

A Multi-Frequency Multi-Resolution Approach for GPR Microwave Imaging of Buried Objects

M. Salucci, L. Poli, and A. Massa

Abstract

This work presents a numerical validation of an innovative inverse scattering (*IS*) technique for the microwave imaging of buried objects through the deterministic inversion of wideband ground penetrating radar (*GPR*) data. The developed methodology is based on an iterative multi-focusing strategy and on a multi-frequency (*MF*) formulation of the *IS* equations. The minimization of the arising *MF* cost function is performed by a customized conjugate gradient (*CG*)-based solver. Some numerical results are presented in order to validate the effectiveness of the developed methodology under different operating conditions, by considering a variation of the shape/material of the unknown objects, as well as a variation of the level of noise on the measured time-domain total field. Moreover, a comparison with respect to a state-of-the-art approach based on a Frequency Hopping (*FH*) strategy is given, as well.

1 Definitions

1.1 Glossary

- D_{inv} : investigation domain;
- D_{obs} : observation domain;
- N : number of discretization cells in D_{ind} ;
- V : number of views;
- M : number of measurement points;
- F : number of frequencies considered for the inversion;
- (x_v, y_v) : coordinates of the v -th source ($v = 1, \dots, V$).
- (x_m^v, y_m^v) : coordinates of the m -th measurement point for the v -th view v , ($m = 1, \dots, M$);
- $\varepsilon_{ra} = \frac{\varepsilon_a}{\varepsilon_0}$: relative electric permittivity for the upper half-space ($y > 0$);
- σ_a : conductivity for the upper half-space ($y > 0$);
- $\varepsilon_{rb} = \frac{\varepsilon_b}{\varepsilon_0}$: background relative electric permittivity;
- σ_b : background conductivity;

2 MF – IMSA – CG vs. FH – IMSA – CG - Performances vs. Noise

2.1 Square-shaped object ($\varepsilon_{r,obj} = 5.0$, $\sigma_{obj} = 10^{-3}$ [S/m])

2.1.1 Parameters

Background

Inhomogeneous and nonmagnetic background composed by two half spaces

- Upper half space ($y > 0$ - air): $\varepsilon_{ra} = 1.0$, $\sigma_a = 0.0$;
- Lower half space ($y < 0$ - soil): $\varepsilon_{rb} = 4.0$, $\sigma_b = 10^{-3}$ [S/m];

Investigation domain (D_{inv})

- Side: $L_{D_{inv}} = 0.8$ [m];
- Barycenter: $(x_{bar}^{D_{inv}}, y_{bar}^{D_{inv}}) = (0.00, -0.4)$ [m];

Time-Domain forward solver ($FDTD - GPRMax2D$)

- Side of the simulated domain: $L = 6$ [m];
- Number of cells: $N^{FDTD} = 750 \times 750 = 5.625 \times 10^5$;
- Side of the $FDTD$ cells $l^{FDTD} = 0.008$ [m];
- Simulation time window: $T^{FDTD} = 20 \times 10^{-9}$ [sec];
- Time step: $\Delta t^{FDTD} = 1.89 \times 10^{-11}$ [sec];
- Number of time samples: $N_t^{FDTD} = 1060$;
- Boundary conditions: perfectly matched layer (PML);
- Source type: Gaussian mono-cycle (first Gaussian pulse derivative, called “Ricker” in $GPRMax2D$)
 - Central frequency: $f_0 = 300$ [MHz];
 - Source amplitude: $A = 1.0$ [A];

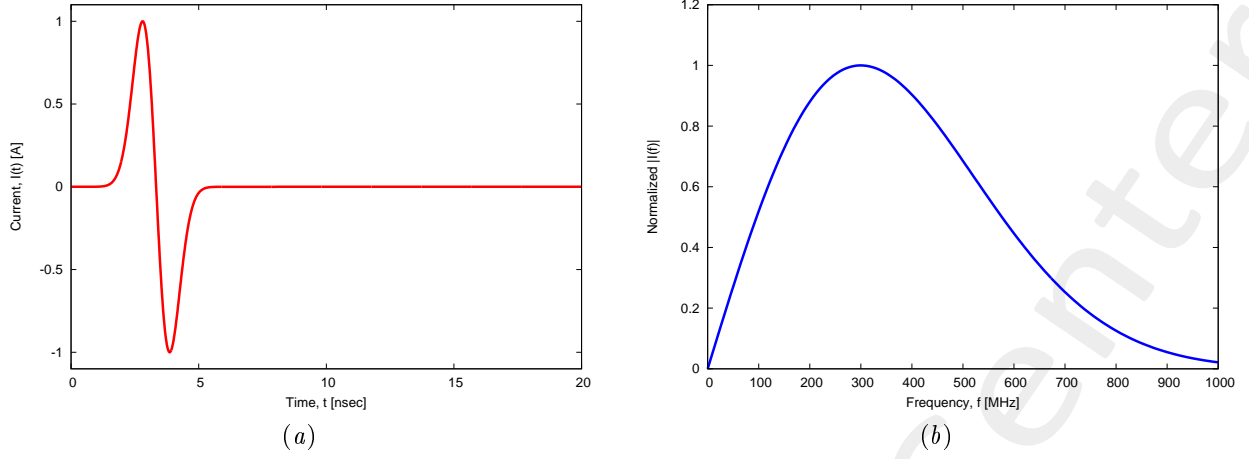


Figure 1: *GPRMax2D* excitation signal. (a) Time pulse, (b) normalized frequency spectrum.

