Planar Arrays Tolerance Analysis Through an Innovative Minkowski-sum based Sensitivity Tool

L. Tenuti, N. Anselmi, P. Rocca, M. Salucci, and A. Massa

Abstract

It is well known that manufacturing errors, mutual coupling effects, mechanical deformations, and climatic changes can cause deviations (both in phase and amplitude) of the excitations in planar phased arrays from their nominal values. As a main consequence, the radiated pattern can differ from the expected one, and the overall system performances can be lower than expected. In this work, an innovative sensitivity tool has been developed to analyze the impact of uncertain but bounded excitation tolerances in planar phased arrays on the radiated pattern. More in detail, the proposed methodology exploits the rules of interval analysis (*IA*) and the Minkowski sum in order to estimate narrower but inclusive bounds than standard Cartesian-based *IA* approaches. Some numerical results are shown, in order to verify the effectiveness of the developed sensitivity tool and to study the influence of excitation tolerances in generating a beam pattern with a given sidelobe level (*SLL*).

1 Numerical Assessment - Analysis vs SLL

GOAL: This section has been developed to study the influence of tolerances on the control points (in amplitude and phase) for generating a beam pattern with given SLL. Accordingly, the test case considers the same antenna geometry, i.e. a planar array with 10×10 elements for which the element excitations have been choosen to generate on the principals planes $u = 0$ and $v = 0$ a Dolph-Chebyshev pattern with SLL equal to -10, -20, -30, -40, -50 and -60 dB. The amplitude and phase tolerances have been choosen as: $\pm 1\%$, $\pm 3\%$ and $\pm 5\%$ for the amplitude and ± 1, ± 3 and ± 5 $[deg]$ for the phase.

Array geometry:

- Uniform planar array: $N \times M = 10 \times 10$.
- Inter-element spacing: $d_x = 0.5$ $[\lambda]$ - $d_y = 0.5$ $[\lambda]$;

Nominal control points:

- Separable distributions:
  - x-axis: Dolph-Chebyshev pattern: $SLL = -10, -20, -30, -40, -50, -60$ $[dB]$.
  - y-axis: Dolph-Chebyshev pattern: $SLL = -10, -20, -30, -40, -50, -60$ $[dB]$.

Tolerances on the control points:

- Amplitude tolerance: $\delta\alpha_n = \pm 1\,\%, \pm 3\,\%, \pm 5\,\%$.
- Phase tolerance: $\delta\beta_n = \pm 1\ [deg], \pm 3\ [deg], \pm 5\ [deg]$.

Minkowski sum parameters:

- Number of sides including polygon: $L = 720$

1.1 $SLL = -10\,dB$

Nominal Pattern

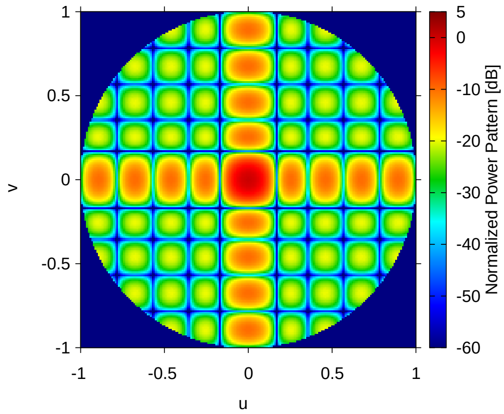


Figure 79:

Nominal Pattern Features

$BW\ [v] - u = 0$	$BW\ [u] - v = 0$	$SLL\ [dB] - u = 0$	$SLL\ [dB] - v = 0$	$PP\ [dB]$
0.156	0.156	-10.0	-10.0	28.28

Table $XXIII$:

