An Innovative *GA*-Based Synthesis Method for the Design of Small-Sized Tiled Planar Sub-Arrayed Phased Arrays

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Abstract

In this work, an innovative methodology for the design of planar sub-arrayed phased arrays composed by irregular arrangements of vertical and horizontal domino-shaped tiles is presented. More precisely, the proposed design method is aimed at optimally synthesizing low and medium size arrays through a suitable customization of mathematical tiling theorems and algorithms. Thanks to the exploitation of a customized genetic algorithm (*GA*)-based optimization strategy, the retrieval of the global optimal solution for the problem of finding the complete tiling affording the minimum side-lobe level (*SLL*) is effectively yielded through the proposed approach. A set of numerical benchmarks is presented in order to assess the proposed sub-arraying technique for small-sized problems.

1 Numerical Validation

1.1 SMALL PROBLEM DIMENSION

1.1.1 Test Case #3: Exhaustive Strategy - 6x6 array

Parameters:

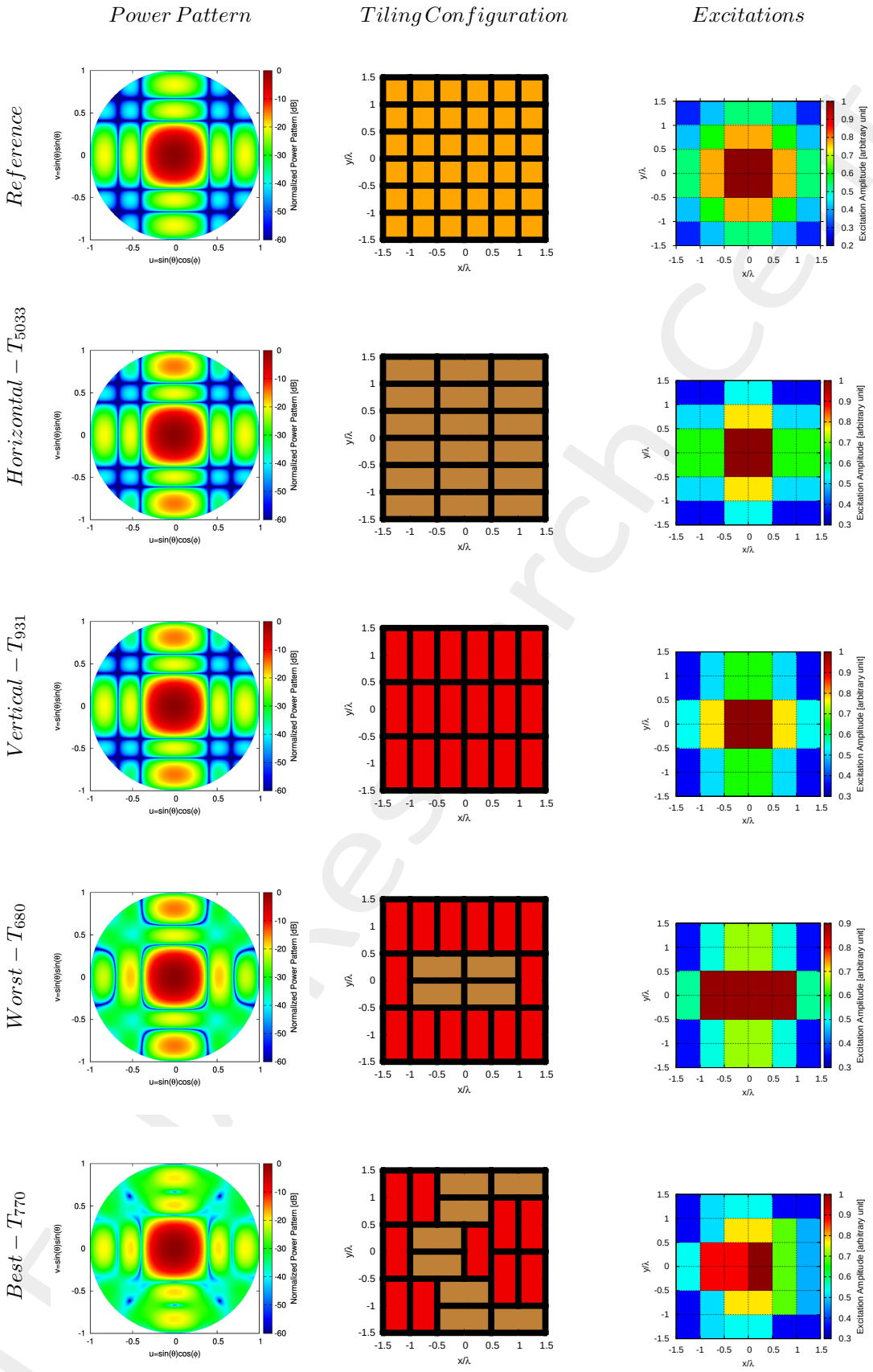
- Total Number of Elements: $M \times N = 6 \times 6 = 36$
- Spacing: $d = \lambda/2$
- Number of Samples along u: 512
- Number of Samples along v: 512
- Steering θ Direction: $\theta_s = 0$
- Steering ϕ Direction: $\phi_s = 0$
- Tile: Domino
- Number of Tiles Types: $L = 2$
  - Horizontal
  - Vertical
- Number of Single Tile Cell Covering: $D_i = 2, i = 1, ..., L$
- Total Number of Configurations: $C_{tot} = 6728$

Cost Function:

- Target SLL: $SLL_{dB}^{TARGET} = -20dB$

$$\Psi(T) = \frac{\left\{SLL\left[P_T(\theta,\phi)\right]_{dB} - SLL_{dB}^{TARGET}\right\}^2}{\left(SLL_{dB}^{TARGET}\right)^2}$$

RESULTS:



$Conf$	**w**
$Horizontal$	101011111111211111110101
$Vertical$	000001111101110111100000
$Worst$	000001101101110110100000
$Best$	000001101111111111111111

Table 1. The words for each selected configuration.

