Efficient Tiling of Large Planar Sub-Arrayed Phased Arrays Through Schemata-Driven Evolutionary Optimization

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Abstract

In this work, the synthesis of large clustered arrays - computationally unaffordable through standard stochastic global optimization techniques - is addressed through an innovative schemata-driven approach. The proposed design methodology is based on the analytic definition of a set of reference tiling arrangements and a customized genetic algorithm (*GA*)-based strategy which is able to effectively and efficiently explore the solution space of the complete tiling configurations. Some representative numerical experiments are presented in order to verify the effectiveness of the developed synthesis technique for the tiling of large planar phased sub-arrays providing optimal side-lobe level (*SLL*) radiation performance.

1 Numerical Validation

1.1 BIG PROBLEM DIMENSION

1.1.1 Test Case #7: GA Strategy - 10x10 array - Schemata Approach

Array Analysis Parameters:

- Total Number of Elements: $M \times N = 10 \times 10 = 100$
- Spacing: $d = \lambda/2$
- Number of Samples along u: 512
- Number of Samples along v: 512
- Steering θ Direction: $\theta_s = 0$
- Steering ϕ Direction: $\phi_s = 0$

Tiling Parameters:

- Tile: Domino
- Number of Tiles Types: $L = 2$
  - Horizontal
  - Vertical
- Number of Single Tile Cell Covering: $D_i = 2, i = 1, ..., L$
- Total Number of Configurations: $C_{tot} = 2.5858 \times 10^{11}$
- Number of Inner Lattice Points: $N_{inn} = 81$

Genetic Algorithm Parameters:

- Number of Unknowns: $U = 243$
- Population Dimension: $P = 176$
- Maximum Number of Iterations: $I = 1000$
- Crossover Probability: $p_{cross} = 0.9$
- Mutation Probability: $p_{mut} = 0.01$
- Diversity Percentage: $p_{div} = 10\%$

Cost Function:

$$\Psi(T) = SLL$$

Schemata Analysis:

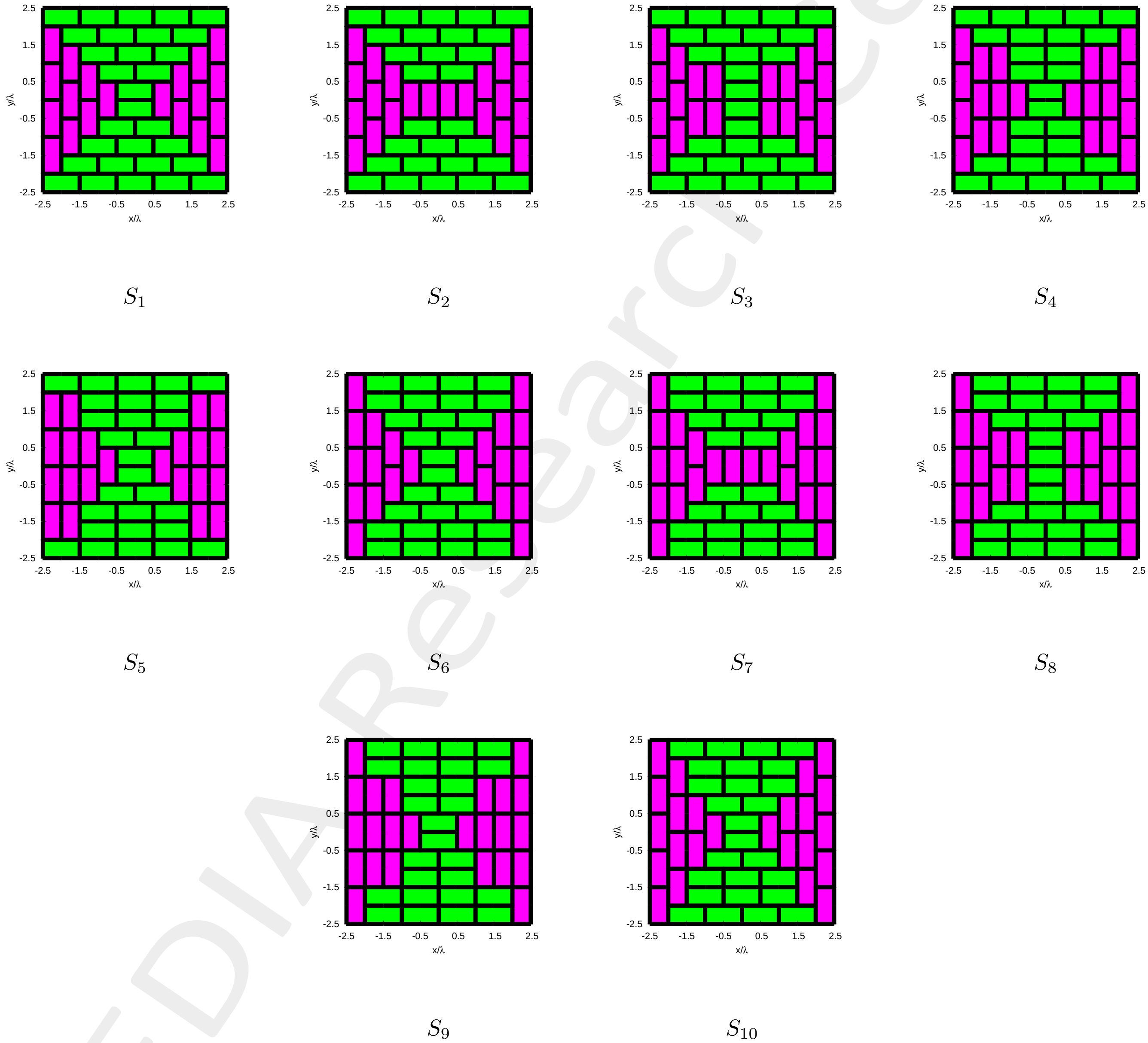


Figure 1. Generated schematas for a 10×10 rectangular region.

