Innovative Bayesian Compressive Sensing Microwave Imaging with Matrix Completion

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Abstract

An innovative microwave imaging methodology to solve the inverse scattering problem in presence of sparse scatterers under the first order Born approximation (Born-I) is proposed. The developed approach is based on the effective integration of a single-task Bayesian compressive sensing (*ST-BCS*) solver with a customized matrix completion (*MC*) strategy. The *BCS-MC* is able to identify the "less reliable" solution coefficients and discard them, by successively competing the retrieved image with improved accuracy, especially when inverting highly blurred scattering data.

Some numerical results are shown in order to verify the effectiveness of the proposed methodology under different operative conditions.

1 Numerical Assessment: Two-Pixel Profile

Goal: show the performances of BCS when dealing with a sparse scatterer

- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$
- $D = 1296$ (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2\times\frac{2\pi}{\lambda}\times\frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2\left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- N scelto in modo da essere vicino a $\#DOF$: $N = 324$ (18×18)

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$
- Full-aspect measurements
- $M \approx 2ka \rightarrow M = 27$

Sources:

- Plane waves
- $V \approx 2ka \rightarrow V = 27$
- Amplitude $A = 1$
- Frequency: 300 MHz $(\lambda = 1)$

Object:

- Two square cylinders of side $\frac{\lambda}{6} = 0.1667$ (single pixel)
- $\varepsilon_r \in \{1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0\}$
- $\sigma = 0$ [S/m]

BCS parameters:

- Initial estimate of the noise: $n_0 = 1.0 \times 10^{-3}$
- Convergenze parameter: $\tau = 1.0 \times 10^{-8}$

MC parameters:

- Threshold: $\eta = 0.2$

RESULTS: $\varepsilon_r = 1.5$

Retrieved Profiles

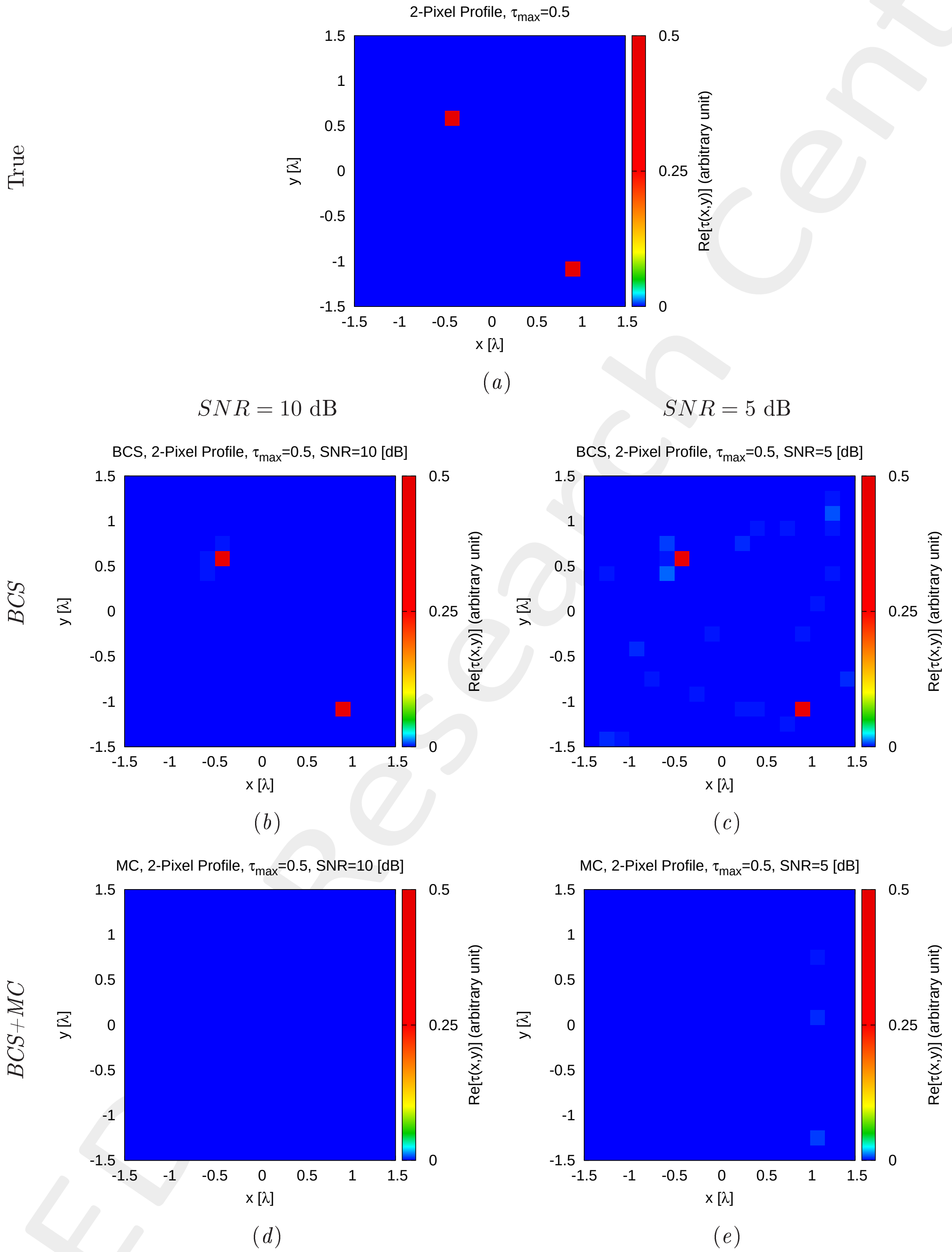

Figure 1: Actual object (a) and reconstructed object by $(b)(c)$ *BCS* and $(c)(d)$ *BCS+MC* when $(b)(d)$ $SNR = 10$ [dB], $(c)(e)$ $SNR = 5$ [dB].

