Dictionary-Based Bayesian Compressive Sensing for Imaging Arbitrary Scatterers

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Abstract

This work deals with an innovative free-space inverse scattering technique. The developed methodology is based on the exploitation of a Bayesian Compressive Sensing (*BCS*) solver and a set (or *dictionary*) of expansion bases. Several *BCS*-regularized reconstructions are performed using the different bases in the dictionary, and the *sparsest* solution is selected as the most *reliable* one. Thanks to such an approach, (*i*) no a-priori information about the unknown scatterers is required, and (*ii*) it is possible to extend the range of applicability of standard BCS-based inversion to objects having arbitrary size and shape. In order to verify the effectiveness of the proposed technique, as well as to test its robustness to noise, some illustrative numerical results are shown in the following.

1 Numerical Results

1.1 Object Exp #0

Goal: To prove the effectiveness of the alphabet based approach using a sparse scatterer w.r.t. exponential basis.

Test Case Description

Object:

- $\varepsilon_{r,max} = 1.03$
- $\sigma = 0$ [S/m]
- Number of Daubechies coefficients: $Nc = 7$

Sources:

- Plane waves
- Amplitude: $A = 1$
- Frequency: 300 MHz ($\lambda = 1$m)
- Number of views: $V = 36$

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- $D = 4096$ (64×64) ($\frac{L_D}{\sqrt{D}} = \frac{\lambda}{16}$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $N = 1024$ (32×32) ($\frac{L_D}{\sqrt{N}} = \frac{\lambda}{8}$)
- $L_D = 4\lambda$

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 4\lambda$
- $M = 36$

M-BCS parameters:

- $a = 1.0 \times 10^{-7}$
- $b = 1.0 \times 10^{-3}$

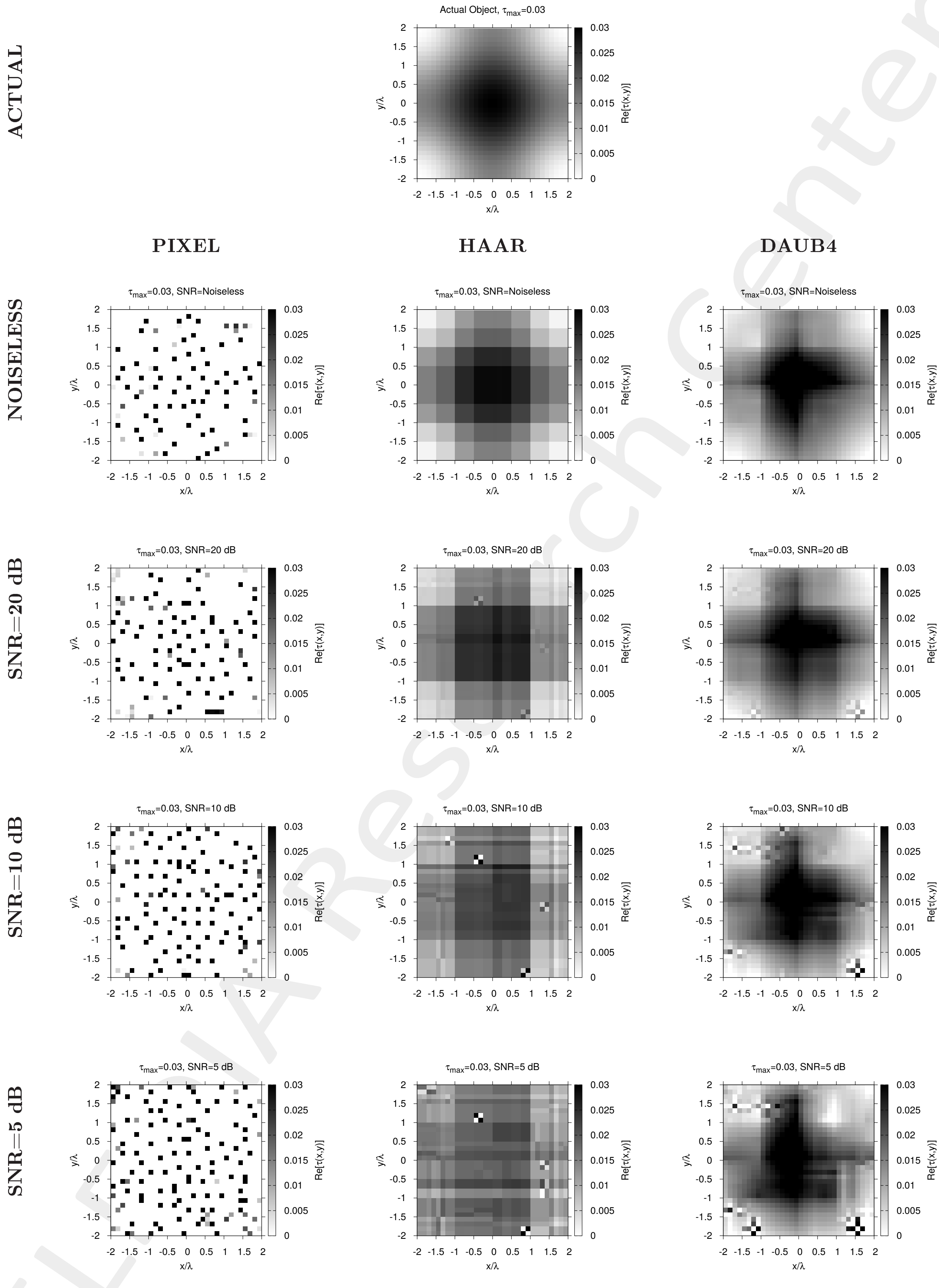

Figure 1: Actual and retrieved object (real part) considering different wavelet expansions.