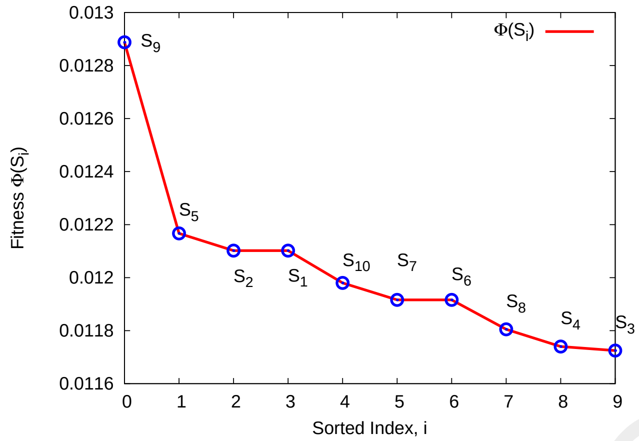


Figure 2. Schematas sorted fitness.

i	$Schemata\, S_i$	$\Psi\,(S_i)$
1	000000000000000000000000000000000000000000000000000000000000000000000000000000000	1.2102×10^{-2}
2	000000000000000000000000000000111000000111000000111000000000000000000000000000000	1.2102×10^{-2}
3	000000000000000000001111100001111100001111100001111100001111100000000000000000000	1.1725×10^{-2}
4	000000000011111110011111110011111110011111110011111110011111110011111110000000000	1.1740×10^{-2}
5	111111111111111111111111111111111111111111111111111111111111111111111111111111111	1.2167×10^{-2}
6	111111111111111111111111111111222111111222111111222111111111111111111111111111111	1.1916×10^{-2}
7	111111111111111111112222211112222211112222211112222211112222211111111111111111111	1.1916×10^{-2}
8	111111111122222221122222221122222221122222221122222221122222221122222221111111111	1.1805×10^{-2}
9	111111111122222221122222221122333221122333221122333221122222221122222221111111111	1.2888×10^{-2}
10	111111111122222221123333321123333321123333321123333321123333321122222221111111111	1.1980×10^{-2}

Table 1. Schemata words

GA Optimization RESULTS:

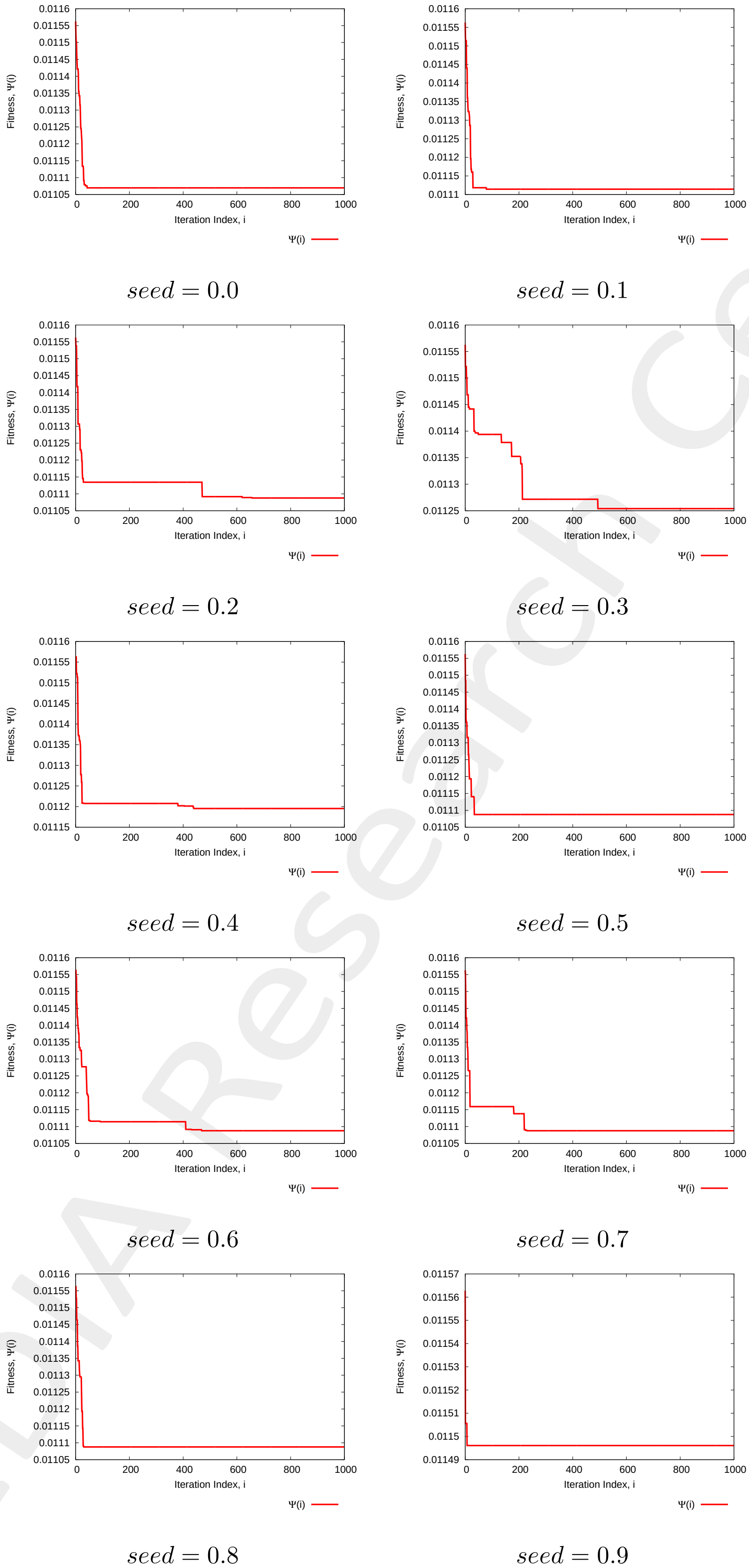


Figure 3. Fitness of the GA simulations for each random seed.