Frequency parameters

- Frequency range: $f \in [f_{min}, f_{max}] = [200.0, 600.0]$ [MHz] [?] (-3 [dB] bandwidth of the Gaussian Monocycle excitation centered at $f_0 = 300$ [MHz]);
- Frequency step: $\Delta f = 100$ [MHz] ($F = 5$ frequency steps in $[f_{min}, f_{max}]$);

f [MHz]	λ_a [m]	λ_b [m]	f^* [MHz]
200.0	1.50	0.75	200.5
300.0	1.00	0.50	297.6
400.0	0.75	0.37	401.1
500.0	0.60	0.30	498.1
600.0	0.50	0.25	601.6

Table 1: Considered frequencies and corresponding wavelength in the upper medium (λ_a , free space) and in the lower medium (λ_b , soil). f^* is the nearest frequency sample available from transformed time-domain data, and represents the real frequency considered by the inversion algorithm.

Scatterer

- Type: square-shaped;
- Barycenter: $(x_{obj}, y_{obj}) = (-0.08, -0.24)$ [m];
- Side: $L_{obj,x} = L_{obj,y} = 0.16$ [m];
- Electromagnetic properties: $\varepsilon_{r,obj} = 5.0$, $\sigma_{obj} = 10^{-3}$ [S/m] ($\sigma_{obj} = \sigma_b$);
- Contrast function: $\tau = 1.0 + j0.0$

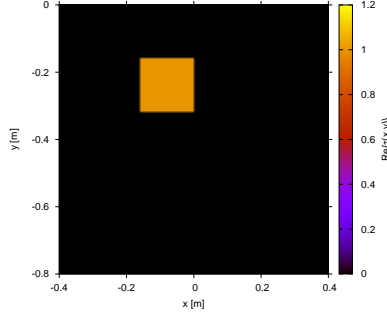


Figure 2: Actual object: offset square cylinder $\tau = 1.0$.

Measurement setup

- Considered frequency: $f_{min} = 200$ [MHz], $\lambda_b = 0.75$ [m].
- $\#DoFs = 2ka = \frac{2\pi}{\lambda_b} L\sqrt{2} = \frac{2\pi}{0.75} 0.8\sqrt{2} \simeq 9.5$;
- Number of views (sources): $V = 10$;
 - $\min \{x_v\} = -0.5$ [m], $\max \{x_v\} = 0.5$ [m];
 - height: $y_v = 0.1$ [m], $\forall v = 1, \dots, V$;
- Number of measurement points: $M = 9$;
 - $\min \{x_m\} = -0.5$ [m], $\max \{x_m\} = 0.5$ [m];
 - height: $y_m = 0.1$ [m], $\forall m = 1, \dots, M$;

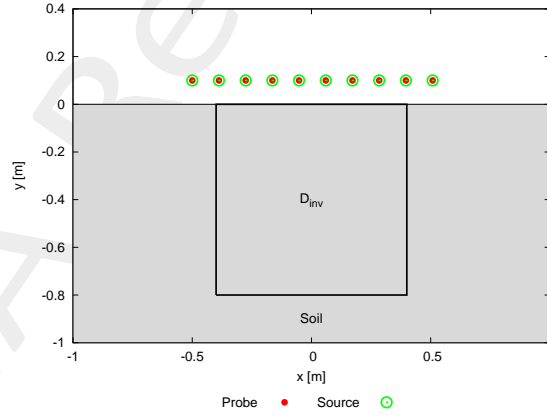


Figure 3: Location of the measurement points ($M = 9$) and of the sources ($V = 10$). Only one source is active for each view.

Inverse solver parameters

- Shared parameters
 - Weight of the state term of the functional: 1.0;
 - Weight of the data term of the functional: 1.0;

- Convergence threshold: 10^{-10} ;
- Variable ranges:
 - * $\varepsilon_r \in [4.0, 5.2]$, $\sigma \in [8.0 \times 10^{-4}, 1.2 \times 10^{-3}]$ [S/m];
 - * $\Re \{E_{tot}^{int}\} \in [-8, 8]$, $\Im \{E_{tot}^{int}\} \in [-8, 8]$;
- Degrees of freedom:
 - * Considered frequency: $f_{min} = 200$ [MHz], $\lambda_b = 0.75$ [m];
 - * $\frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda_b} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda_b}\right)^2 = 4\pi^2 \left(\frac{0.8}{0.75}\right)^2 \simeq 44.87$;
- Number of cells: $N = 49 = 7 \times 7$;
- Maximum number of *IMSA* steps: $S = 4$;
- Side ratio threshold: $\eta_{th} = 0.2$;
- ***MF – IMSA – CG* parameters**
 - Maximum number of iterations: $I = 200$;
- ***FH – IMSA – CG* parameters**
 - Maximum number of iterations: $I = 400$;

Signal to noise ratio (on $E_{tot}(t)$)

- $SNR = \{50, 40, 30, 20\}$ [dB] + Noiseless data.

