Microwave Imaging of Sparse Objects through Matrix Completion

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Abstract

In this work, a novel approach to solve the inverse scattering (*IS*) problem to image sparse and weak targets is presented. Towards this aim, the *2D-TM IS* problem at hand is formulated within the first-order Born approximation and solved by integrating a Bayesian compressive sensing (*BCS*) solver with a customized matrix completion (*MC*) procedure. If from the one hand the *BCS* allows the exploitation of *sparseness priors* to regularize the *IS* problem, on the other hand the *MC* allows to enhance the reconstruction quality when dealing with highly noisy scattering data, allowing to filter out the less reliable solution coefficients and to complete the retrieved dielectric profile image.

1 Preliminary Numerical Assessment

Goal: show the performances of BCS when dealing with a sparse scatterer

- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$
- $D = 1296$ (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2\times\frac{2\pi}{\lambda}\times\frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2\left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- N scelto in modo da essere vicino a $\#DOF$: $N = 324$ (18×18)

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$
- Full-aspect measurements
- $M \approx 2ka \rightarrow M = 27$

Sources:

- Plane waves
- $V \approx 2ka \rightarrow V = 27$
- Amplitude $A = 1$
- Frequency: 300 MHz ($\lambda = 1$)

Object:

- Square cylinder of side $\frac{\lambda}{6} = 0.1667$ (single pixel)
- $\varepsilon_r \in \{1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0\}$
- $\sigma = 0$ [S/m]

BCS parameters:

- Initial estimate of the noise: $n_0 = 1.0 \times 10^{-3}$
- Convergenze parameter: $\tau = 1.0 \times 10^{-8}$

MC parameters:

