Thinning Planar Phased Arrays by Means of Hybrid Analytical-Evolutionary Optimization

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Abstract

This work deals with the design of planar thinned phased arrays. An innovative 2D hybrid thinning procedure is proposed to combine the efficiency of analytical almost difference sets (*ADS*s) with the effectiveness of genetic algorithms (*GA*s) to enable faster convergence rates and improved side-lobe suppression with respect to classical thinning strategies. The performance of the proposed *ADSGA* method are carefully assessed by means of selected numerical results. Moreover, a comparison with competitive state-of-the-art approaches addressing the same design problem is shown, as well.

1 Problem II-b - PSL Minimization in Array Synthesis (Comparison ADSGA vs. Kopilovich [1] & Donelli [2])

In order to determine an optimal thinned configuration starting from the (usually) sub-optimal ADS arrangement with a given aperture size N_{ADS} and thinning factor ν_{ADS}, let us formulate the following constrained optimization problem

$$Min\left(F\{\rho\}\right) = \frac{max_{(u,v)\notin R_m^2}\left\{\left|S(u,v)\right|^2\right\}}{\left|S(0,0)\right|^2}$$

subject to $K \neq K_{ADS}$ ($K > K_{ADS}$) and $N_x \neq N_{x-ADS}$ and $N_y \neq N_{y-ADS}$ ($N > N_{x-ADS}$ and $N_y > N_{y-ADS}$) to be solved through ADSGA.

In such a case, the GA fitness function is defined as the PSL of the array while the constraints force the array to kept its descriptive parameters.

- PSL: Kopilovich
- Initialization: Random vs Hybrid
- Fitness: PSL and Thinning

$$\Psi(i) = \frac{\alpha}{PSL^i_{Kopilovich}} + \beta\nu^i$$

where i is associated to the i-th trial solution

RESULTS: $P = 6$, $Q = 6$, $K_{Kopilovich} = 15$

Setting Parameters of Algorithms

GA Parameters

- Chromosome Dimension $C = 36\, bits$
- Population Dimension $S = 20$
- Max Iteration number $K_{max} = 5000$

FFT Parameters

- $FFT\ Theta = 64$
- $FFT\ Phi = 64$

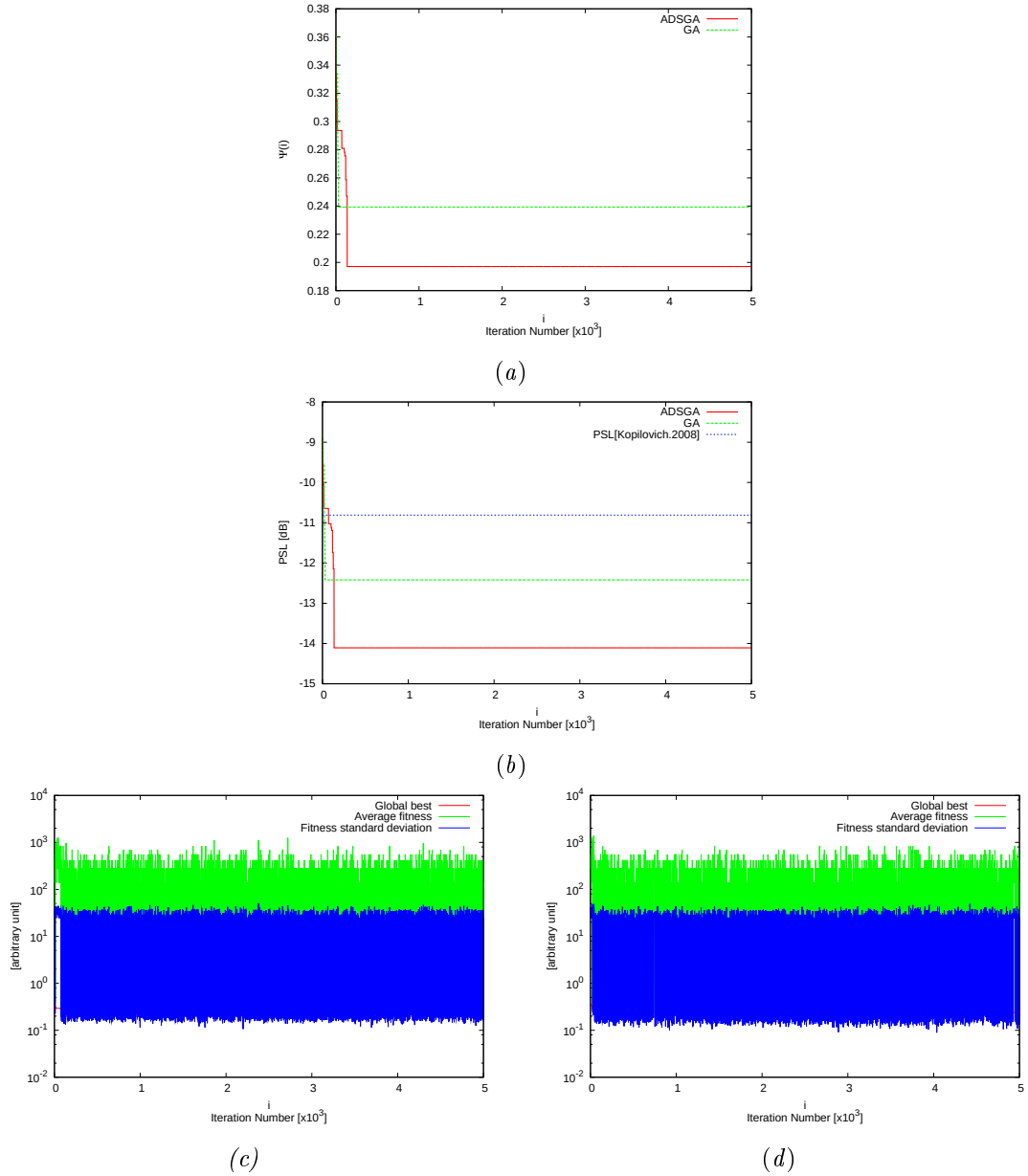


Figure 1.

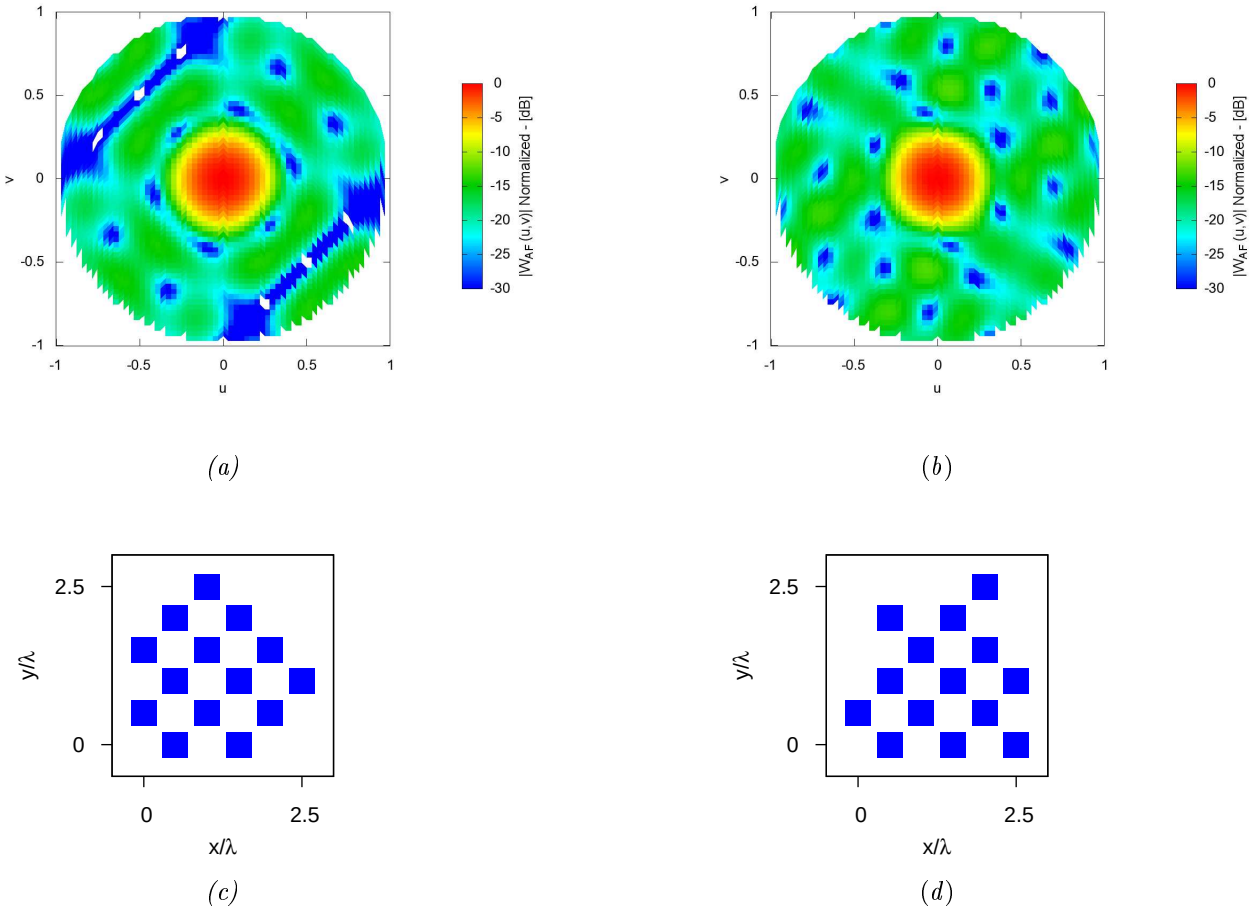
Figure 1: ADSGA approach (c), GA approach (d)

Array Parameters Starting Geometry

- Number of total cells $N = 25$
- Dimension X: 5
- Dimension Y: 5

Array Parameters Final Geometry

- Number of total cells $N = 36$
- Dimension X: 6
- Dimension Y: 6



(a) (b) (c) (d)

Figure 2.