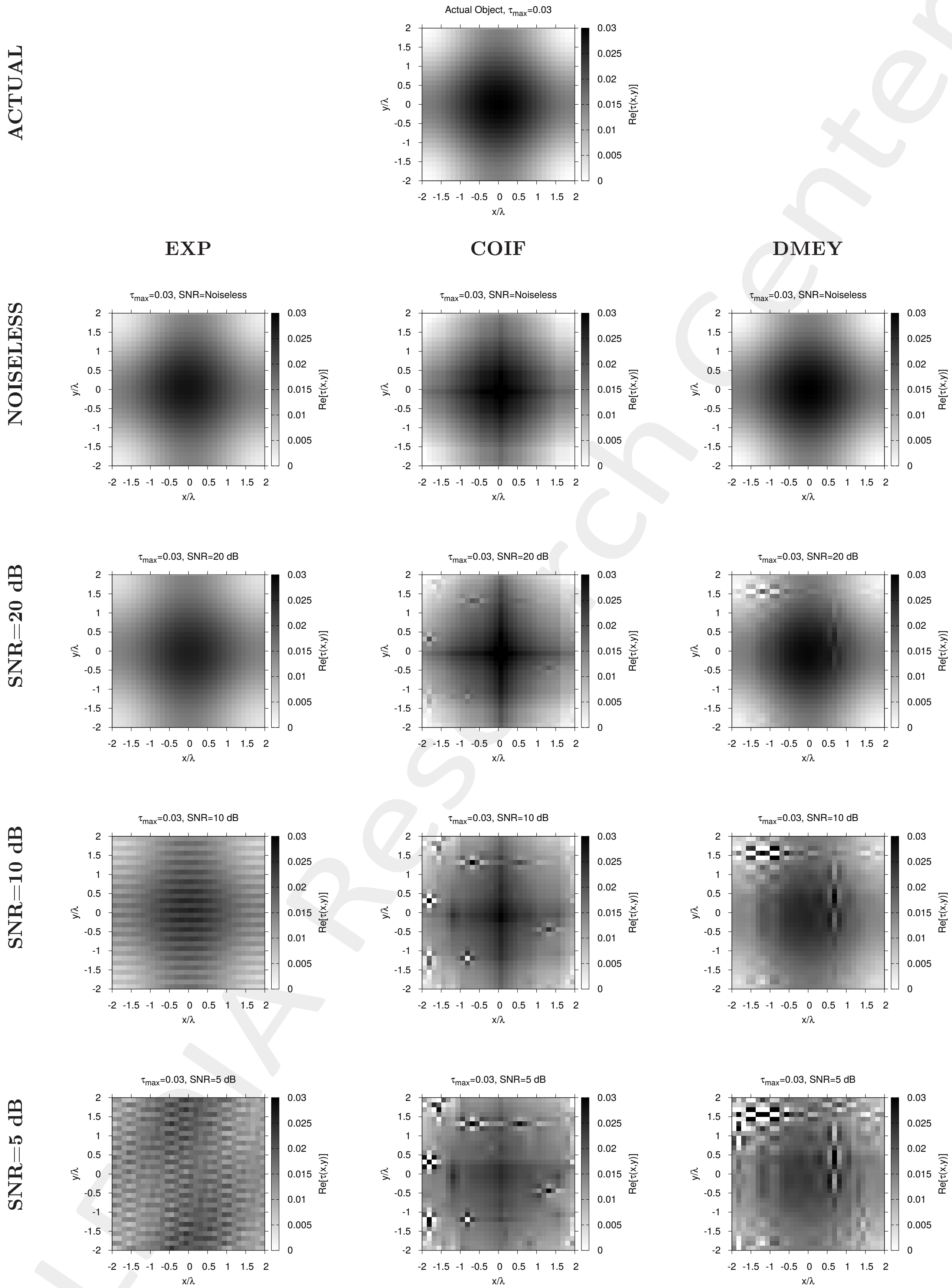

Figure 2: Actual and retrieved object (real part) considering different wavelet expansions.