SNR on $E_{tot}(t)$ [dB]	Av. SNR on $E_{scatt}(f)$ [dB]
50	35
40	25
30	15
20	5

Table 2: Average *SNR* measured on the scattered field in frequency domain.

2.1.2 Results

Final reconstructions ($@f_{max} = 600$ [MHz])

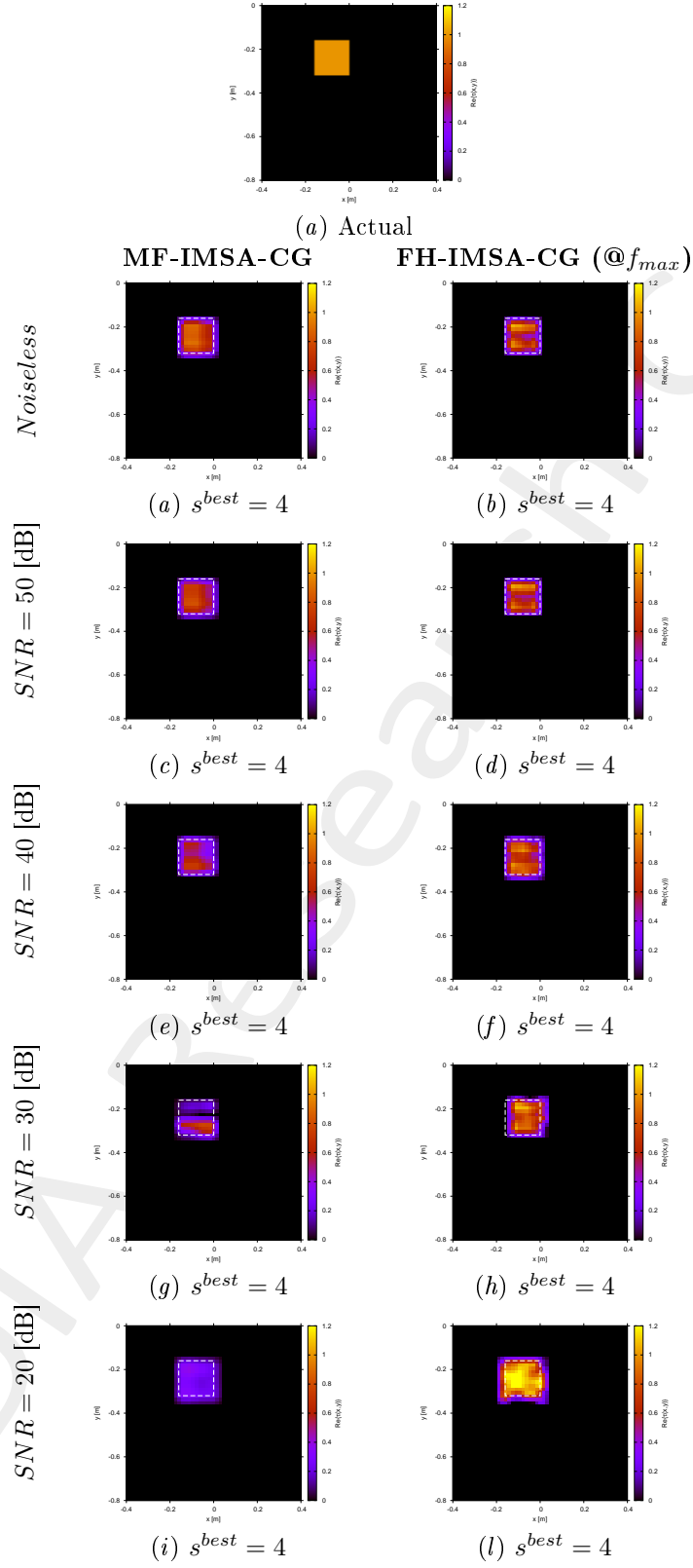


Figure 4: *MF – IMSA – CG* vs. *FH – IMSA – CG*: Retrieved dielectric profiles at the *IMSA* convergence step (s^{best}).

Reconstruction Error (@ $f_{max} = 600$ [MHz])

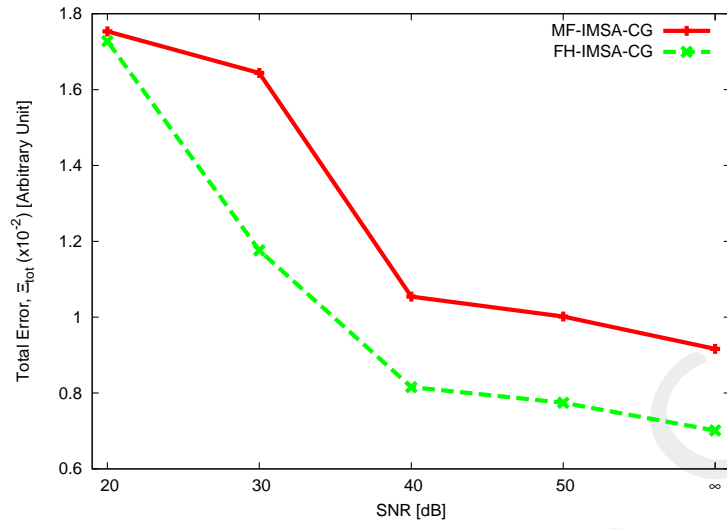


Figure 5: *MF-IMSA-CG* vs. *FH-IMSA-CG*: Total reconstruction error vs. *SNR*.