Retrieved Profiles

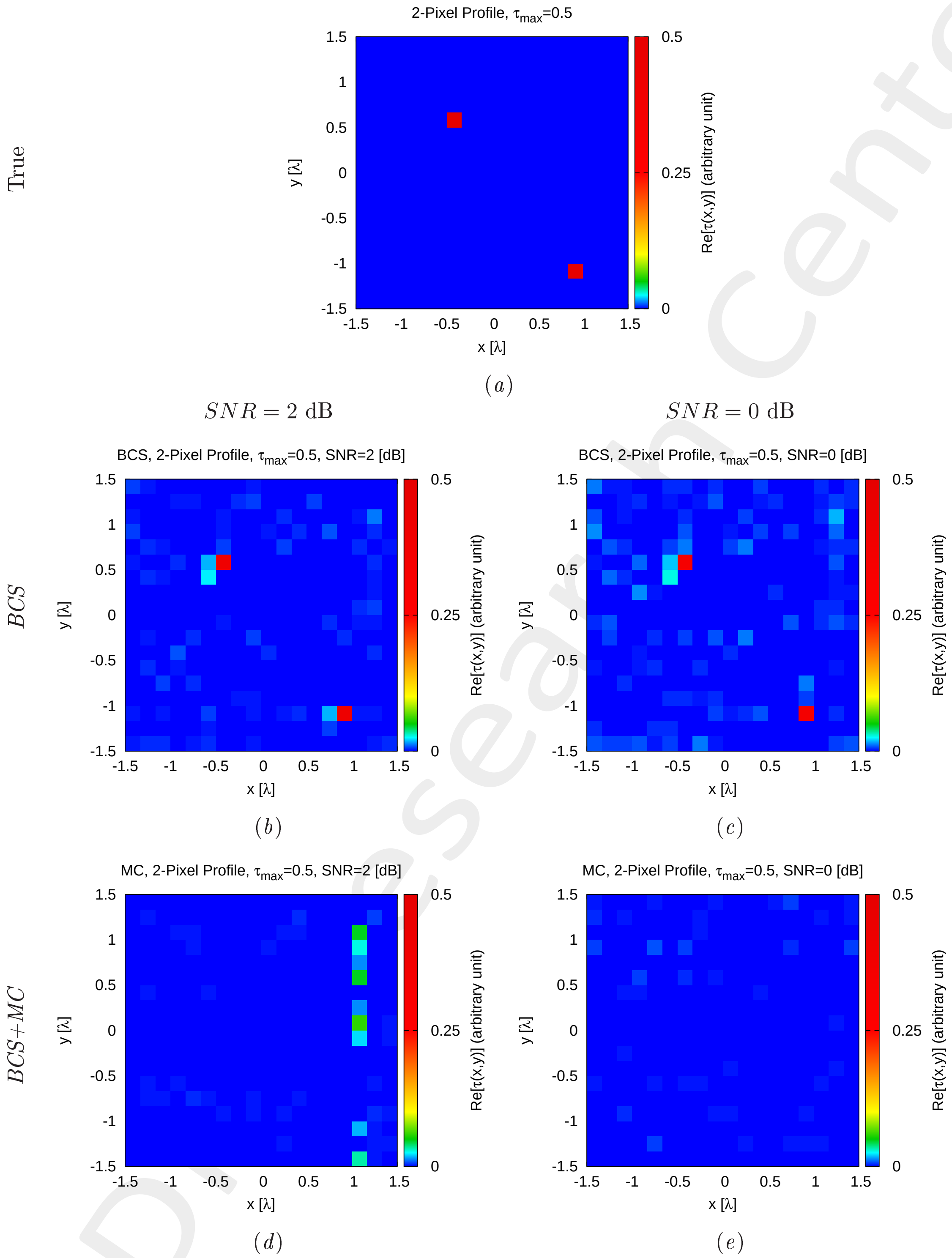

Figure 2: Actual object (a) and reconstructed object by $(b)(c)$ *BCS* and $(c)(d)$ *BCS+MC* when $(b)(d)$ $SNR = 2$ [dB], $(c)(e)$ $SNR = 0$ [dB].