- Threshold: $\eta = 0.2$

RESULTS: $\varepsilon_r = 1.5$

Retrieved Profiles

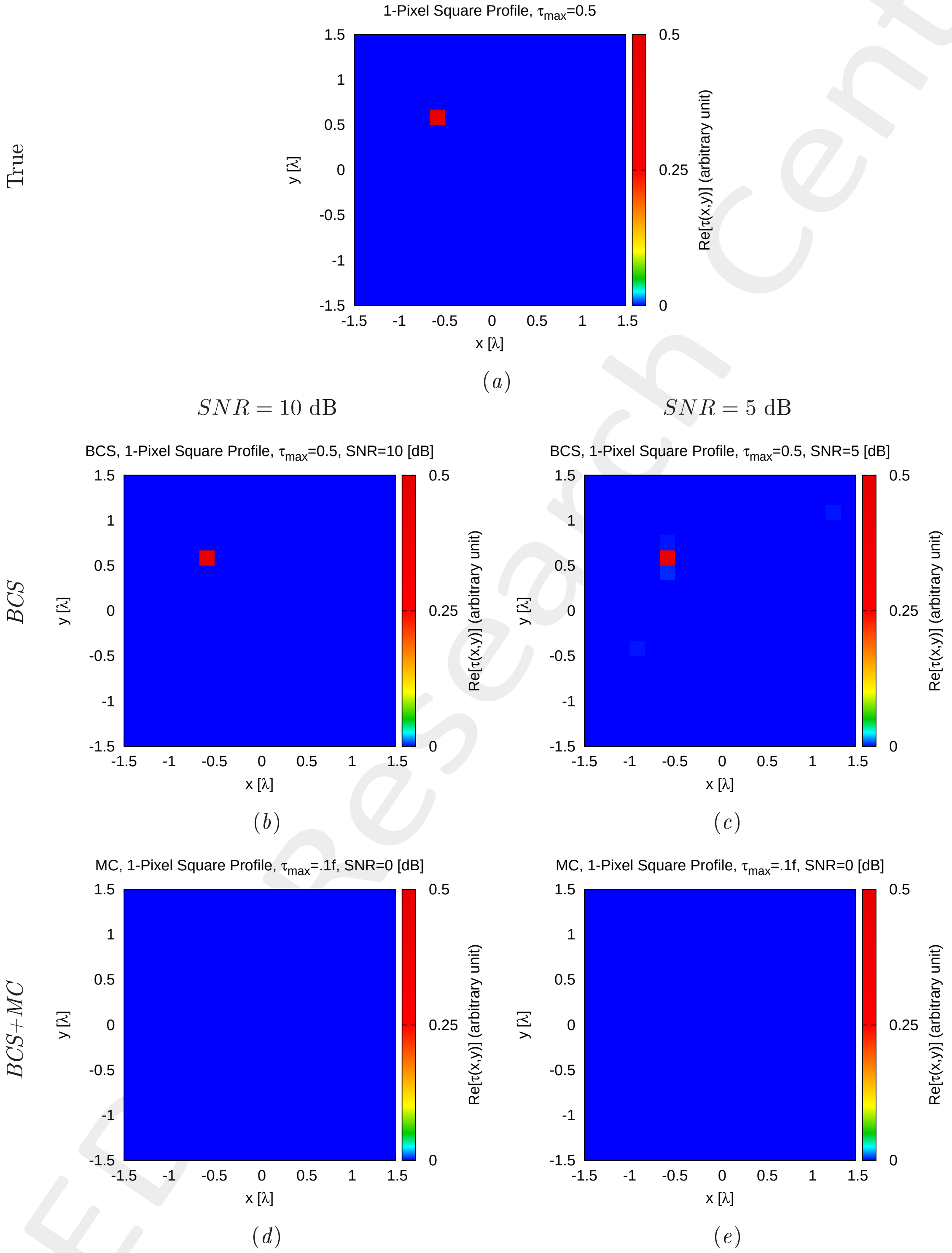
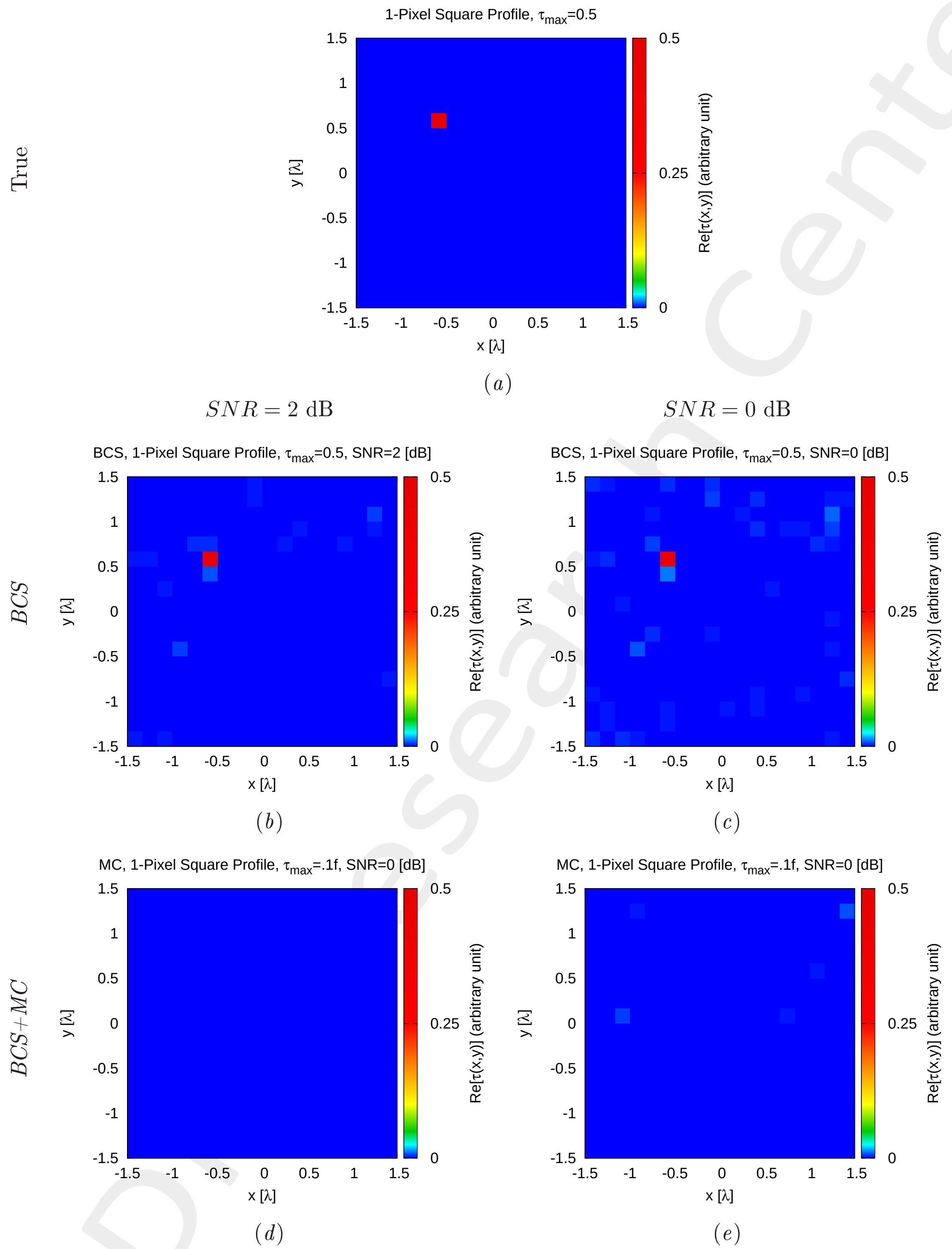