1.1.1 Amplitude Tolerance $\delta\alpha_n = 0\,\%$

3D Pattern

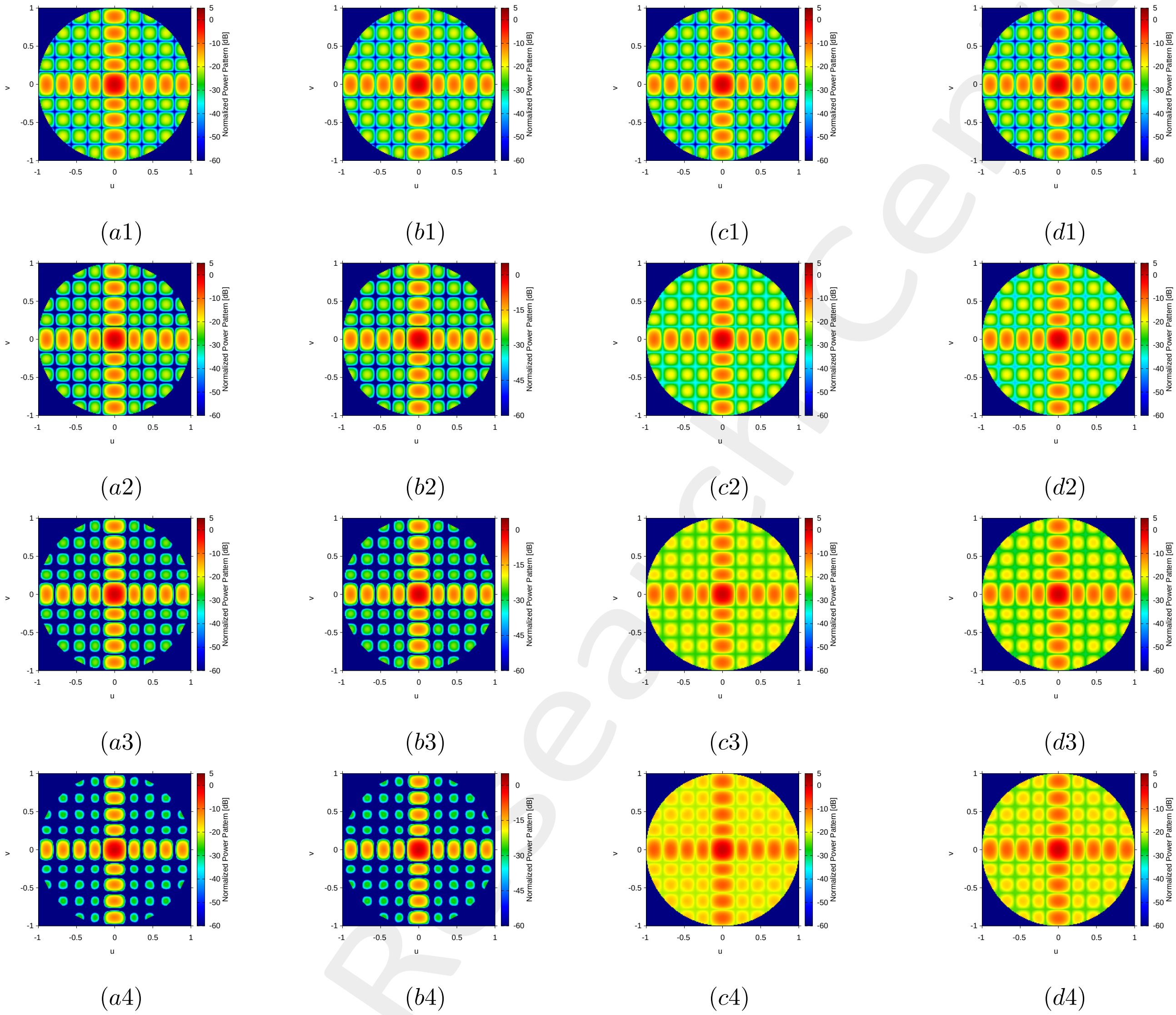


Figure 80: Infimum of the power Cartesian (a) and Minkowski sum (b), Supremum of the power pattern Cartesian (c) and Minkoski sum (d), 1- $\delta\beta_n = 0$ $[deg]$, 2 - $\delta\beta_n = \pm1$ $[deg]$, 3 $\delta\beta_n = \pm3$ $[deg]$, 4 $\delta\beta_n = \pm5$ $[deg]$