Retrieved Profiles

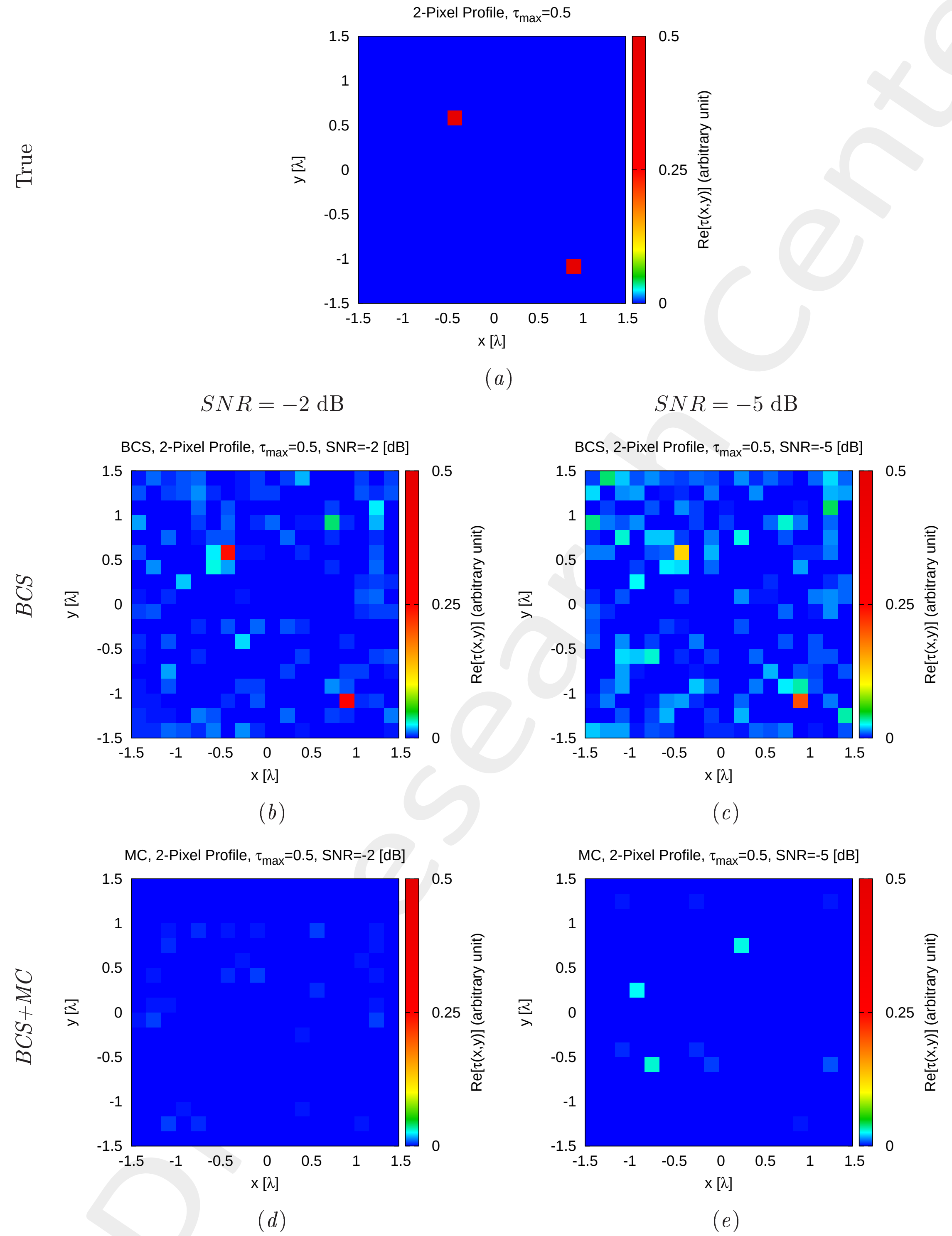

Figure 3: Actual object (a) and reconstructed object by $(b)(c)$ *BCS* and $(c)(d)$ *BCS+MC* when $(b)(d)$ $SNR = -2$ [dB], $(c)(e)$ $SNR = -5$ [dB].