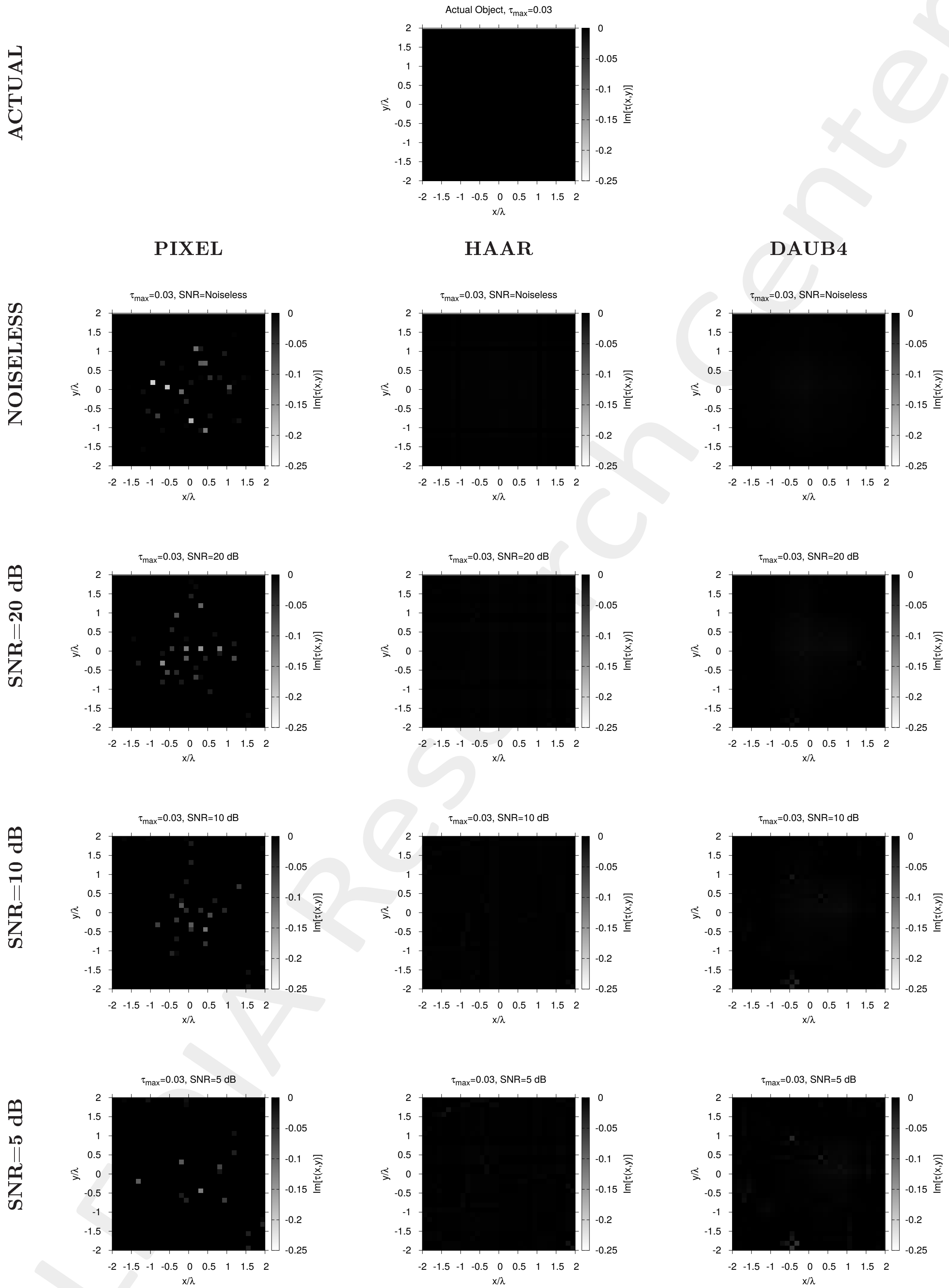

Figure 3: Actual and retrieved object (imaginary part) considering different wavelet expansions.