Pattern: Cuts on the plane

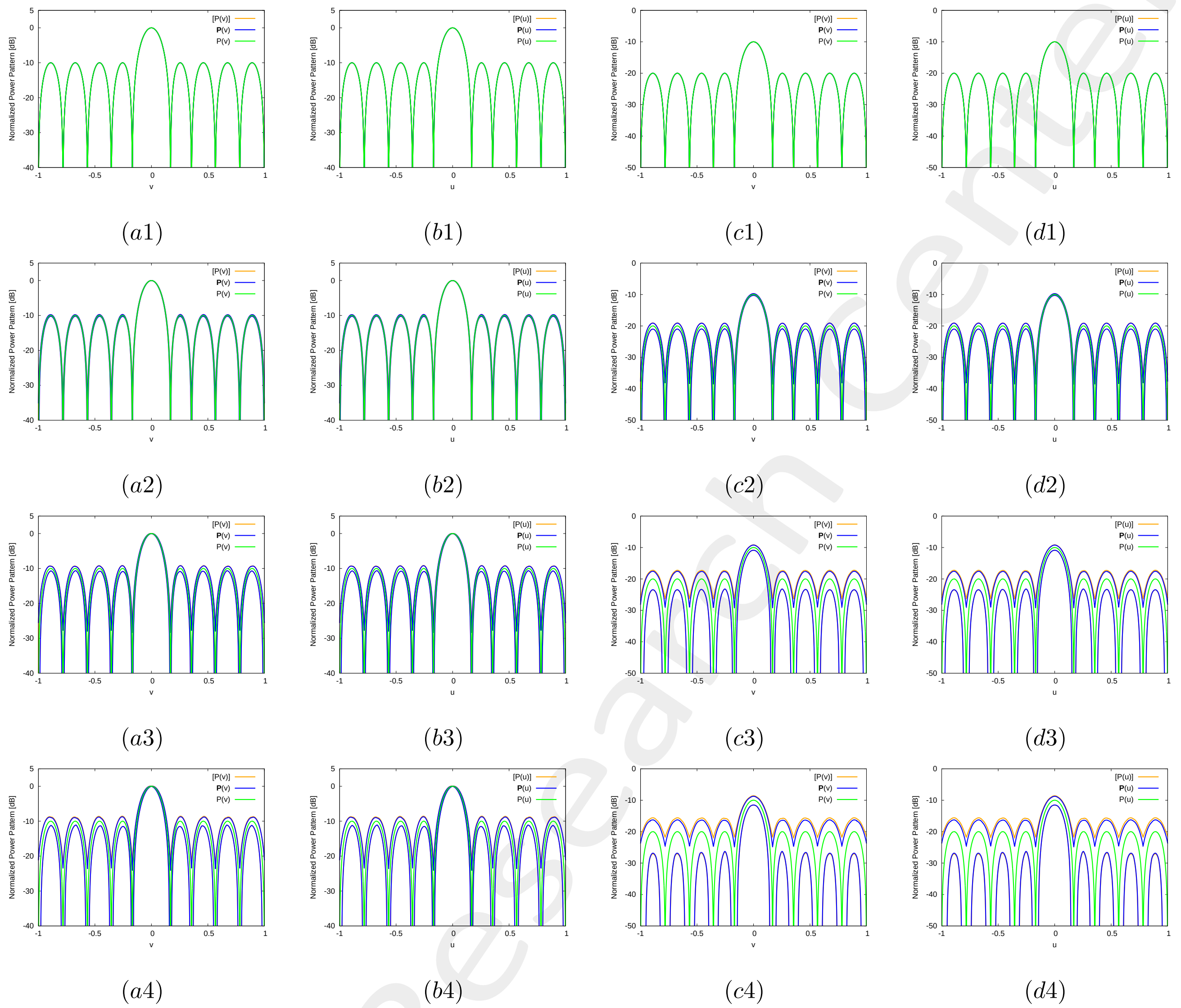


Figure 81: Interval Pattern cut on the plane $u = 0$ (a) cut on the plane $v = 0$ (b) cut on the plane $u = 0.255$ (c) and cut on the plane $v = 0.255$ (d), 1- $\delta\beta_n = 0$ $[deg]$, 2 - $\delta\beta_n = \pm 1$ $[deg]$, 3 $\delta\beta_n = \pm 3$ $[deg]$, 4 $\delta\beta_n = \pm 5$ $[deg]$