Confidence Levels

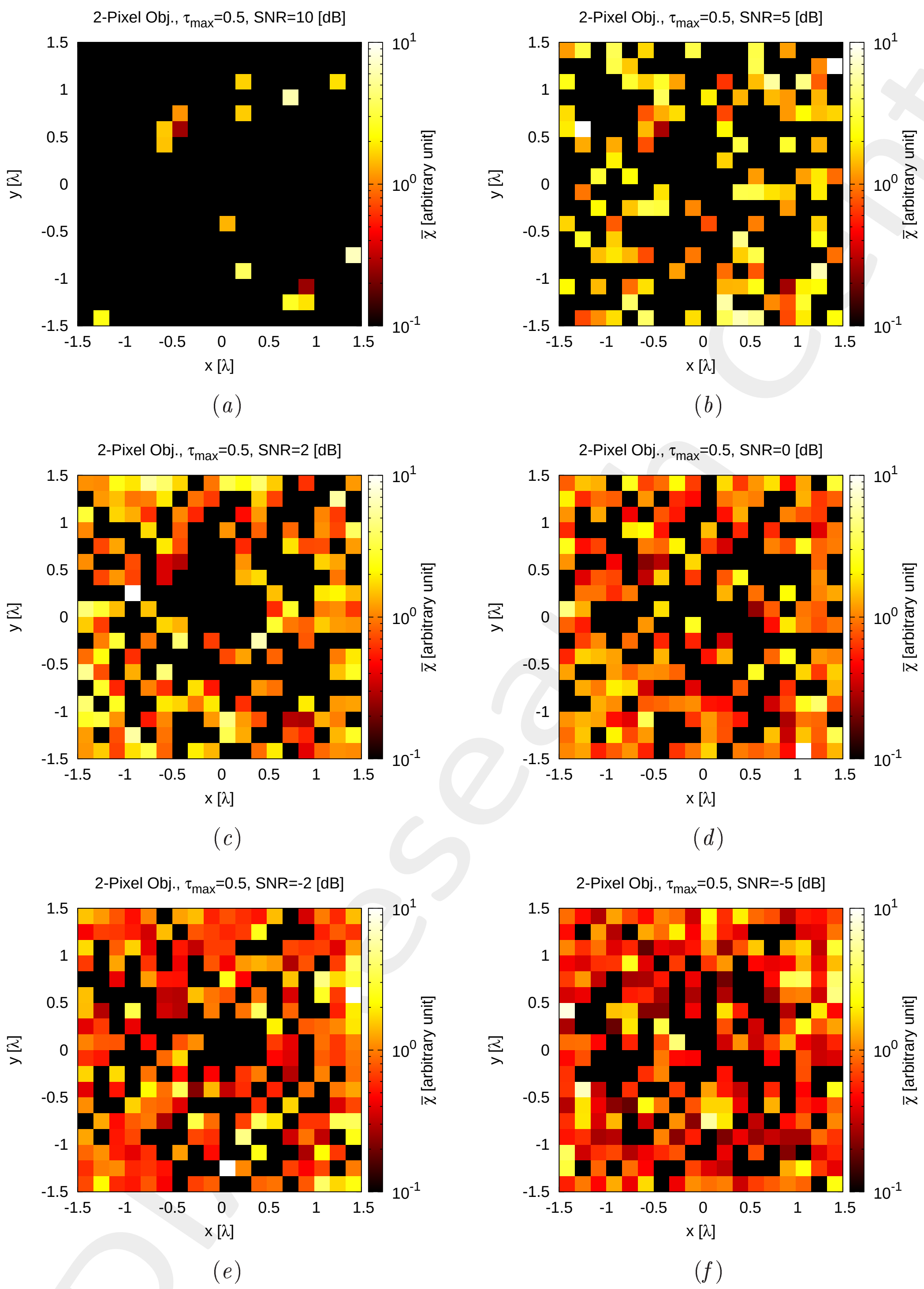

Figure 4: Confidence Levels when (a) $SNR = 10$ [dB], (b) $SNR = 5$ [dB], (c) $SNR = 2$ [dB], (d) $SNR = 0$ [dB], (e) $SNR = -2$ [dB], (f) $SNR = -5$ [dB].

RESULTS: $\varepsilon_r = 2.0$

Retrieved Profiles

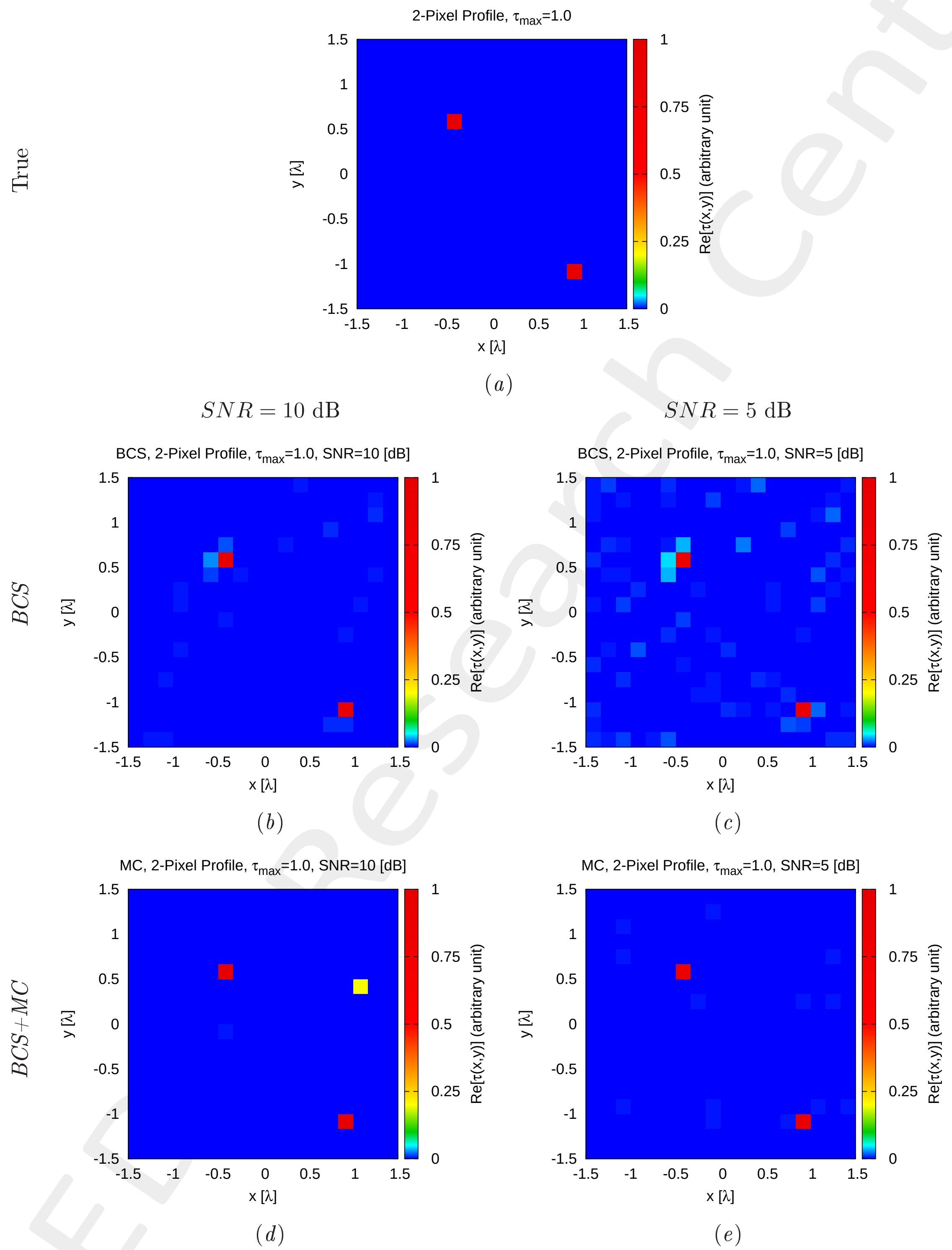

Figure 5: Actual object (a) and reconstructed object by $(b)(c)$ BCS and $(c)(d)$ BCS+MC when $(b)(d)$ $SNR =$ 10 [dB], $(c)(e)$ $SNR = 5$ [dB].