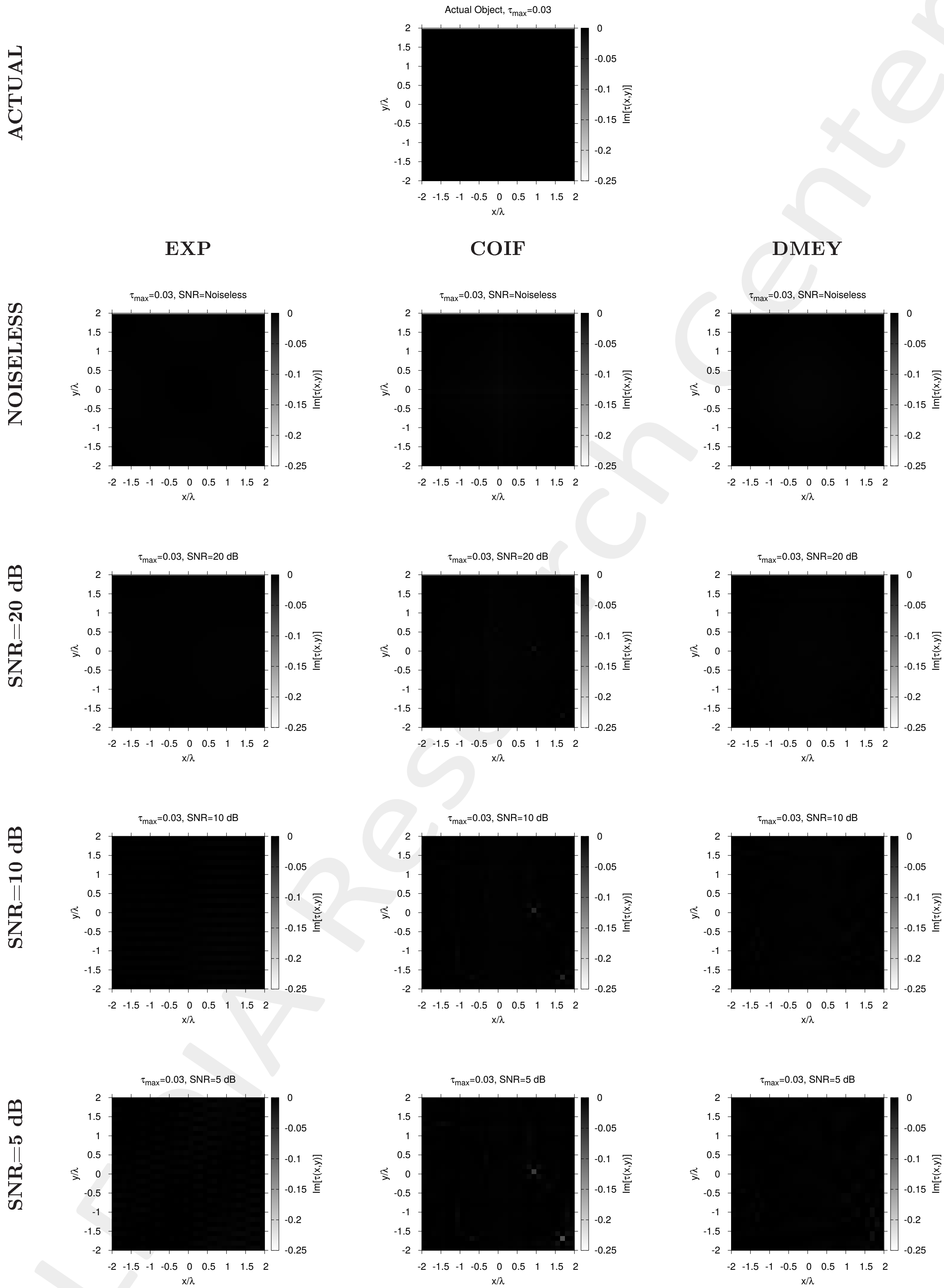

Figure 4: Actual and retrieved object (imaginary part) considering different wavelet expansions.

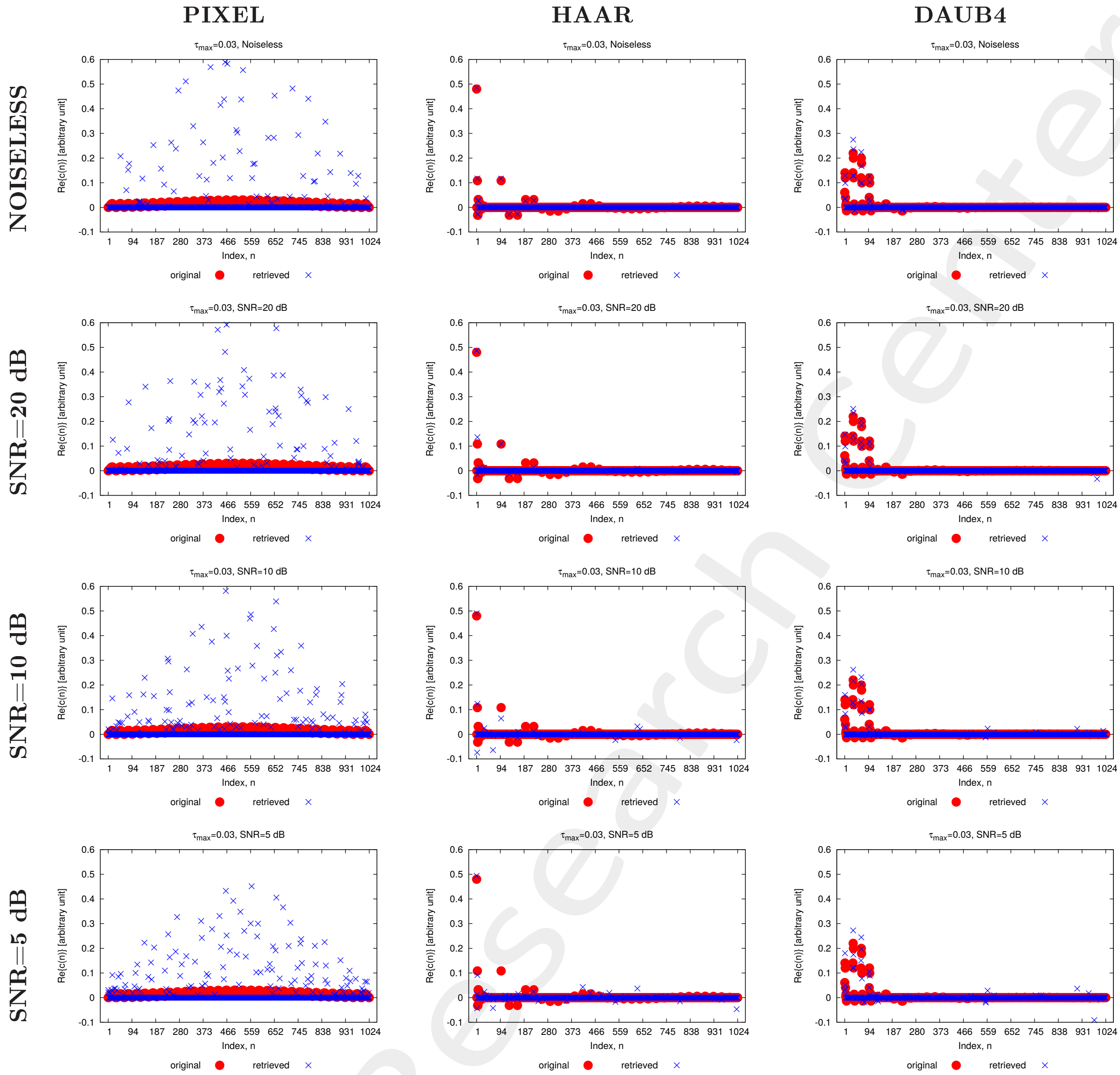

Figure 5: Real part of the actual and retrieved coefficients considering different wavelet expansions.