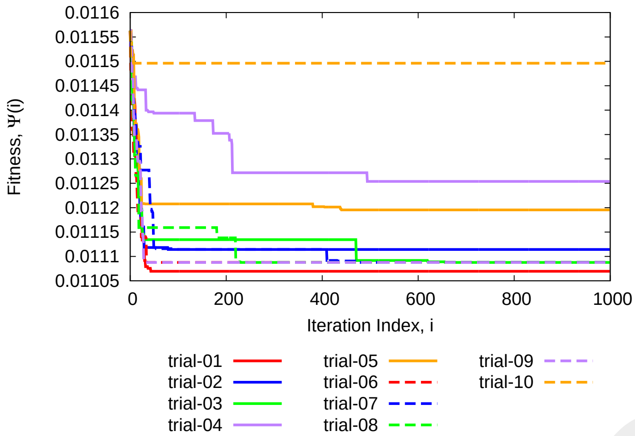


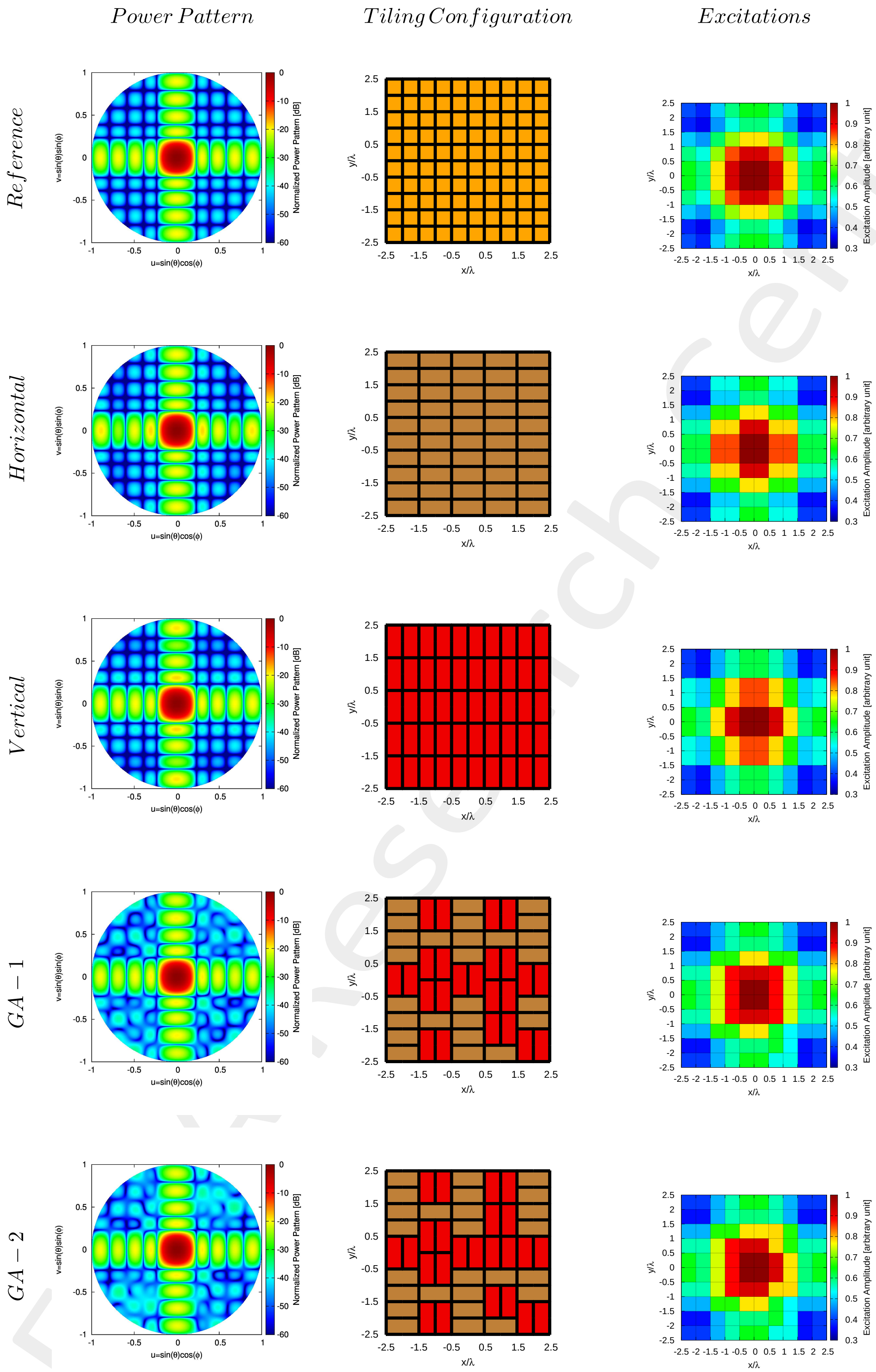
Figure 4. Fitness of the GA simulation: statistic simulation results.

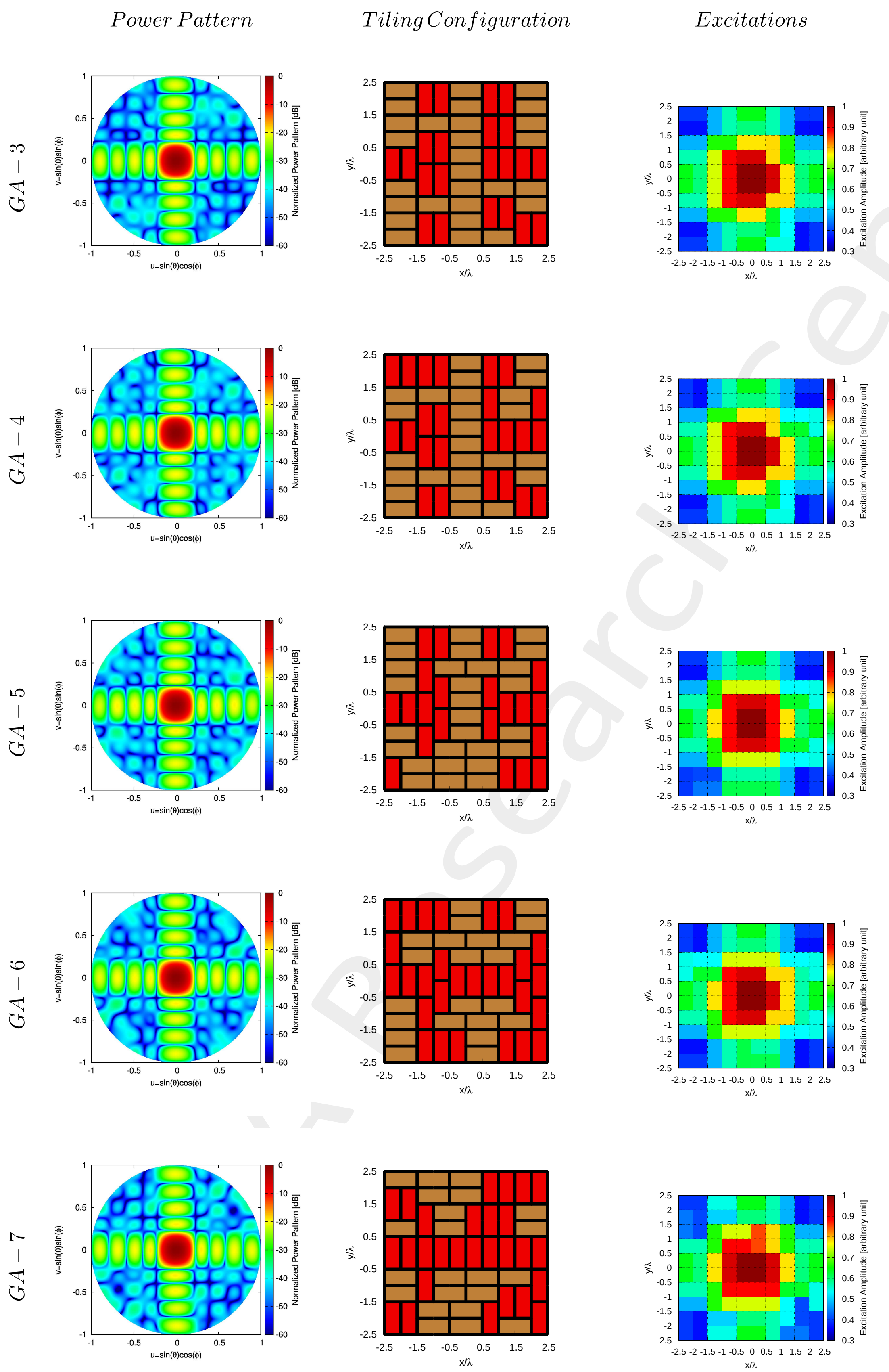
Solution	$\mathbf{w} - GA$	*Seed*	$\Psi\left(T_{GA}\right)$
$GA-1$	1010001011111111110122222101122222111122222111122222111222222101111111111001000101	$\{0.0\}$	1.10696×10^{-2}
$GA-2$	1010001011111111110122222101122222111122222111122222111221111101111111111001000101	$\{0.5\}$	1.10877×10^{-2}
$GA-3$	1010001011111111110122222101122222111122322111122222111221111101111111111001000101	$\{0.2, 0.6, 0.7, 0.8\}$	1.10877×10^{-2}
$GA-4$	1010001001111111110122222101122222111122322111122222111221111101111111011001000001	$\{0.1\}$	1.11143×10^{-2}
$GA-5$	0010001010111111110111111100111111110111211110111111110111111101101110110000000001	$\{0.4\}$	1.11954×10^{-2}
$GA-6$	1010000000111111011011111110111111111011111111011111111011000110110000011000000001	$\{0.3\}$	1.12539×10^{-2}
$GA-7$	0010001111111111111111111111112221111112221111112222110111111100001111111000000100	$\{0.9\}$	1.14961×10^{-2}