Figure 2: ADSGA approach (*a*)-(*c*), GA approach (*b*)-(*d*)

RESULTS: $P = 6$, $Q = 6$, $K_{Kopilovich} = 21$

Setting Parameters of Algorithms

GA Parameters

- Chromosome Dimension $C = 36\, bits$
- Population Dimension $S = 20$
- Max Iteration number $K_{max} = 5000$

FFT Parameters

- $FFT\ Theta = 64$
- $FFT\ Phi = 64$

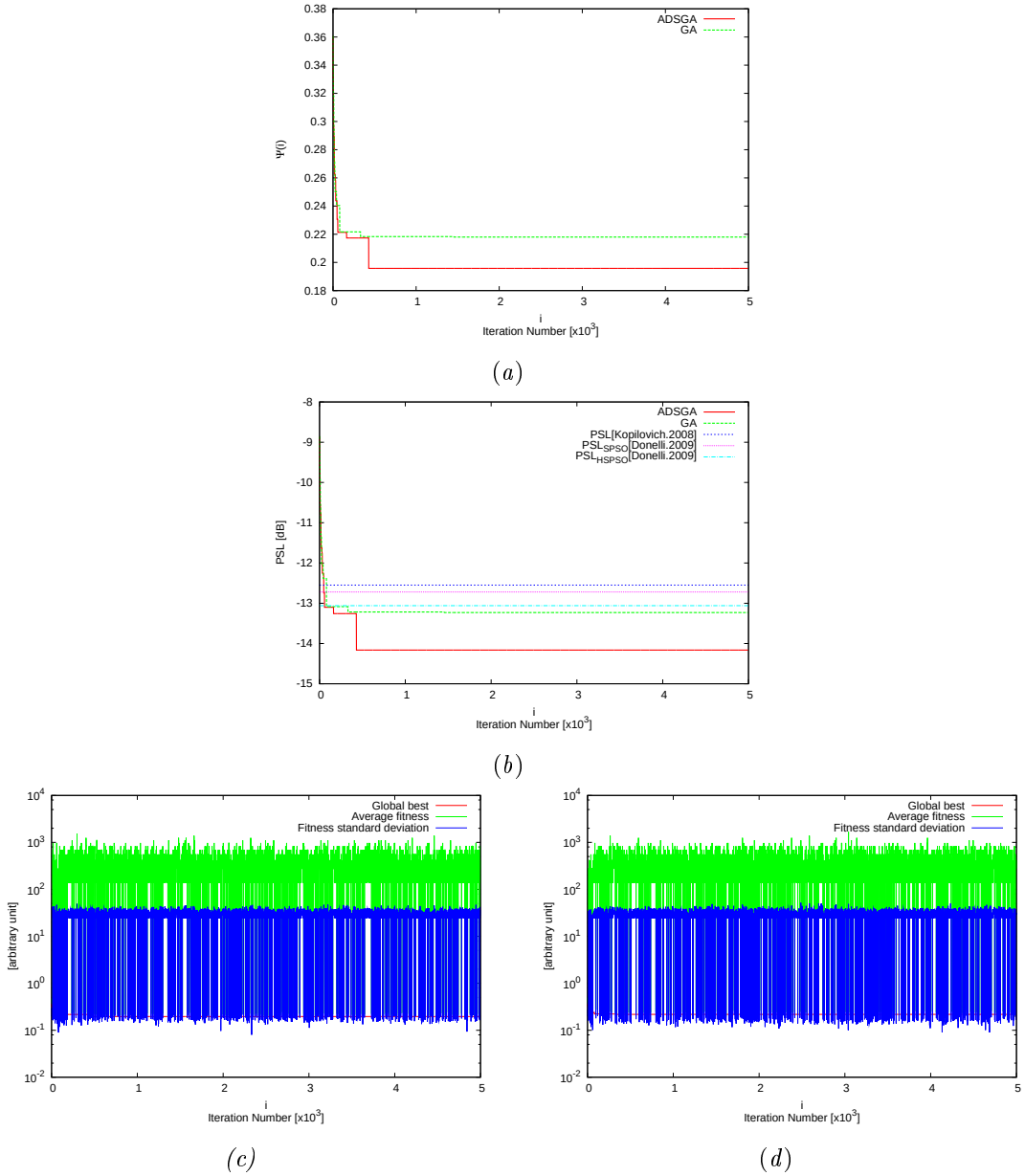


Figure 3.

Figure 3: ADSGA approach (c), GA approach (d)

Array Parameters Starting Geometry

- Number of total cells $N = 25$
- Dimension X: 5
- Dimension Y: 5

Array Parameters Final Geometry

- Number of total cells $N = 36$
- Dimension X: 6
- Dimension Y: 6

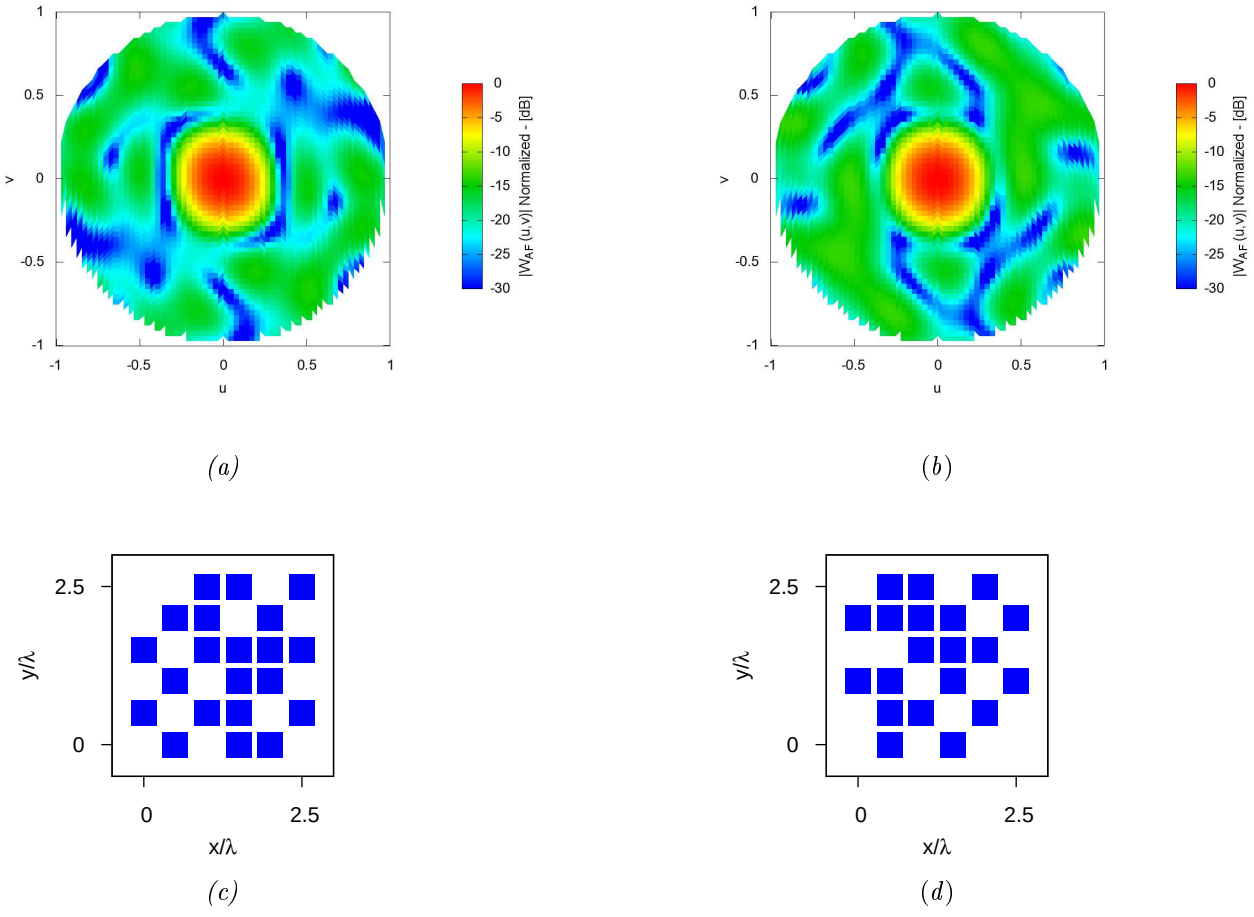


Figure 4.

Figure 4: ADSGA approach (a)-(c), GA approach (b)-(d)

RESULTS: $P = 8$, $Q = 8$, $K_{Kopilovich} = 28$

Setting Parameters of Algorithms

GA Parameters

- Chromosome Dimension $C = 64\,bits$
- Population Dimension $S = 20$
- Max Iteration number $K_{max} = 5000$

FFT Parameters

- $FFT\ Theta = 128$
- $FFT\ Phi = 128$

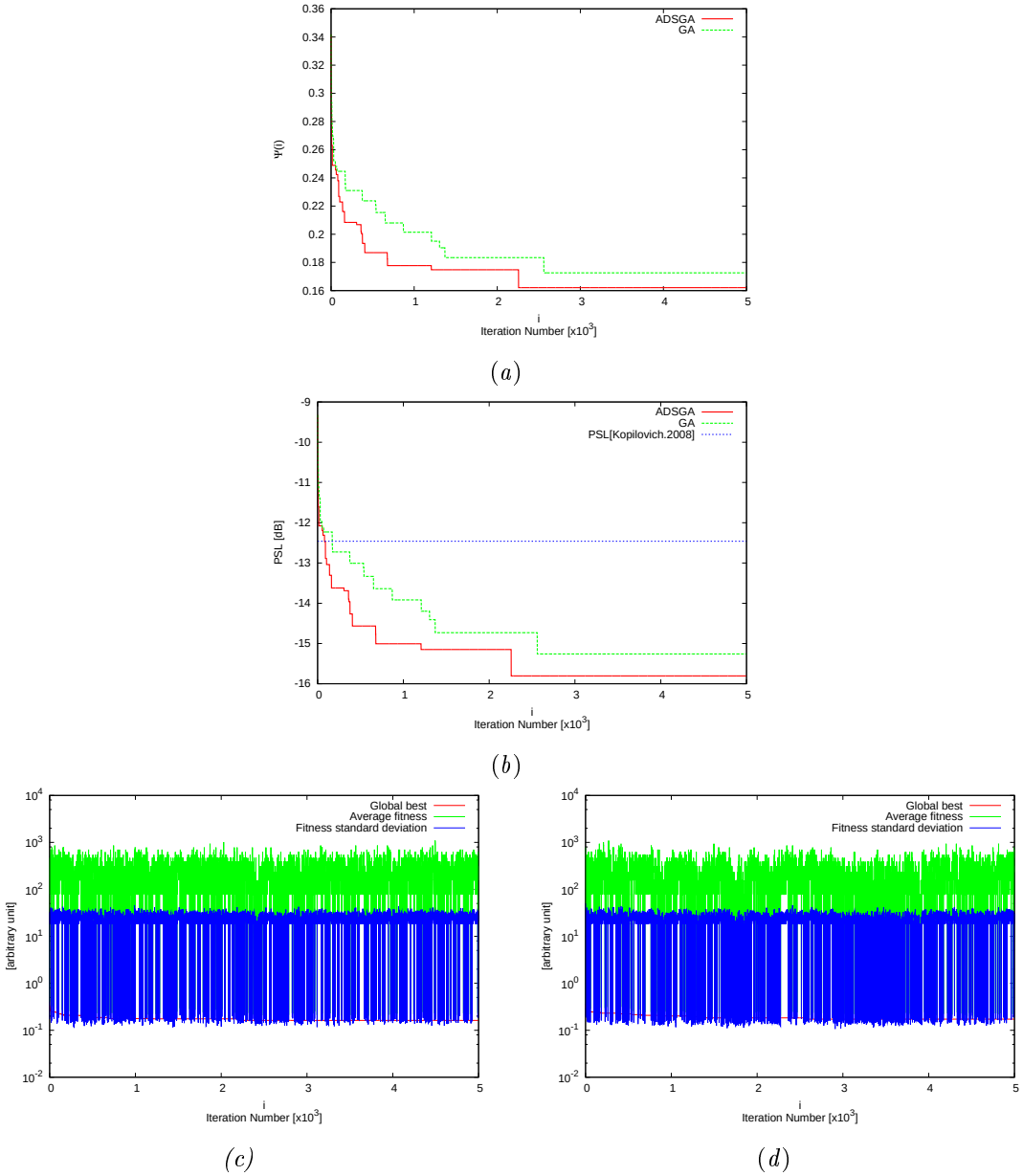


Figure 5.