	SLL [dB]	D [dBi]	$HPBW_{az}$ [deg]	$HPBW_{el}$ [deg]	$\Psi(T)$
$Reference$	-20.0	19.87	19.46	19.46	0.0
$Horizontal$	-14.50	19.84	19.46	18.63	7.569×10^{-2}
$Vertical$	-14.50	19.84	18.63	19.45	7.569×10^{-2}
$Worst$	-14.50	19.87	19.25	18.63	7.569×10^{-2}
$Best$	-18.60	19.89	19.25	19.08	4.921×10^{-3}

Table 2. Pattern descriptors and fitness values for the presented solutions.

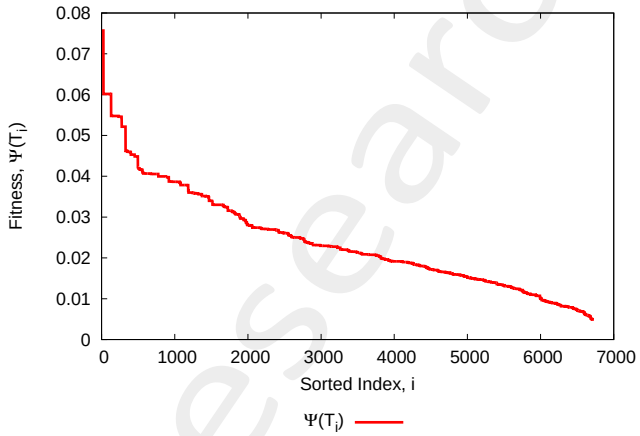
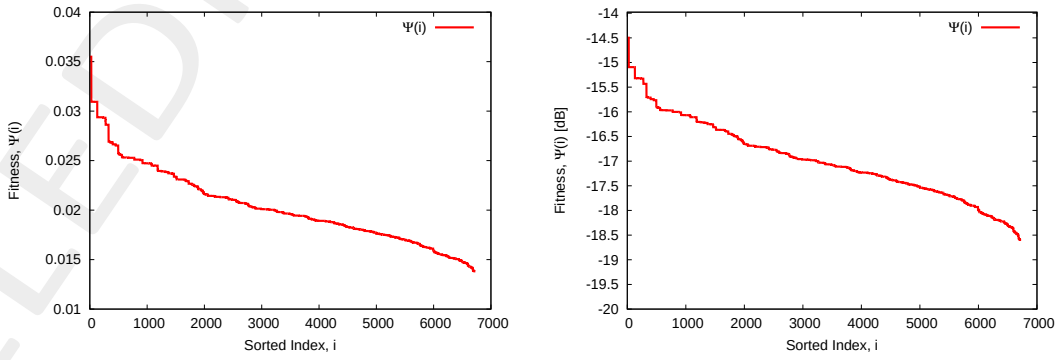


Figure 2. Fitness. Ordered solutions w.r.t. the fitness value.

Re-defined fitness:

$$\Psi(T) = SLL$$

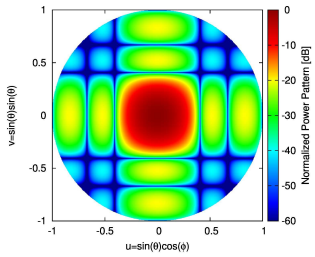
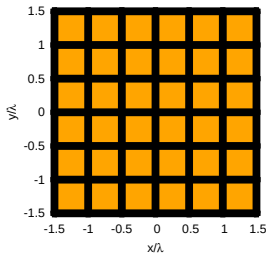
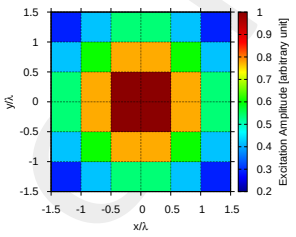
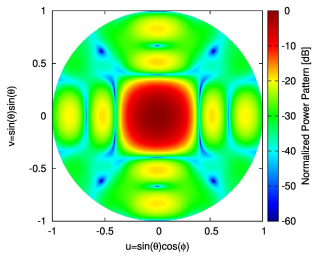
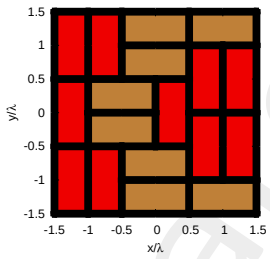
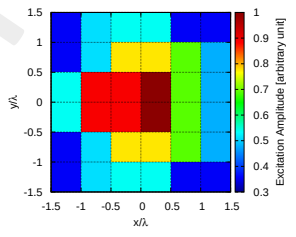
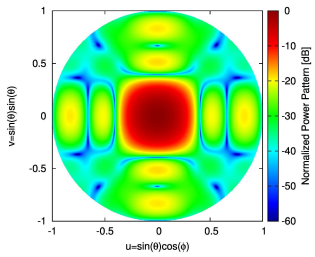
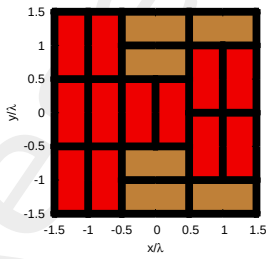
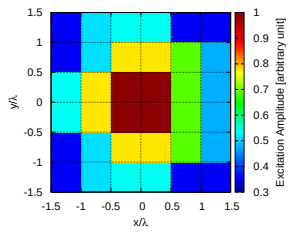
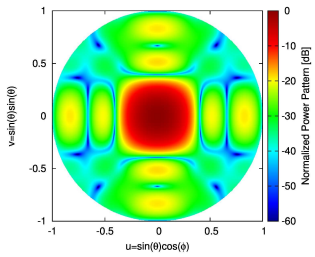
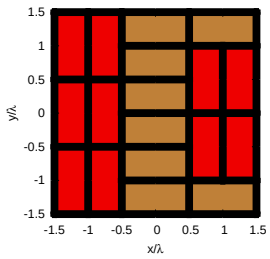
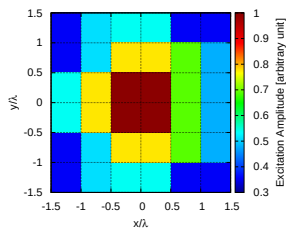
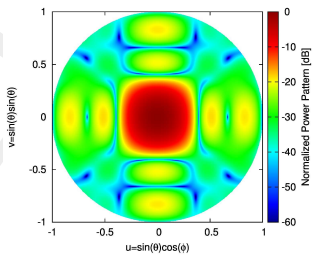
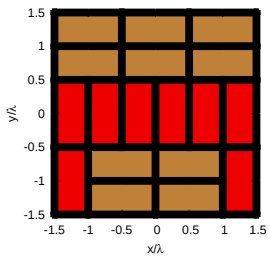
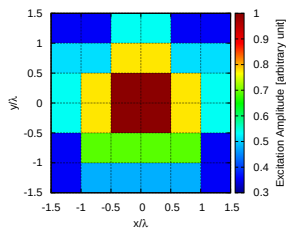