2.2 Circular void ($\varepsilon_{r,obj} = 1.0$, $\sigma_{obj} = 0.0$ [S/m])

2.2.1 Parameters

Background

Inhomogeneous and nonmagnetic background composed by two half spaces

- Upper half space ($y > 0$ - air): $\varepsilon_{ra} = 1.0$, $\sigma_a = 0.0$;
- Lower half space ($y < 0$ - soil): $\varepsilon_{rb} = 4.0$, $\sigma_b = 10^{-3}$ [S/m];

Investigation domain (D_{inv})

- Side: $L_{D_{inv}} = 0.8$ [m];
- Barycenter: $(x_{bar}^{D_{inv}}, y_{bar}^{D_{inv}}) = (0.00, -0.4)$ [m];

Time-Domain forward solver ($FDTD - GPRMax2D$)

- Side of the simulated domain: $L = 6$ [m];
- Number of cells: $N^{FDTD} = 750 \times 750 = 5.625 \times 10^5$;
- Side of the $FDTD$ cells $l^{FDTD} = 0.008$ [m];
- Simulation time window: $T^{FDTD} = 20 \times 10^{-9}$ [sec];
- Time step: $\Delta t^{FDTD} = 1.89 \times 10^{-11}$ [sec];
- Number of time samples: $N_t^{FDTD} = 1060$;
- Boundary conditions: perfectly matched layer (PML);
- Source type: Gaussian mono-cycle (first Gaussian pulse derivative, called “Ricker” in $GPRMax2D$)
 - Central frequency: $f_0 = 300$ [MHz];
 - Source amplitude: $A = 1.0$ [A];

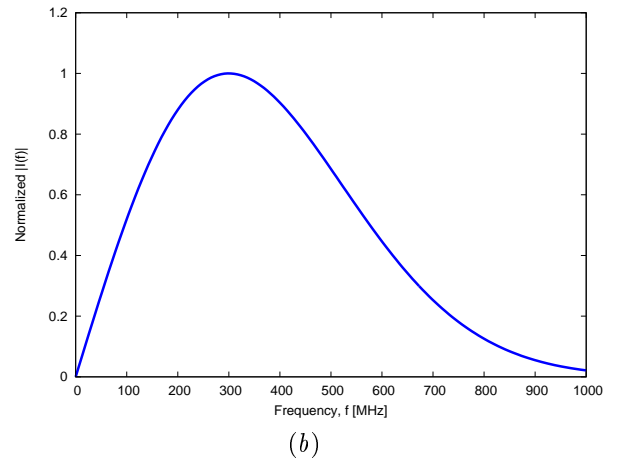
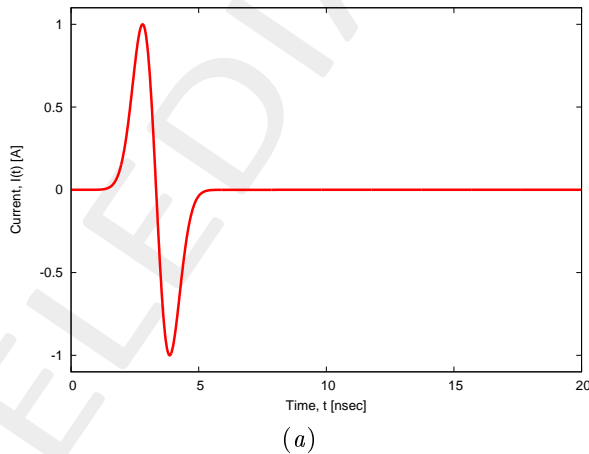


Figure 6: $GPRMax2D$ excitation signal. (a) Time pulse, (b) normalized frequency spectrum.

Frequency parameters

- Frequency range: $f \in [f_{min}, f_{max}] = [200.0, 600.0]$ [MHz] [?] (-3 [dB] bandwidth of the Gaussian Monocycle excitation centered at $f_0 = 300$ [MHz]);
- Frequency step: $\Delta f = 100$ [MHz] ($F = 5$ frequency steps in $[f_{min}, f_{max}]$);

f [MHz]	λ_a [m]	λ_b [m]	f^* [MHz]
200.0	1.50	0.75	200.5
300.0	1.00	0.50	297.6
400.0	0.75	0.37	401.1
500.0	0.60	0.30	498.1
600.0	0.50	0.25	601.6

Table 3: Considered frequencies and corresponding wavelength in the upper medium (λ_a , free space) and in the lower medium (λ_b , soil). f^* is the nearest frequency sample available from transformed time-domain data, and represents the real frequency considered by the inversion algorithm.