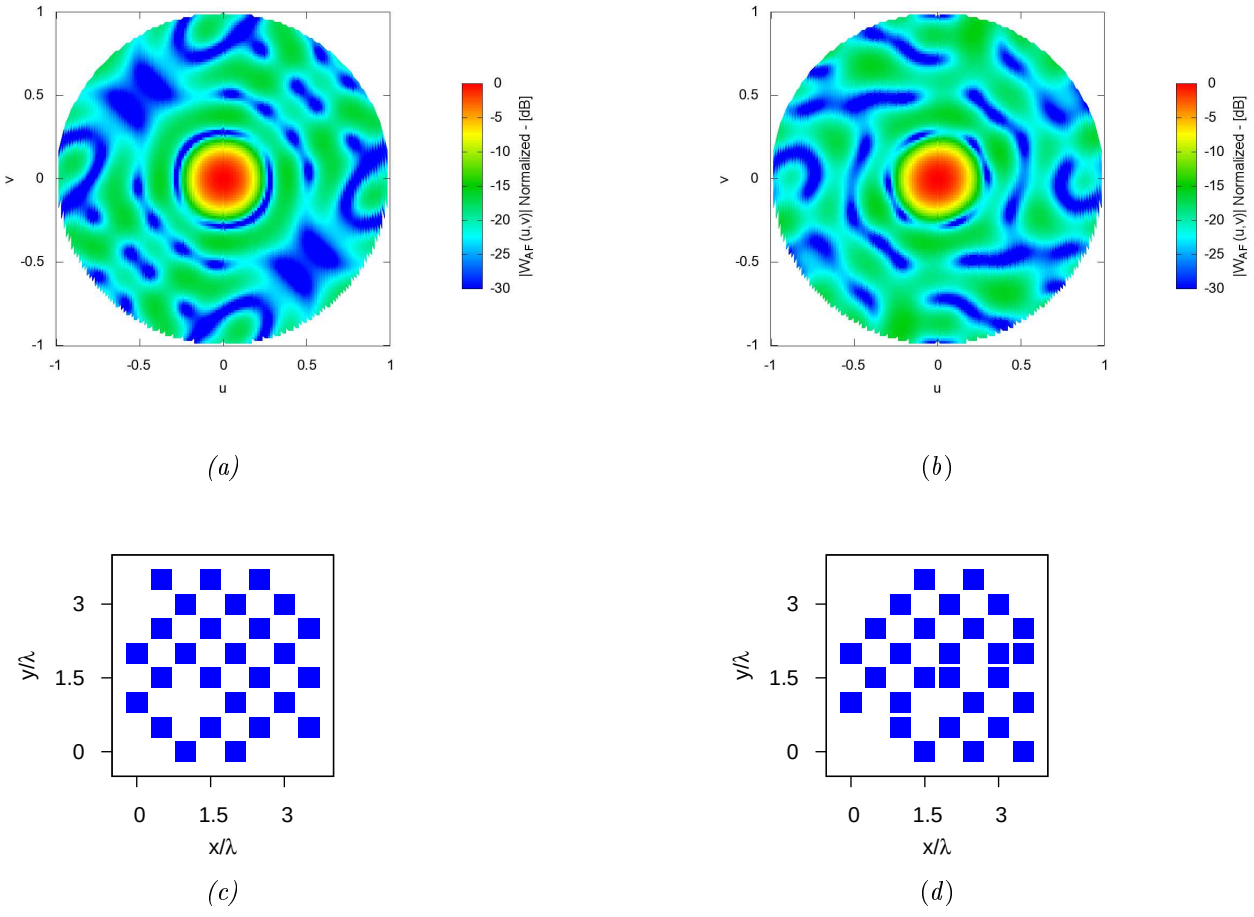
Figure 5: ADSGA approach (c), GA approach (d)

Array Parameters Starting Geometry

- Number of total cells $N = 49$
- Dimension X: 7
- Dimension Y: 7

Array Parameters Final Geometry

- Number of total cells $N = 64$
- Dimension X: 8
- Dimension Y: 8



(a) (b)

(c) (d)

Figure 6.

Figure 6: ADSGA approach (*a*)-(*c*), GA approach (*b*)-(*d*)

RESULTS: $P = 8$, $Q = 8$, $K_{Kopilovich} = 36$

Setting Parameters of Algorithms

GA Parameters

- Chromosome Dimension $C = 64\,bits$
- Population Dimension $S = 20$
- Max Iteration number $K_{max} = 5000$

FFT Parameters

- $FFT\ Theta = 128$
- $FFT\ Phi = 128$

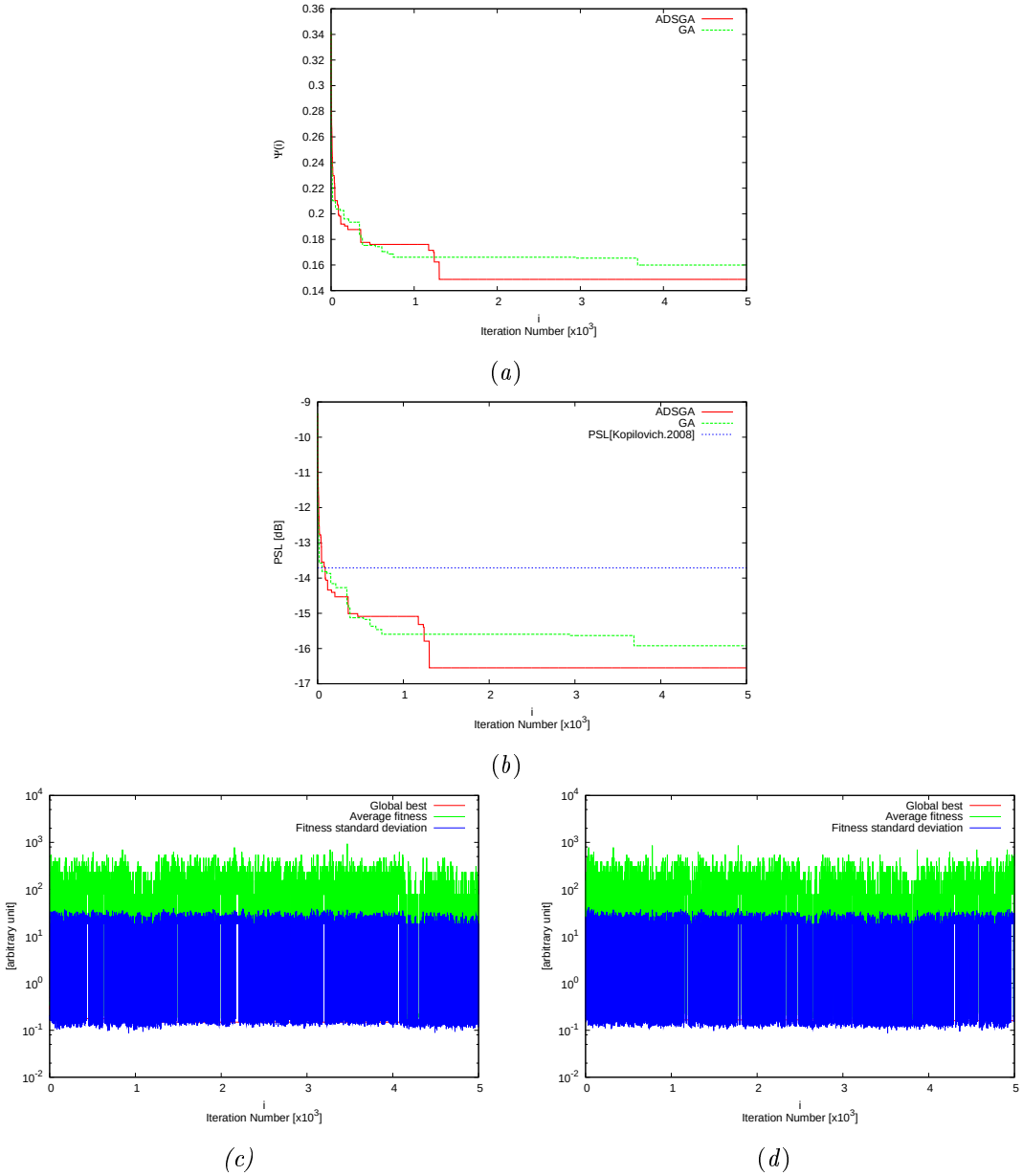


Figure 7.

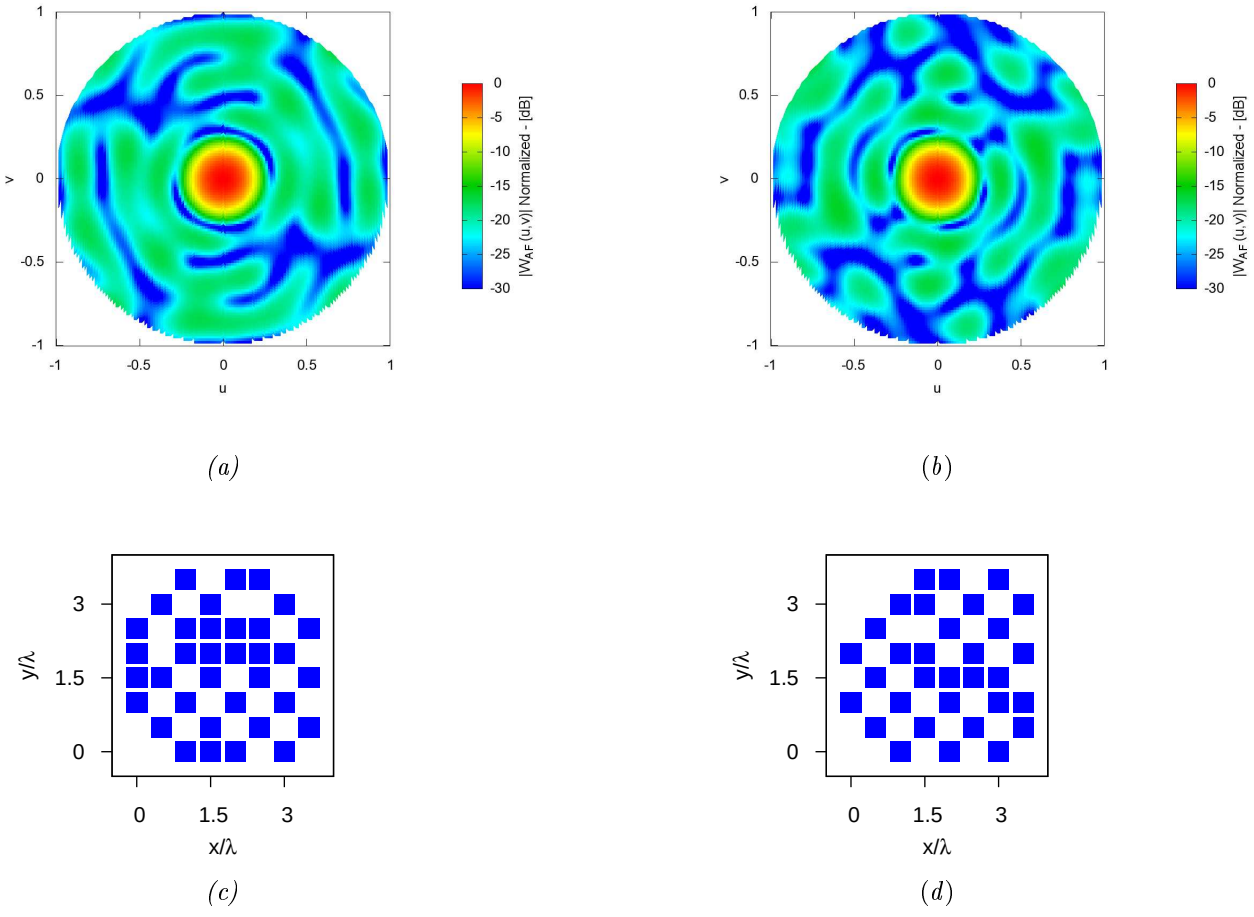
Figure 7: ADSGA approach (c), GA approach (d)

Array Parameters Starting Geometry

- Number of total cells $N = 49$
- Dimension X: 7
- Dimension Y: 7

Array Parameters Final Geometry

- Number of total cells $N = 64$
- Dimension X: 8
- Dimension Y: 8



(a) (b) (c) (d)

Figure 8.