Figure 1: Actual object (a) and reconstructed object by $(b)(c)$ *BCS* and $(c)(d)$ *BCS+MC* when $(b)(d)$ $SNR = 10$ [dB], $(c)(e)$ $SNR = 5$ [dB].

Retrieved Profiles



(a)

(b)

(c)

(d)

(e)

Figure 2: Actual object (a) and reconstructed object by $(b)(c)$ *BCS* and $(c)(d)$ *BCS+MC* when $(b)(d)$ $SNR = 2$ [dB], $(c)(e)$ $SNR = 0$ [dB].

Retrieved Profiles

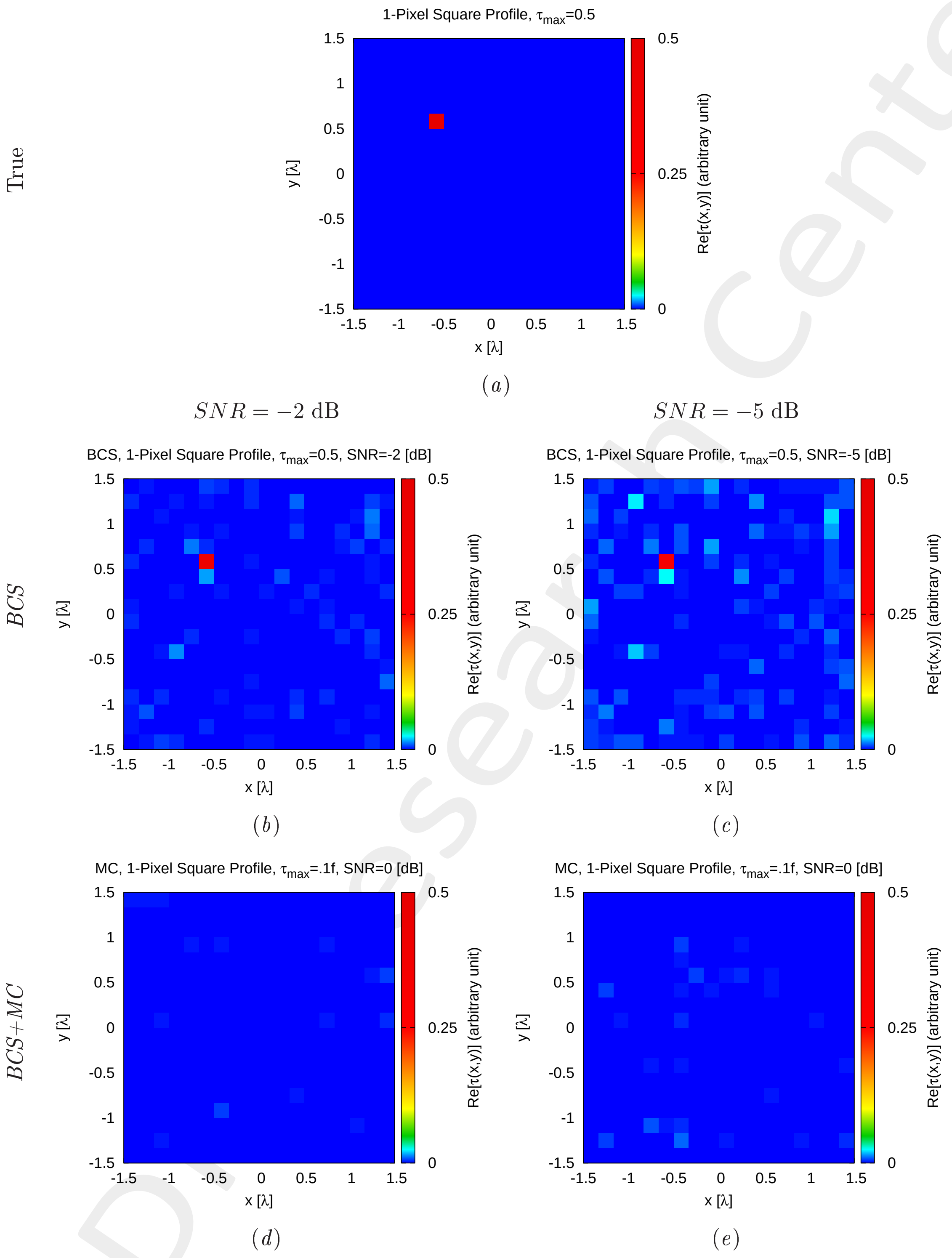
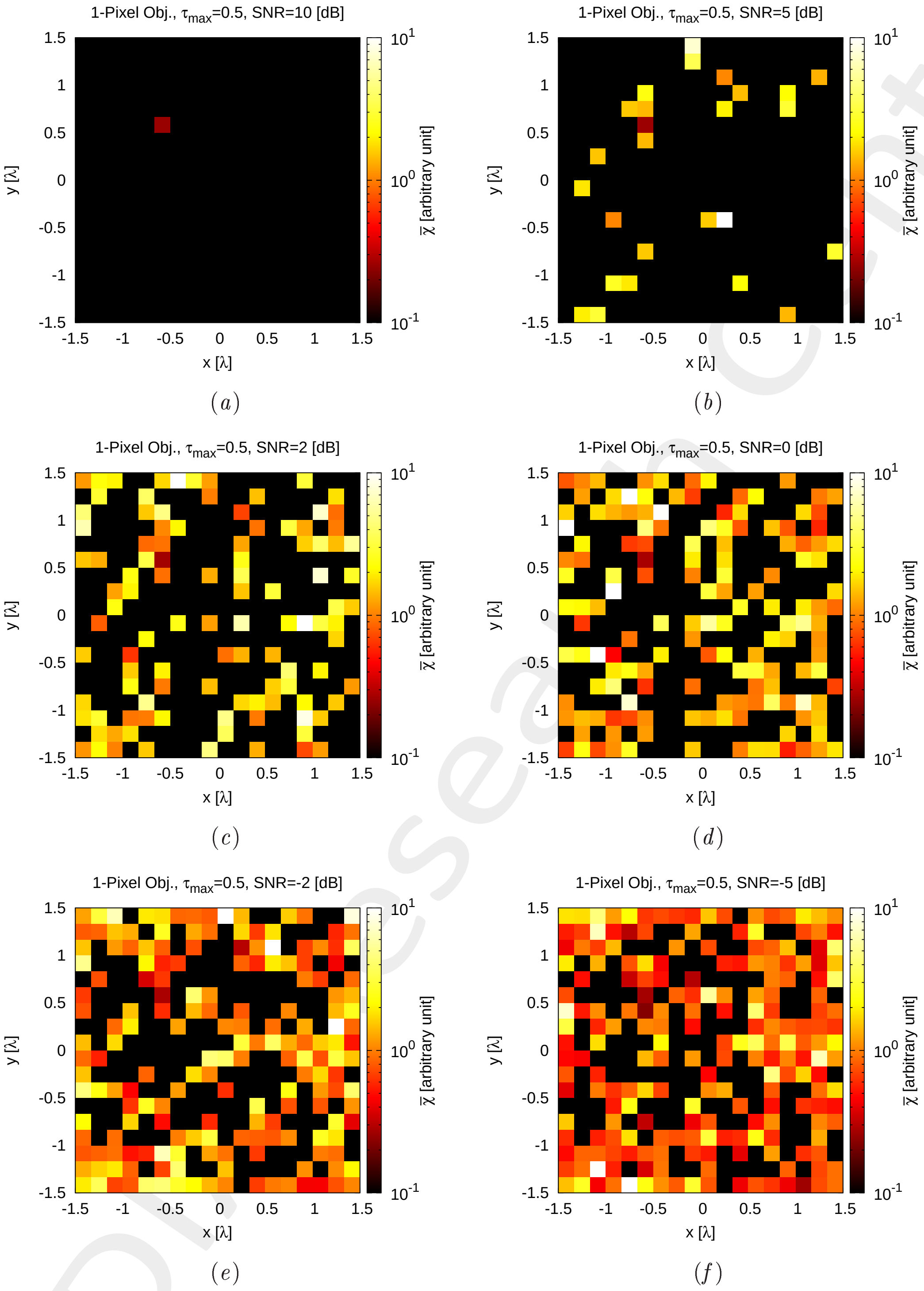


Figure 3: Actual object (a) and reconstructed object by $(b)(c)$ *BCS* and $(c)(d)$ *BCS+MC* when $(b)(d)$ $SNR = -2$ [dB], $(c)(e)$ $SNR = -5$ [dB].