Scatterer

- Type: Circular void;
- Barycenter: $(x_{obj}, y_{obj}) = (-0.16, -0.4)$ [m];
- Radius: $r_{obj} = 0.08$ [m];
- Electromagnetic properties: $\varepsilon_{r,obj} = 1.0$, $\sigma_{obj} = 0.0$ [S/m];
- Contrast function: $\tau = -3.0 + j0.03$ (@ $f_{max} = 600$ [MHz]);

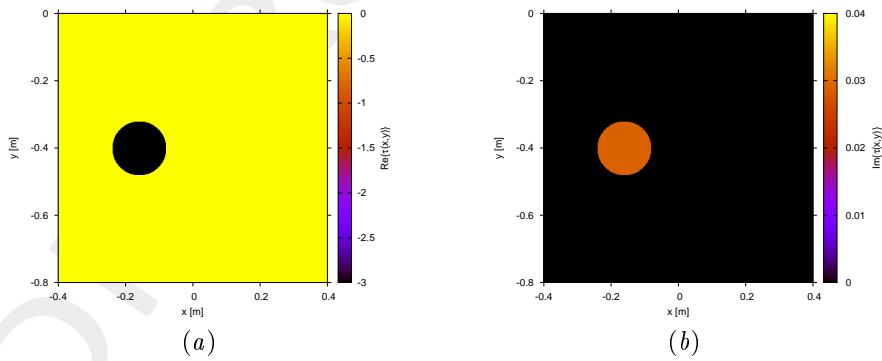


Figure 7: Actual object (a) real part and (b) imaginary part @ $f_{max} = 600$ [MHz].

Measurement setup

- Considered frequency: $f_{min} = 200$ [MHz], $\lambda_b = 0.75$ [m].
- $\#DoFs = 2ka = \frac{2\pi}{\lambda_b} L\sqrt{2} = \frac{2\pi}{0.75} 0.8\sqrt{2} \simeq 9.5$;
- Number of views (sources): $V = 10$;

- $\min \{x_v\} = -0.5$ [m], $\max \{x_v\} = 0.5$ [m];
- height: $y_v = 0.1$ [m], $\forall v = 1, \dots, V$;
- Number of measurement points: $M = 9$;
- $\min \{x_m\} = -0.5$ [m], $\max \{x_m\} = 0.5$ [m];
- height: $y_m = 0.1$ [m], $\forall m = 1, \dots, M$;

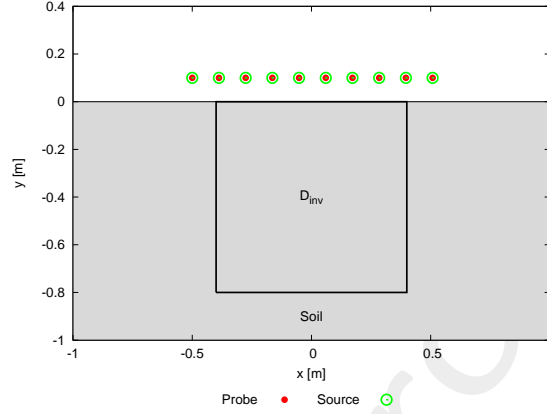


Figure 8: Location of the measurement points ($M = 9$) and of the sources ($V = 10$). Only one source is active for each view.

Inverse solver parameters

- **Shared parameters**

- Weight of the state term of the functional: 1.0;
- Weight of the data term of the functional: 1.0;
- Convergence threshold: 10^{-10} ;
- Variable ranges:
 - * $\varepsilon_r \in [1.0, 4.0]$, $\sigma \in [0.0, 1.0 \times 10^{-3}]$ [S/m];
 - * $\Re \{E_{tot}^{int}\} \in [-8, 8]$, $\Im \{E_{tot}^{int}\} \in [-8, 8]$;
- Degrees of freedom:
 - * Considered frequency: $f_{min} = 200$ [MHz], $\lambda_b = 0.75$ [m];
 - * $\frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda_b} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda_b}\right)^2 = 4\pi^2 \left(\frac{0.8}{0.75}\right)^2 \simeq 44.87$;
- Number of cells: $N = 49 = 7 \times 7$;
- Maximum number of *IMSA* steps: $S = 4$;
- Side ratio threshold: $\eta_{th} = 0.2$;

- **MF – IMSA – CG parameters**

– Maximum number of iterations: $I = 200$;

• ***FH – IMSA – CG* parameters**

– Maximum number of iterations: $I = 400$;

Signal to noise ratio (on $E_{tot}(t)$)

- $SNR = \{50, 40\}$ [dB] + Noiseless data.

SNR on $E_{tot}(t)$ [dB]	Av. SNR on $E_{scatt}(f)$ [dB]
50	45
40	35

Table 4: Average SNR measured on the scattered field in frequency domain.