Figure 8: ADSGA approach (*a*)-(*c*), GA approach (*b*)-(*d*)

RESULTS: $P = 12$, $Q = 12$, $K_{Kopilovich} = 66$

Setting Parameters of Algorithms

GA Parameters

- Chromosome Dimension $C = 144\, bits$
- Population Dimension $S = 40$
- Max Iteration number $K_{max} = 5000$

FFT Parameters

- $FFT\ Theta = 128$
- $FFT\ Phi = 128$

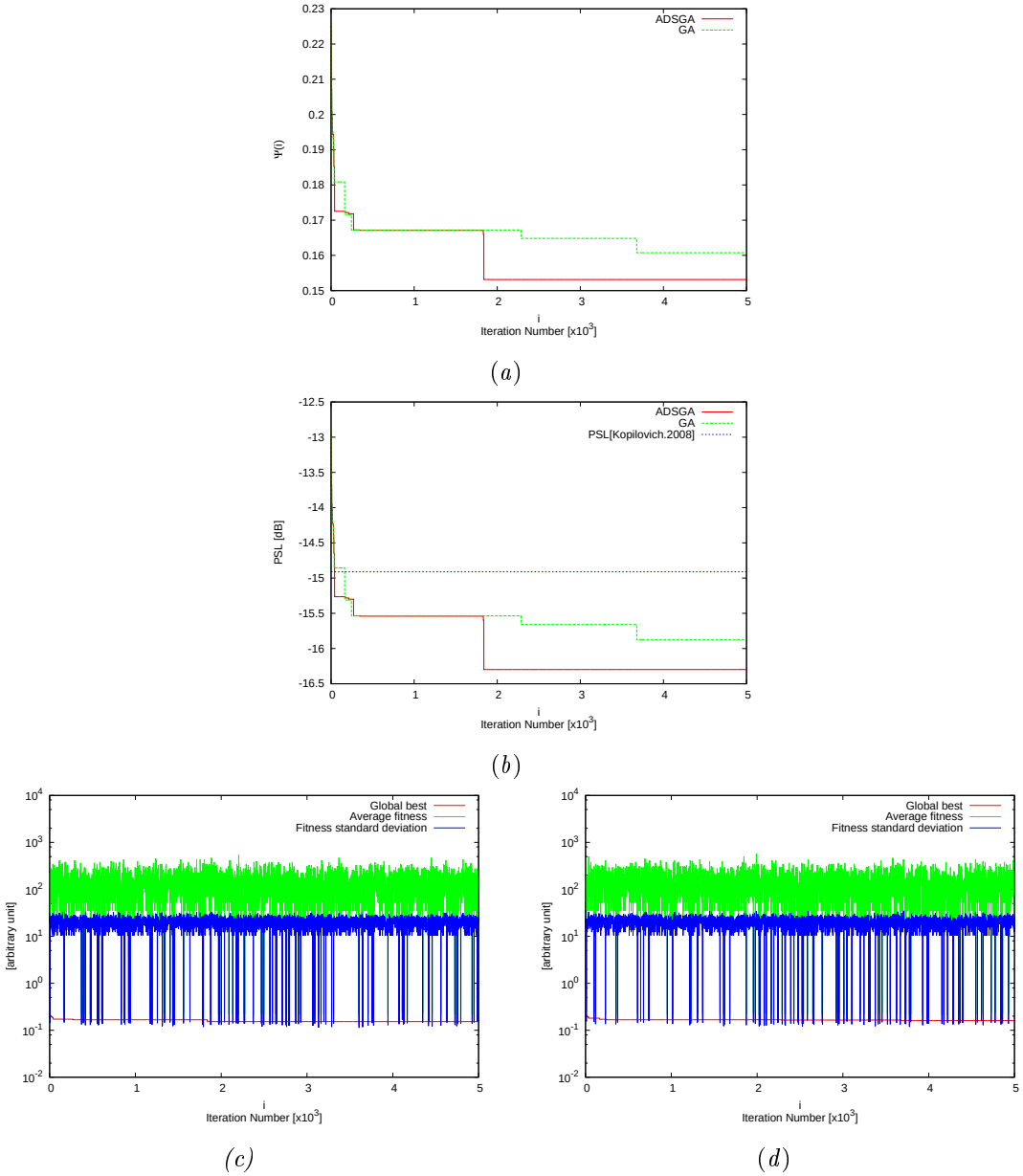


Figure 9.

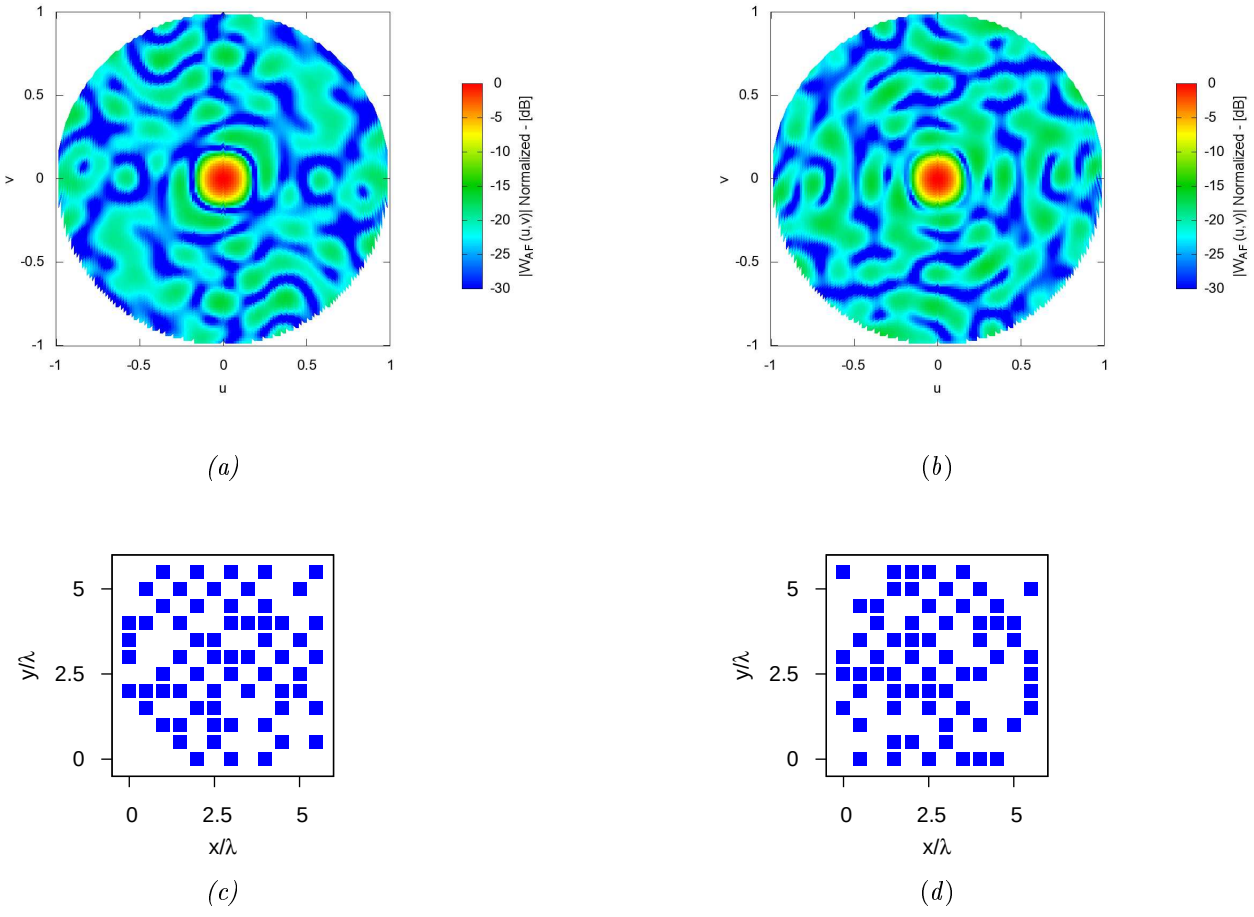
Figure 9: ADSGA approach (c), GA approach (d)

Array Parameters Starting Geometry

- Number of total cells $N = 121$
- Dimension X: 11
- Dimension Y: 11

Array Parameters Final Geometry

- Number of total cells $N = 144$
- Dimension X: 12
- Dimension Y: 12



(a) (b)

(c) (d)

Figure 10.

Figure 10: ADSGA approach (*a*)-(*c*), GA approach (*b*)-(*d*)

RESULTS: $P = 12$, $Q = 12$, $K_{Kopilovich} = 78$

Setting Parameters of Algorithms

GA Parameters

- Chromosome Dimension $C = 144\, bits$
- Population Dimension $S = 40$
- Max Iteration number $K_{max} = 5000$

FFT Parameters

- $FFT\ Theta = 128$
- $FFT\ Phi = 128$

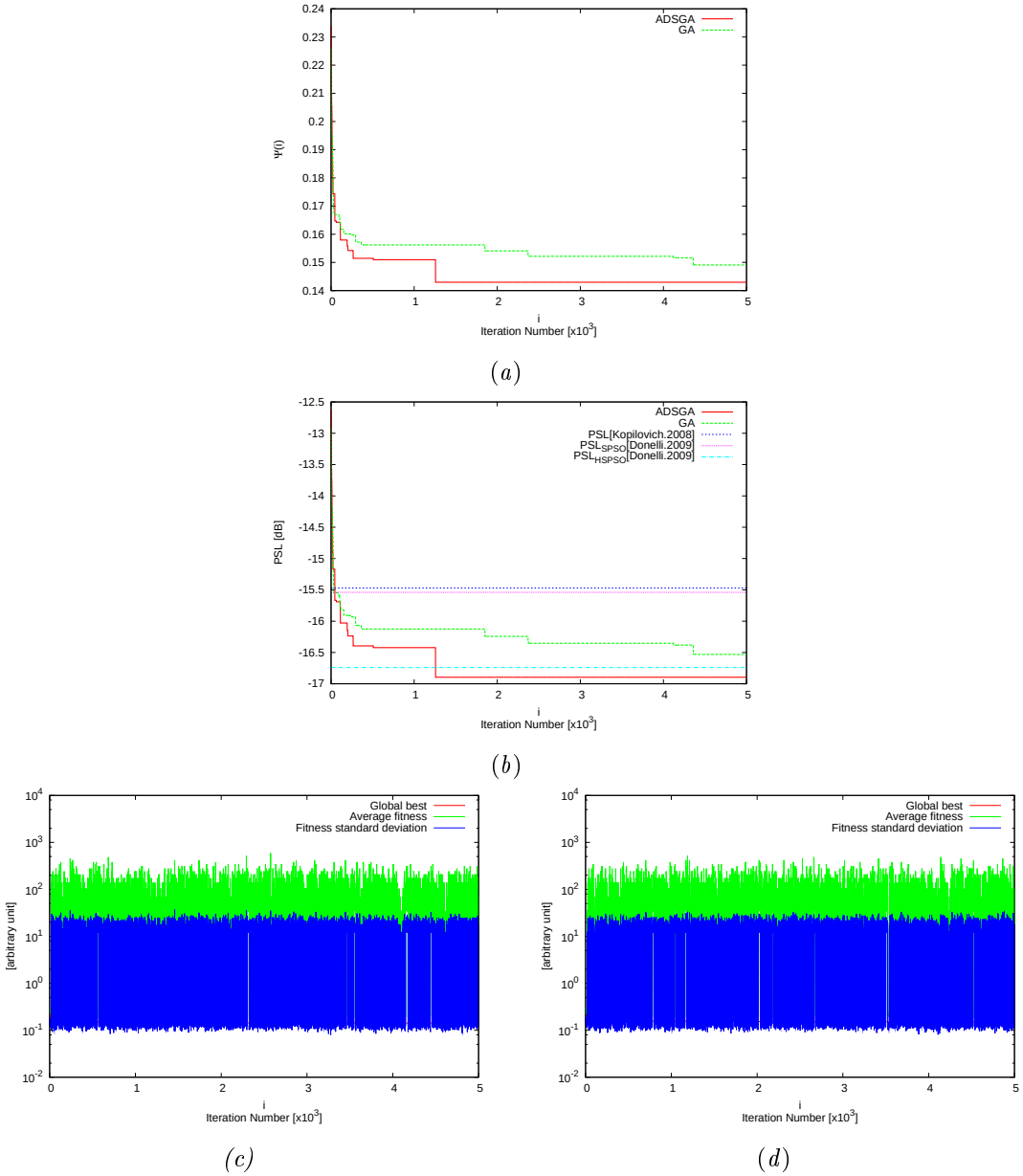


Figure 11.