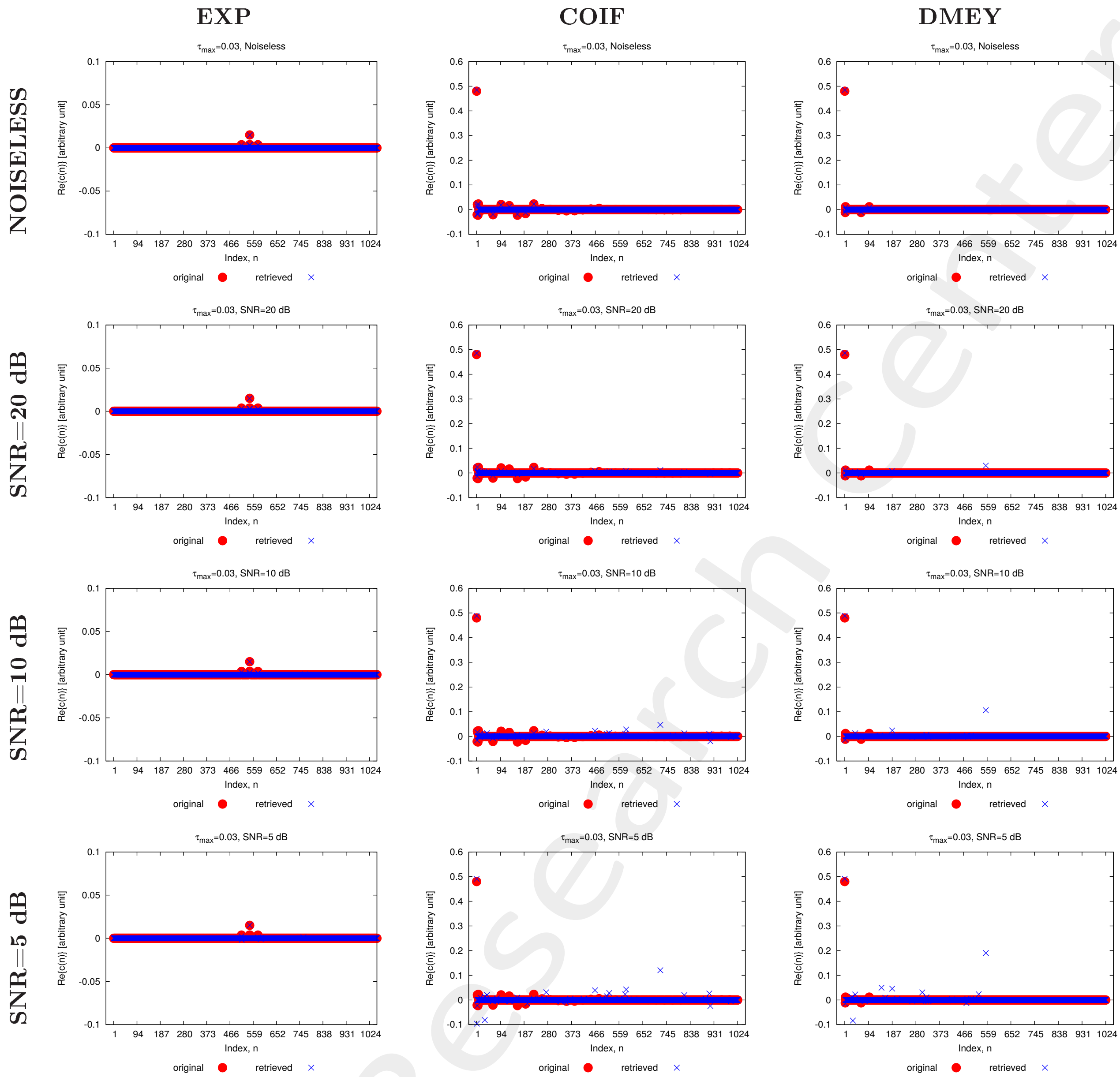

Figure 6: Real part of the actual and retrieved coefficients considering different wavelet expansions.