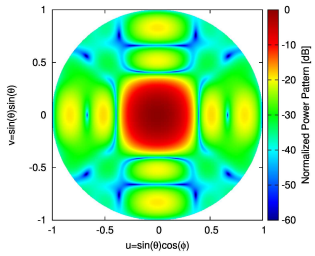
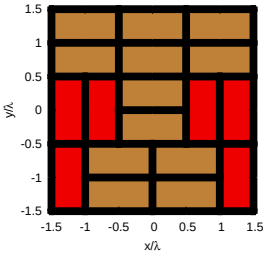
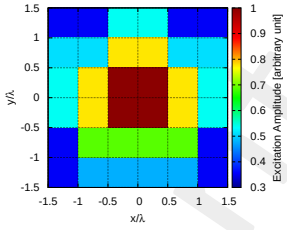
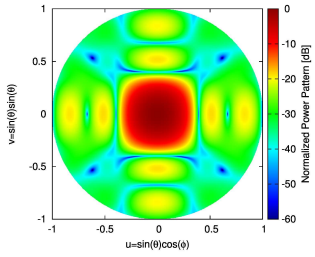
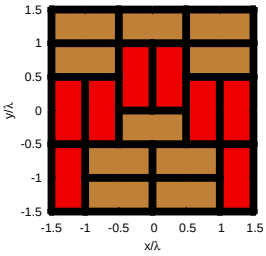
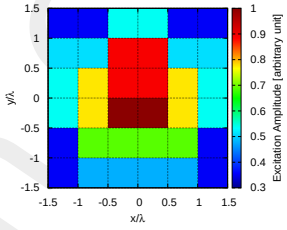
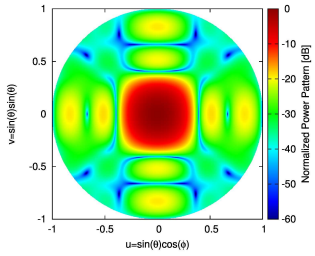
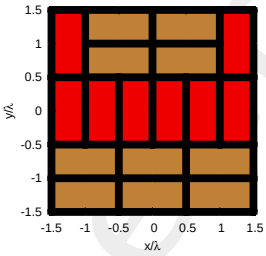
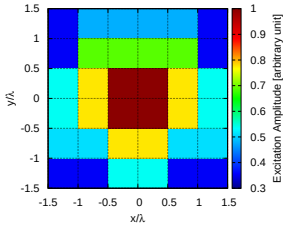
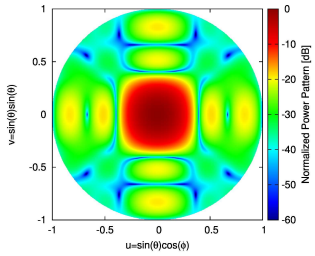
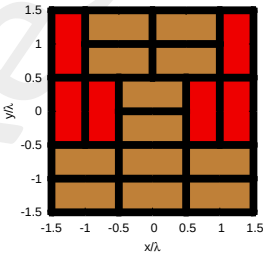
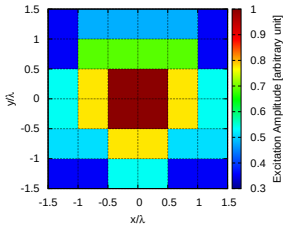
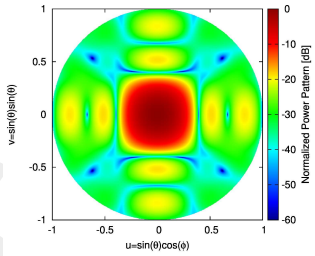
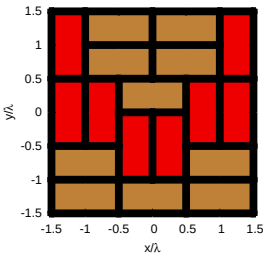
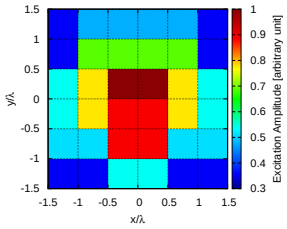
(a) (b)