Confidence Levels

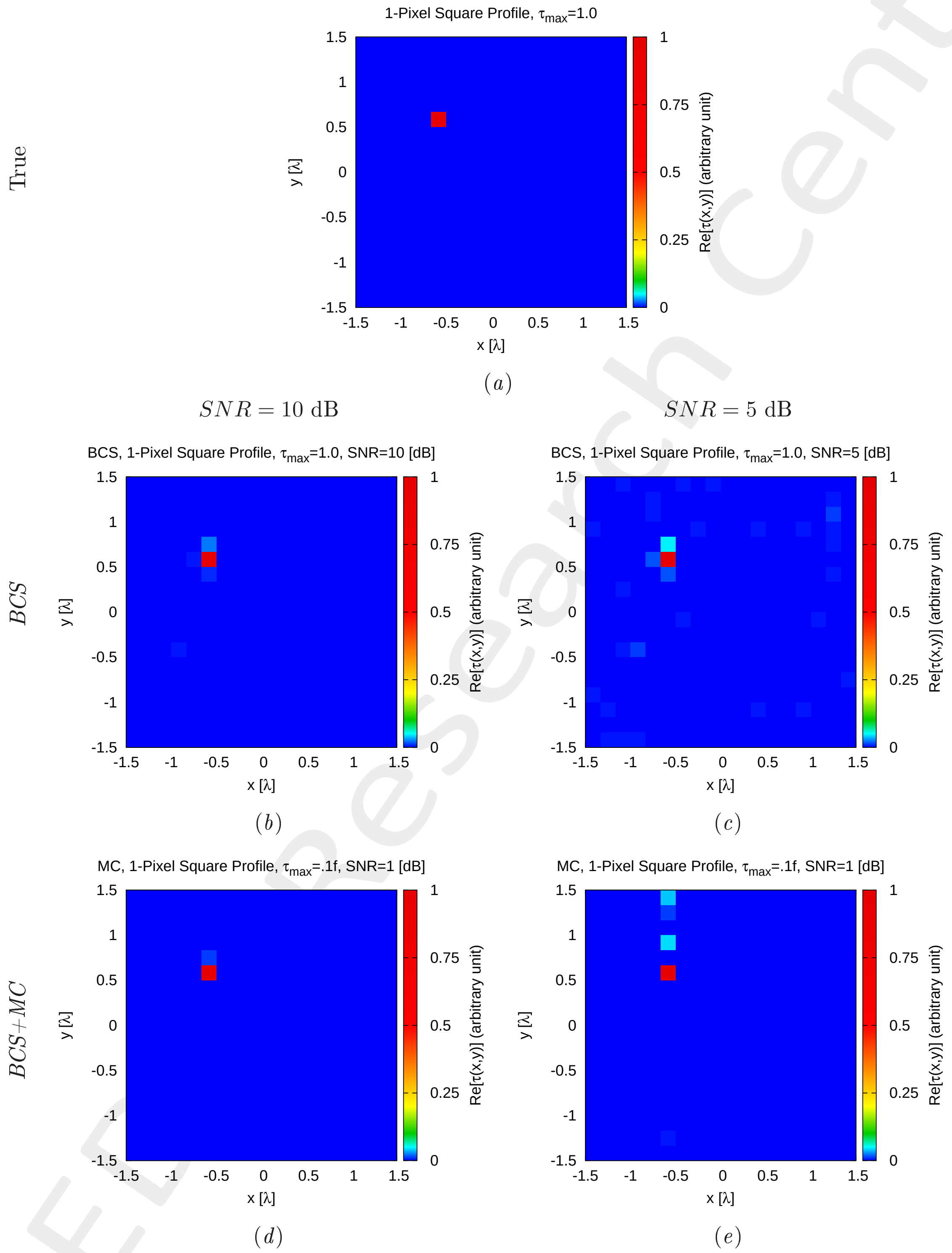


(a) (b) (c) (d) (e) (f)

Figure 4: Confidence Levels when (a) $SNR = 10$ [dB], (b) $SNR = 5$ [dB], (c) $SNR = 2$ [dB], (d) $SNR = 0$ [dB], (e) $SNR = -2$ [dB], (f) $SNR = -5$ [dB].