1.1.2 Amplitude Tolerance $\delta\alpha_n = 1\,\%$

3D Pattern

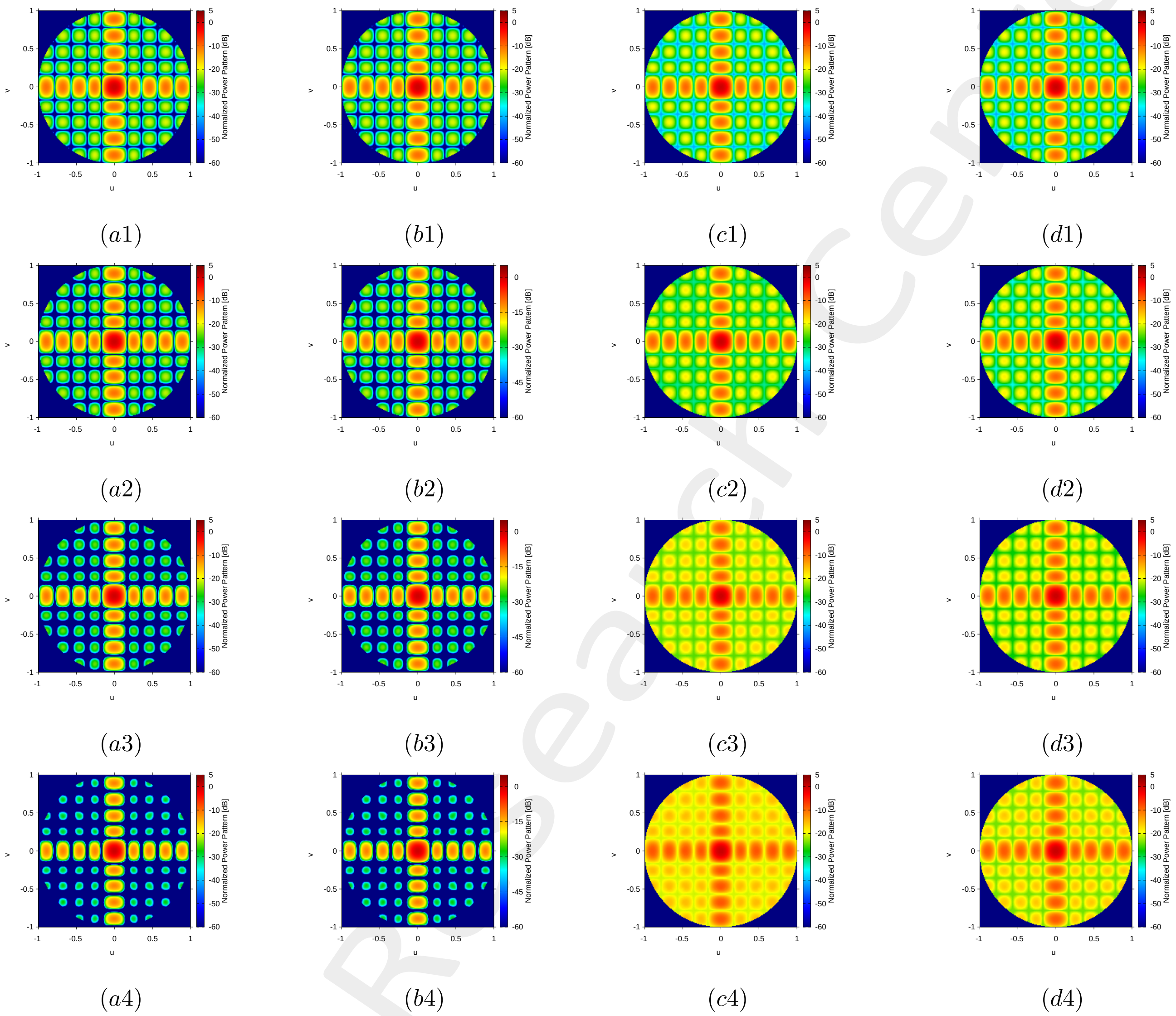


Figure 82: Infimum of the power Cartesian (a) and Minkowski sum (b), Supremum of the power pattern Cartesian (c) and Minkoski sum (d), 1 - $\delta\beta_n = 0$ $[deg]$, 2 - $\delta\beta_n = \pm 1$ $[deg]$, 3 - $\delta\beta_n = \pm 3$ $[deg]$, 4 - $\delta\beta_n = \pm 5$ $[deg]$