Retrieved Profiles

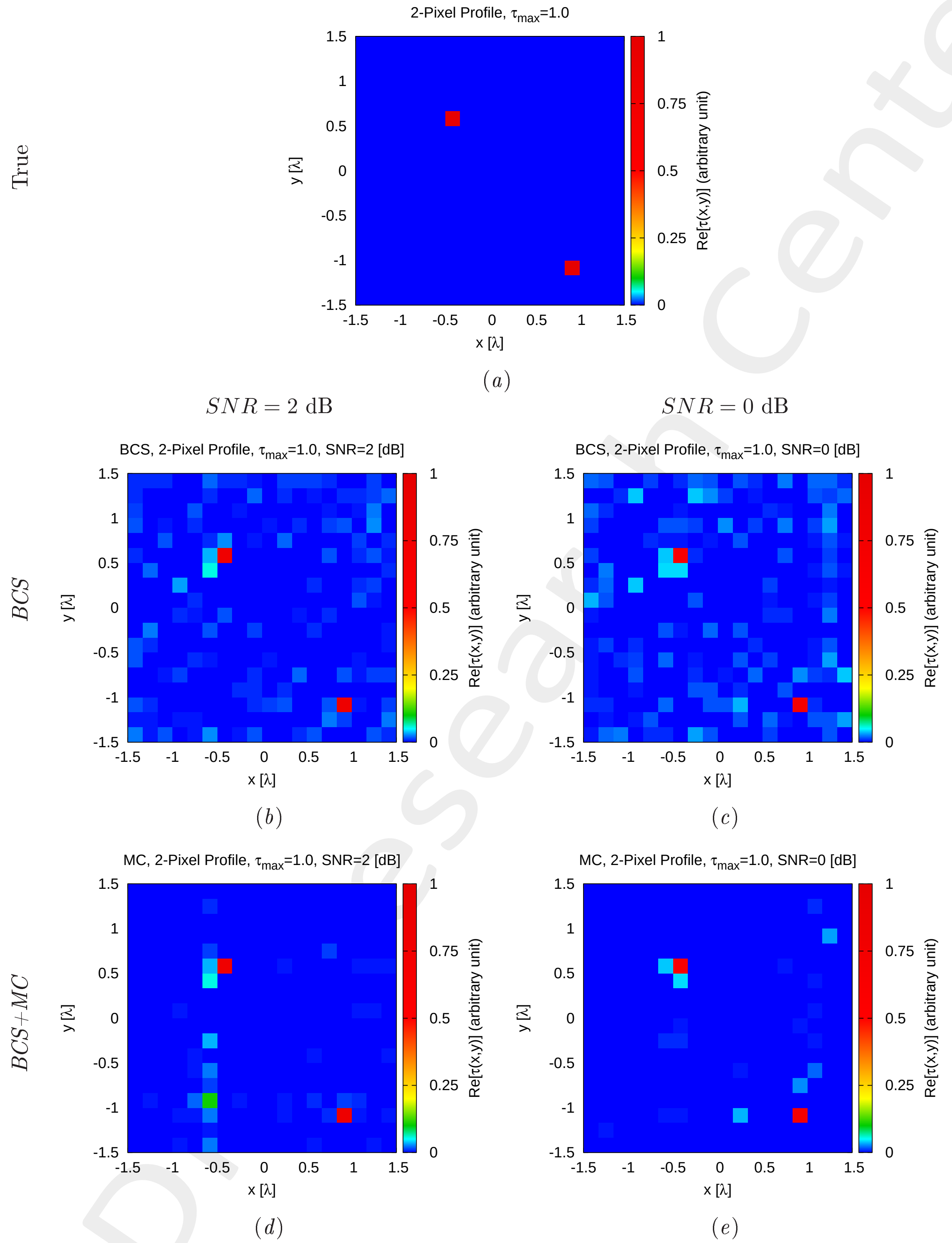

Figure 6: Actual object (a) and reconstructed object by $(b)(c)$ *BCS* and $(c)(d)$ *BCS+MC* when $(b)(d)$ $SNR = 2$ [dB], $(c)(e)$ $SNR = 0$ [dB].