RESULTS: $\varepsilon_r = 2.0$

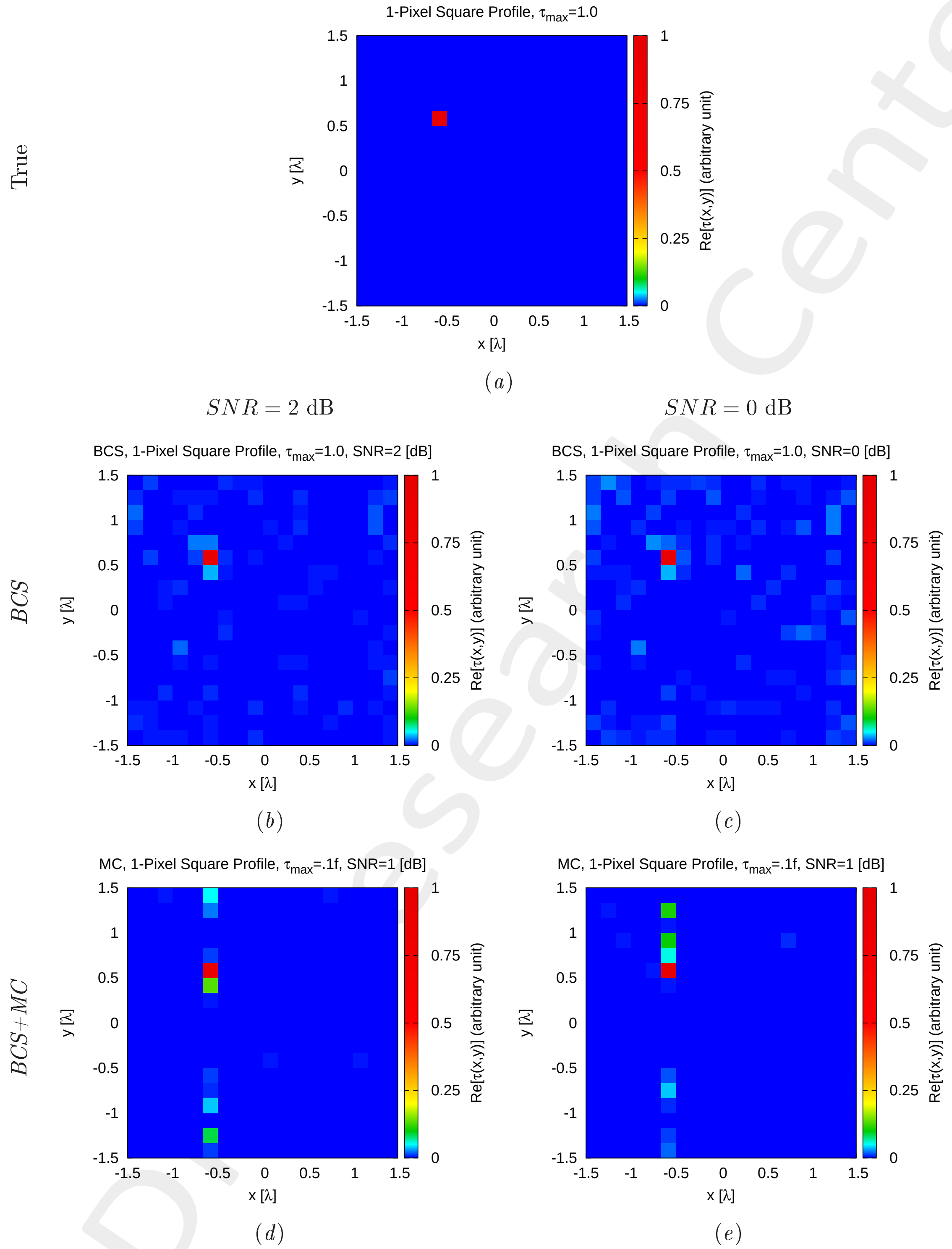
Retrieved Profiles



(a) (b) (c) (d) (e)

Figure 5: Actual object (a) and reconstructed object by $(b)(c)$ BCS and $(c)(d)$ $BCS{+}MC$ when $(b)(d)$ $SNR = 10$ [dB], $(c)(e)$ $SNR = 5$ [dB].

