An Innovative Particle Swarm Optimization-Based Approach for GPR Microwave Imaging

M. Salucci, L. Poli, N. Anselmi, and A. Massa

Abstract

This work presents an innovative microwave imaging technique for accurate and robust subsurface imaging. The proposed approach is based on the integration of a customized particle swarm optimization (*PSO*) algorithm within the iterative multi-scaling approach (*IMSA*), and exploits multiple frequency components extracted from ground penetrating radar (*GPR*) wideband data. The solution of the arising inverse scattering problem is yielded within a multi-frequency (*MF*) approach, allowing to exploit the intrinsic frequency diversity of GPR measurements in order to add information and mitigate the ill-posedness and non-linearity issues. Some numerical experiments are shown in order to assess the effectiveness of the proposed *MF-IMSA-PSO* method when dealing with the retrieval of unknown buried scatterers having different shape. Moreover, a comparison to a competitive state-of-the-art deterministic approach is shown, in order to highlight the benefits of exploiting a global optimization algorithm in minimizing the *MF* cost function.

1 Definitions

1.1 Glossary

- SF: Single-Frequency;
- FH: Frequency-Hopping;
- MF: Multi-Frequency;
- P: Swarm dimension;
- U: Total number of unknowns;
- S: Maximum number of $IMSA$ zooming steps;
- s^{best}: Last performed $IMSA$ zooming step ($s^{best} \leq S$);
- η_{th}: $IMSA$ zooming threshold;
- D_{inv}: Investigation domain;
- D_{obs}: Observation domain;
- L: Side of the investigation domain;
- N: Number of discretization cells in D_{ind};
- V: Number of views;
- M: Number of measurement points;
- F: Number of frequencies considered for the inversion;
- $\mathbf{r}^{(v)} = \left(x^{(v)}, y^{(v)}\right)$: Coordinates of the v-th source ($v = 1, \dots, V$).
- $\mathbf{r}_m^{(v)} = \left(x_m^{(v)}, y_m^{(v)}\right)$: Coordinates of the m-th measurement point for the v-th view v, ($m = 1, \dots, M$);
- $\varepsilon_{ra} = \frac{\varepsilon_a}{\varepsilon_0}$: Relative electric permittivity for the upper half-space ($y > 0$);
- σ_a: Conductivity for the upper half-space ($y > 0$);
- $\varepsilon_{rb} = \frac{\varepsilon_b}{\varepsilon_0}$: Background relative electric permittivity;
- σ_b: Background conductivity;
- $E_{inc}^{(v)}(\mathbf{r}_n; f)$: Measured internal incident field inside the n-th cell, for the v-th view at frequency f;
- $\widetilde{E}_{inc}^{(v)}(\mathbf{r}_n; f)$: Computed internal incident field inside the n-th cell, for the v-th view at frequency f;
- $E_{scatt}^{(v)}\left(\mathbf{r}_m^{(v)}; f\right)$: Measured external scattered by the m-th measurement point, for the v-th view at frequency f;
- $\widetilde{E}_{scatt}^{(v)}\left(\mathbf{r}_m^{(v)}; f\right)$: Measured external scattered by the m-th measurement point, for the v-th view at frequency f.

1.2 Contrast function

The contrast function at frequency f is defined as

$$\tau(\mathbf{r}; f) = \frac{\varepsilon_{eq}(\mathbf{r}) - \varepsilon_{eqb}}{\varepsilon_0} = [\varepsilon_r(\mathbf{r}) - \varepsilon_{rb}] + j\left[\frac{\sigma_b - \sigma(\mathbf{r})}{2\pi f \varepsilon_0}\right]$$

where

- $\mathbf{r} = (x, y)$: position vector;
- $\Re\{\tau(\mathbf{r}; f)\} = [\varepsilon_r(\mathbf{r}) - \varepsilon_{rb}]$;
- $\Im\{\tau(\mathbf{r}; f)\} = \left[\frac{\sigma_b - \sigma(\mathbf{r})}{2\pi f \varepsilon_0}\right]$;
- $\varepsilon_{eq}(\mathbf{r}) = \varepsilon_0 \varepsilon_r(\mathbf{r}) - j\frac{\sigma(\mathbf{r})}{2\pi f}$;
- $\varepsilon_{eqb} = \varepsilon_0 \varepsilon_{rb} - j\frac{\sigma_b}{2\pi f}$;
- $\varepsilon_r(\mathbf{r})$: relative electric permittivity at position $\mathbf{r}$;
- $\sigma(\mathbf{r})$: conductivity at position $\mathbf{r}$;

NOTE: we assume that $\varepsilon_r(\mathbf{r})$ and $\sigma(\mathbf{r})$ are **not frequency dependent** (non-dispersive mediums).

1.2.1 Contrast function and reference frequency f_{ref} (MF approaches)

The contrast function at a generic frequency f can be expressed by means of the contrast function computed for a selected reference frequency

$$f = f_{ref} \tag{1}$$

as follows

$$\tau(\mathbf{r}; f) = \Re\{\tau(\mathbf{r}; f_{ref})\} + j\frac{f_{ref}}{f}\Im\{\tau(\mathbf{r}; f_{ref})\}. \tag{2}$$

This allows to reduce the number of unknowns when dealing with multi-frequency techniques, since we can just consider the contrast function at the reference frequency.

1.3 Cost function & unknowns

1.3.1 Multi-Frequency (MF) approaches

These approaches jointly consider data at F frequencies. The functional minimized by the inversion algorithm is defined as

$$\Phi\left(\mathbf{x}\right)=\Phi_{state}\left(\mathbf{x}\right)+\Phi_{data}\left(\mathbf{x}\right) \tag{3}$$

where $\Phi_{state}\left(\mathbf{x}\right)$ and $\Phi_{data}\left(\mathbf{x}\right)$ are respectively the data and state terms of the cost function, defined as

$$\Phi_{state}\left(\mathbf{x}\right)=\frac{\sum_{j=1}^{F}\sum_{v=1}^{V}\sum_{n=1}^{N}\left|E_{inc}^{(v)}\left(\mathbf{r}_n;f_j\right)-\widetilde{E}_{inc}^{(v)}\left(\mathbf{r}_n;f_j\right)\right|^2}{\sum_{j=1}^{F}\sum_{v=1}^{V}\sum_{n=1}^{N}\left|E_{inc}^{(v)}\left(\mathbf{r}_n;f_j\right)\right|^2} \tag{4}$$

$$\Phi_{data}=\frac{\sum_{j=1}^{F}\sum_{v=1}^{V}\sum_{m=1}^{M}\left|E_{scatt}^{(v)}\left(\mathbf{r}_m^{(v)};f_j\right)-\widetilde{E}_{scatt}^{(v)}\left(\mathbf{r}_m^{(v)};f_j\right)\right|^2}{\sum_{j=1}^{F}\sum_{v=1}^{V}\sum_{m=1}^{M}\left|E_{scatt}^{(v)}\left(\mathbf{r}_m^{(v)};f_j\right)\right|^2} \tag{5}$$

The unknowns of the inversion problem are

$$\mathbf{x}=\left\{\tau\left(\mathbf{r};f_{ref}\right);E_{tot}^{(v)}\left(\mathbf{r}_n;f_j\right)\right\}\qquad n=1,...,N;v=1,...,V;j=1,...,F. \tag{6}$$

The total number of unknowns for MF-based approaches is then given by

$$U_{MF}=2N\left(1+VF\right). \tag{7}$$

1.4 Reconstruction errors

The following integral error is defined

$$\Xi_{reg}=\frac{1}{N_{reg}}\sum_{n=1}^{N_{reg}}\frac{\left|\tau_n^{act}-\tau_n^{rec}\right|}{\left|\tau_n^{act}+1\right|} \tag{8}$$

where reg indicates if the error computation covers

- the overall investigation domain ($reg\Rightarrow tot$),
- the actual scatterer support ($reg\Rightarrow int$),
- or the background region ($reg\Rightarrow ext$).