Table 2. GA solutions

Seed	t_{tot}	K
0.0	1.50×10^4	42
0.1	1.55×10^4	78
0.2	1.49×10^4	655
0.3	1.47×10^4	494
0.4	1.43×10^4	437
0.5	1.52×10^4	33
0.6	1.45×10^4	468
0.7	1.42×10^4	229
0.8	1.45×10^4	29
0.9	1.49×10^4	6

Table 5. Timings and number of iterations for convergence (K).


Power Pattern
Tiling Configuration
Excitations
Reference
Horizontal
Vertical
GA − 1
GA − 2
v=sin(θ)sin(φ)
u=sin(θ)cos(φ)
Normalized Power Pattern [dB]
y/λ
x/λ
Excitation Amplitude [arbitrary unit]


Power Pattern
Tiling Configuration
Excitations
GA − 3
GA − 4
GA − 5
GA − 6
GA − 7
v=sin(θ)sin(φ)
u=sin(θ)cos(φ)
Normalized Power Pattern [dB]
y/λ
x/λ
Excitation Amplitude [arbitrary unit]

	SLL [dB]	D [dBi]	$HPBW_{az}$ [deg]	$HPBW_{el}$ [deg]	$\Psi(T)$
$Reference$	-20.0	24.44	11.19	11.19	1.00×10^{-2}
$Horizontal$	-18.4826	24.4540	11.1547	11.1861	1.41821×10^{-2}
$Vertical$	-18.4826	24.4540	11.1861	11.1547	1.41821×10^{-2}
$GA-1$	-19.5587	24.4659	11.1978	11.1794	1.10696×10^{-2}
$GA-2$	-19.5517	24.4641	11.1972	11.1762	1.10877×10^{-2}
$GA-3$	-19.5517	24.4641	11.1972	11.1762	1.10877×10^{-2}
$GA-4$	-19.5412	24.4672	11.1772	11.1797	1.11143×10^{-2}
$GA-5$	-19.5096	24.4690	11.1573	11.1833	1.11954×10^{-2}
$GA-6$	-19.4870	24.4687	11.1574	11.1925	1.12539×10^{-2}
$GA-7$	-19.3945	24.4606	11.1686	11.1582	1.14961×10^{-2}

Table 2. Pattern descriptors and fitness values for the presented solutions.

1.1.2 Test Case #8: GA Strategy - 16x16 array - Schemata Approach

Array Analysis Parameters:

- Total Number of Elements: $M \times N = 16 \times 16 = 256$
- Spacing: $d = \lambda/2$
- Number of Samples along u: 512
- Number of Samples along v: 512
- Steering θ Direction: $\theta_s = 0$
- Steering ϕ Direction: $\phi_s = 0$

Tiling Parameters:

- Tile: Domino
- Number of Tiles Types: $L = 2$
  - Horizontal
  - Vertical
- Number of Single Tile Cell Covering: $D_i = 2, i = 1, ..., L$
- Total Number of Configurations: $C_{tot} = 2.4449 \times 10^{30}$
- Number of Inner Lattice Points: $N_{inn} = 225$

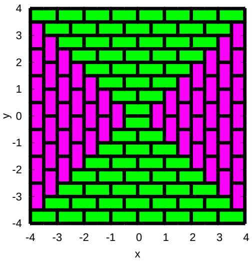
Genetic Algorithm Parameters:

- Number of Unknowns: $U = 675$
- Population Dimension: $P = 464$
- Maximum Number of Iterations: $I = 1000$
- Crossover Probability: $p_{cross} = 0.9$
- Mutation Probability: $p_{mut} = 0.01$
- Diversity Percentage: $p_{div} = 10\%$

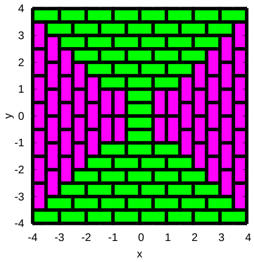
Cost Function:

$$\Psi(T) = SLL$$

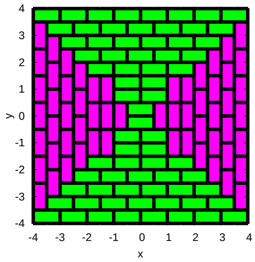
Schemata Analysis:



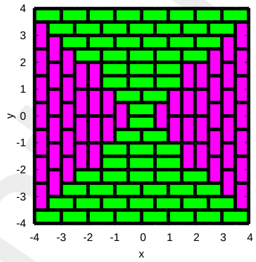
S_1



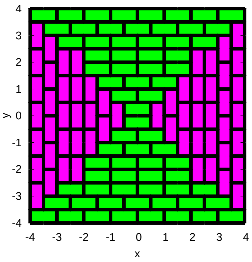
S_2



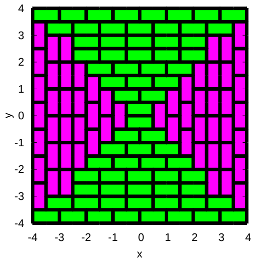
S_3



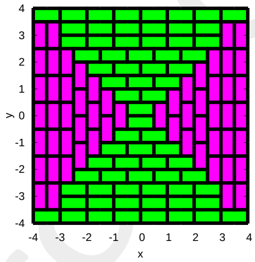
S_4



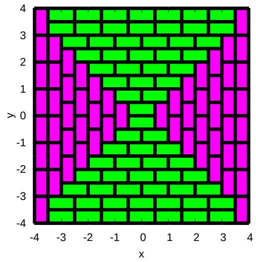
S_5



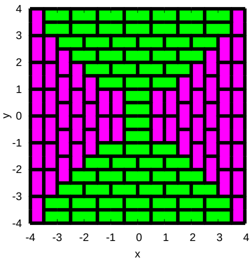
S_6



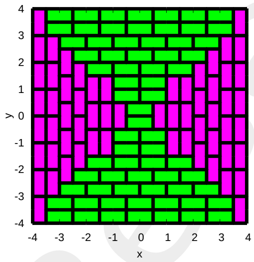
S_7



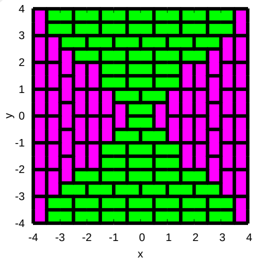
S_8



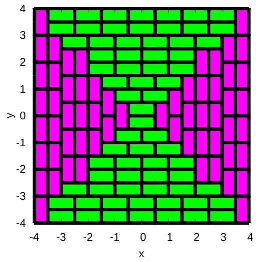
S_9



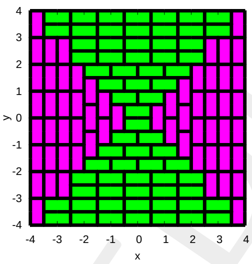
S_{10}



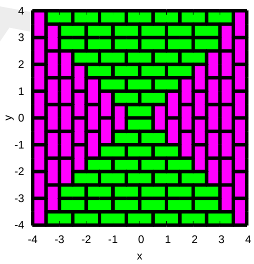
S_{11}



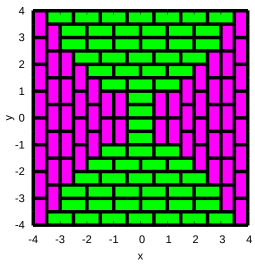
S_{12}



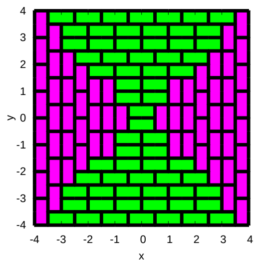
S_{13}



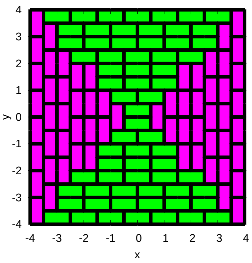
S_{14}



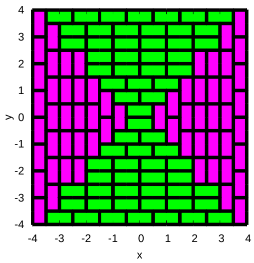
S_{15}



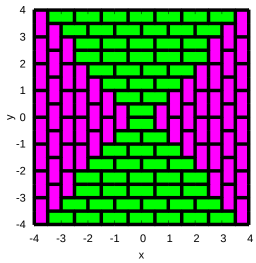
S_{16}



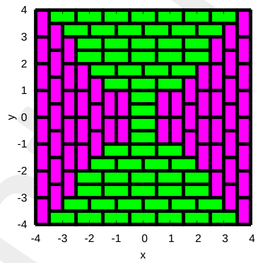
S_{17}



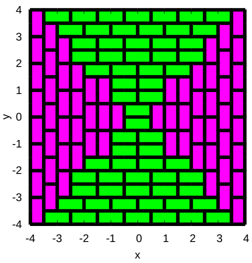
S_{18}



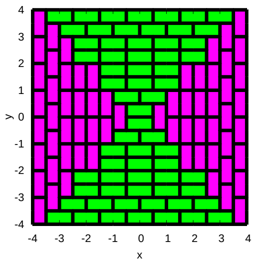
S_{19}



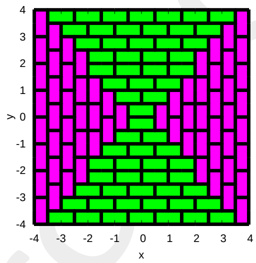
S_{20}



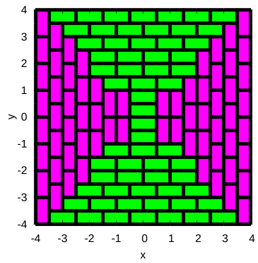
S_{21}



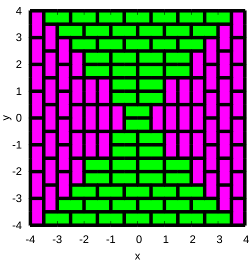
S_{22}



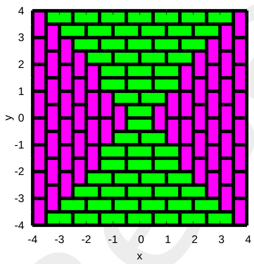
S_{23}



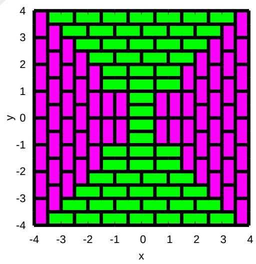
S_{24}



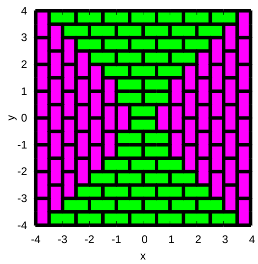
S_{25}



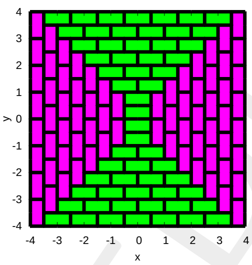
S_{26}



S_{27}



S_{28}



S_{29}

Figure 1. Generated schematas for a 16×16 rectangular region.