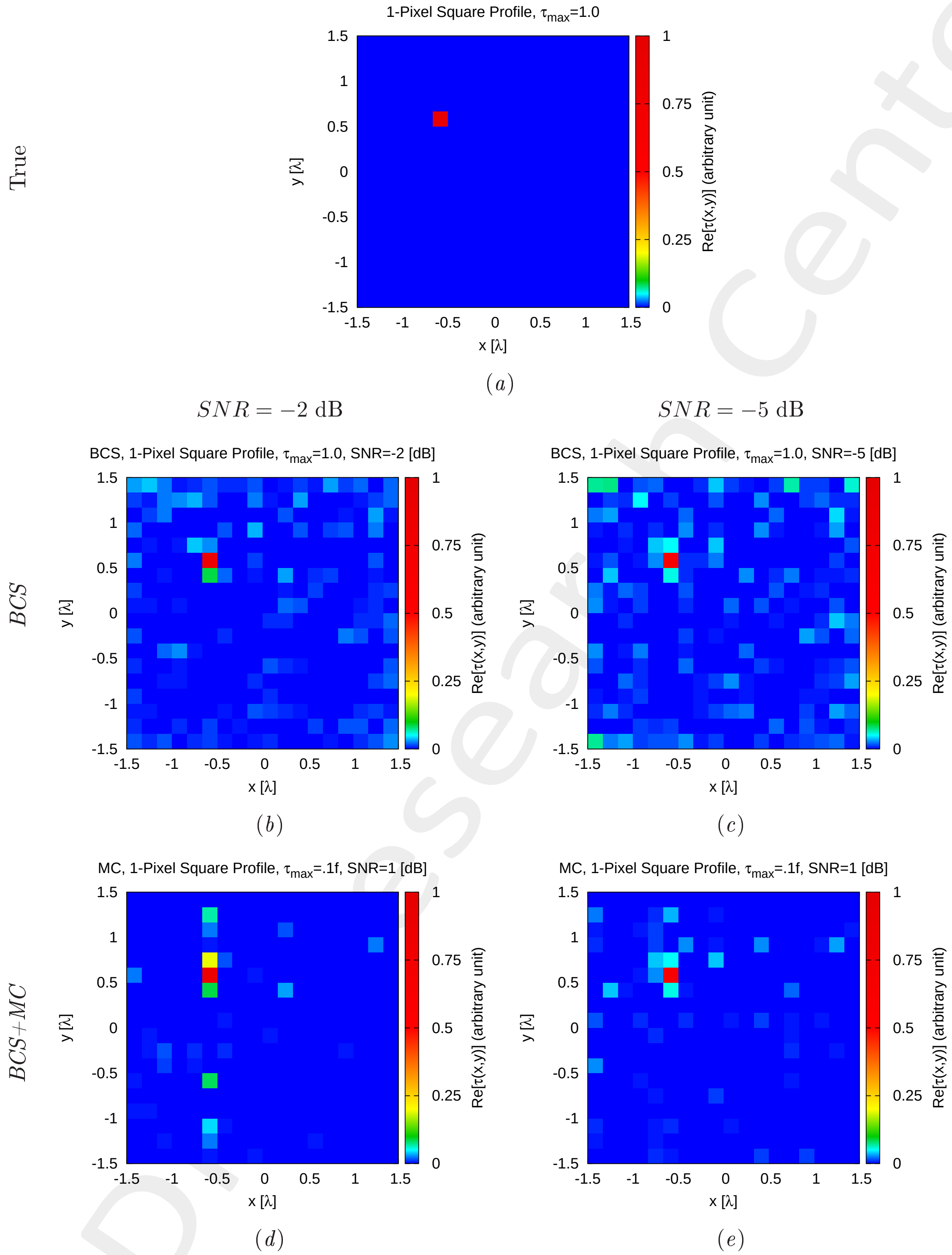
Retrieved Profiles



(a) (b) (c) (d) (e)

Figure 6: Actual object (a) and reconstructed object by $(b)(c)$ *BCS* and $(c)(d)$ *BCS+MC* when $(b)(d)$ $SNR = 2$ [dB], $(c)(e)$ $SNR = 0$ [dB].