2.2.2 Results

Final reconstructions ($@f_{max} = 600$ [MHz])

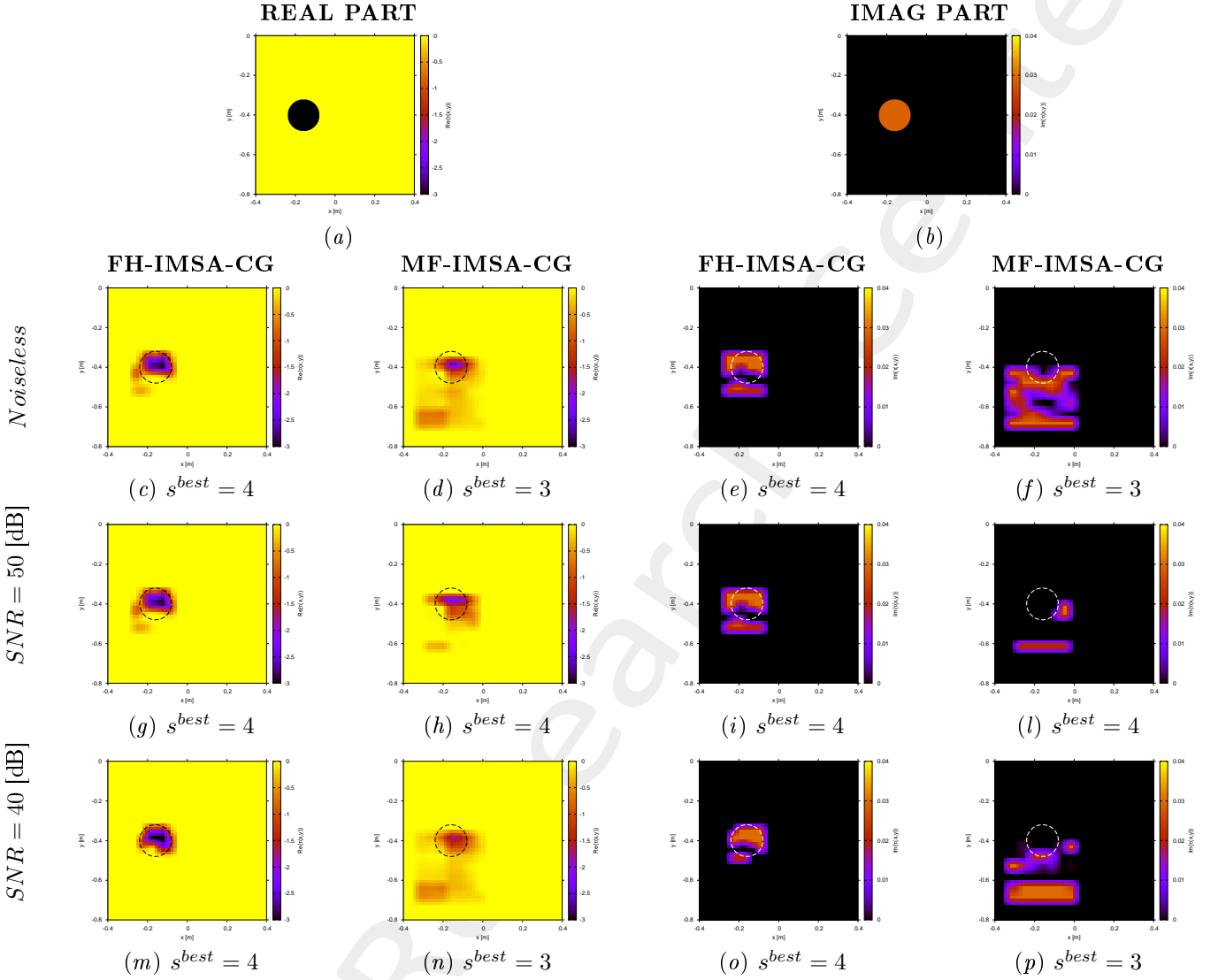


Figure 9: *MF-IMSA-PSO* vs. *MF-IMSA-CG*: Retrieved dielectric profiles at the *IMSA* convergence step (s^{best}).

2.3 Two-Square object ($\varepsilon_{r,obj} = 5.0$, $\sigma_{obj} = 10^{-3}$ [S/m])

2.3.1 Parameters

Background

Inhomogeneous and nonmagnetic background composed by two half spaces

- Upper half space ($y > 0$ - air): $\varepsilon_{ra} = 1.0$, $\sigma_a = 0.0$;
- Lower half space ($y < 0$ - soil): $\varepsilon_{rb} = 4.0$, $\sigma_b = 10^{-3}$ [S/m];

Investigation domain (D_{inv})

- Side: $L_{D_{inv}} = 0.8$ [m];
- Barycenter: $(x_{bar}^{D_{inv}}, y_{bar}^{D_{inv}}) = (0.00, -0.4)$ [m];

Time-Domain forward solver ($FDTD - GPRMax2D$)