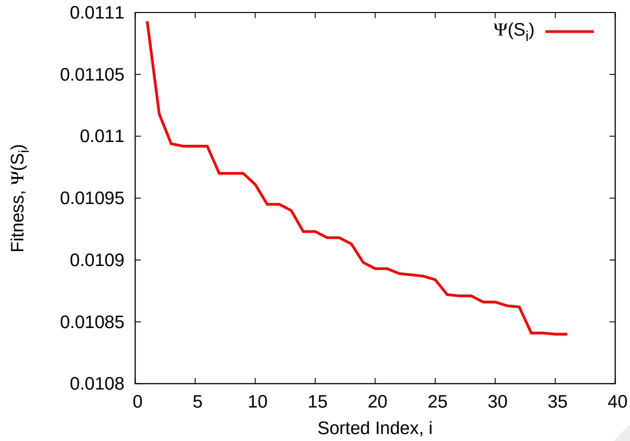
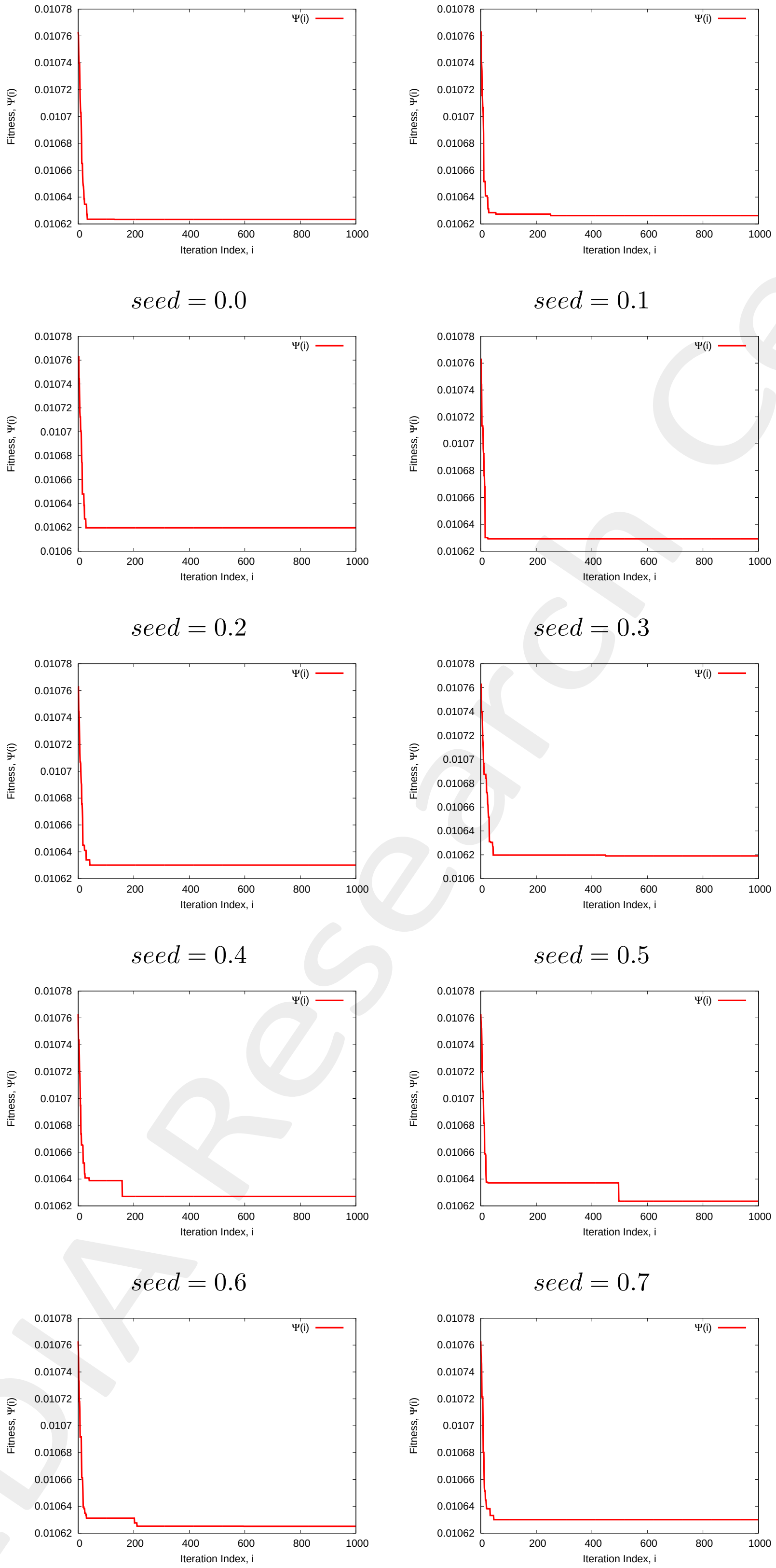


Figure 2. Schematas sorted fitness.

i	$\Psi(S_i)$	i	$\Psi(S_i)$	i	$\Psi(S_i)$
1	1.0992×10^{-2}	11	1.0918×10^{-2}	21	1.0887×10^{-2}
2	1.0970×10^{-2}	12	1.0898×10^{-2}	22	1.1093×10^{-2}
3	1.0945×10^{-2}	13	1.0884×10^{-2}	23	1.0871×10^{-2}
4	1.0940×10^{-2}	14	1.0841×10^{-2}	24	1.0840×10^{-2}
5	1.0961×10^{-2}	15	1.0863×10^{-2}	25	1.0918×10^{-2}
6	1.0994×10^{-2}	16	1.0888×10^{-2}	26	1.0923×10^{-2}
7	1.1018×10^{-2}	17	1.0893×10^{-2}	27	1.0945×10^{-2}
8	1.0866×10^{-2}	18	1.0872×10^{-2}	28	1.0970×10^{-2}
9	1.0889×10^{-2}	19	1.0840×10^{-2}	29	1.0992×10^{-2}
10	1.0913×10^{-2}	20	1.0862×10^{-2}	–	–

Table 1. Fitness of the schematas

GA Optimization RESULTS:



$seed = 0.0$ $seed = 0.1$

$seed = 0.2$ $seed = 0.3$

$seed = 0.4$ $seed = 0.5$

$seed = 0.6$ $seed = 0.7$

$seed = 0.8$ $seed = 0.9$

Figure 3. Fitness of the GA simulations for each random seed.