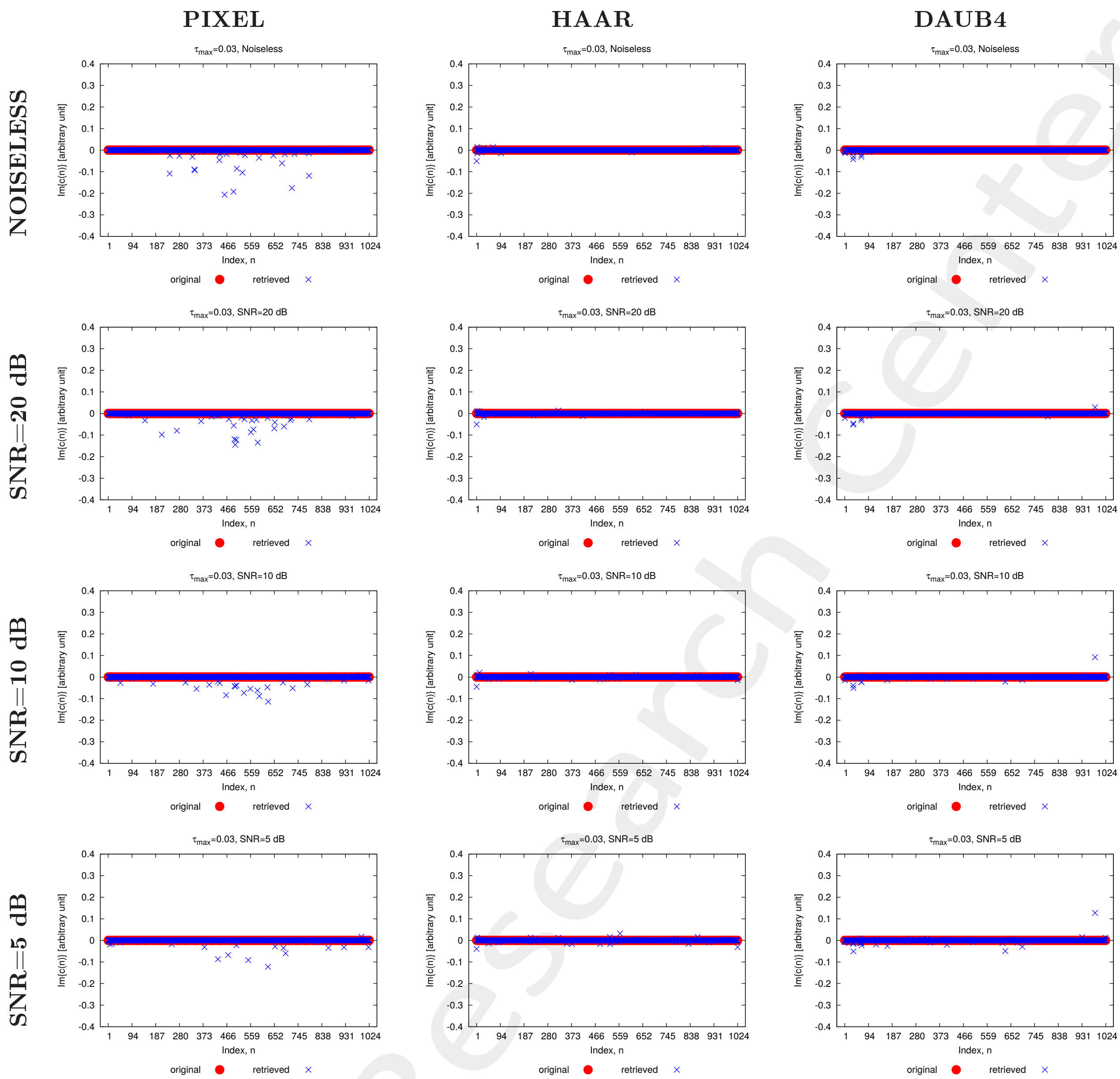

Figure 7: Imaginary part of the actual and retrieved coefficients considering different wavelet expansions.