Retrieved Profiles



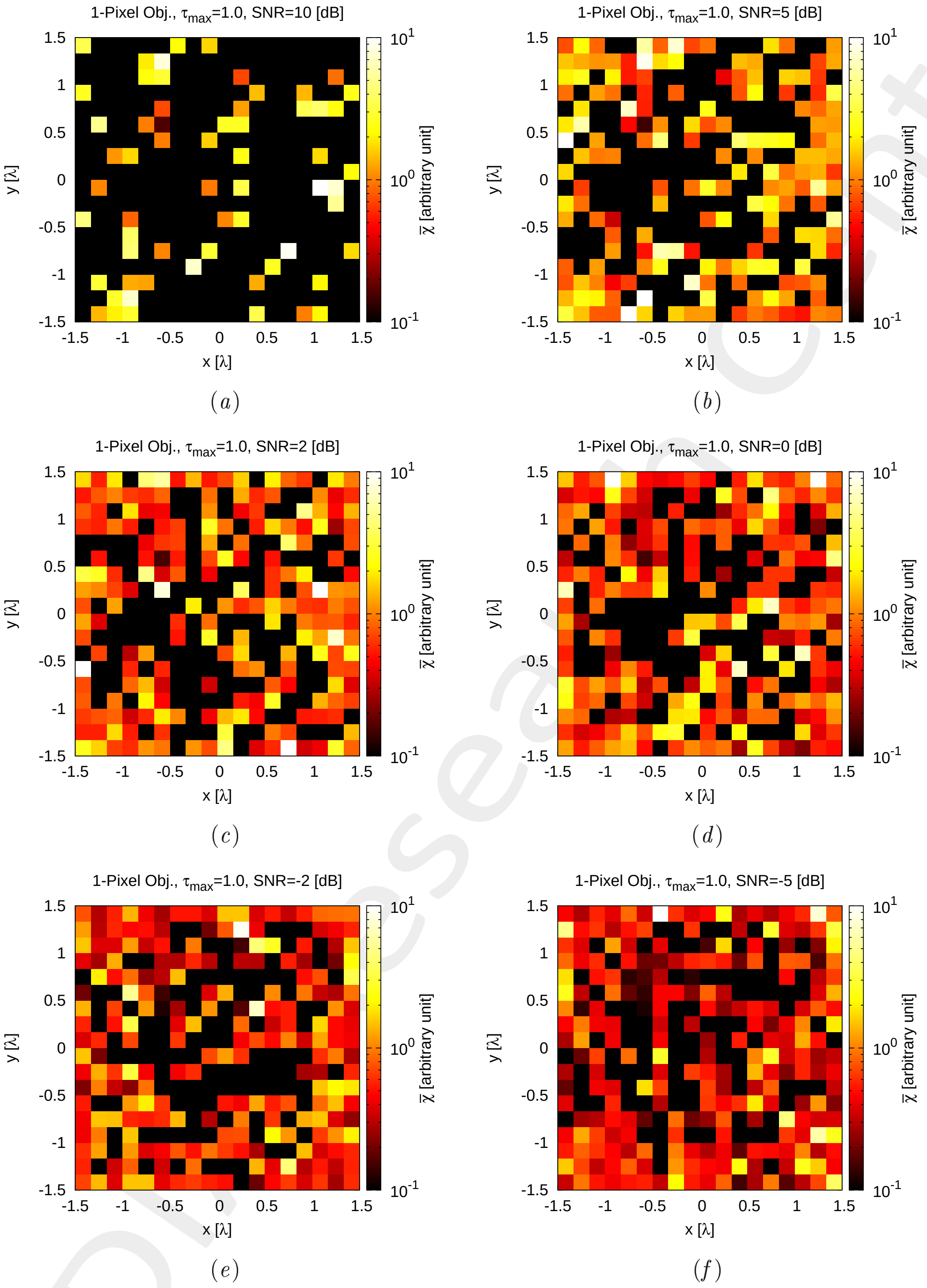
(a)

(b) (c)

(d) (e)

Figure 7: Actual object (a) and reconstructed object by $(b)(c)$ *BCS* and $(c)(d)$ *BCS+MC* when $(b)(d)$ $SNR = -2$ [dB], $(c)(e)$ $SNR = -5$ [dB].

Confidence Levels



(*a*) (*b*)

(*c*) (*d*)

(*e*) (*f*)

Figure 8: Confidence Levels when (*a*) $SNR = 10$ [dB], (*b*) $SNR = 5$ [dB], (*c*) $SNR = 2$ [dB], (*d*) $SNR = 0$ [dB], (*e*) $SNR = -2$ [dB], (*f*) $SNR = -5$ [dB].

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