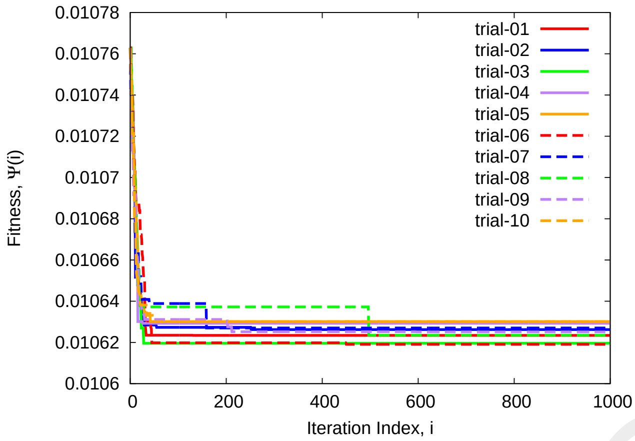


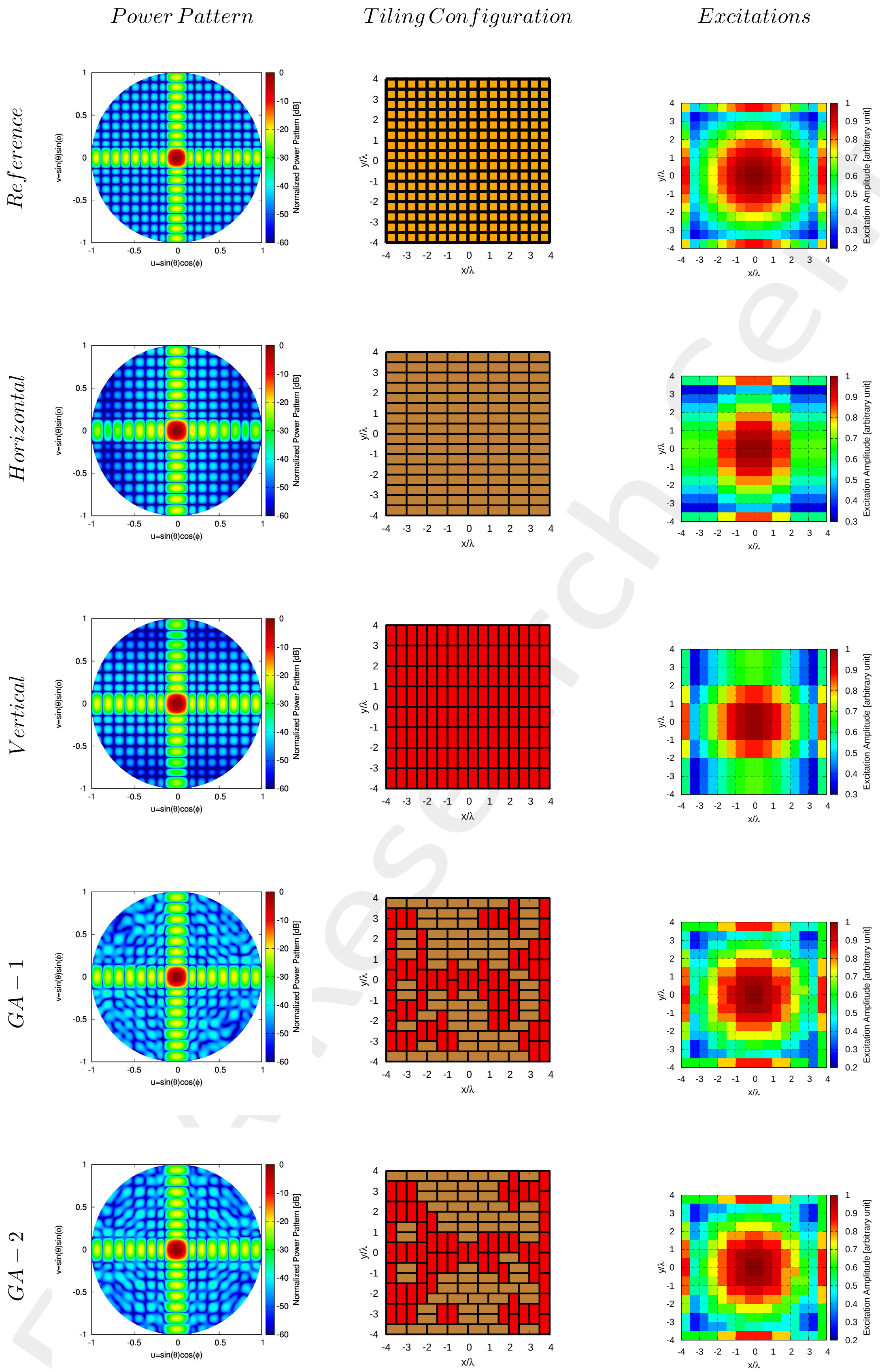
Figure 4. Fitness of the GA simulation: statistic simulation results.