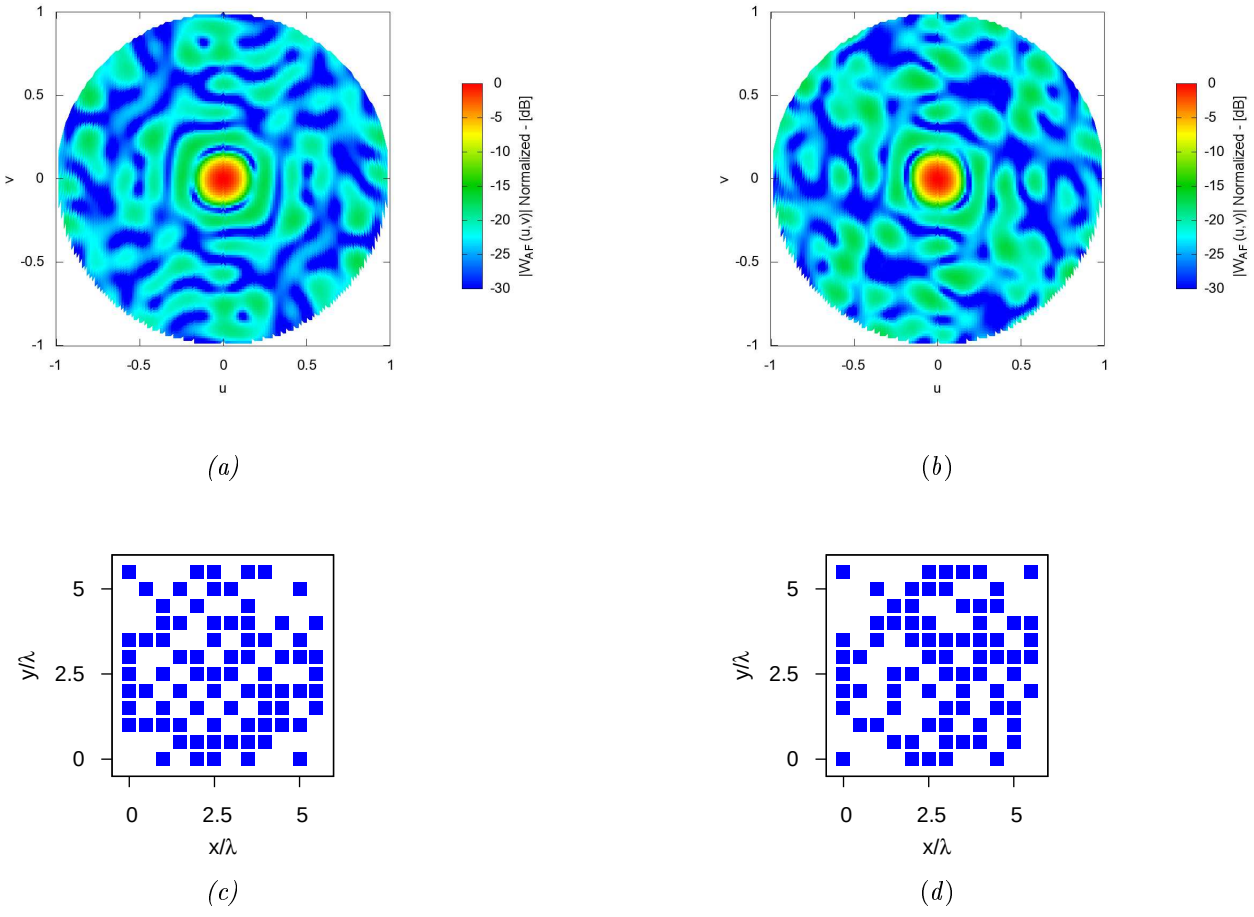
Figure 11: ADSGA approach (c), GA approach (d)

Array Parameters Starting Geometry

- Number of total cells $N = 121$
- Dimension X: 11
- Dimension Y: 11

Array Parameters Final Geometry

- Number of total cells $N = 144$
- Dimension X: 12
- Dimension Y: 12



(a) (b)

(c) (d)

Figure 12.

Figure 12: ADSGA approach (a)-(c), GA approach (b)-(d)

RESULTS: $P = 16$, $Q = 16$, $K_{Kopilovich} = 120$

Setting Parameters of Algorithms

GA Parameters

- Chromosome Dimension $C = 256\, bits$
- Population Dimension $S = 40$
- Max Iteration number $K_{max} = 5000$

FFT Parameters

- $FFT\ Theta = 256$
- $FFT\ Phi = 256$

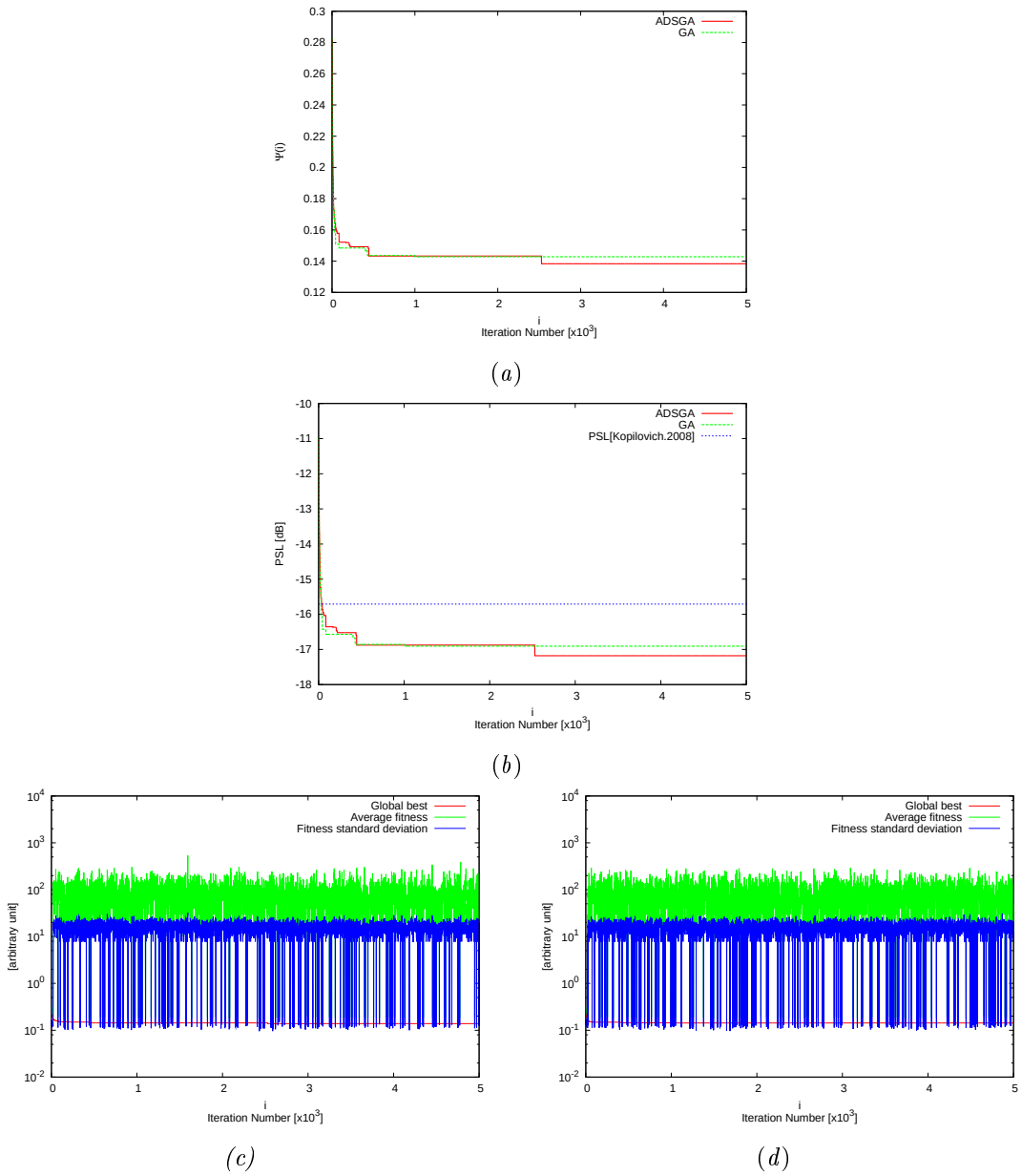


Figure 13.

Figure 13: ADSGA approach (c), GA approach (d)