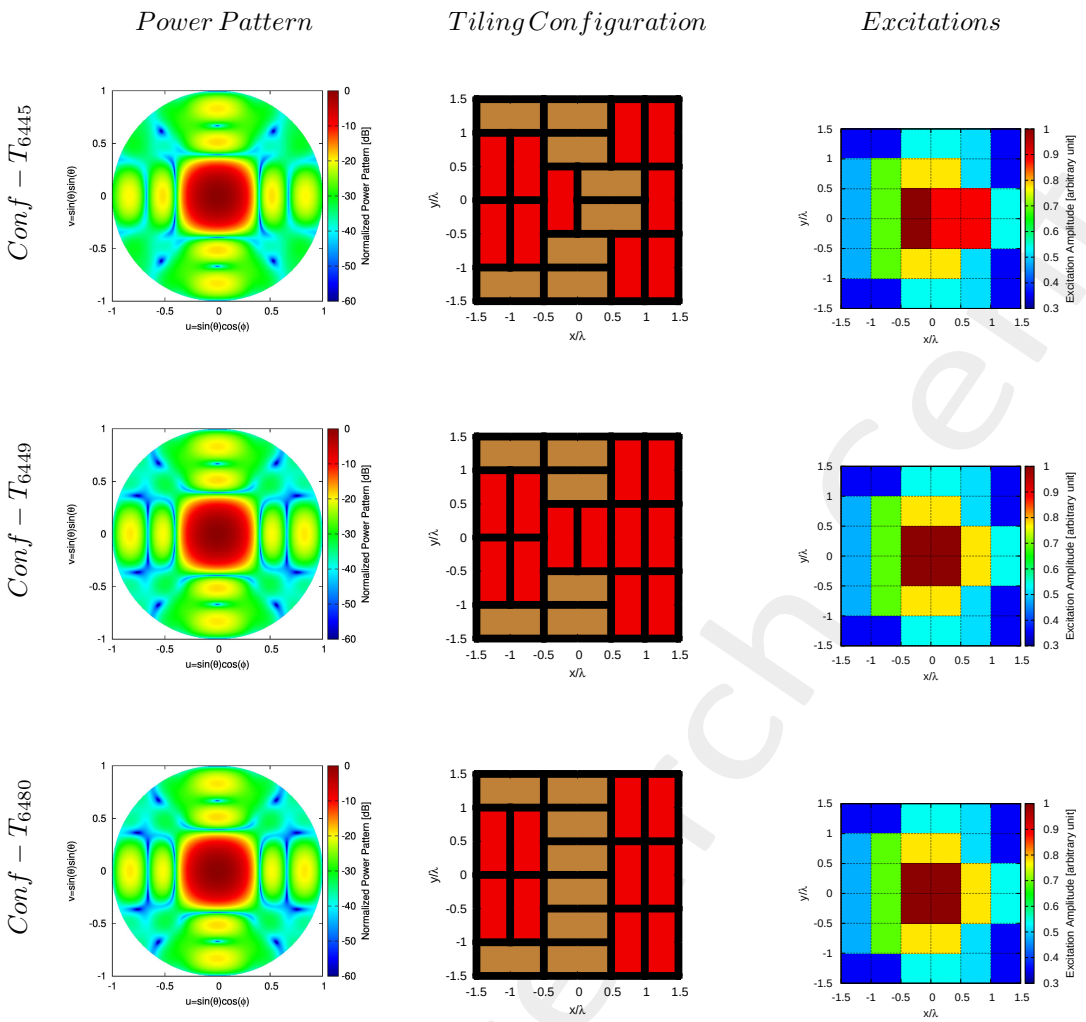
Figure 2. Fitness behaviour in linear (a) and decibel (b) scales.

Optimal Solutions

- The following configurations are all optimal solutions, i.e. all of them shares with the "best" solution the same fitness value $\Psi(T) = 4.9 \times 10^{-3}$, thus the same $SLL = -18.60dB$

	Power Pattern	*Tiling Configuration*	*Excitations*
Reference			
$Best - T_{770}$			
$Conf - T_{1077}$			
$Conf - T_{1108}$			
$Conf - T_{1370}$			

	Power Pattern	*Tiling Configuration*	*Excitations*
$Conf - T_{1417}$			
$Conf - T_{1435}$			
$Conf - T_{3691}$			
$Conf - T_{3709}$			
$Conf - T_{3727}$			



$Conf$	**w**
$Best - T_{770}$	00000110111111111111111111
$Conf - T_{1077}$	00000111111111111111111111
$Conf - T_{1108}$	00000111111112111111111111
$Conf - T_{1370}$	00001011110111101111100001
$Conf - T_{1417}$	00001011110121101111100001
$Conf - T_{1435}$	00001011110122101111100001
$Conf - T_{3691}$	10000111101111011111010000
$Conf - T_{3709}$	10000111101121011111010000
$Conf - T_{3727}$	10000111101221011111010000
$Conf - T_{6445}$	11111111111111111101100000
$Conf - T_{6449}$	11111111111111111111100000
$Conf - T_{6480}$	11111111111112111111100000

Table 3. The words for each selected configuration.

1.1.2 Test Case #4: GA Strategy - 6x6 array - Schemata Approach

NOTE: with respect to the previous test cases the cost function is re-defined as: $\Psi(T) = SLL$

Array Analysis Parameters:

- Total Number of Elements: $M \times N = 6 \times 6 = 36$
- Spacing: $d = \lambda/2$
- Number of Samples along u: 512
- Number of Samples along v: 512
- Steering θ Direction: $\theta_s = 0$
- Steering ϕ Direction: $\phi_s = 0$

Tiling Parameters:

- Tile: Domino
- Number of Tiles Types: $L = 2$
  - Horizontal
  - Vertical
- Number of Single Tile Cell Covering: $D_i = 2, i = 1, ..., L$
- Total Number of Configurations: $C_{tot} = 6728$
- Number of Inner Points: $N_{inn} = 25$