Pattern: Cuts on the plane

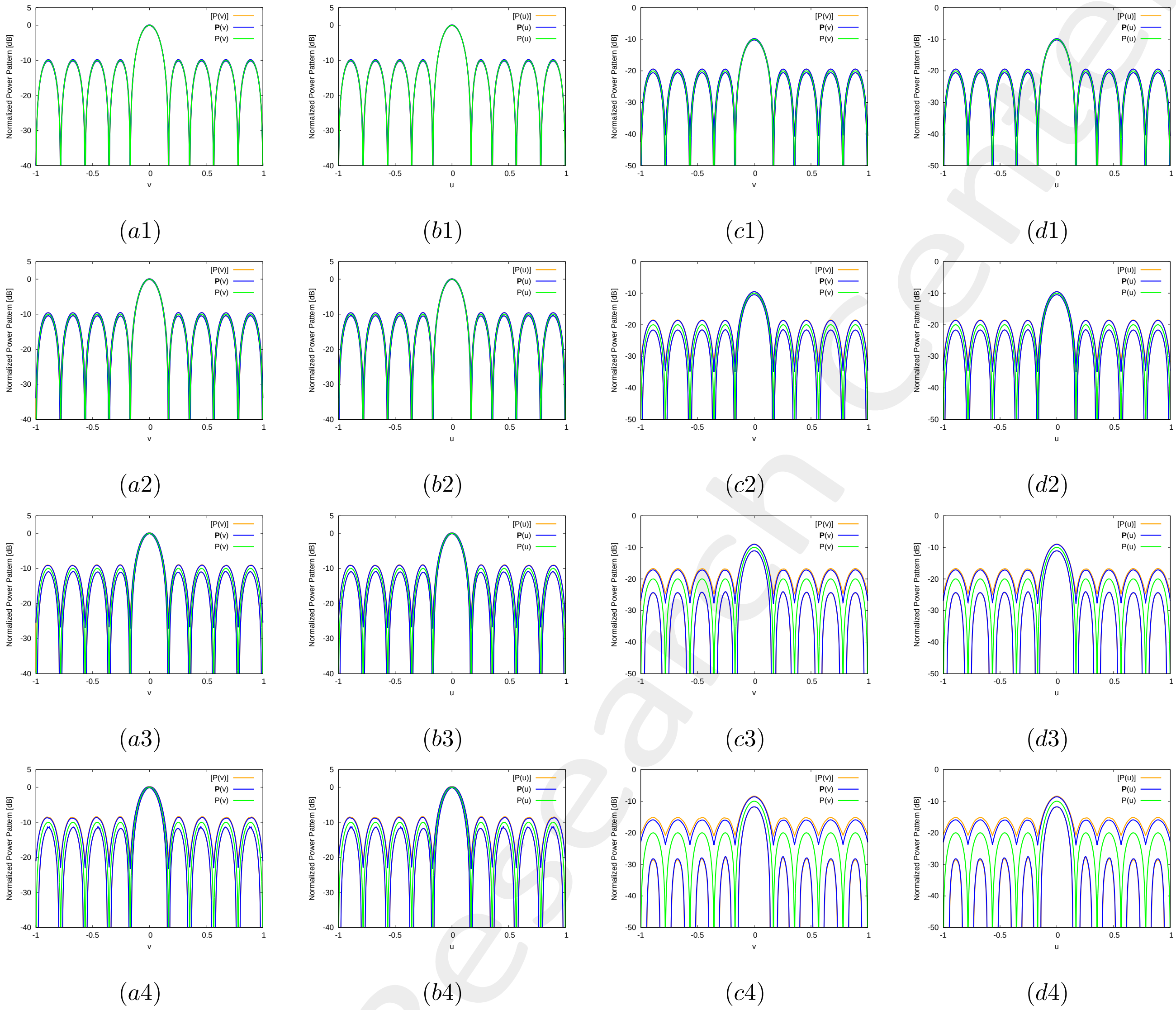


Figure 83: Interval Pattern cut on the plane $u = 0$ (a) cut on the plane $v = 0$ (b) cut on the plane $u = 0.255$ (c) and cut on the plane $v = 0.255$ (d), 1 - $\delta\beta_n = 0$ $[deg]$, 2 - $\delta\beta_n = \pm 1$ $[deg]$, 3 $\delta\beta_n = \pm 3$ $[deg]$, 4 - $\delta\beta_n = \pm 5$ $[deg]$