2 Numerical Validation

2.1 O-Shaped object ($\varepsilon_{r,obj} = 5.0$, $\sigma_{obj} = 10^{-3}$ [S/m])

2.1.1 Parameters

Background

Inhomogeneous and nonmagnetic background composed by two half spaces

- Upper half space ($y > 0$ - air): $\varepsilon_{ra} = 1.0$, $\sigma_a = 0.0$;
- Lower half space ($y < 0$ - soil): $\varepsilon_{rb} = 4.0$, $\sigma_b = 10^{-3}$[S/m];

Investigation domain (D_{inv})

- Side: $L_{D_{inv}} = 0.8$ [m];
- Barycenter: $\left(x_{bar}^{D_{inv}}, y_{bar}^{D_{inv}}\right) = (0.00, -0.4)$ [m];

Time-Domain forward solver ($FDTD$ - $GPRMax2D$**)**

- Side of the simulated domain: $L = 6$ [m];
- Number of cells: $N^{FDTD} = 750 \times 750 = 5.625 \times 10^5$;
- Side of the $FDTD$ cells $l^{FDTD} = 0.008$ [m];
- Simulation time window: $T^{FDTD} = 20 \times 10^{-9}$ [sec];
- Time step: $\Delta t^{FDTD} = 1.89 \times 10^{-11}$ [sec];
- Number of time samples: $N_t^{FDTD} = 1060$;
- Boundary conditions: perfectly matched layer (PML);
- Source type: Gaussian mono-cycle (first Gaussian pulse derivative, called "Ricker" in $GPRMax2D$)
  - Central frequency: $f_0 = 300$ [MHz];
  - Source amplitude: $A = 1.0$ [A];

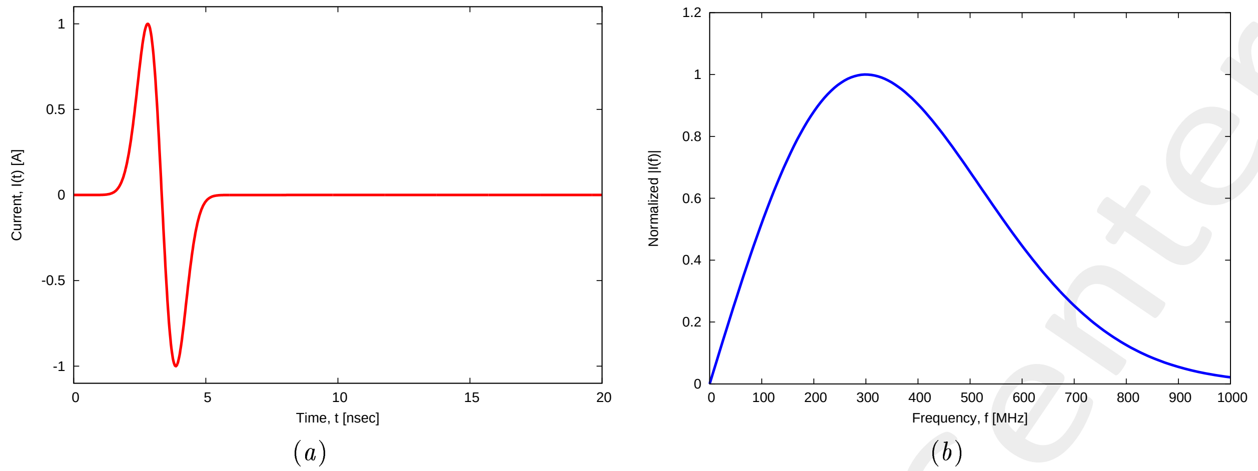


Figure 1: *GPRMax2D* excitation signal. (a) Time pulse, (b) normalized frequency spectrum.

Frequency parameters

- Frequency range: $f \in [f_{min}, f_{max}] = [200.0, 600.0]$ [MHz] (-3 [dB] bandwidth of the Gaussian Monocycle excitation centered at $f_0 = 300$ [MHz]);
- Frequency step: $\Delta f = 100$ [MHz] ($F = 5$ frequency steps in $[f_{min}, f_{max}]$);

f [MHz]	λ_a [m]	λ_b [m]	f^* [MHz]
200.0	1.50	0.75	200.5
300.0	1.00	0.50	297.6
400.0	0.75	0.37	401.1
500.0	0.60	0.30	498.1
600.0	0.50	0.25	601.6

Table 1: Considered frequencies and corresponding wavelength in the upper medium (λ_a, free space) and in the lower medium (λ_b, soil). f^* is the nearest frequency sample available from transformed time-domain data, and represents the real frequency considered by the inversion algorithm.

Scatterer

- Type: O-shaped;
- Barycenter: $(x_{obj}, y_{obj}) = (0.12, -0.36)$ [m];
- Side (external): $L_{obj,x} = L_{obj,y} = 0.24$ [m];
- Electromagnetic properties: $\varepsilon_{r,obj} = 5.0$, $\sigma_{obj} = 10^{-3}$ [S/m] ($\sigma_{obj} = \sigma_b$);
- Contrast function: $\tau = 1.0 + j0.0$

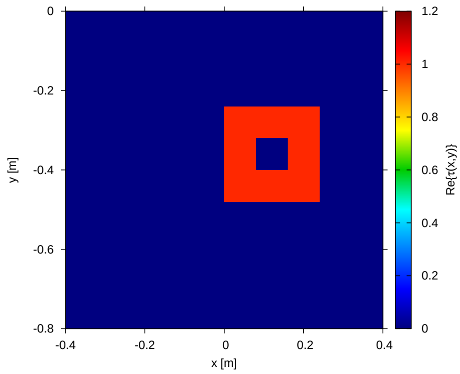


Figure 2: Actual object ($\tau = 1.0$).

Measurement setup

- Considered frequency: $f_{min} = 200$ [MHz], $\lambda_b = 0.75$ [m]. [1]
- $\#DoFs = 2ka = \frac{2\pi}{\lambda_b} L\sqrt{2} = \frac{2\pi}{0.75} 0.8\sqrt{2} \simeq 9.5$;
- Number of views (sources): $V = 10$;
  - $\min\{x_v\} = -0.5$ [m], $\max\{x_v\} = 0.5$ [m];
  - height: $y_v = 0.1$ [m], $\forall v = 1, \ldots, V$;
- Number of measurement points: $M = 9$;
  - $\min\{x_m\} = -0.5$ [m], $\max\{x_m\} = 0.5$ [m];
  - height: $y_m = 0.1$ [m], $\forall m = 1, \ldots, M$;

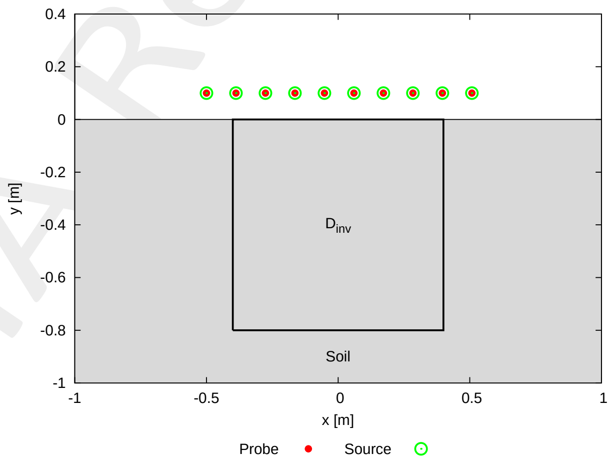


Figure 3: Location of the measurement points ($M = 9$) and of the sources ($V = 10$). Only one source is active for each view.

Inverse solver parameters

- **Shared parameters**

[1]**NOTE:** This choice is done in order to keep the number of unknowns lower than 5000.

- Number of unknowns: $U = 2N(1 + VF) = 4998$;
- Weight of the state term of the functional: 1.0;
- Weight of the data term of the functional: 1.0;
- Weight of the penalty term of the functional: 0.0;
- Convergence threshold: 10^{-10};
- Variable ranges:
  - $\varepsilon_r \in [4.0, 5.2]$, $\sigma \in \left[8.0 \times 10^{-4}, 1.2 \times 10^{-3}\right]$ [S/m];
  - $\Re\left\{E_{tot}^{int}\right\} \in [-8, 8]$, $\Im\left\{E_{tot}^{int}\right\} \in [-8, 8]$;
- Degrees of freedom:
  - Considered frequency: $f_{min} = 200$ [MHz], $\lambda_b = 0.75$ [m];
  - $\frac{(2ka)^2}{2} = \frac{\left(2\times\frac{2\pi}{\lambda_b}\times\frac{L\sqrt{2}}{2}\right)^2}{2} = 4\pi^2\left(\frac{L}{\lambda_b}\right)^2 = 4\pi^2\left(\frac{0.8}{0.75}\right)^2 \simeq 44.87$;
- Number of cells: $N = 49 = 7 \times 7$;
- Maximum number of $IMSA$ steps: $S = 4$;
- Side ratio threshold: $\eta_{th} = 0.2$;

- $MF - IMSA - PSO$ **parameters**
  - Maximum number of iterations: $I = 20000$;
  - Swarm dimension: $P = \frac{5}{100} \times U = 250$;
  - $C_1 = C_2 = 2.0$;
  - Inertial weight: $w = 0.4$;
  - Velocity clamping: enabled;
- $MF - IMSA - CG$ **parameters**
  - Maximum number of iterations: $I = 200$;

Signal to noise ratio (on $E_{tot}(t)$)

- $SNR = \{50, 40, 30, 20\}$ [dB] + Noiseless data.