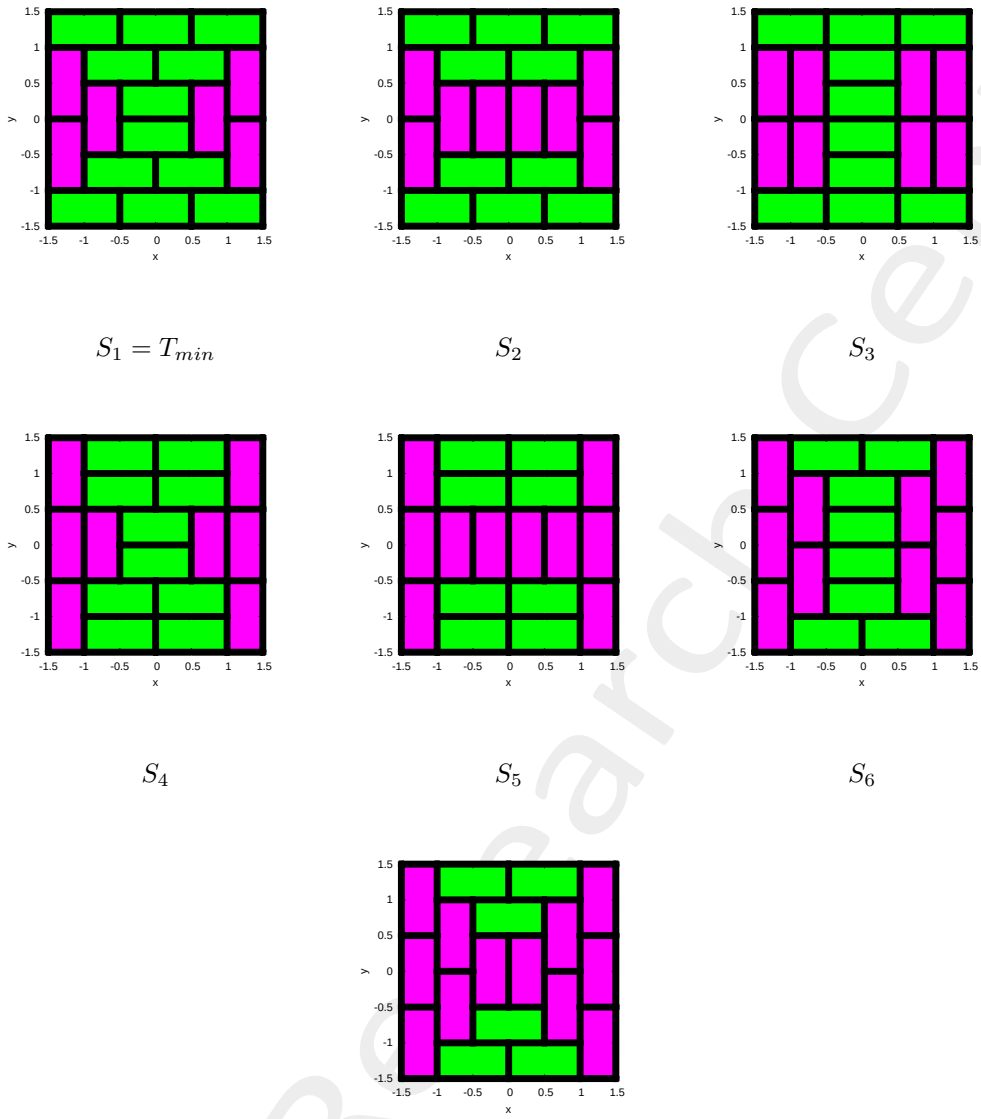
Genetic Algorithm Parameters:

- Number of Unknowns: $U = 50$
- Population Dimension: $P = 52$
- Maximum Number of Iterations: $I = 1000$
- Crossover Probability: $p_{cross} = 0.9$
- Mutation Probability: $p_{mut} = 0.01$
- Diversified Percentage: $p_{div} = 10\%$

Cost Function:

$$\Psi(T) = SLL$$

Schemata Analysis:

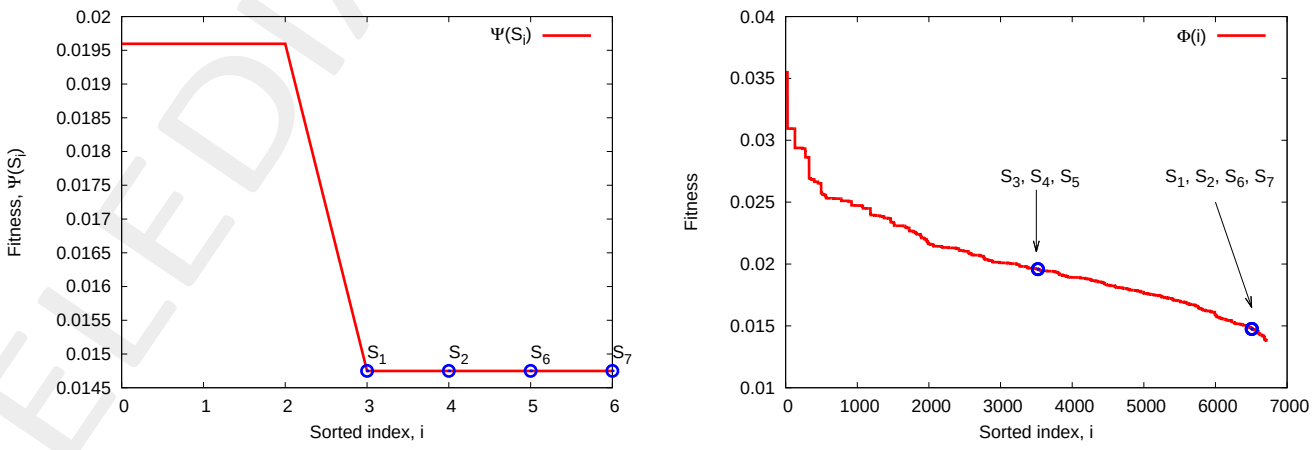


$S_1 = T_{min}$ S_2 S_3

S_4 S_5 S_6

$S_7 = T_{max}$

Figure 1. Generated schematas for a 6×6 rectangular region.



(a) (b)

Figure 2. Schematas sorted fitness (a), and comparison with the exhaustive fitness analysis (b).