1.1.3 Amplitude Tolerance $\delta\alpha_n = 3\,\%$

3D Pattern

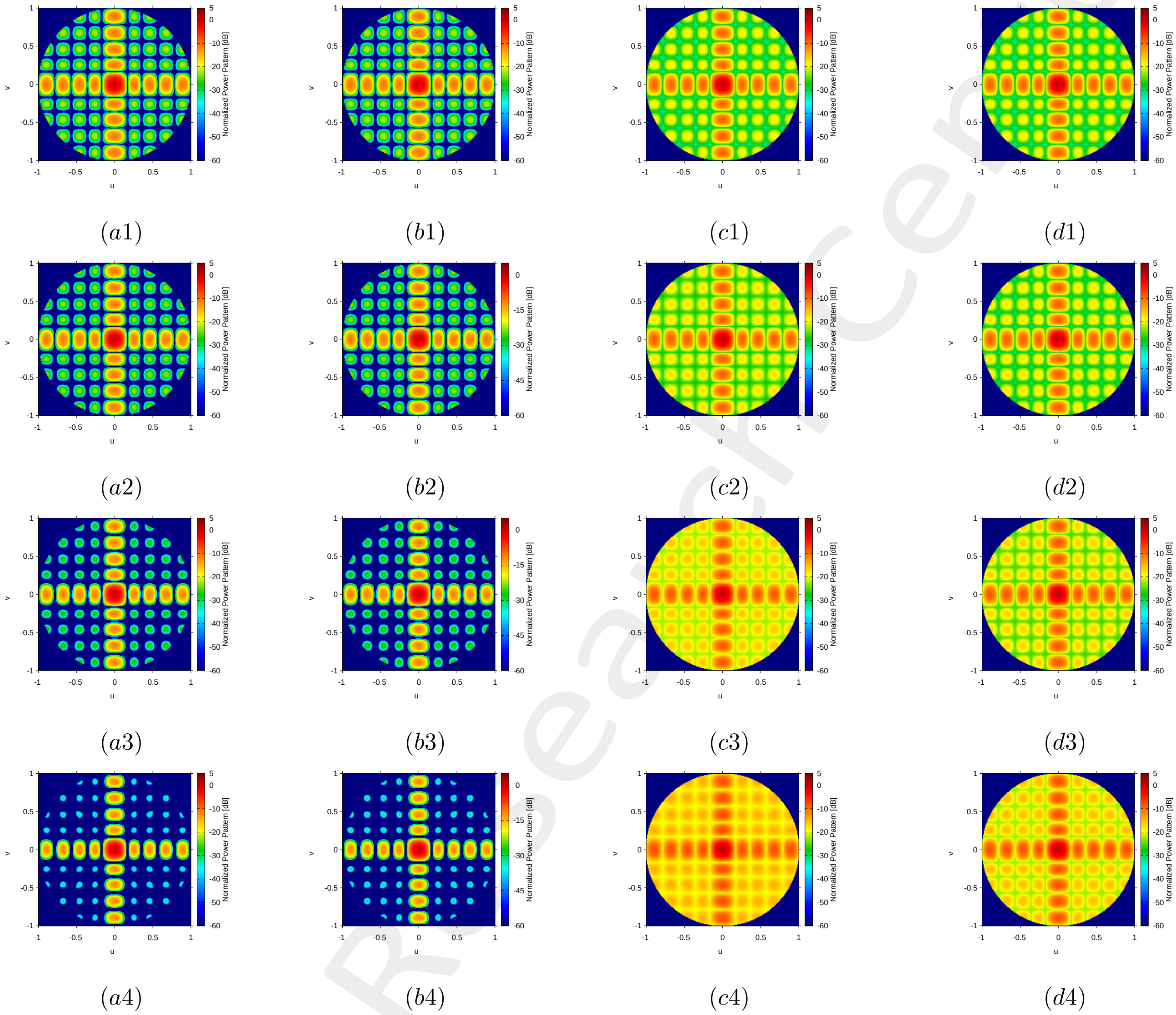


Figure 84: Infimum of the power Cartesian (a) and Minkowski sum (b), Supremum of the power pattern Cartesian (c) and Minkoski sum (d), 1- $\delta\beta_n = 0$ $[deg]$, 2 - $\delta\beta_n = \pm1$ $[deg]$, 3 - $\delta\beta_n = \pm3$ $[deg]$, 4 - $\delta\beta_n = \pm5$ $[deg]$