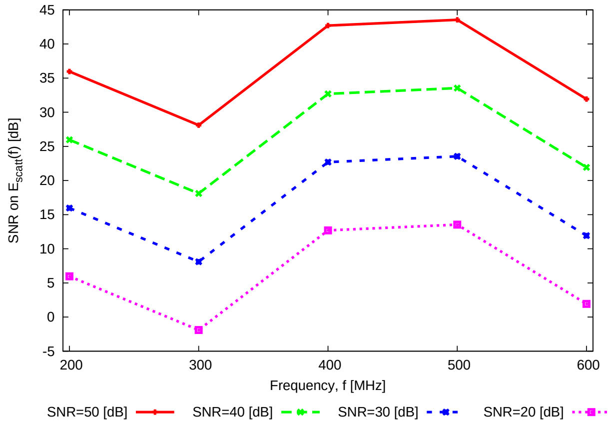


Figure 4: Signal to noise ratio (SNR [dB]) computed on the external scattered field in frequency domain for each considered frequency and for each input SNR on the measured total field in time-domain.

SNR on $E_{tot}(t)$ [dB]	Av. SNR on $E_{scatt}(f)$ [dB]
50	36.4
40	26.4
30	16.4
20	6.4

Table 2: Average SNR measured on the scattered field in frequency domain.

2.1.2 $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Final reconstructions

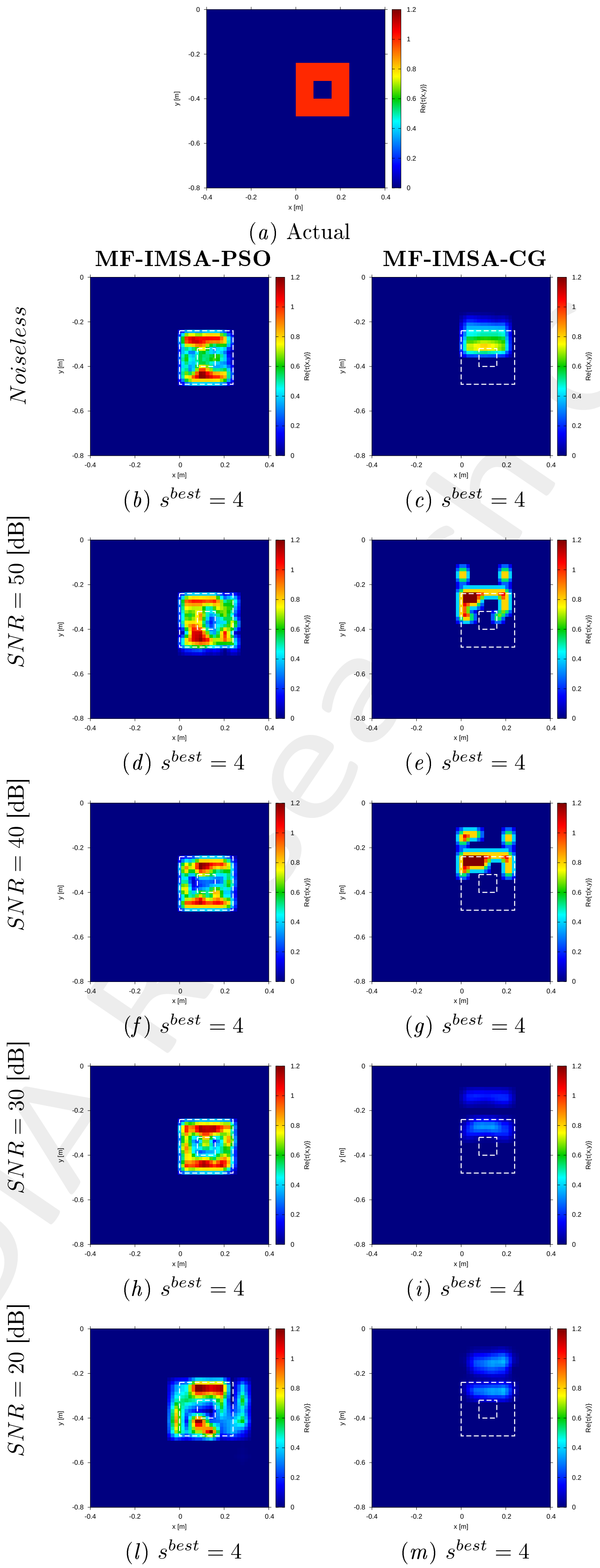


Figure 5: $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Retrieved dielectric profiles at the $IMSA$ convergence step (s^{best}).

2.1.3 $MF-IMSA-PSO$: Intermediate reconstructions

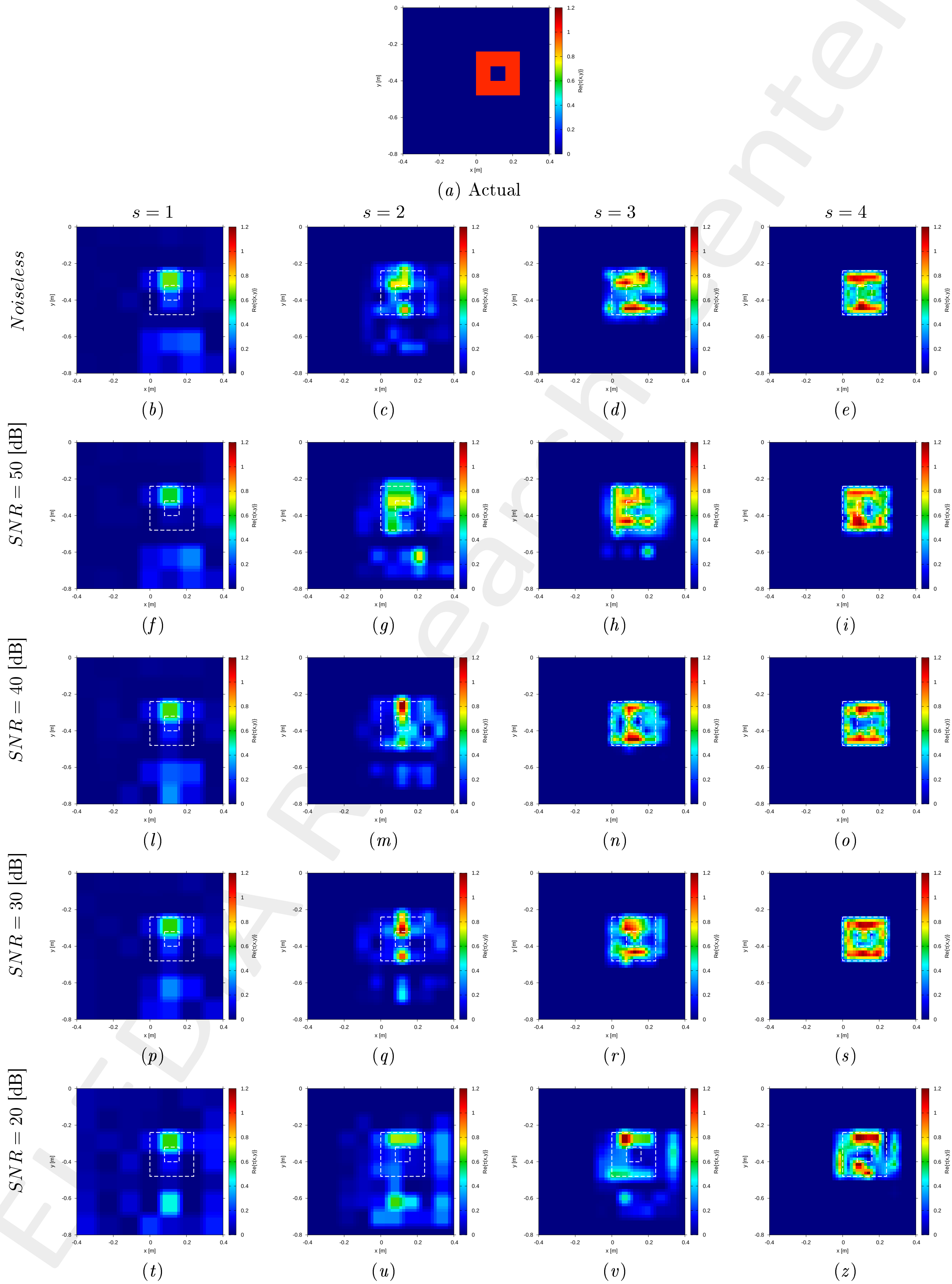


Figure 6: $MF-IMSA-PSO$: Retrieved dielectric profiles at each $IMSA$ step.