- Side of the simulated domain: $L = 6$ [m];
- Number of cells: $N^{FDTD} = 750 \times 750 = 5.625 \times 10^5$;
- Side of the $FDTD$ cells $l^{FDTD} = 0.008$ [m];
- Simulation time window: $T^{FDTD} = 20 \times 10^{-9}$ [sec];
- Time step: $\Delta t^{FDTD} = 1.89 \times 10^{-11}$ [sec];
- Number of time samples: $N_t^{FDTD} = 1060$;
- Boundary conditions: perfectly matched layer (PML);
- Source type: Gaussian mono-cycle (first Gaussian pulse derivative, called “Ricker” in $GPRMax2D$)
 - Central frequency: $f_0 = 300$ [MHz];
 - Source amplitude: $A = 1.0$ [A];

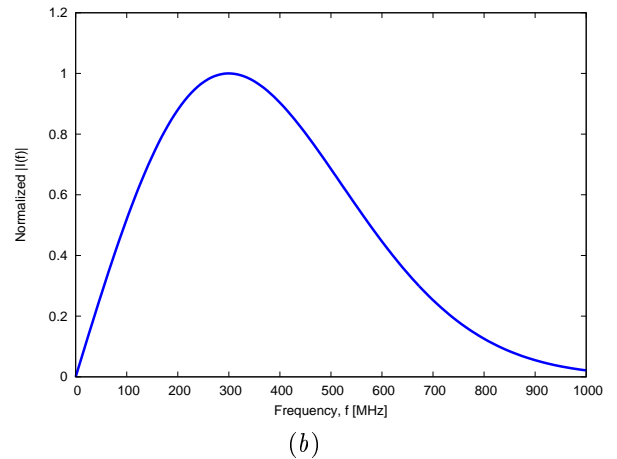
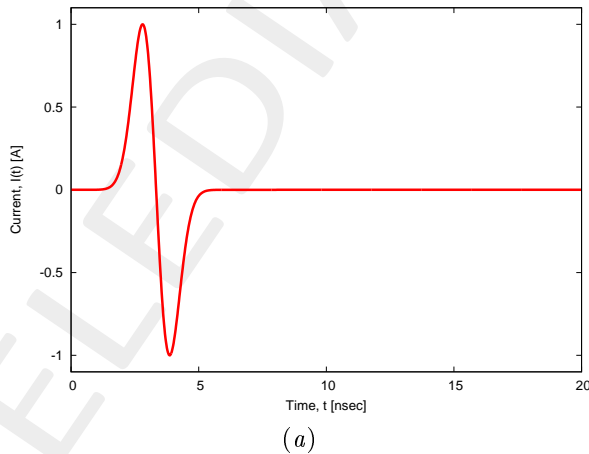


Figure 10: $GPRMax2D$ excitation signal. (a) Time pulse, (b) normalized frequency spectrum.

Frequency parameters

- Frequency range: $f \in [f_{min}, f_{max}] = [200.0, 600.0]$ [MHz] [?] (-3 [dB] bandwidth of the Gaussian Monocycle excitation centered at $f_0 = 300$ [MHz]);
- Frequency step: $\Delta f = 100$ [MHz] ($F = 5$ frequency steps in $[f_{min}, f_{max}]$);

f [MHz]	λ_a [m]	λ_b [m]	f^* [MHz]
200.0	1.50	0.75	200.5
300.0	1.00	0.50	297.6
400.0	0.75	0.37	401.1
500.0	0.60	0.30	498.1
600.0	0.50	0.25	601.6

Table 5: Considered frequencies and corresponding wavelength in the upper medium (λ_a , free space) and in the lower medium (λ_b , soil). f^* is the nearest frequency sample available from transformed time-domain data, and represents the real frequency considered by the inversion algorithm.

Scatterer

- Type: Two squares;
- Electromagnetic properties: $\varepsilon_{r,obj} = 5.0$, $\sigma_{obj} = 10^{-3}$ [S/m] ($\sigma_{obj} = \sigma_b$);
- Contrast function: $\tau = 1.0 + j0.0$

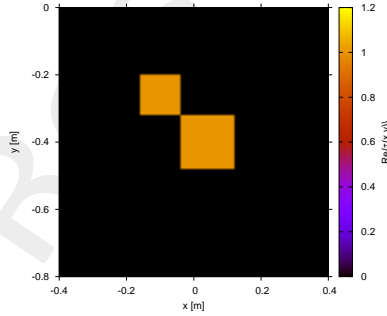


Figure 11: Actual object ($\tau = 1.0$).

Measurement setup

- Considered frequency: $f_{min} = 200$ [MHz], $\lambda_b = 0.75$ [m].
- $\#DoFs = 2ka = \frac{2\pi}{\lambda_b} L\sqrt{2} = \frac{2\pi}{0.75} 0.8\sqrt{2} \simeq 9.5$;
- Number of views (sources): $V = 10$;
 - $\min\{x_v\} = -0.5$ [m], $\max\{x_v\} = 0.5$ [m];
 - height: $y_v = 0.1$ [m], $\forall v = 1, \dots, V$;
- Number of measurement points: $M = 9$;

- $\min \{x_m\} = -0.5$ [m], $\max \{x_m\} = 0.5$ [m];
- height: $y_m = 0.1$ [m], $\forall m = 1, \dots, M$;