Array Parameters Starting Geometry

- Number of total cells $N = 169$
- Dimension X: 13
- Dimension Y: 13

Array Parameters Final Geometry

- Number of total cells $N = 256$
- Dimension X: 16
- Dimension Y: 16

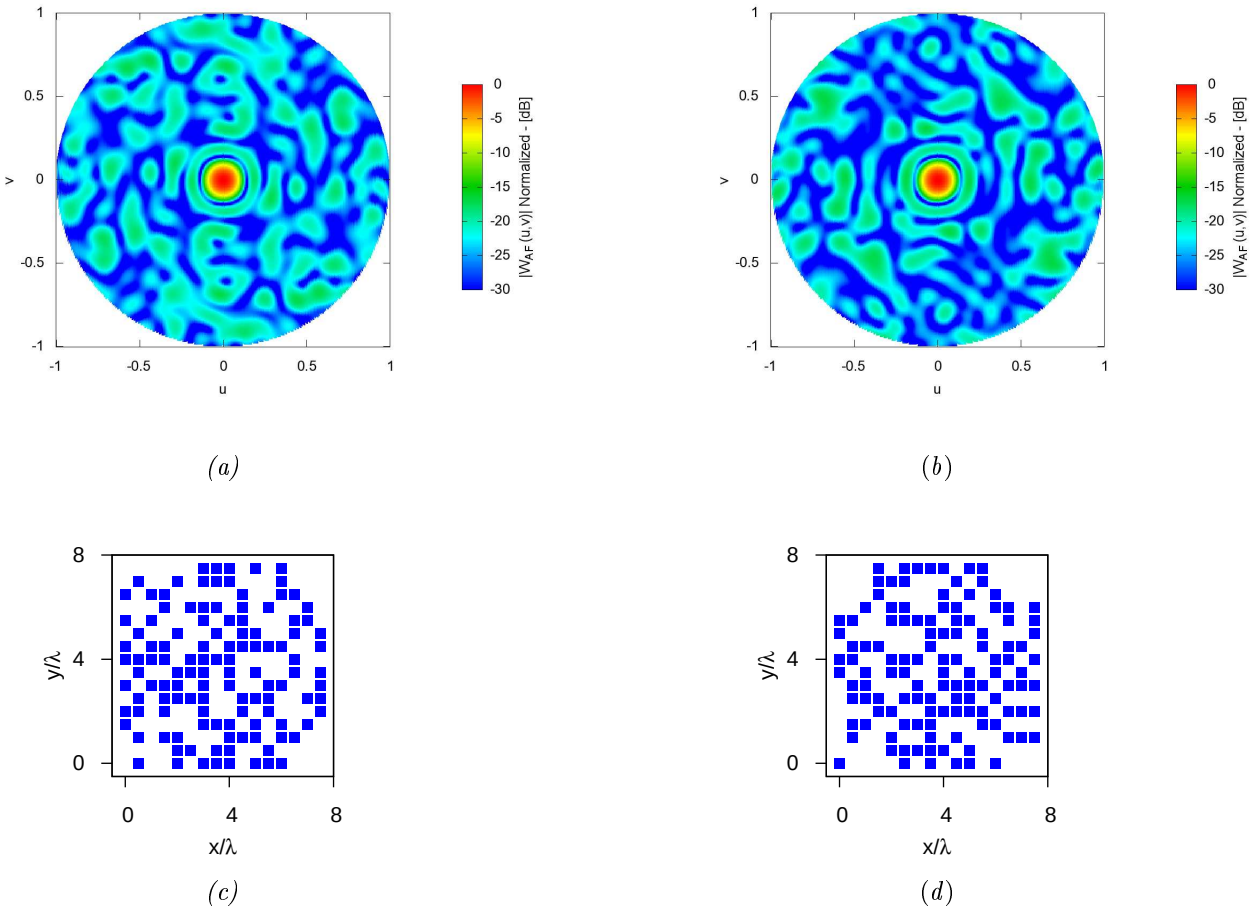


Figure 14.

Figure 14: ADSGA approach (a)-(c), GA approach (b)-(d)

RESULTS: $P = 16$, $Q = 16$, $K_{Kopilovich} = 136$

Setting Parameters of Algorithms

GA Parameters

- Chromosome Dimension $C = 256\, bits$
- Population Dimension $S = 40$
- Max Iteration number $K_{max} = 5000$

FFT Parameters

- $FFT\ Theta = 256$
- $FFT\ Phi = 256$

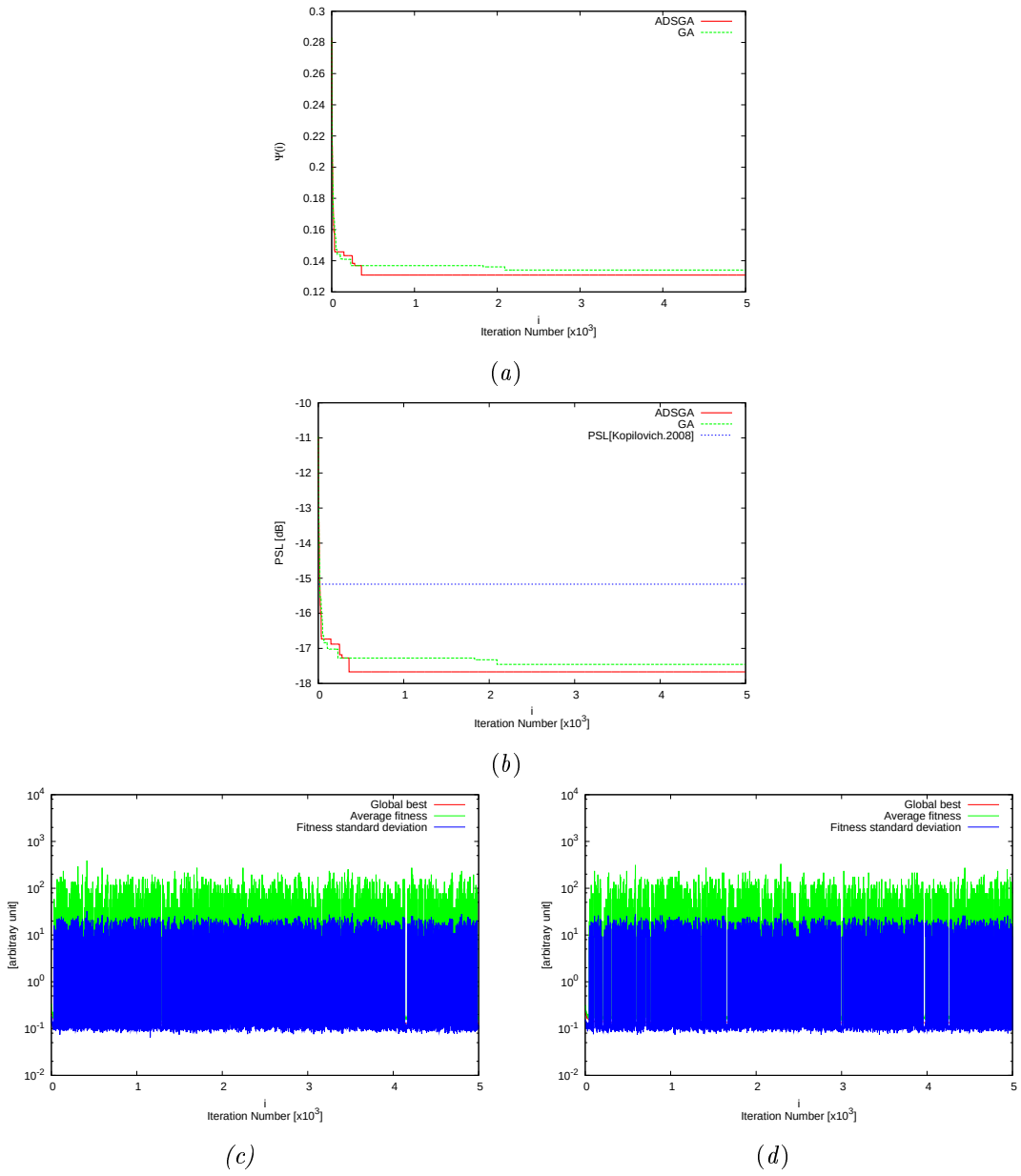


Figure 15.

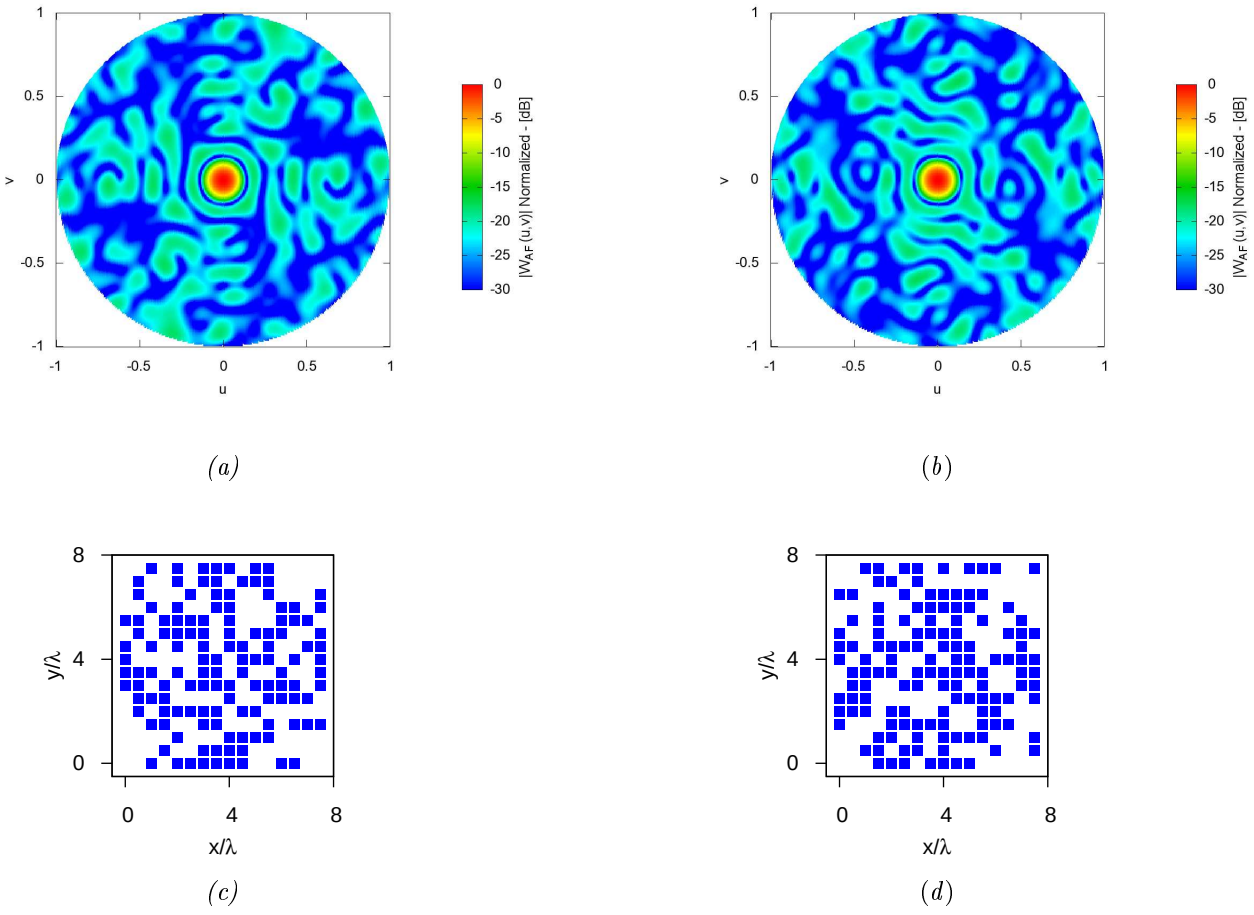
Figure 15: ADSGA approach (c), GA approach (d)

Array Parameters Starting Geometry

- Number of total cells $N = 169$
- Dimension X: 13
- Dimension Y: 13

Array Parameters Final Geometry

- Number of total cells $N = 256$
- Dimension X: 16
- Dimension Y: 16



(a) (b) (c) (d)

Figure 16.