2.1.4 $MF-IMSA-CG$: Intermediate reconstructions

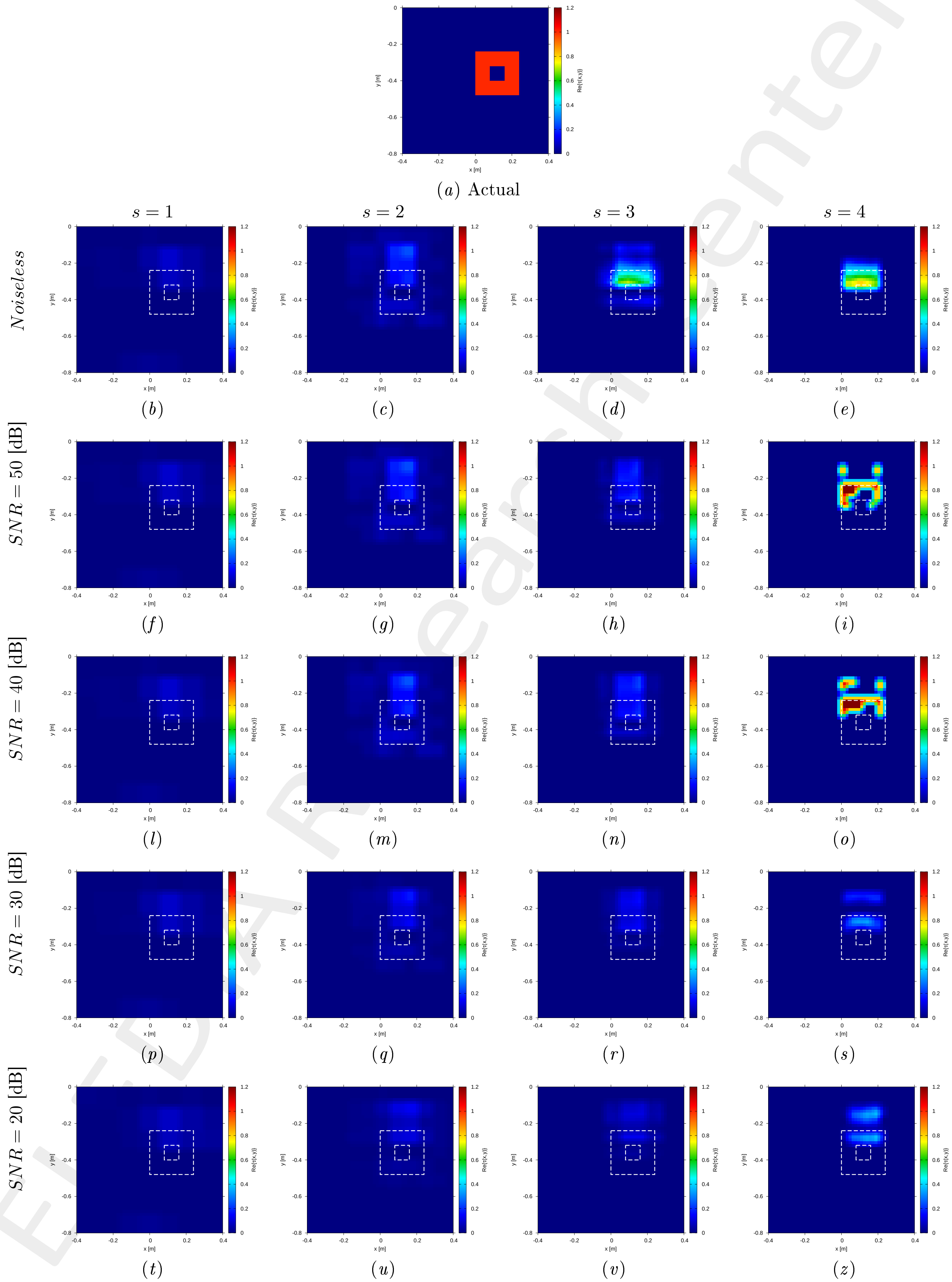


Figure 7: $MF-IMSA-CG$: Retrieved dielectric profiles at each $IMSA$ step.

2.1.5 Reconstruction errors

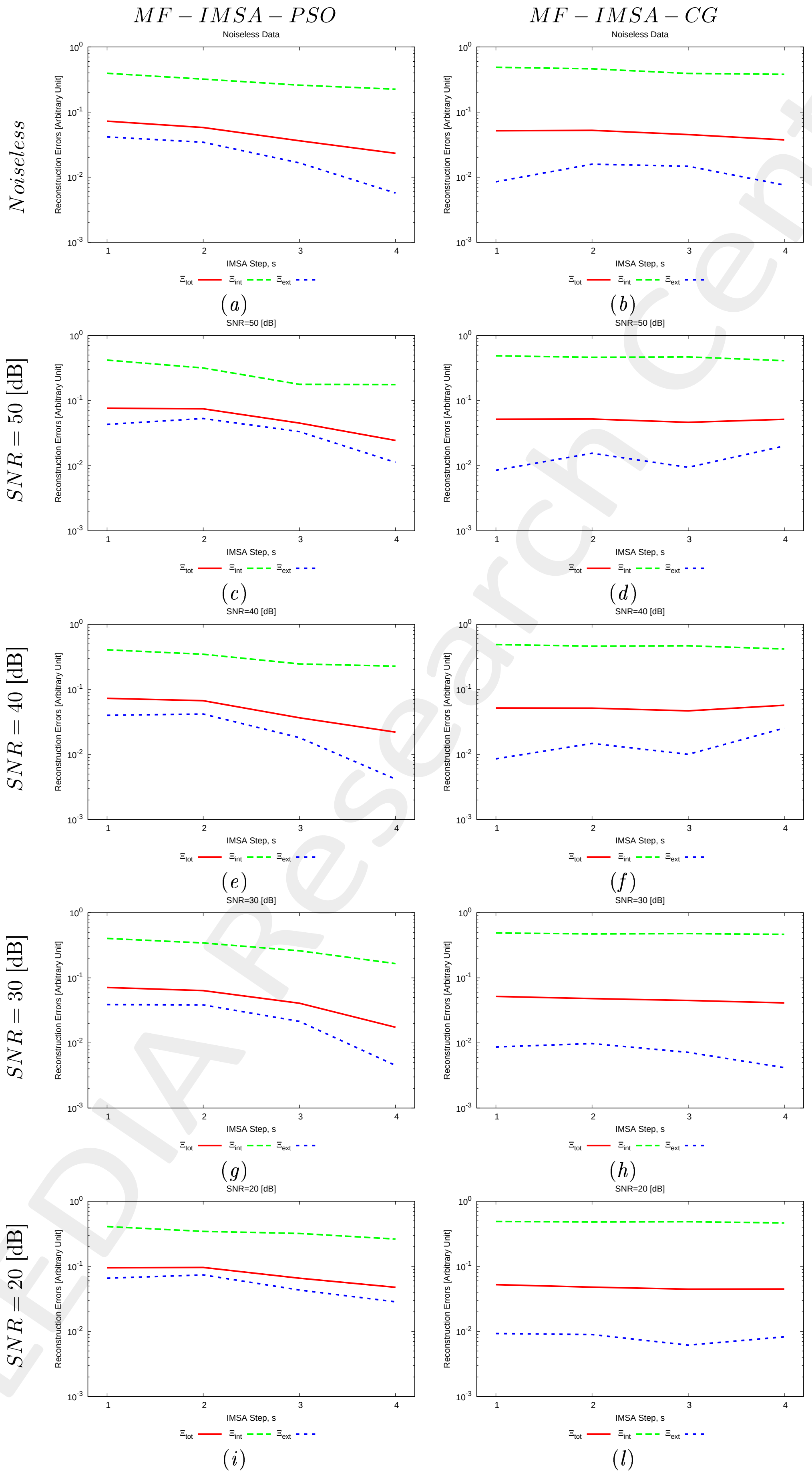


Figure 8: $MF - IMSA - PSO$ vs. $MF - IMSA - CG$: Reconstruction error evolution.