Retrieved Profiles

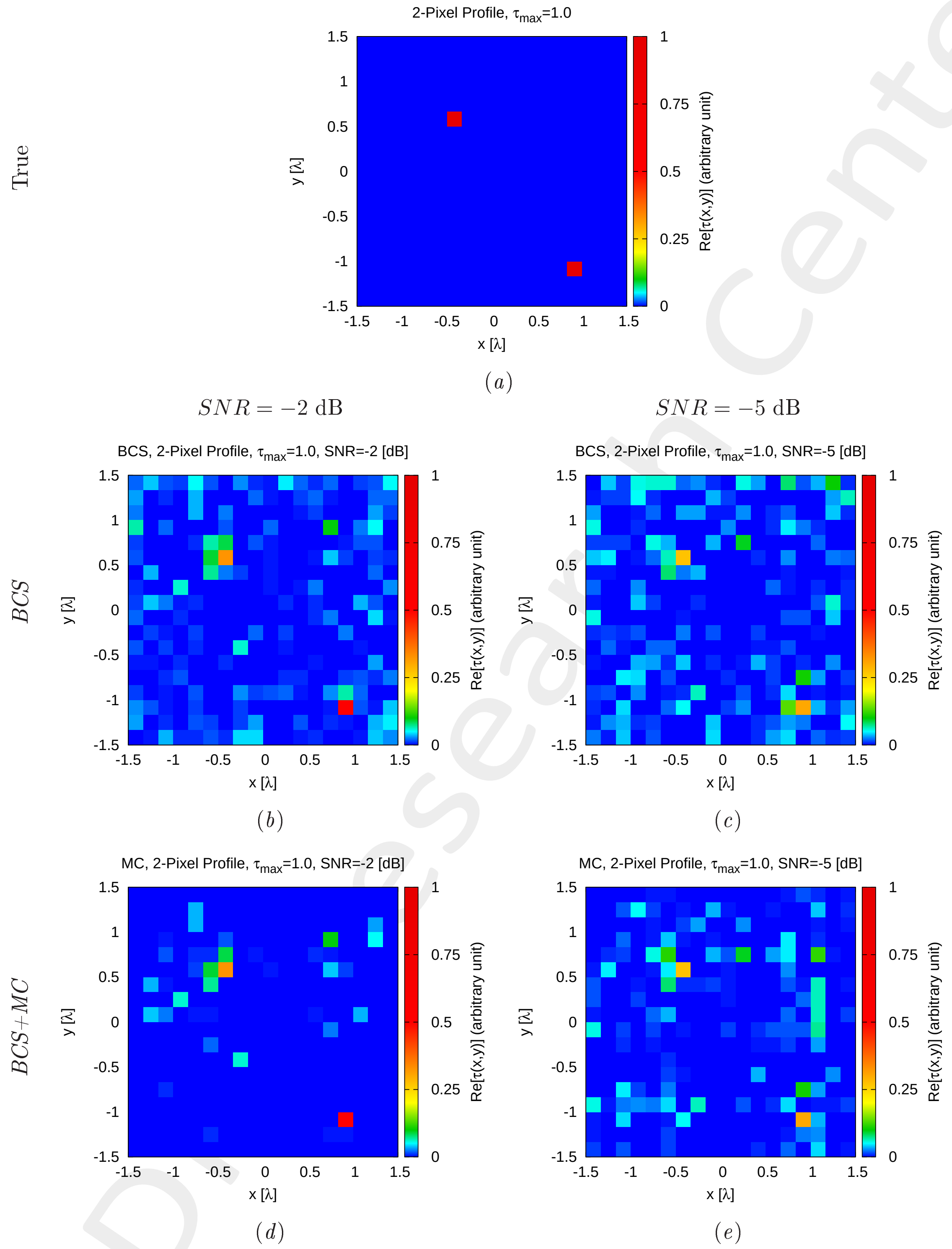

Figure 7: Actual object (a) and reconstructed object by $(b)(c)$ *BCS* and $(c)(d)$ *BCS+MC* when $(b)(d)$ $SNR = -2$ [dB], $(c)(e)$ $SNR = -5$ [dB].