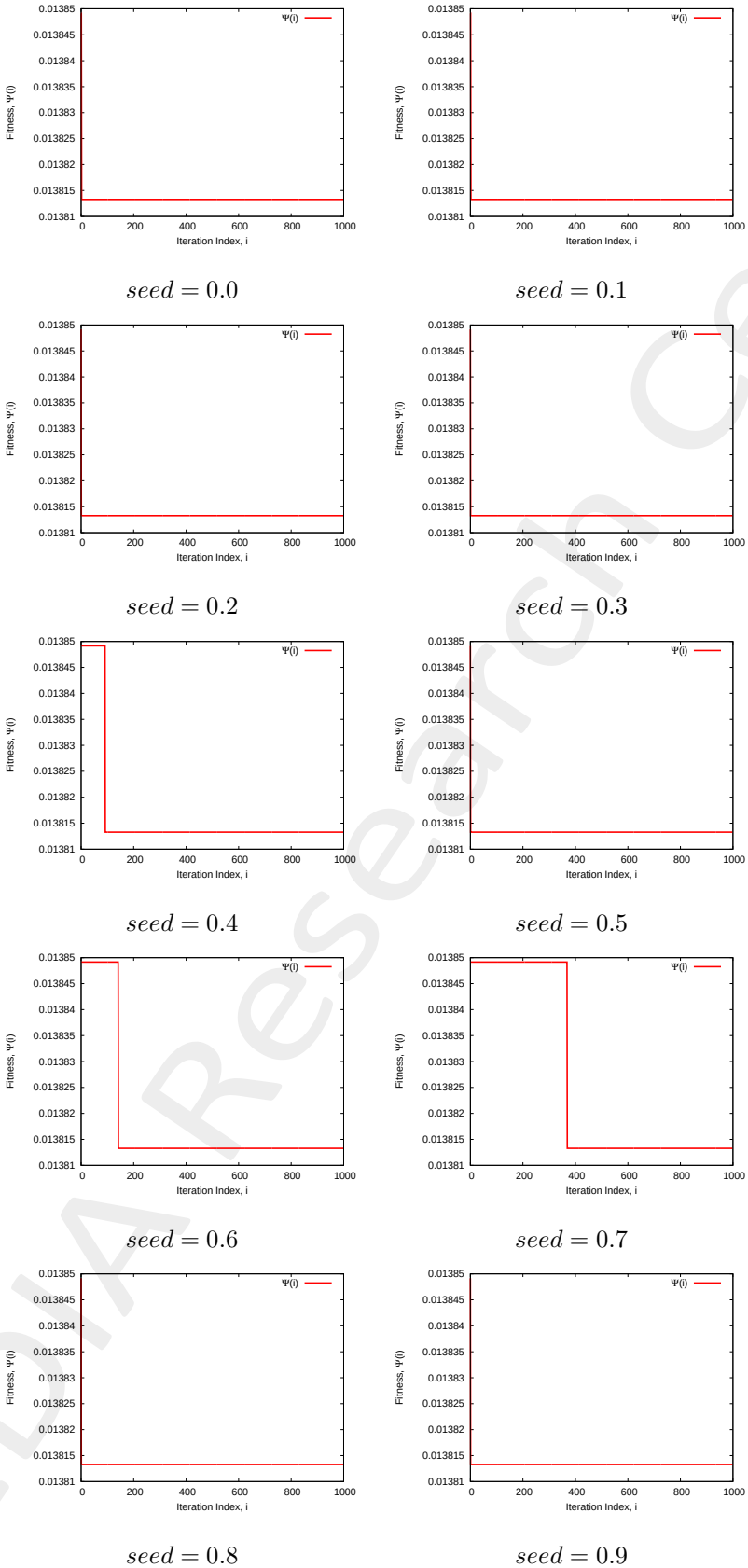
i	$Schemata\, S_i$	$\Psi\,(S_i)$
1	000000000000000000000000	1.4649×10^{-2}
2	000000000000010000000000	1.4649×10^{-2}
3	000000111001110011100000	1.9596×10^{-2}
4	111111111111111111111111	1.9596×10^{-2}
5	111111111112111111111111	1.9596×10^{-2}
6	111111222112221122211111	1.4649×10^{-2}
7	111111222112321122211111	1.4649×10^{-2}

Table 1. Schematas words

Observations:

- $S_1$and S_7 are the minimal (T_{min}) and the maximal (T_{max}) tiling configurations respectively.

GA Optimization RESULTS:



$seed = 0.0$ $seed = 0.1$

$seed = 0.2$ $seed = 0.3$

$seed = 0.4$ $seed = 0.5$

$seed = 0.6$ $seed = 0.7$

$seed = 0.8$ $seed = 0.9$

Figure 3. Fitness of the GA simulations for each random seed.

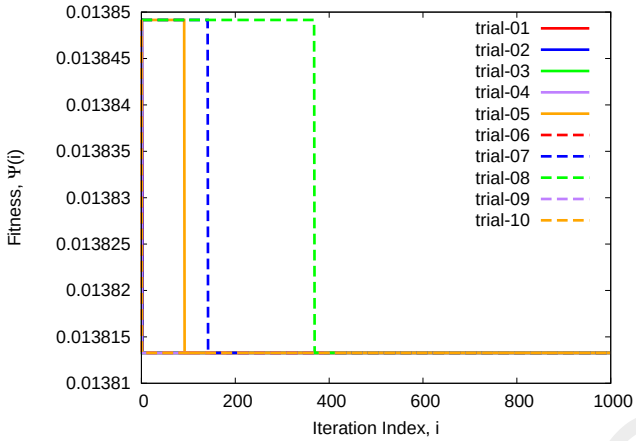


Figure 4. Fitness of the GA simulations.