Pattern: Cuts on the plane

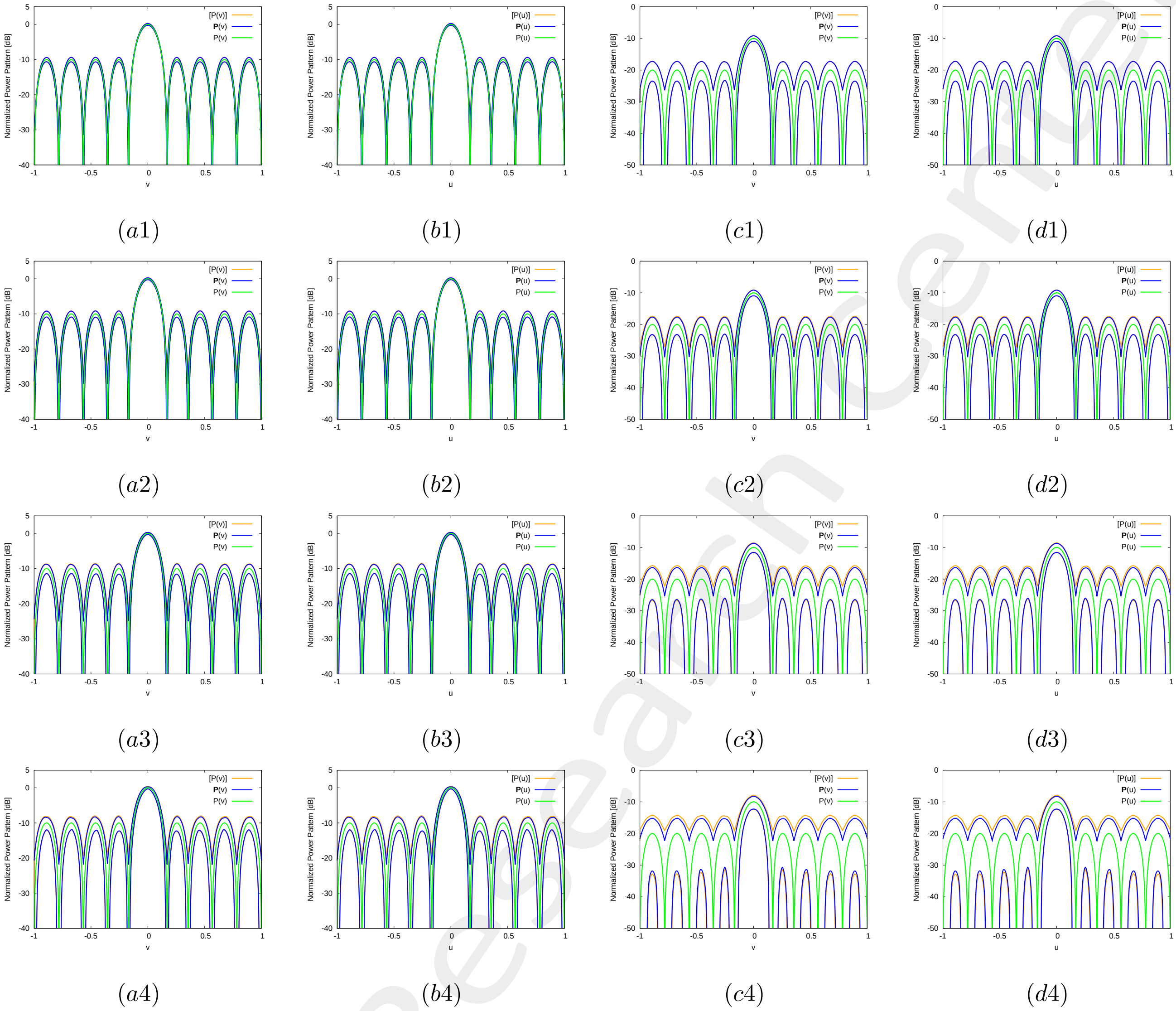


Figure 85: Interval Pattern cut on the plane $u = 0$ (a) cut on the plane $v = 0$ (b) cut on the plane $u = 0.255$ (c) and cut on the plane $v = 0.255$ (d), 1- $\delta\beta = 0$ $[deg]$, 2 - $\delta\beta_n = \pm 1$ $[deg]$, 3 - $\delta\beta_n = \pm 3$ $[deg]$, 4 - $\delta\beta_n = \pm 5$ $[deg]$