2.1.6 Evolution of the cost function

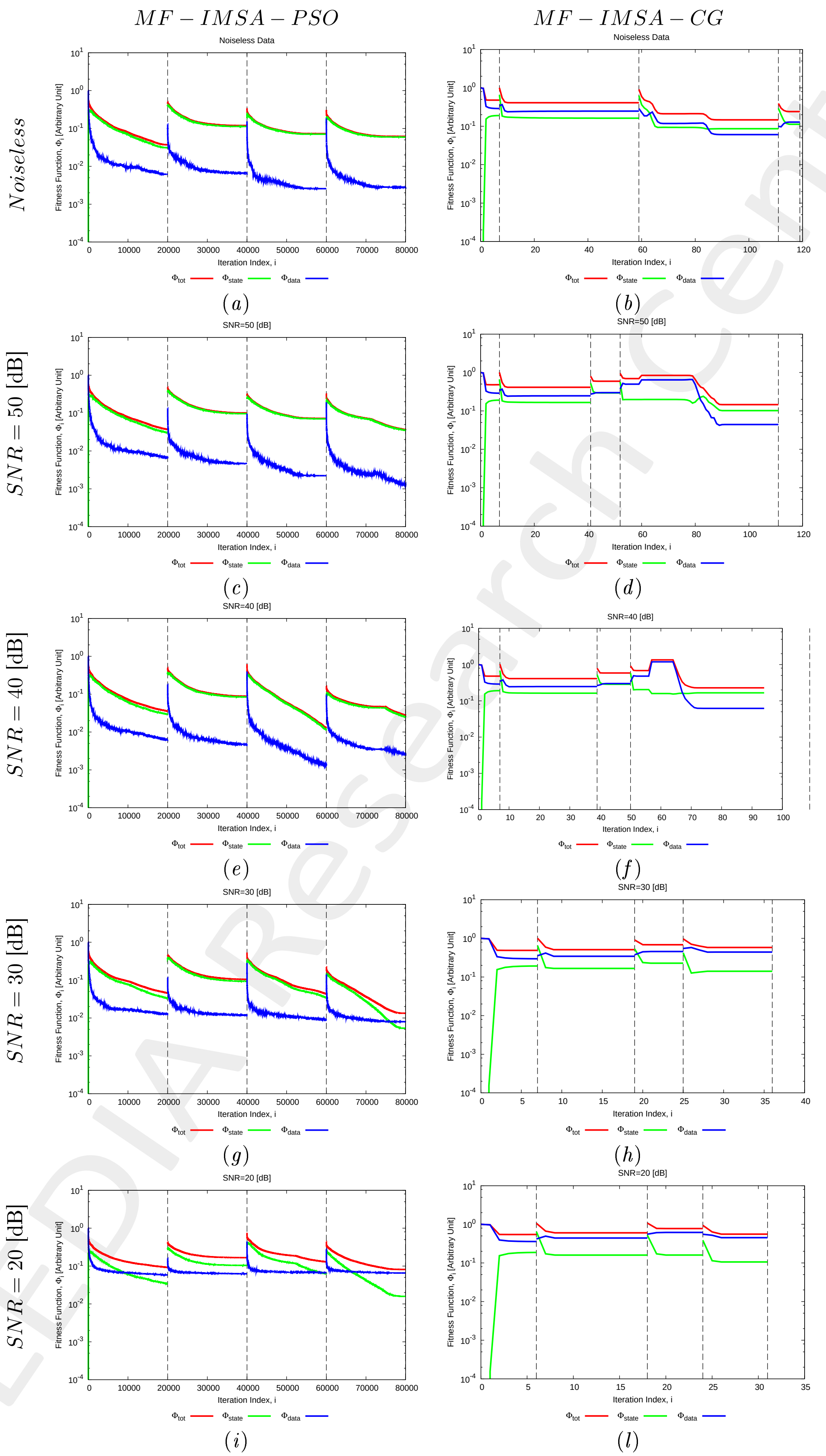


Figure 9: $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Evolution of the cost function.

2.2 Circular object ($\varepsilon_{r,obj} = 5.0$, $\sigma_{obj} = 10^{-3}$ [S/m])

2.2.1 Parameters

Background

Inhomogeneous and nonmagnetic background composed by two half spaces

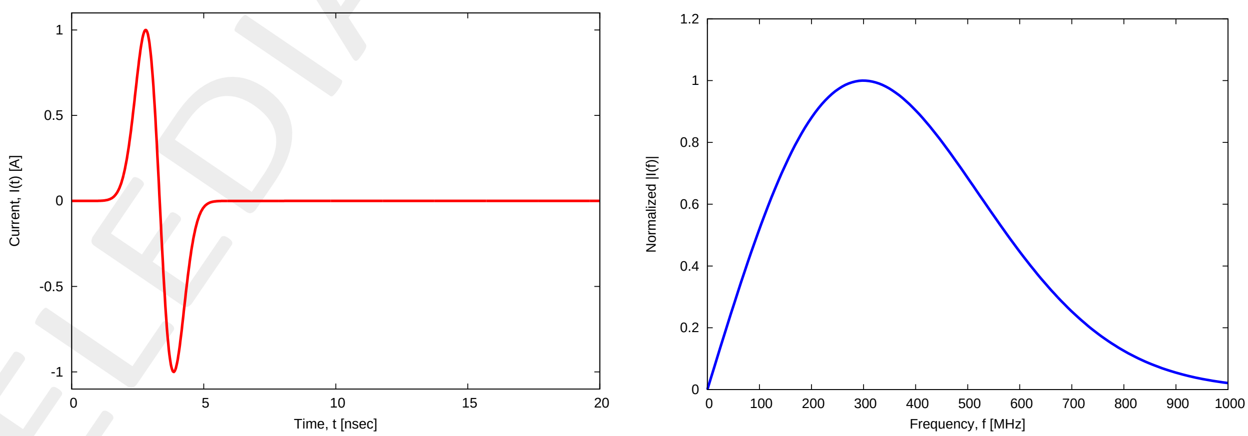
- Upper half space ($y > 0$ - air): $\varepsilon_{ra} = 1.0$, $\sigma_a = 0.0$;
- Lower half space ($y < 0$ - soil): $\varepsilon_{rb} = 4.0$, $\sigma_b = 10^{-3}$[S/m];

Investigation domain (D_{inv})

- Side: $L_{D_{inv}} = 0.8$ [m];
- Barycenter: $\left(x_{bar}^{D_{inv}}, y_{bar}^{D_{inv}}\right) = (0.00, -0.4)$ [m];

Time-Domain forward solver ($FDTD$ - $GPRMax2D$)

- Side of the simulated domain: $L = 6$ [m];
- Number of cells: $N^{FDTD} = 750 \times 750 = 5.625 \times 10^5$;
- Side of the $FDTD$ cells $l^{FDTD} = 0.008$ [m];
- Simulation time window: $T^{FDTD} = 20 \times 10^{-9}$ [sec];
- Time step: $\Delta t^{FDTD} = 1.89 \times 10^{-11}$ [sec];
- Number of time samples: $N_t^{FDTD} = 1060$;
- Boundary conditions: perfectly matched layer (PML);
- Source type: Gaussian mono-cycle (first Gaussian pulse derivative, called "Ricker" in $GPRMax2D$)
  - Central frequency: $f_0 = 300$ [MHz];
  - Source amplitude: $A = 1.0$ [A];



(a) (b)

Figure 10: $GPRMax2D$ excitation signal. (a) Time pulse, (b) normalized frequency spectrum.

Frequency parameters

- Frequency range: $f \in [f_{min}, f_{max}] = [200.0, 600.0]$ [MHz] (-3 [dB] bandwidth of the Gaussian Monocycle excitation centered at $f_0 = 300$ [MHz]);
- Frequency step: $\Delta f = 100$ [MHz] ($F = 5$ frequency steps in $[f_{min}, f_{max}]$);

f [MHz]	λ_a [m]	λ_b [m]	f^* [MHz]
200.0	1.50	0.75	200.5
300.0	1.00	0.50	297.6
400.0	0.75	0.37	401.1
500.0	0.60	0.30	498.1
600.0	0.50	0.25	601.6

Table 3: Considered frequencies and corresponding wavelength in the upper medium (λ_a, free space) and in the lower medium (λ_b, soil). f^* is the nearest frequency sample available from transformed time-domain data, and represents the real frequency considered by the inversion algorithm.