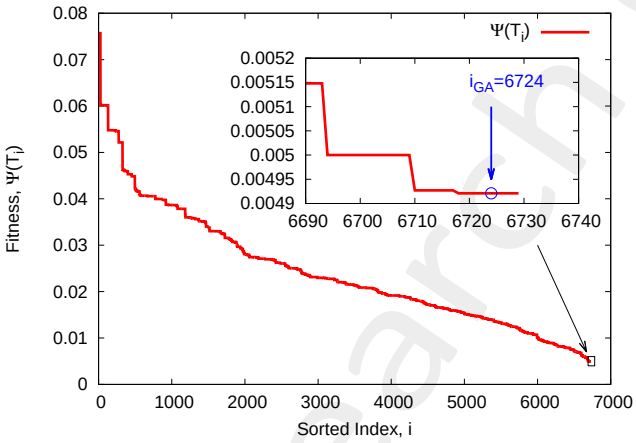
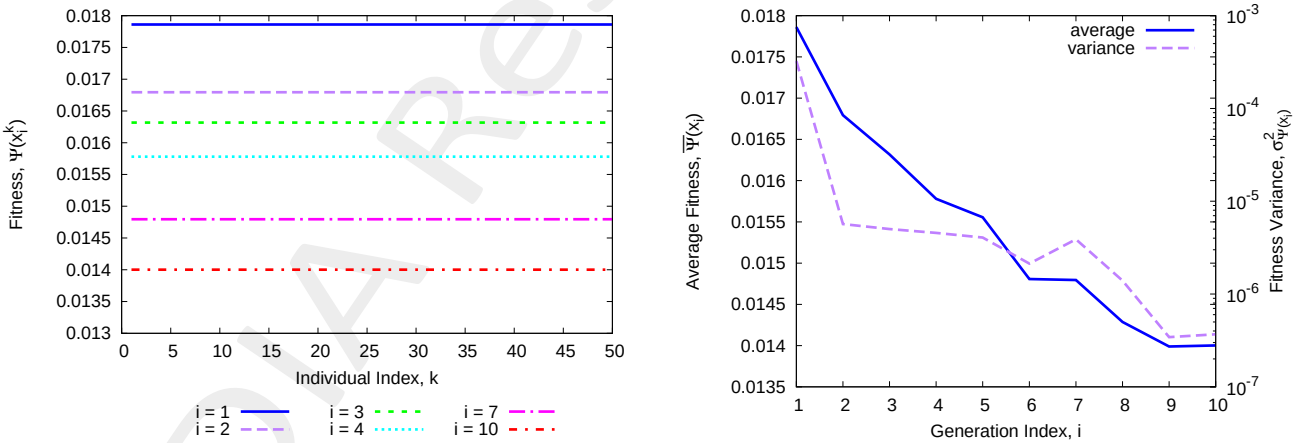
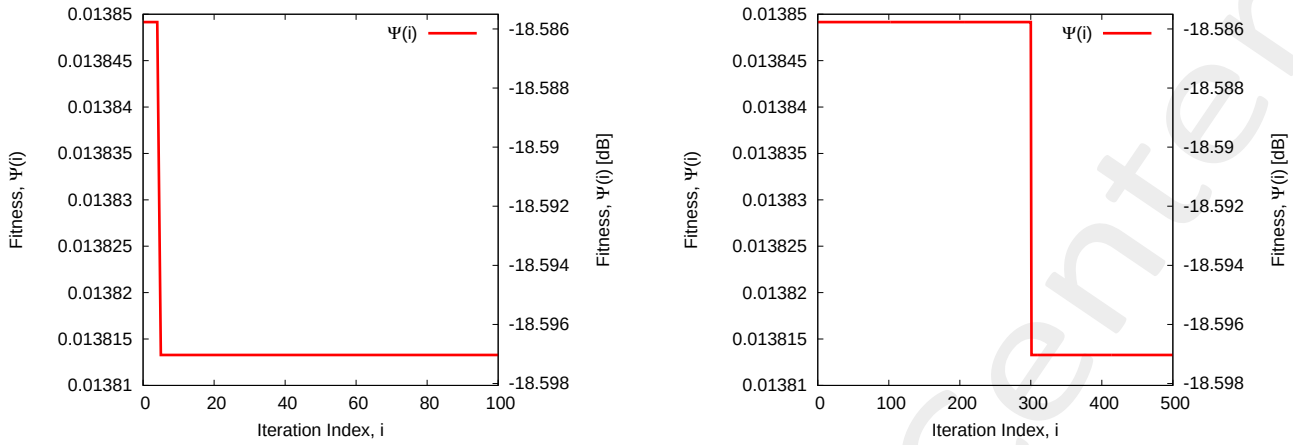


Figure 5. Ordered fitness values of the Exhaustive Strategy compared with the GA solution - case $seed = 0.0$.



(a) (b)

Figure 6. Average fitness of the generations (a), and statistics (average and variance) (b) - case $seed = 0.0$.



(a) (b)

Figure 7. Fitness of the GA simulation using: (a) the schemata $S_{[1-7]} = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7\}$ for the initial population generation, and (b) the sub-optimal set of schemata $S_{[3-4-5]} = \{S_3, S_4, S_5\}$.

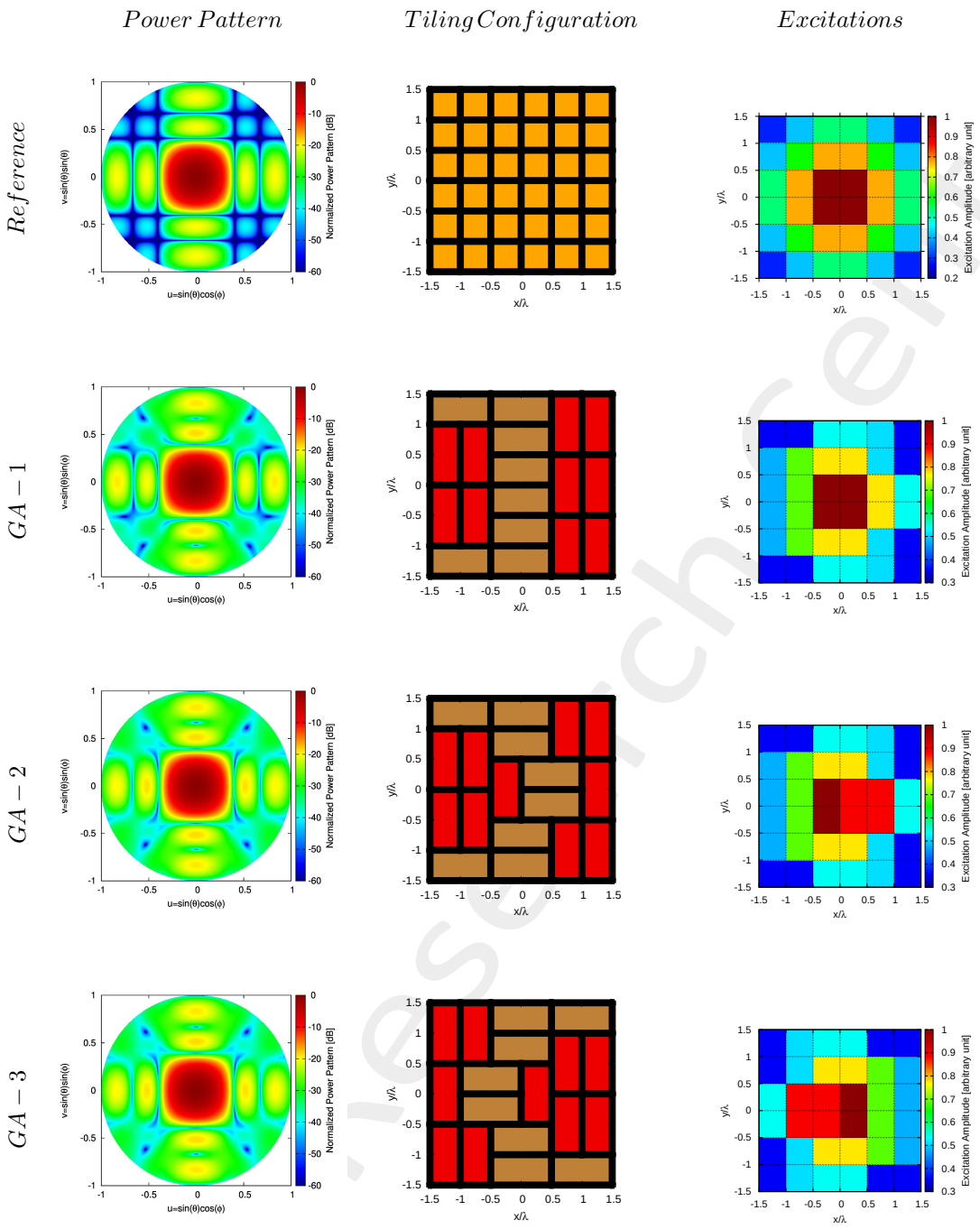
- **Observation:** considering only the sub-optimal schemata does not change the dynamic of the obtained fitness. The initial population still contains some "good" individuals, even if only sub-optimal schematas are used as "starting points" for individuals generation.