Figure 16: ADSGA approach (*a*)-(*c*), GA approach (*b*)-(*d*)

RESULTS: $P = 24$, $Q = 24$, $K_{Kopilovich} = 276$

Setting Parameters of Algorithms

GA Parameters

- Chromosome Dimension $C = 576\, bits$
- Population Dimension $S = 60$
- Max Iteration number $K_{max} = 5000$

FFT Parameters

- $FFT\ Theta = 256$
- $FFT\ Phi = 256$

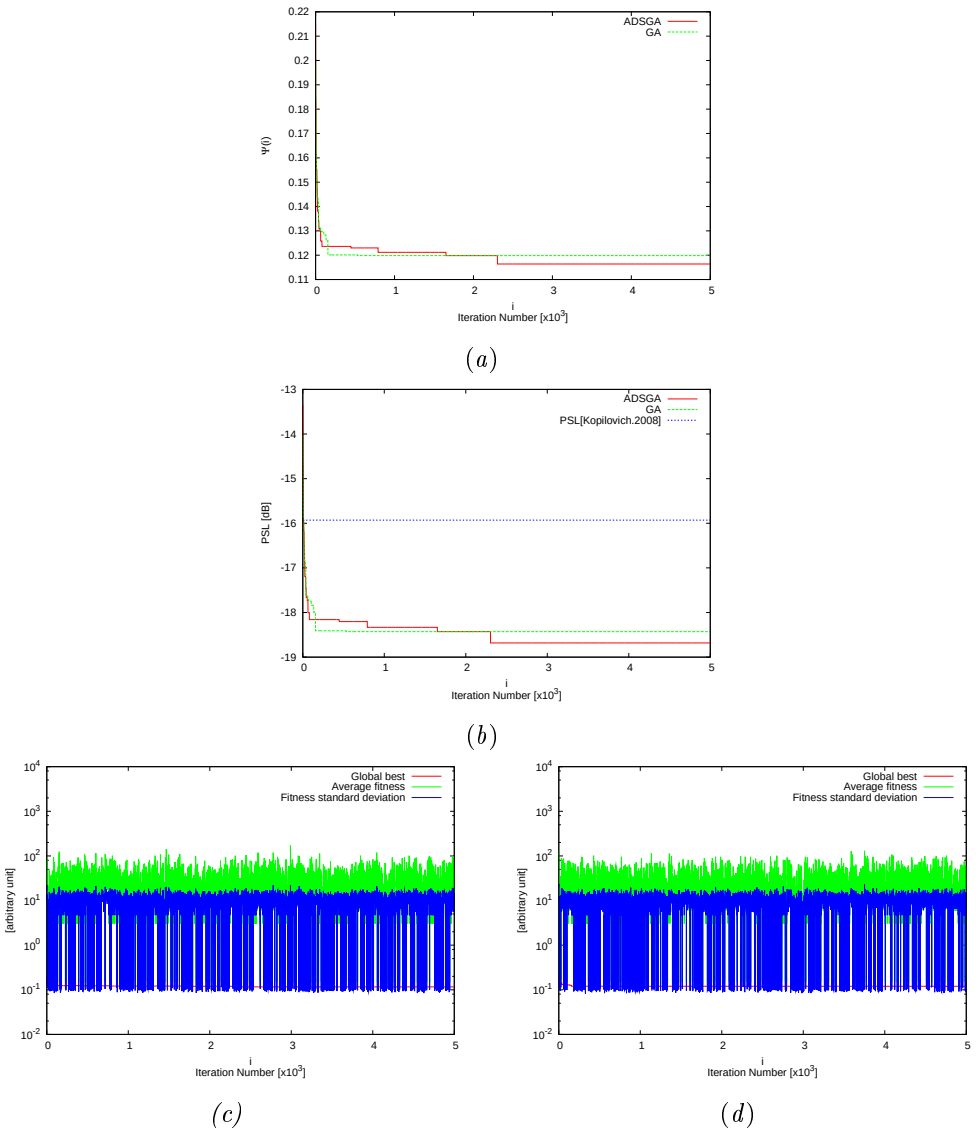


Figure 17.

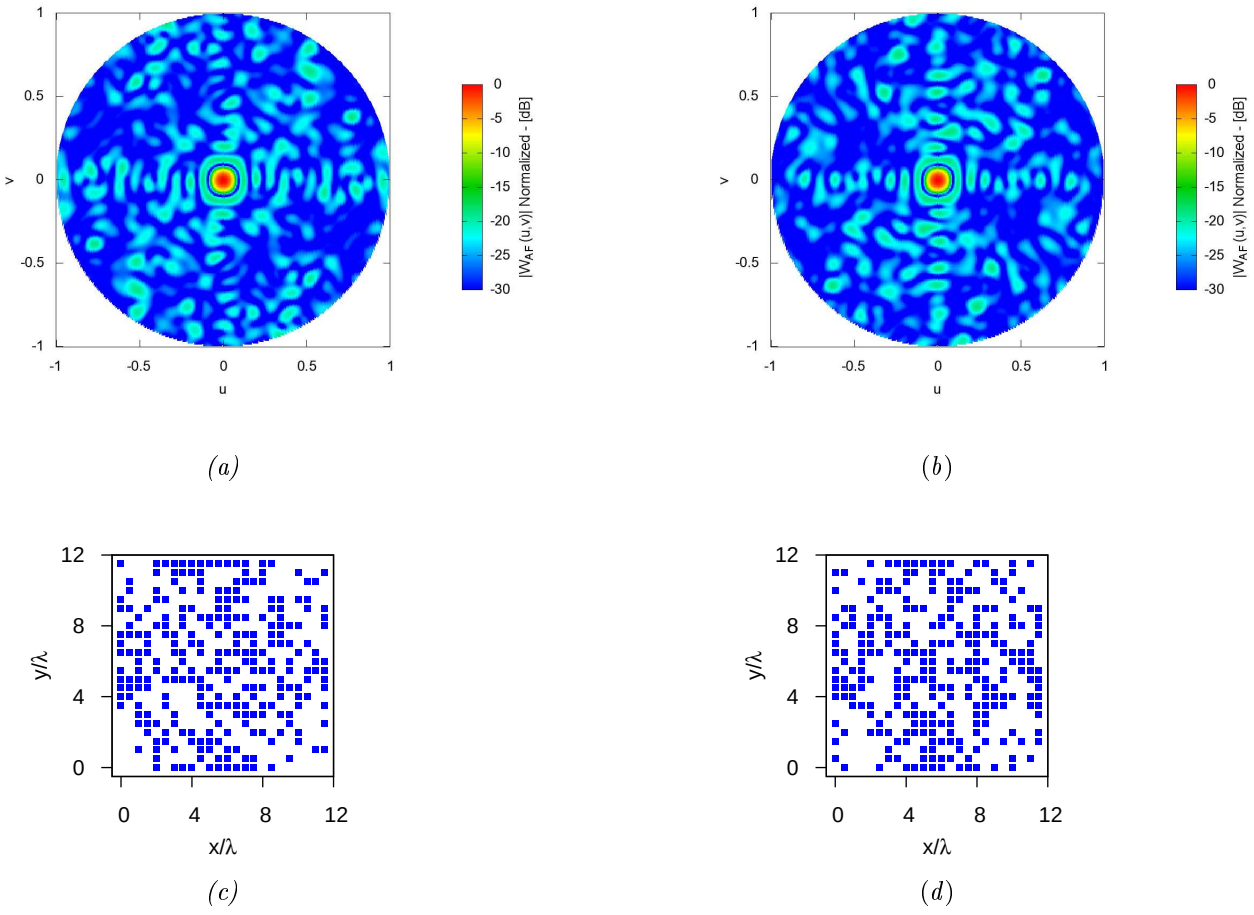
Figure 17: ADSGA approach (c), GA approach (d)

Array Parameters Starting Geometry

- Number of total cells $N = 529$
- Dimension X: 23
- Dimension Y: 23

Array Parameters Final Geometry

- Number of total cells $N = 576$
- Dimension X: 24
- Dimension Y: 24



(a) (b) (c) (d)

Figure 18.

Figure 18: ADSGA approach (*a*)-(*c*), GA approach (*b*)-(*d*)

RESULTS: $P = 24$, $Q = 24$, $K_{Kopilovich} = 300$

Setting Parameters of Algorithms

GA Parameters

- Chromosome Dimension $C = 576\, bits$
- Population Dimension $S = 60$
- Max Iteration number $K_{max} = 5000$

FFT Parameters

- $FFT\ Theta = 256$
- $FFT\ Phi = 256$

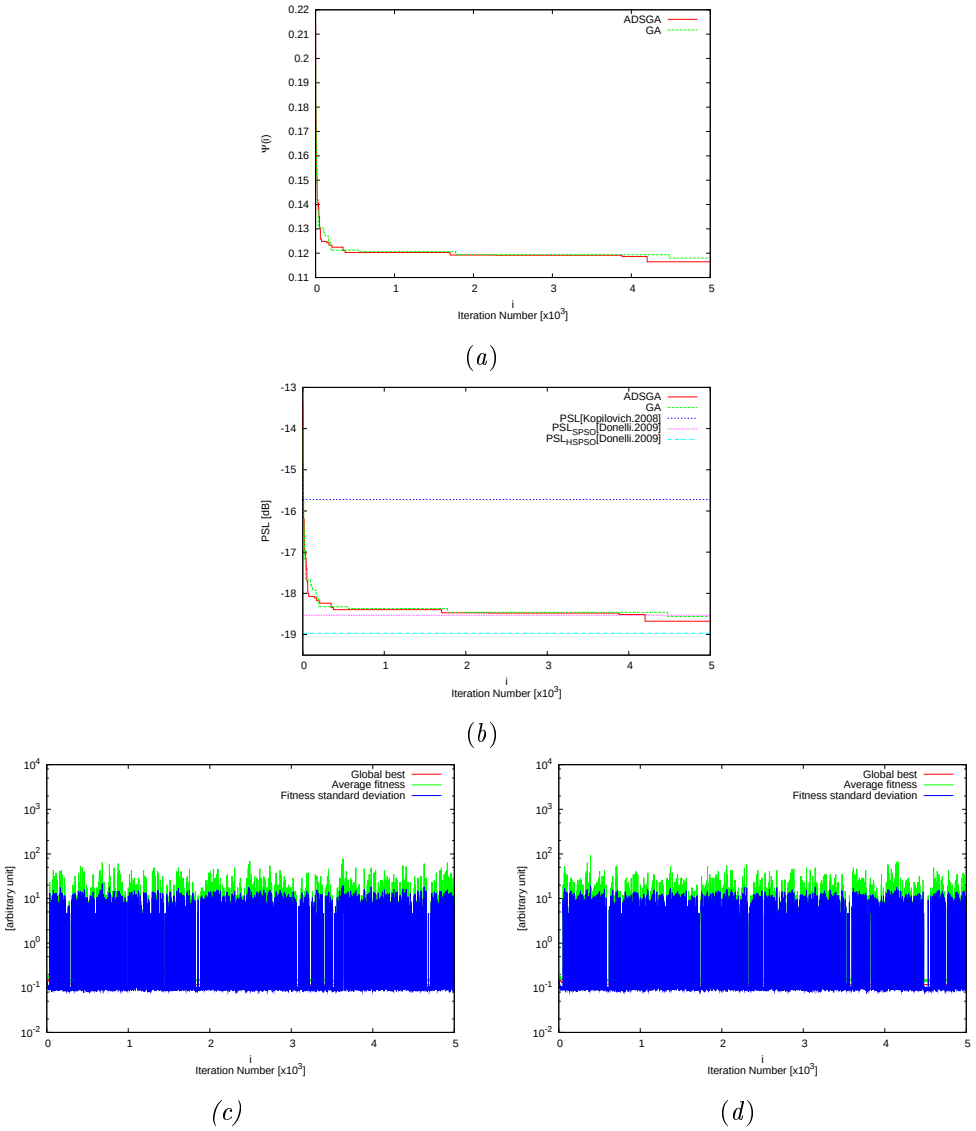


Figure 19.