Scatterer

- Type: Circular;
- Barycenter: $(x_{obj}, y_{obj}) = (-0.16, -0.4)$ [m];
- Radius: $r_{obj} = 0.08$ [m];
- Electromagnetic properties: $\varepsilon_{r,obj} = 5.0$, $\sigma_{obj} = 10^{-3}$ [S/m] ($\sigma_{obj} = \sigma_b$);
- Contrast function: $\tau = 1.0 + j0.0$

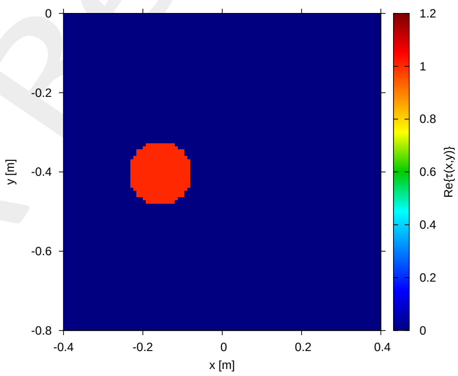


Figure 11: Actual object ($\tau = 1.0$).

Measurement setup

- Considered frequency: $f_{min} = 200$ [MHz], $\lambda_b = 0.75$ [m]. [2]
- $\#DoFs = 2ka = \frac{2\pi}{\lambda_b} L\sqrt{2} = \frac{2\pi}{0.75} 0.8\sqrt{2} \simeq 9.5$;
- Number of views (sources): $V = 10$;

[2]**NOTE:** This choice is done in order to keep the number of unknowns lower than 5000.

- $\min\{x_v\} = -0.5$ [m], $\max\{x_v\} = 0.5$ [m];
- height: $y_v = 0.1$ [m], $\forall v = 1, \dots, V$;

- Number of measurement points: $M = 9$;
  - $\min\{x_m\} = -0.5$ [m], $\max\{x_m\} = 0.5$ [m];
  - height: $y_m = 0.1$ [m], $\forall m = 1, \dots, M$;

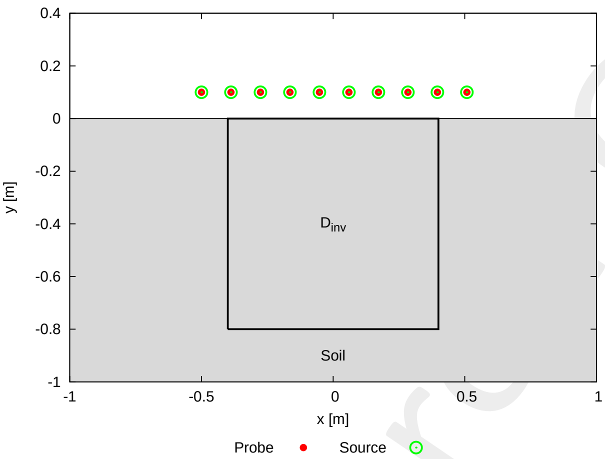


Figure 12: Location of the measurement points ($M = 9$) and of the sources ($V = 10$). Only one source is active for each view.

Inverse solver parameters

- **Shared parameters**
  - Number of unknowns: $U = 2N(1 + VF) = 4998$;
  - Weight of the state term of the functional: 1.0;
  - Weight of the data term of the functional: 1.0;
  - Weight of the penalty term of the functional: 0.0;
  - Convergence threshold: 10^{-10};
  - Variable ranges:
    - $\varepsilon_r \in [4.0, 5.2]$, $\sigma \in \left[8.0 \times 10^{-4}, 1.2 \times 10^{-3}\right]$ [S/m];
    - $\Re\left\{E_{tot}^{int}\right\} \in [-8, 8]$, $\Im\left\{E_{tot}^{int}\right\} \in [-8, 8]$;
  - Degrees of freedom:
    - Considered frequency: $f_{min} = 200$ [MHz], $\lambda_b = 0.75$ [m];
    - $\frac{(2ka)^2}{2} = \frac{\left(2 \times \frac{2\pi}{\lambda_b} \times \frac{L\sqrt{2}}{2}\right)^2}{2} = 4\pi^2 \left(\frac{L}{\lambda_b}\right)^2 = 4\pi^2 \left(\frac{0.8}{0.75}\right)^2 \simeq 44.87$;
  - Number of cells: $N = 49 = 7 \times 7$;
  - Maximum number of $IMSA$ steps: $S = 4$;

– Side ratio threshold: $\eta_{th} = 0.2$;

- $MF - IMSA - PSO$ **parameters**

  – Maximum number of iterations: $I = 20000$;

  – Swarm dimension: $P = \frac{5}{100} \times U = 250$;

  – $C_1 = C_2 = 2.0$;

  – Inertial weight: $w = 0.4$;

  – Velocity clamping: enabled;

- $MF - IMSA - CG$ **parameters**

  – Maximum number of iterations: $I = 200$;

Signal to noise ratio (on $E_{tot}(t)$)

- $SNR = \{50, 40, 30, 20\}$ [dB] + Noiseless data.

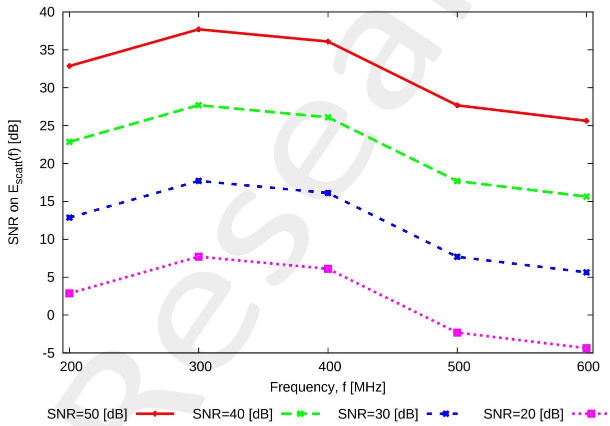


Figure 13: Signal to noise ratio (SNR [dB]) computed on the external scattered field in frequency domain for each considered frequency and for each input SNR on the measured total field in time-domain.

SNR on $E_{tot}(t)$ [dB]	Av. SNR on $E_{scatt}(f)$ [dB]
50	32
40	22
30	12
20	2

Table 4: Average SNR measured on the scattered field in frequency domain.

2.2.2 $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Final reconstructions

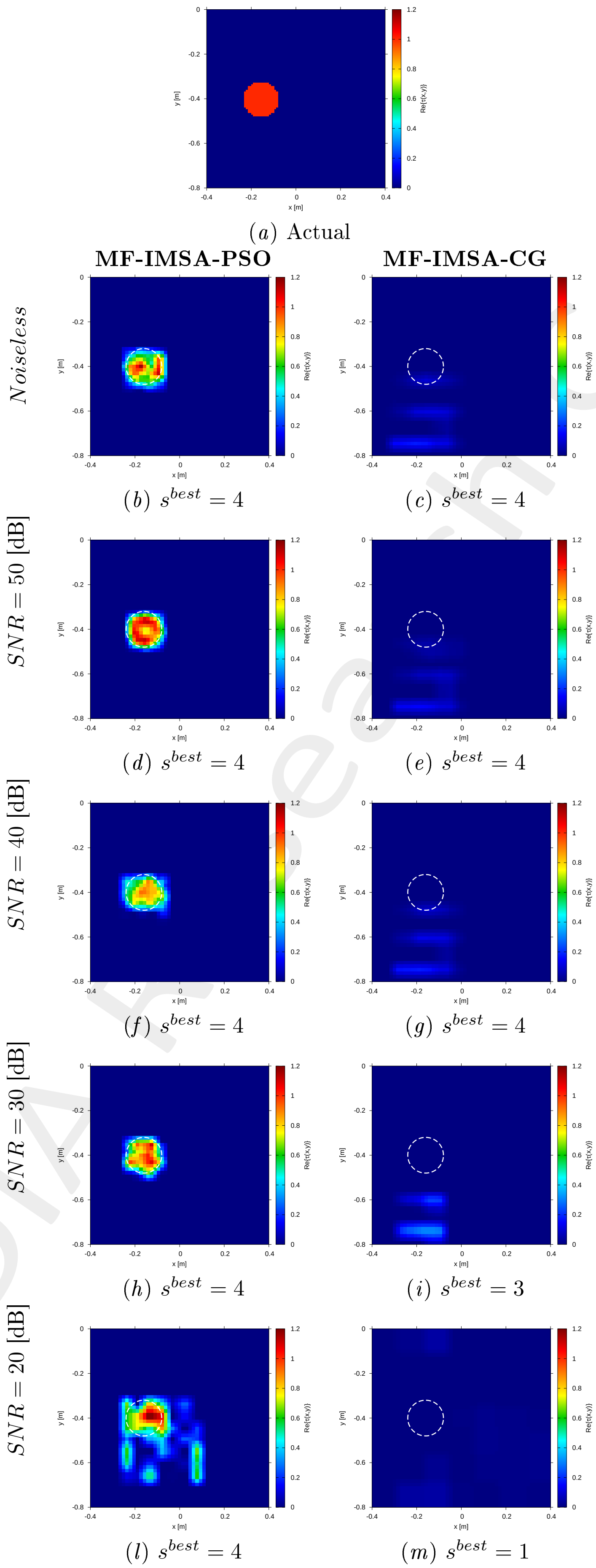


Figure 14: $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Retrieved dielectric profiles at the $IMSA$ convergence step (s^{best}).