Conf	$\mathbf{w} - Exhaustive$	*Solution*	$\mathbf{w} - GA$	*Seed*
$Best - T_{770}$	000001101111111111111111	$GA - 1$	111111111112111111100000	$\{0.0, 0.3, 0.6, 0.7, 0.9\}$
$Conf - T_{1077}$	000001111111111111111111	$GA - 2$	111111111111111101100000	$\{0.1, 0.2, 0.4, 0, 5\}$
$Conf - T_{1108}$	000001111111211111111111	$GA - 3$	000001101111111111111111	$\{0.8\}$
$Conf - T_{1370}$	000010111101111010111100001			
$Conf - T_{1417}$	000010111101211010111100001			
$Conf - T_{1435}$	000010111101221010111100001			
$Conf - T_{3691}$	100001111011110111010000			
$Conf - T_{3709}$	100001111011210111010000			
$Conf - T_{3727}$	100001111012210111010000			
$Conf - T_{6445}$	111111111111111101100000			
$Conf - T_{6449}$	111111111111111111100000			
$Conf - T_{6480}$	111111111112111111100000			

Table 3. The global optimal solutions and GA solutions words.

$Seed$	t_{tot} [s]	K
0.0	4.73×10^3	3
0.1	4.77×10^3	3
0.2	4.61×10^3	2
0.3	4.68×10^3	2
0.4	4.73×10^3	91
0.5	4.74×10^3	2
0.6	4.71×10^3	140
0.7	4.86×10^3	369
0.8	4.79×10^3	2
0.9	4.95×10^3	2

Table 5. Timings and number of iterations for convergence (K).



	SLL [dB]	D [dBi]	$HPBW_{az}$ [deg]	$HPBW_{el}$ [deg]	$\Psi(T)$
$Reference$	-20.0	19.87	19.46	19.46	1.0000×10^{-2}
$GA-1$	-18.5971	19.88	19.35	19.08	1.3813×10^{-2}
$GA-2$	-18.5971	19.88	19.25	19.08	1.3813×10^{-2}
$GA-3$	-18.5971	19.88	19.25	19.08	1.3813×10^{-2}

Table 4. Pattern descriptors and fitness values for the presented solutions.

Schemata Analysis

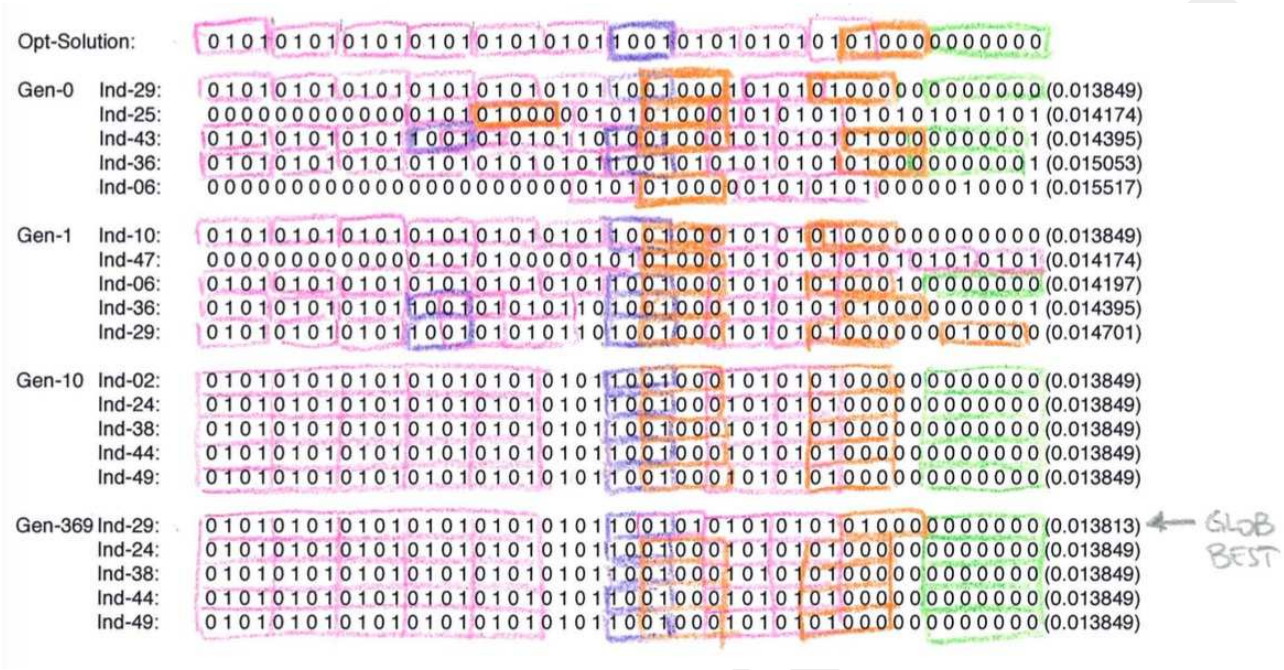


Figure 8. Schemata Analysis.

Observations:

- All the selected schematas are heavily present in the initial population (in the picture only the fittest individuals are reported)
- In the initial generations the schematas are randomly positioned among the binary strings, then at convergence the schematas are positioned as in the global best individual.

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