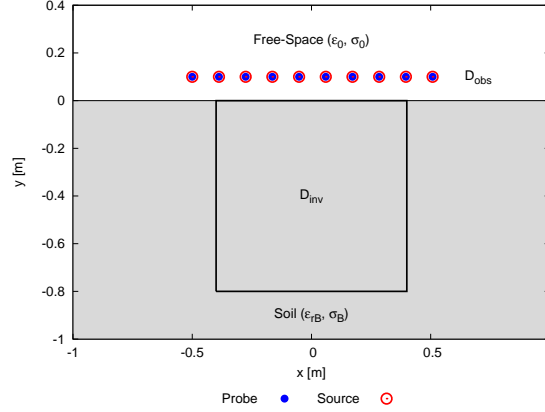


Figure 12: Location of the measurement points ($M = 9$) and of the sources ($V = 10$). Only one source is active for each view.

Inverse solver parameters

- **Shared parameters**

- Weight of the state term of the functional: 1.0;
- Weight of the data term of the functional: 1.0;
- Convergence threshold: 10^{-10} ;
- Variable ranges:
 - * $\varepsilon_r \in [4.0, 5.2]$, $\sigma \in [8.0 \times 10^{-4}, 1.2 \times 10^{-3}]$ [S/m];
 - * $\Re \{E_{tot}^{int}\} \in [-8, 8]$, $\Im \{E_{tot}^{int}\} \in [-8, 8]$;
- Degrees of freedom:
 - * Considered frequency: $f_{min} = 200$ [MHz], $\lambda_b = 0.75$ [m];
 - * $\frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda_b} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda_b}\right)^2 = 4\pi^2 \left(\frac{0.8}{0.75}\right)^2 \simeq 44.87$;
- Number of cells: $N = 49 = 7 \times 7$;
- Maximum number of *IMSA* steps: $S = 4$;
- Side ratio threshold: $\eta_{th} = 0.2$;

- **MF – IMSA – CG parameters**

- Maximum number of iterations: $I = 200$;

- **FH – IMSA – CG parameters**

- Maximum number of iterations: $I = 400$;

Signal to noise ratio (on $E_{tot}(t)$)

- $SNR = \{50, 40\}$ [dB] + Noiseless data.

SNR on $E_{tot}(t)$ [dB]	Av. SNR on $E_{scatt}(f)$ [dB]
50	36
40	26

Table 6: Average SNR measured on the scattered field in frequency domain.

2.3.2 Results

Final reconstructions ($@f_{max} = 600$ [MHz])

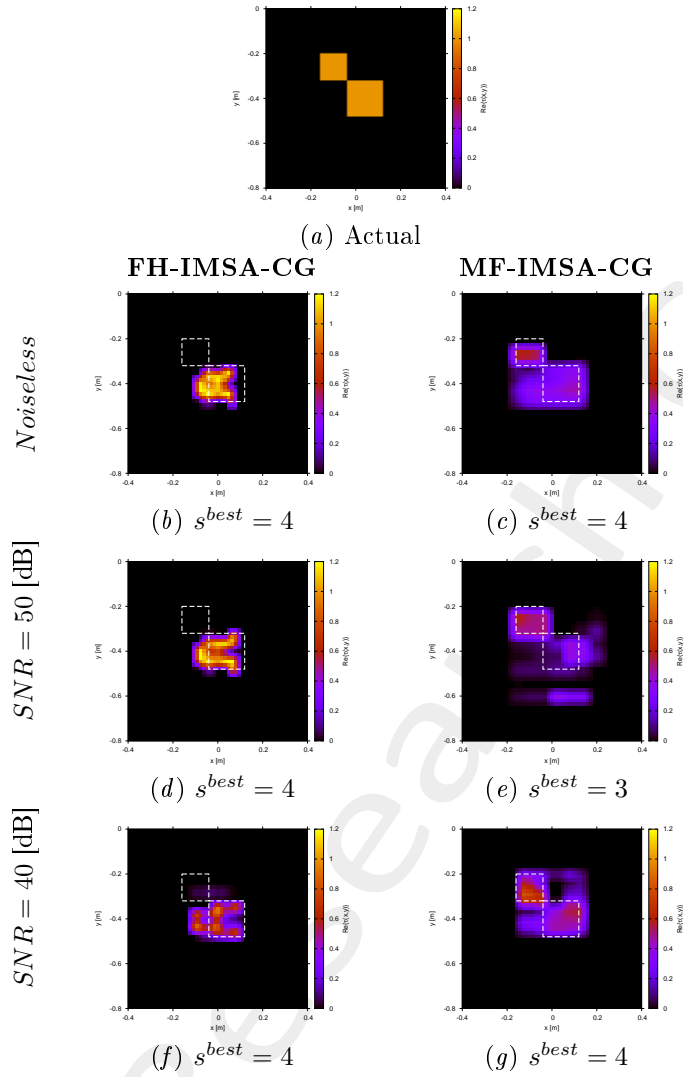


Figure 13: *FH-IMSA-CG* vs. *MF-IMSA-CG*: Retrieved dielectric profiles at the *IMSA* convergence step (s^{best}) $@f_{max} = 600$ [MHz].

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