Confidence Levels

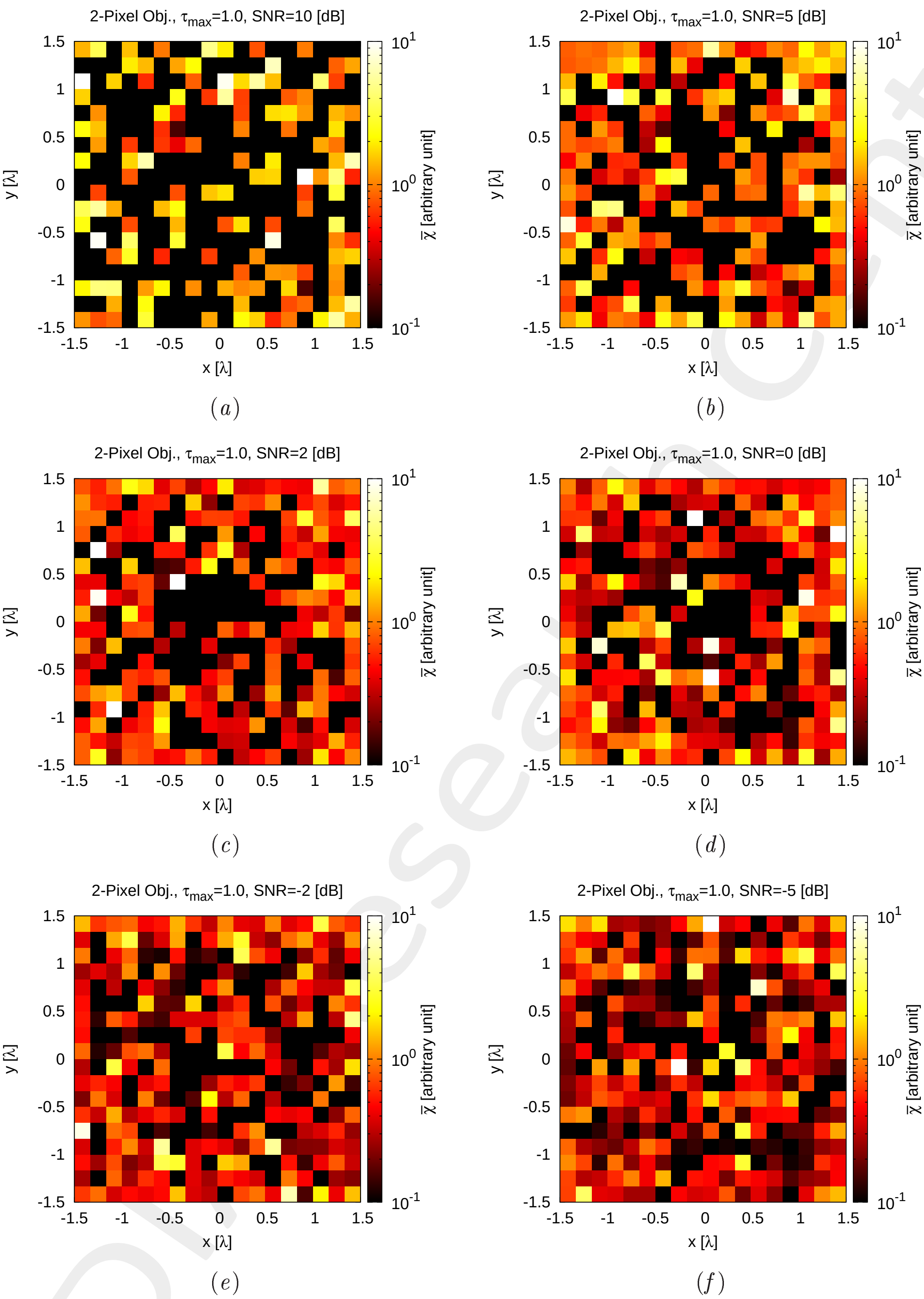

Figure 8: Confidence Levels when (a) $SNR = 10$ [dB], (b) $SNR = 5$ [dB], (c) $SNR = 2$ [dB], (d) $SNR = 0$ [dB], (e) $SNR = -2$ [dB], (f) $SNR = -5$ [dB].

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