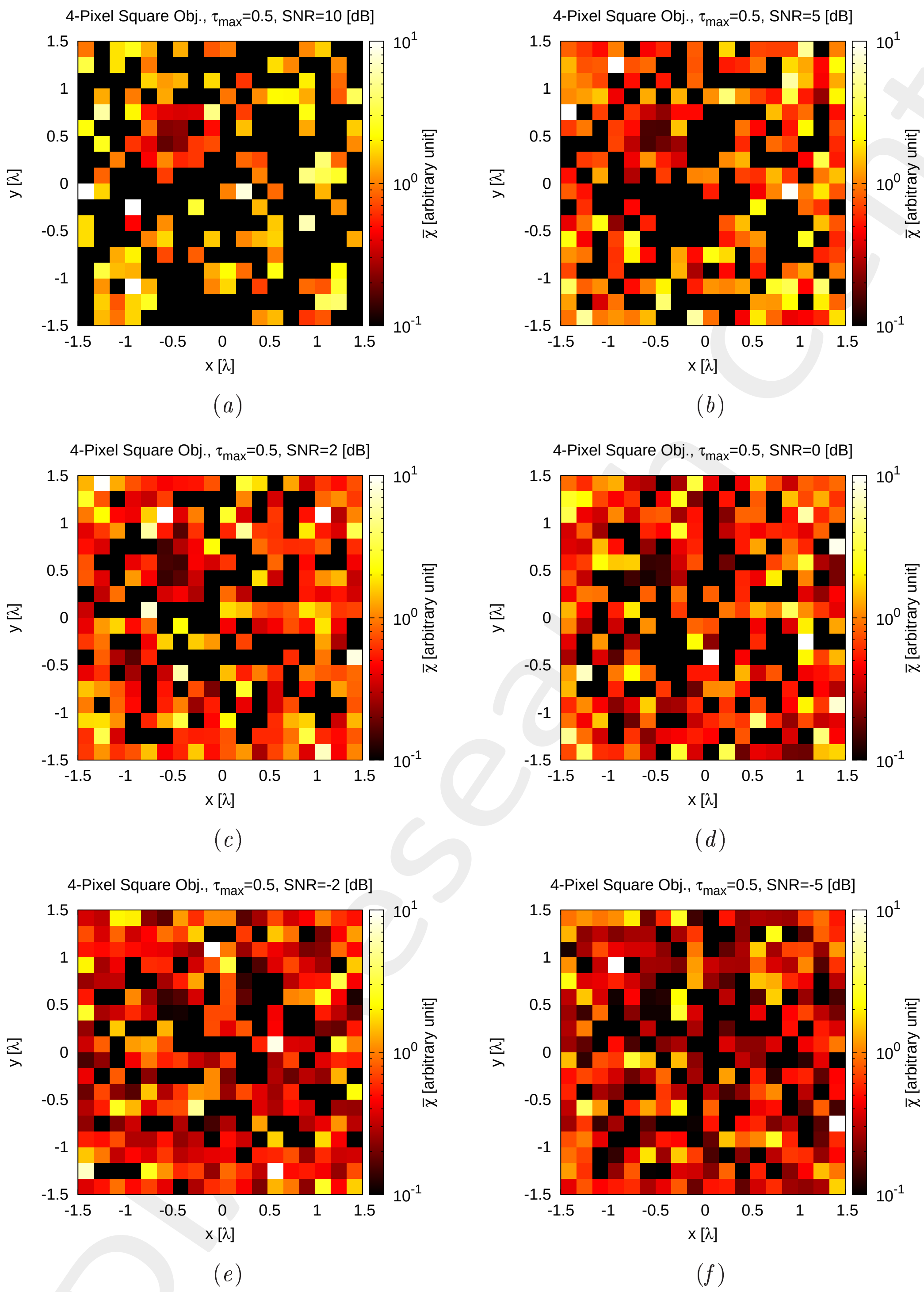

Figure 8: Confidence Levels when (a) $SNR = 10$ [dB], (b) $SNR = 5$ [dB], (c) $SNR = 2$ [dB], (d) $SNR = 0$ [dB], (e) $SNR = -2$ [dB], (f) $SNR = -5$ [dB].

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