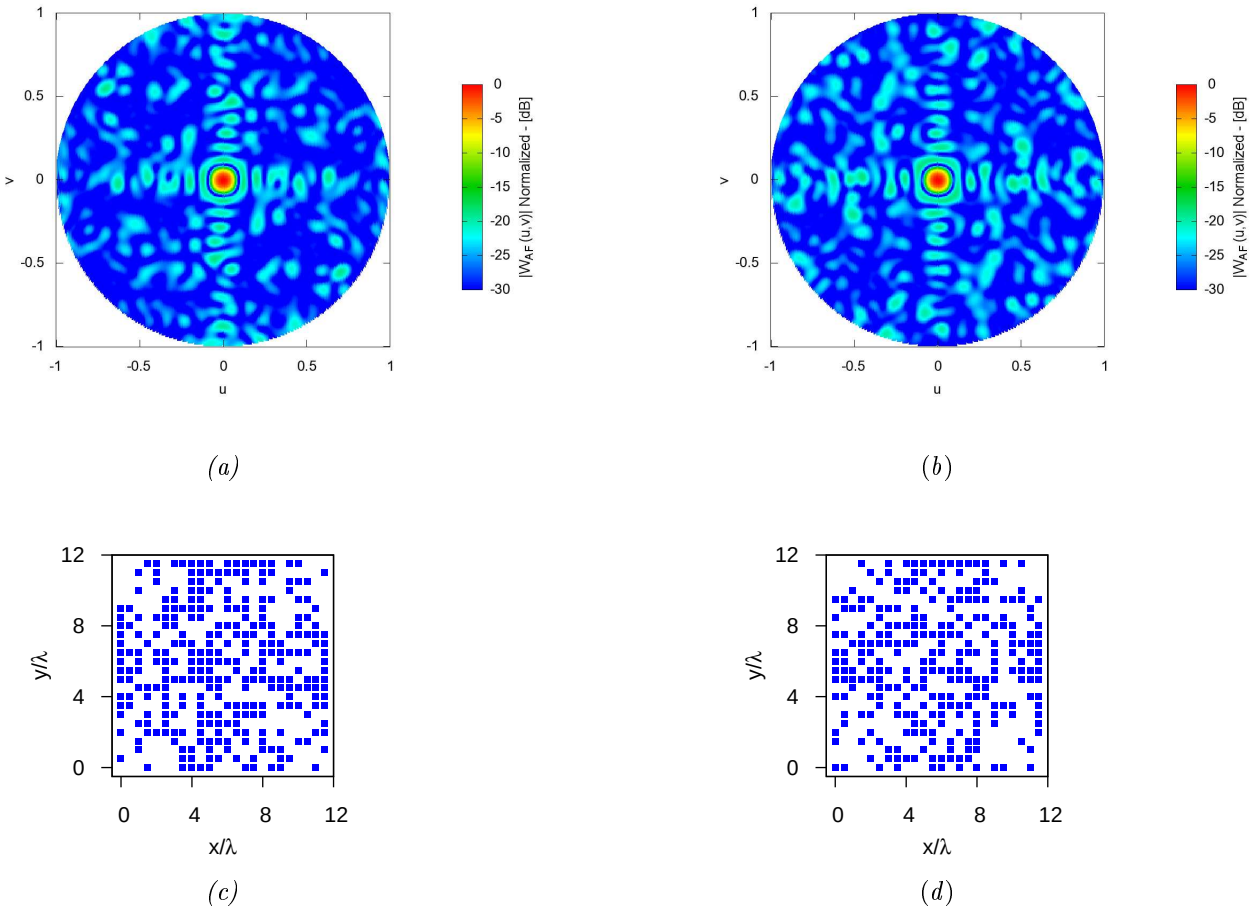
Figure 19: ADSGA approach (c), GA approach (d)

Array Parameters Starting Geometry

- Number of total cells $N = 529$
- Dimension X: 23
- Dimension Y: 23

Array Parameters Final Geometry

- Number of total cells $N = 576$
- Dimension X: 24
- Dimension Y: 24



(a) (b) (c) (d)

Figure 20.

Figure 20: ADSGA approach (*a*)-(*c*), GA approach (*b*)-(*d*)

Risultati Sintesi ADSGA vs GA

$ADS-Dimension$	$Array-Dimesion$	$ADSGA$	GA	$ADSGA$	GA
PXQ	N_xXN_y	$\nu[\%]$	$\nu[\%]$	$PSL[dB]$	$PSL[dB]$
$5X5-A$	$6X6-A$	0.388	0.388	-14.11	-12.42
$5X5-B$	$6X6-B$	0.583	0.555	-14.16	-13.23
$7X7-A$	$8X8-A$	0.421	0.437	-15.81	-15.25
$7X7-B$	$8X8-B$	0.546	0.500	-16.55	-15.92
$11X11-A$	$12X12-A$	0.444	0.451	-16.30	-15.87
$11X11-B$	$12X12-B$	0.541	0.534	-16.90	-16.53
$13X13-A$	$16X16-A$	0.464	0.468	-17.18	-16.90
$13X13-B$	$16X16-B$	0.500	0.515	-17.45	-17.67
$23X23-A$	$24X24-A$	0453	0.468	-18.68	-18.42
$23X23-B$	$24X24-B$	0.505	0.467	-18.68	-18.56

Table I.a

$ADS-Dimension$	$Array-Dimesion$	$ADSGA$	GA
PXQ	N_xXN_y	$BW\ (U_m=V_m)$	$BW\ (U_m=V_m)$
$5X5-A$	$6X6-A$	0.333	0.333
$5X5-B$	$6X6-B$	0.333	0.333
$7X7-A$	$8X8-A$	0.250	0.250
$7X7-B$	$8X8-B$	0.250	0.250
$11X11-A$	$12X12-A$	0.166	0.166
$11X11-B$	$12X12-B$	0.166	0.166
$13X13-A$	$16X16-A$	0.125	0.125
$13X13-B$	$16X16-B$	0.125	0.125
$23X23-A$	$24X24-A$	0.0833	0.0833
$23X23-B$	$24X24-B$	0.0833	0.0833

Table I.b

$Array - Dimesion$		$[Kopilovich]$ [1]	$[Kopilovich][1]$
N_xXN_y		$\nu[\%]$	$PSL[dB]$
$6X6 - A$		0.417	-10.18
$6X6 - B$		0.583	-12.55
$8X8 - A$		0.438	-12.46
$8X8 - B$		0.562	-13.71
$12X12 - A$		0.458	-14.91
$12X12 - B$		0.542	-15.47
$16X16 - A$		0.469	-15.71
$16X16 - B$		0.531	-15.17
$24X24 - A$		0.479	-15.93
$24X24 - B$		0.521	-15.72

Table II

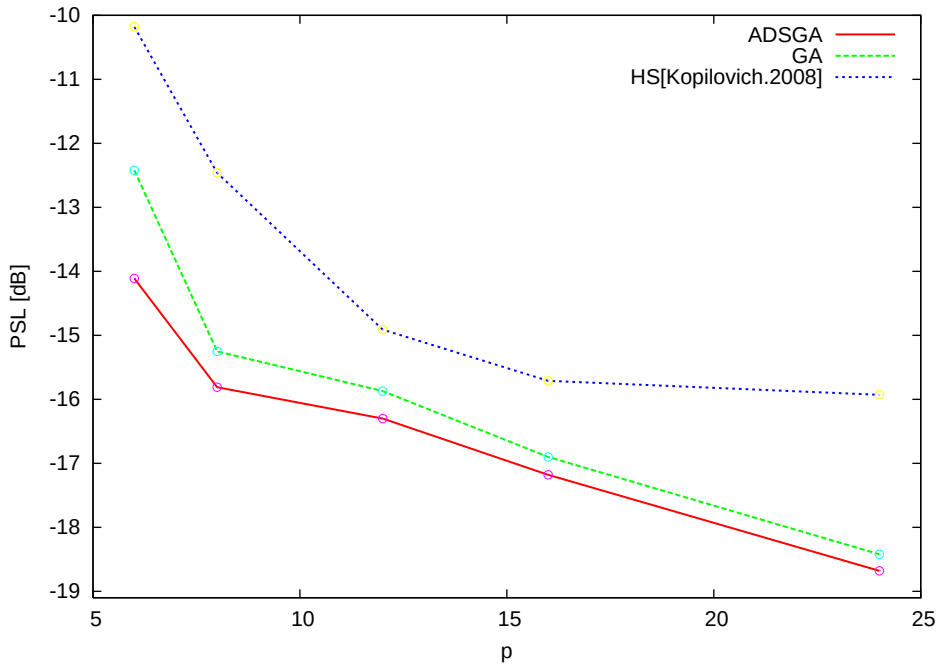
Riepilogo Sintesi Reference [2]

$Array - Dimesion$		$SPSO - [Donelli][2]$	$HSPSO - [Donelli][2]$
N_xXN_y		$\nu[\%]$	$\nu[\%]$
$6X6 - A$		-	-
$6X6 - B$		0.50	0.42
$8X8 - A$		-	-
$8X8 - B$		-	-
$12X12 - A$		-	-
$12X12 - B$		0.44	0.48
$16X16 - A$		-	-
$16X16 - B$		-	-
$24X24 - A$		-	-
$24X24 - B$		0.43	0.44

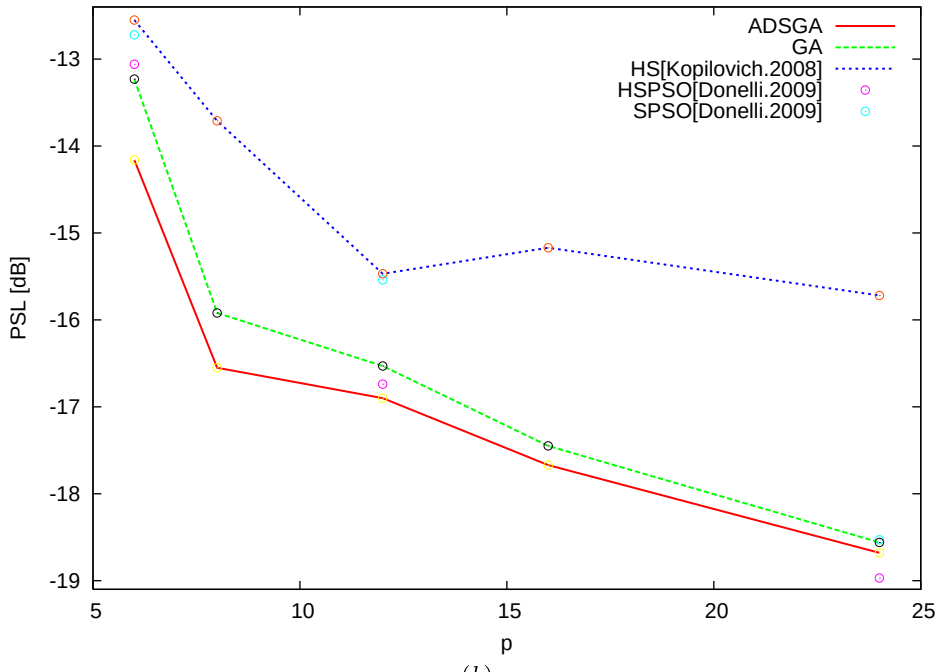
Table III

$Array - Dimesion$		$SPSO - [Donelli][2]$	$HSPSO - [Donelli][2]$
N_xXN_y		$PSL[dB]$	$PSL[dB]$
$6X6 - A$		-	-
$6X6 - B$		-12.72	-13.06
$8X8 - A$		-	-
$8X8 - B$		-	-
$12X12 - A$		-	-
$12X12 - B$		-15.54	-16.74
$16X16 - A$		-	-
$16X16 - B$		-	-
$24X24 - A$		-	-
$24X24 - B$		-18.53	-18.97

Table IV



(a)



(b)

Figure 21.

NOTA **Figure 21:**

- Casistica Densità Elementi Attivi Array "A": Figure 7-(a)
- Casistica Densità Elementi Attivi Array "B": Figure 7-(b)

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