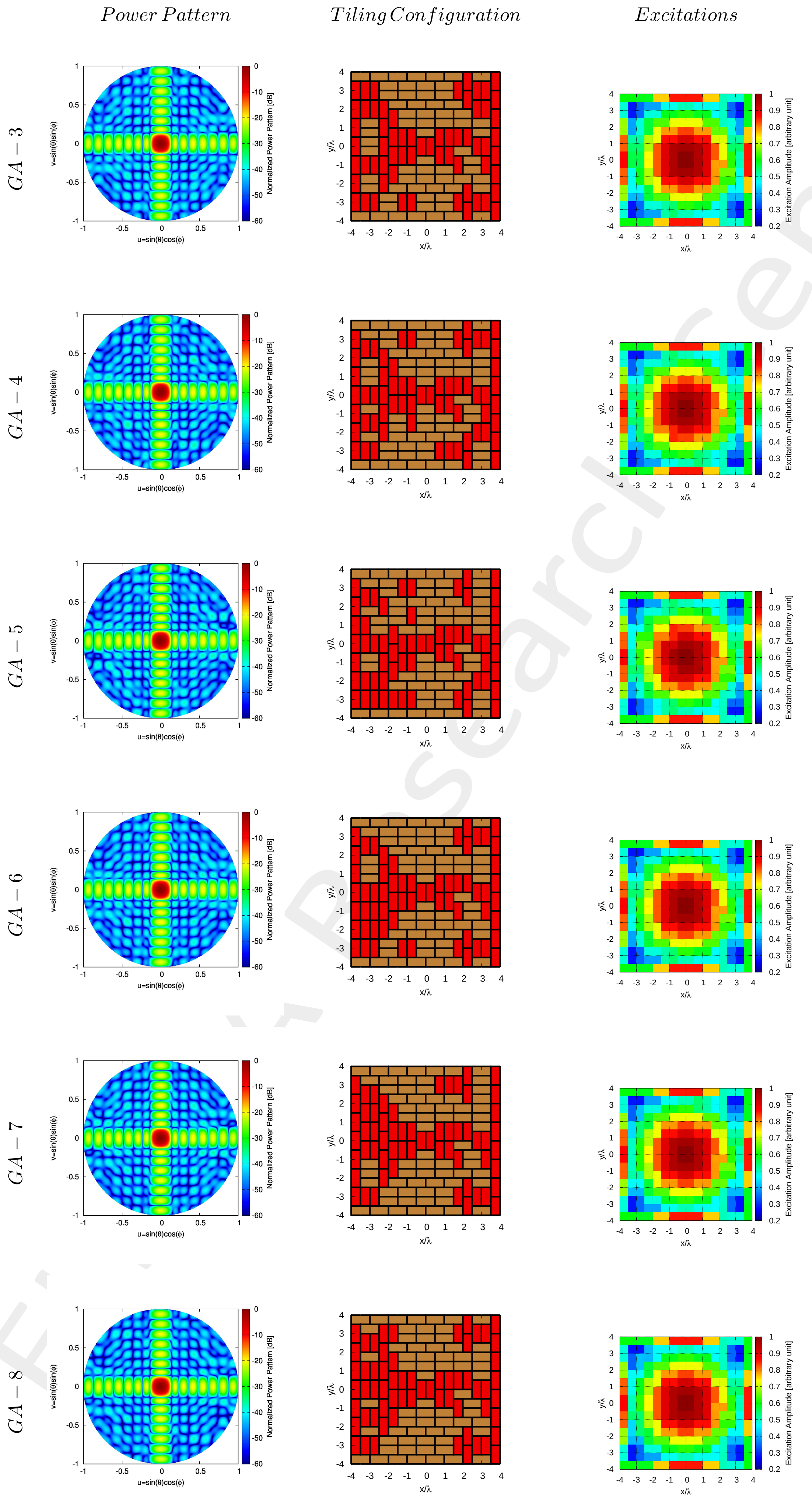
Solution	*Seed*	$\Psi(T_{GA})$
$GA-1$	$\{0.5\}$	1.06191×10^{-2}
$GA-2$	$\{0.2\}$	1.06196×10^{-2}
$GA-3$	$\{0.0, 0.7\}$	1.06234×10^{-2}
$GA-4$	$\{0.8\}$	1.06251×10^{-2}
$GA-5$	$\{0.1\}$	1.06262×10^{-2}
$GA-6$	$\{0.6\}$	1.06270×10^{-2}
$GA-7$	$\{0.3\}$	1.06292×10^{-2}
$GA-8$	$\{0.4, 0.9\}$	1.06301×10^{-2}

Table 2. GA solutions

Seed	t_{tot} [s]	K
0.0	3.56×10^4	130
0.1	3.51×10^4	251
0.2	3.59×10^4	27
0.3	3.52×10^4	25
0.4	3.55×10^4	41
0.5	3.52×10^4	450
0.6	3.55×10^4	158
0.7	3.58×10^4	496
0.8	3.51×10^4	596
0.9	3.56×10^4	46

Table 3. Timings and number of iterations for convergence (K).


Power Pattern
Tiling Configuration
Excitations
Reference
Horizontal
Vertical
GA − 1
GA − 2
v=sin(θ)sin(φ)
u=sin(θ)cos(φ)
Normalized Power Pattern [dB]
y/λ
x/λ
Excitation Amplitude [arbitrary unit]

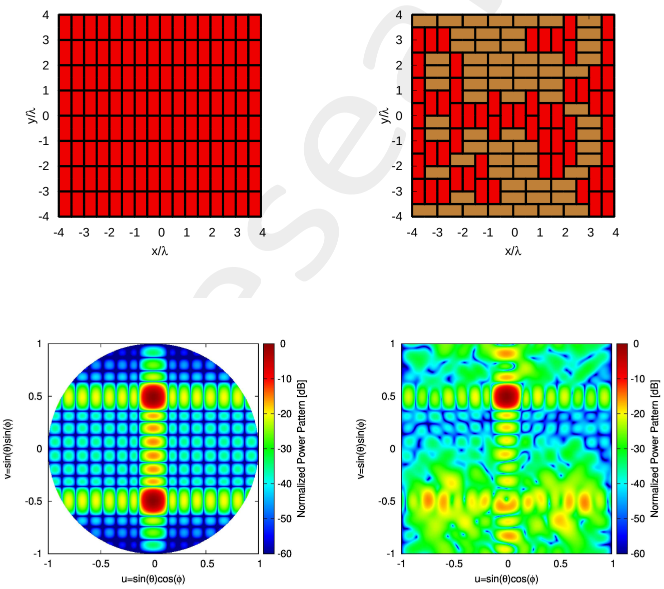


$Power\ Pattern$
$Tiling\ Configuration$
$Excitations$
$GA-3$
$GA-4$
$GA-5$
$GA-6$
$GA-7$
$GA-8$
u=sin(θ)cos(φ)
v=sin(θ)sin(φ)
Normalized Power Pattern [dB]
x/λ
y/λ
Excitation Amplitude [arbitrary unit]

	SLL [dB]	D [dBi]	$HPBW_{az}$ [deg]	$HPBW_{el}$ [deg]	$\Psi(T)$
$Reference$	-20.0	29.30	6.03	6.03	1.00×10^{-2}
$Horizontal$	-18.5654	28.5534	6.8224	6.7823	1.39×10^{-2}
$Vertical$	-18.5654	28.5534	6.7823	6.8224	1.39×10^{-2}
$GA-1$	-19.7391	28.4608	6.7680	6.7748	1.06191×10^{-2}
$GA-2$	-19.7389	28.4645	6.7696	6.7760	1.06196×10^{-2}
$GA-3$	-19.7374	28.4655	6.7695	6.7758	1.06234×10^{-2}
$GA-4$	-19.7367	28.4635	6.7690	6.7757	1.06251×10^{-2}
$GA-5$	-19.7362	28.4621	6.7687	6.7750	1.06262×10^{-2}
$GA-6$	-19.7359	28.4616	6.7700	6.7765	1.06270×10^{-2}
$GA-7$	-19.7350	28.4605	6.7694	6.7776	1.06292×10^{-2}
$GA-8$	-19.7346	28.4597	6.7706	6.7769	1.06301×10^{-2}

Table 1. Pattern descriptors and fitness values for the presented solutions.

Steered Beam



(a) (b)

Table 1. Comparison between the steered pattern ($\theta = 30° - \phi = 90°$) for (a) the "trivial" vertical tiling, (b) the optimized GA tiling.

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