2.2.3 $MF-IMSA-PSO$: Intermediate reconstructions

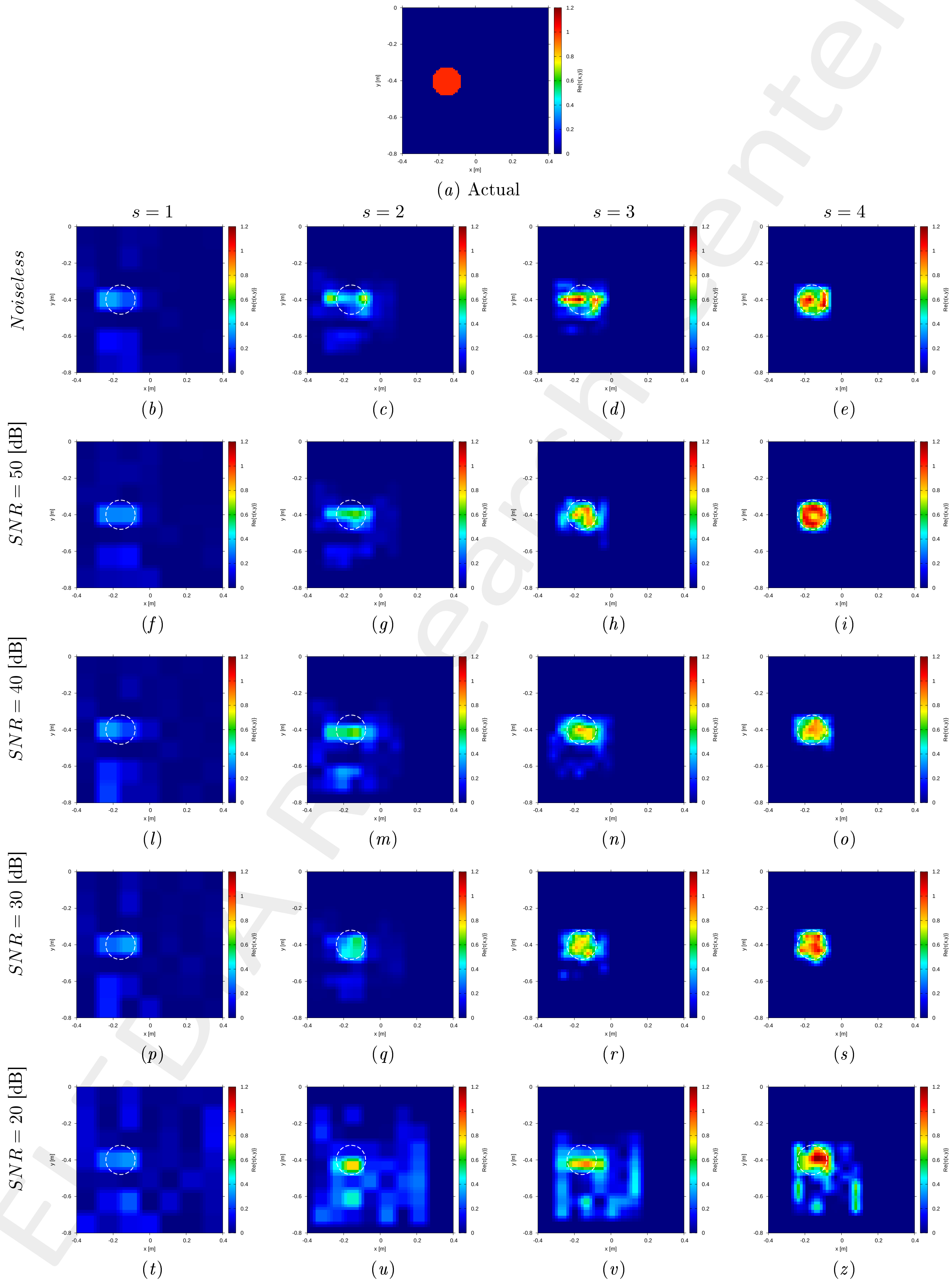


Figure 15: $MF-IMSA-PSO$: Retrieved dielectric profiles at each $IMSA$ step.

2.2.4 $MF-IMSA-CG$: Intermediate reconstructions

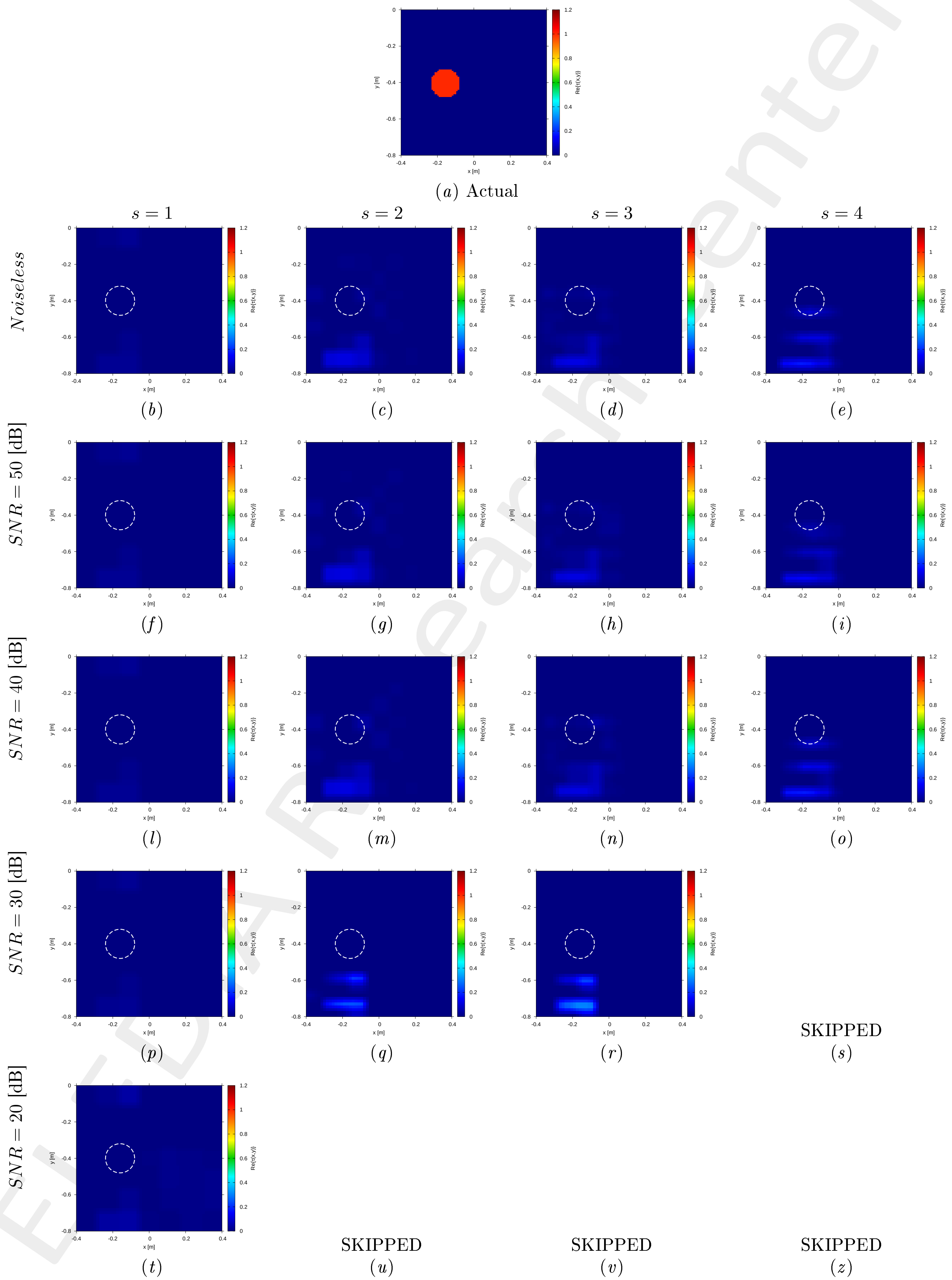


Figure 16: $MF-IMSA-CG$: Retrieved dielectric profiles at each $IMSA$ step.