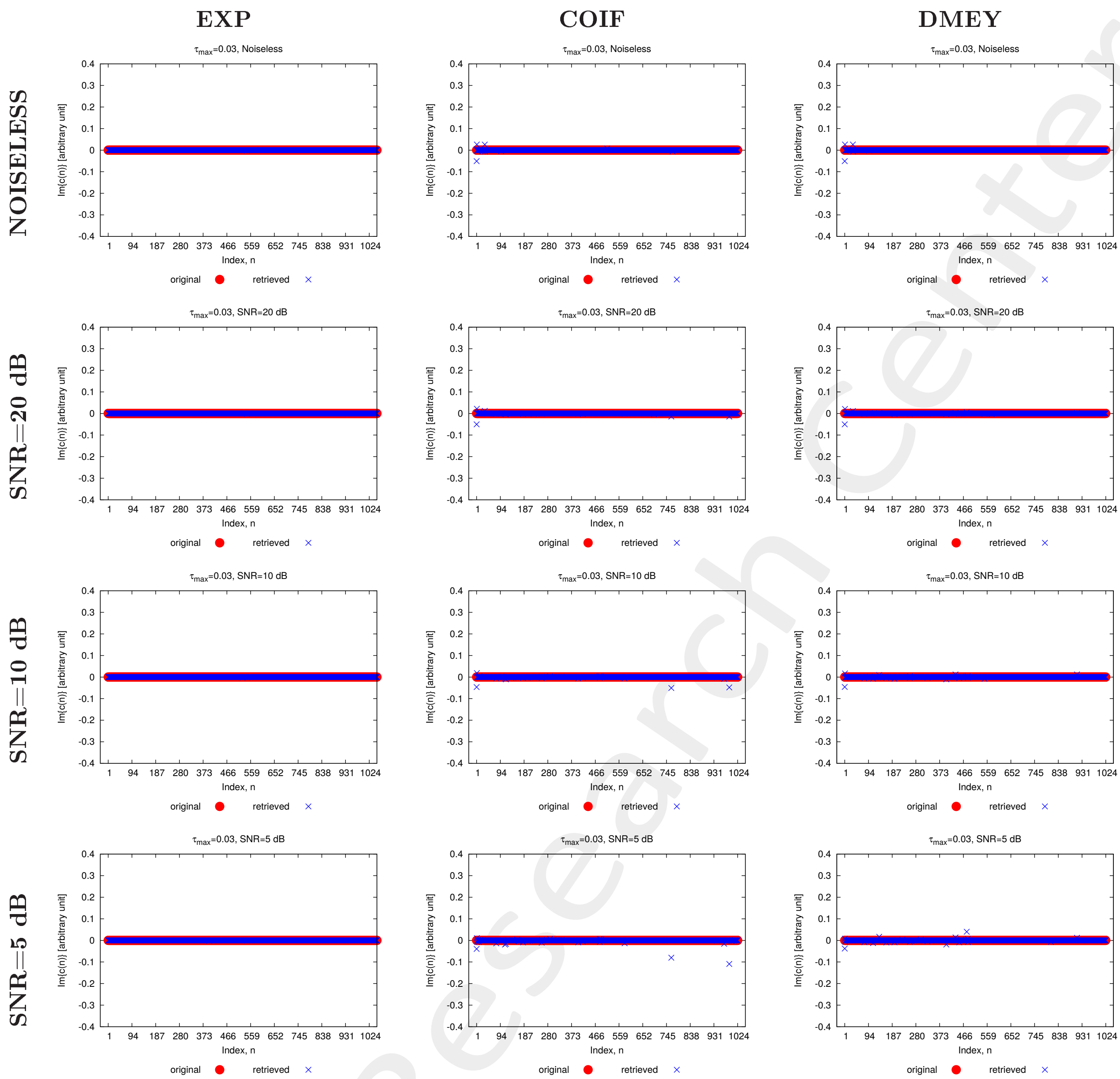

Figure 8: Imaginary part of the actual and retrieved coefficients considering different wavelet expansions.

Coefficients Analysis:

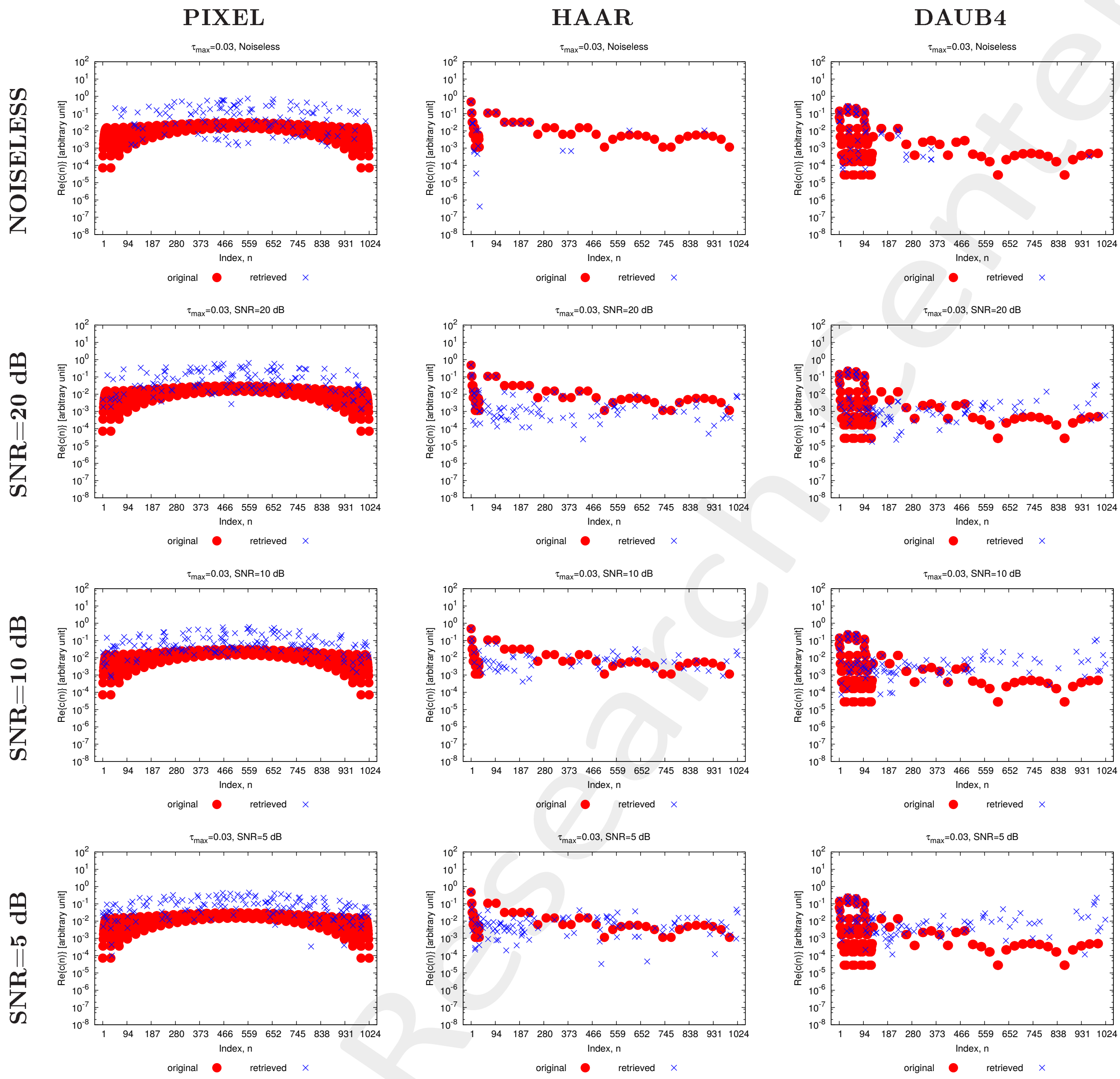

Figure 9: Imaginary part of the actual and retrieved coefficients considering different wavelet expansions.

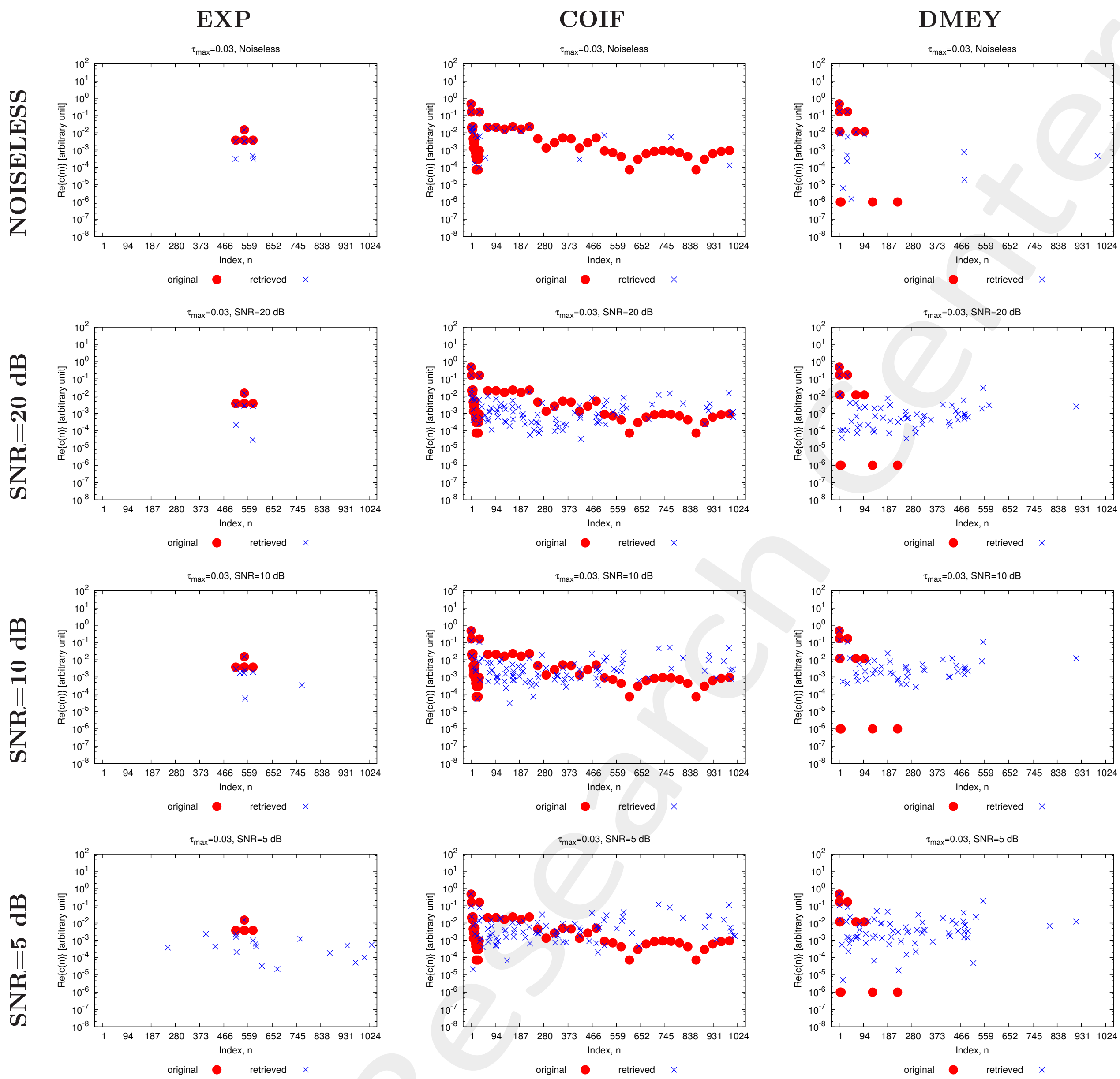

Figure 10: Imaginary part of the actual and retrieved coefficients considering different wavelet expansions.

SNR [dB]	*Pixel*	*Haar*	*Daub4*	*Coiflet*	*DMayer*	*Exp*
Noiseless	119	25	41	25	15	**8**
20	134	90	95	95	61	**7**
10	147	58	101	97	49	**8**
5	153	102	95	91	69	**17**

Table 1: Number of the retrieved non-zero coefficients, using different wavelet functions.

More information on the topics of this document can be found in the following list of references.

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