2.2.5 Reconstruction errors

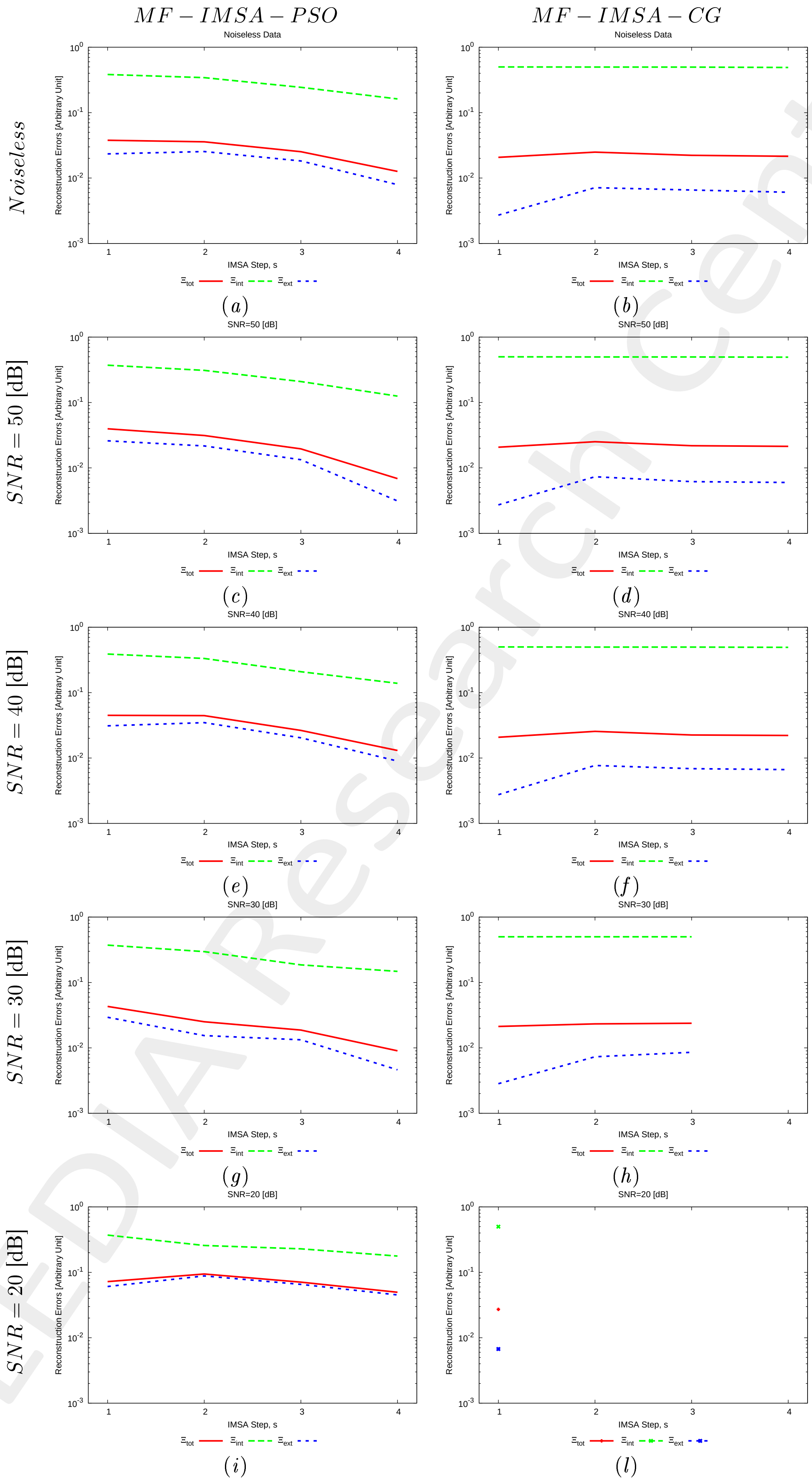


Figure 17: $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Reconstruction error evolution.

2.2.6 Evolution of the cost function

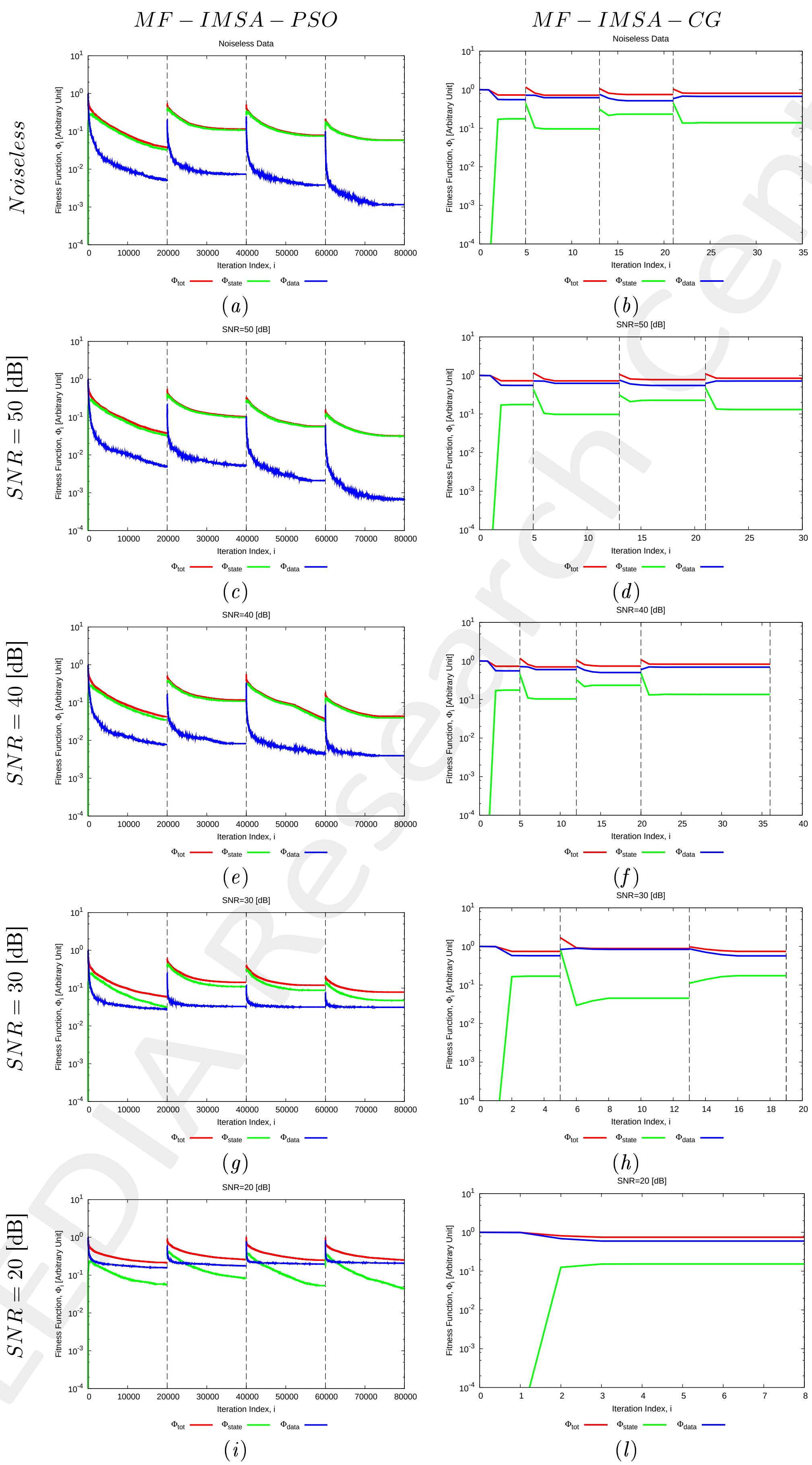


Figure 18: $MF-IMSA-PSO$ vs. $MF-IMSA-CG$: Evolution of the cost function.

3 Conclusions

The reported numerical validation indicates that

- The $MF-IMSA-PSO$ provides better reconstruction with respect to the $MF-IMSA-CG$ thanks to the exploitation of a stochastic global search algorithm;
- The $MF-IMSA-PSO$ provides accurate results also when data is blurred with a non-negligible amount of noise;
- The $MF-IMSA-PSO$ is able to provide accurate reconstructions for objects of different size and shape.

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