1.1.4 Amplitude Tolerance $\delta\alpha_n = 5\,\%$

3D Pattern

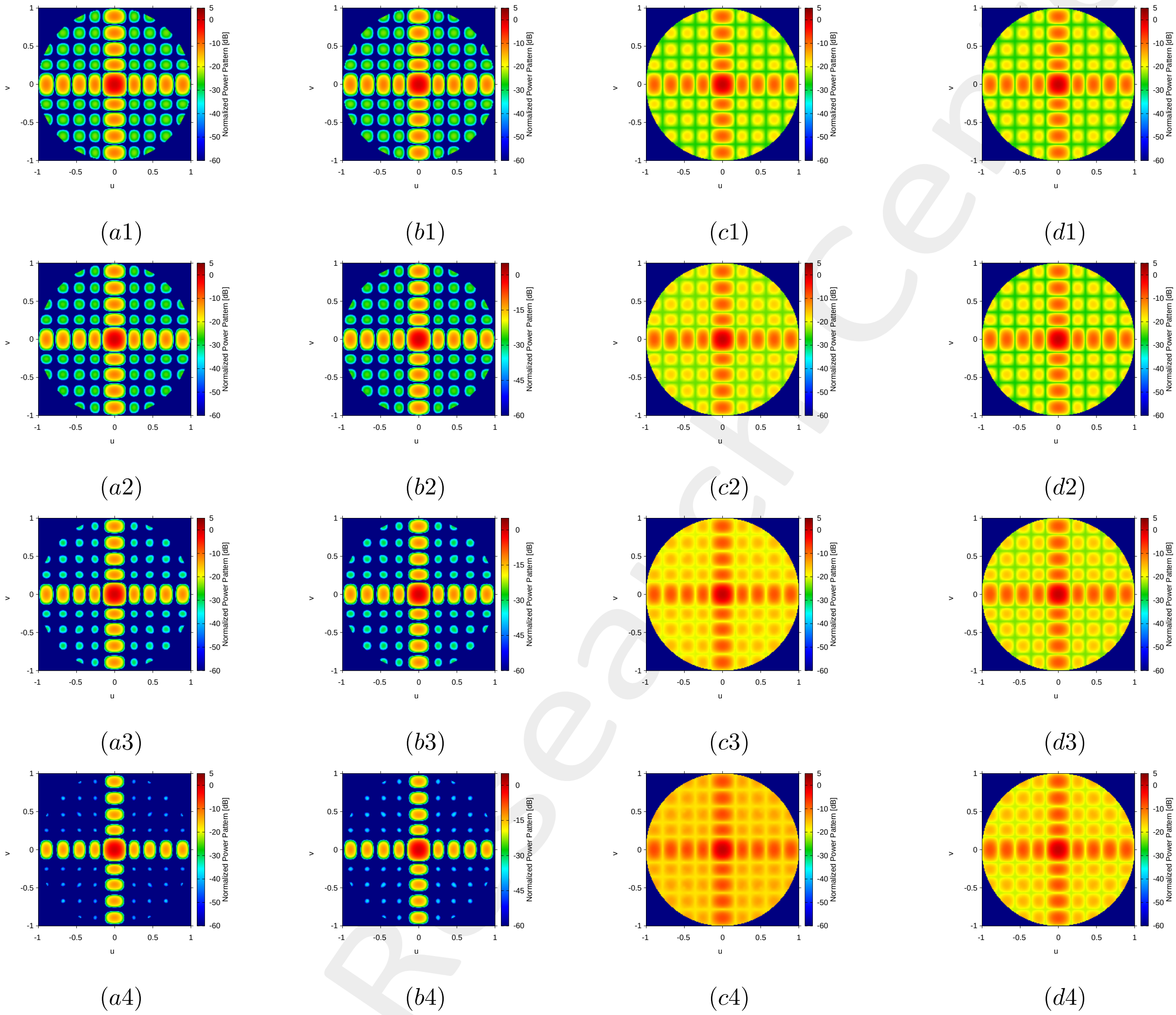


Figure 86: Infimum of the power Cartesian (a) and Minkowski sum (b), Supremum of the power pattern Cartesian (c) and Minkoski sum (d), 1- $\delta\beta_n = 0$ $[deg]$, 2 - $\delta\beta_n = \pm 1$ $[deg]$, 3 - $\delta\beta_n = \pm 3$ $[deg]$, 4 - $\delta\beta_n = 5$ $[deg]$

Pattern: Cuts on the plane

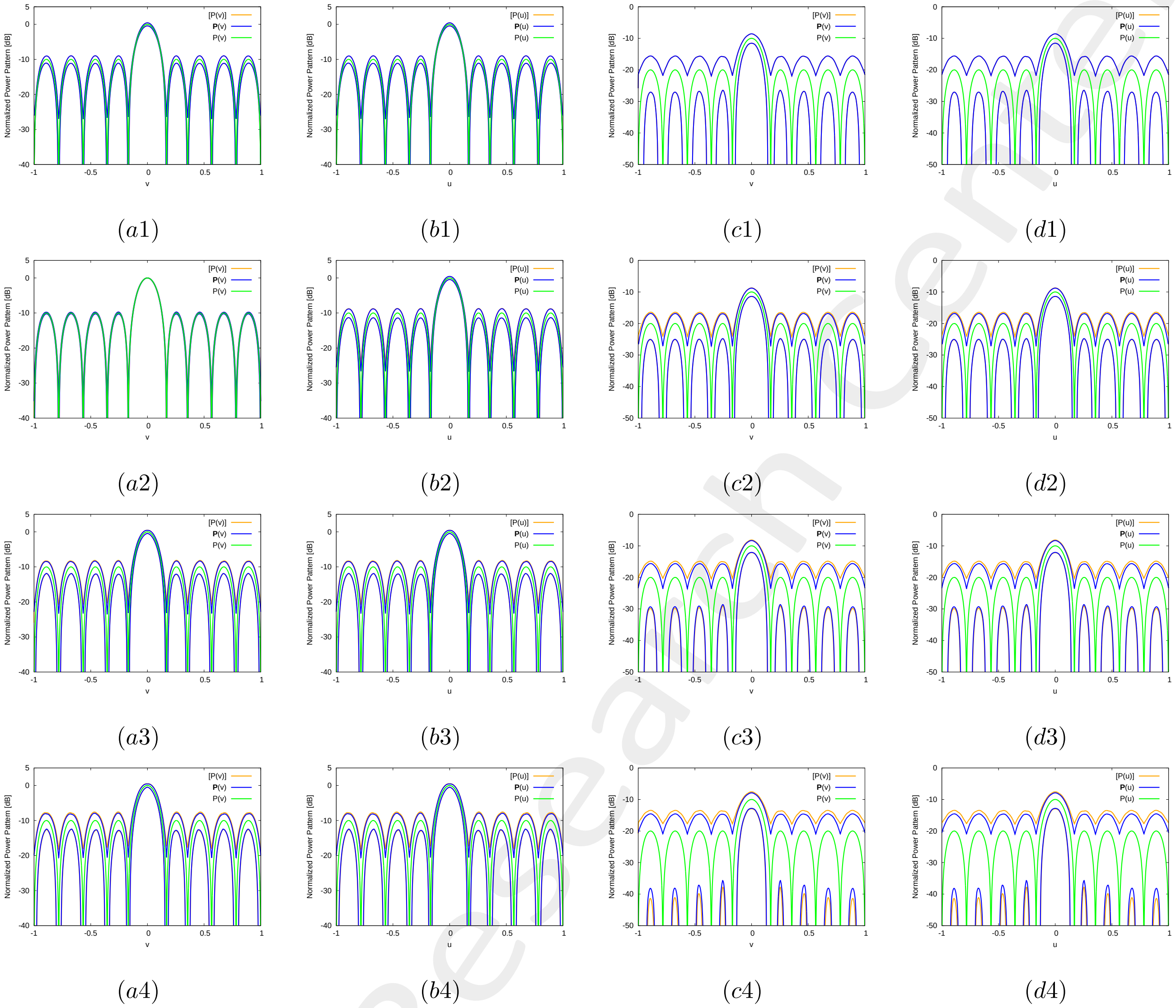


Figure 87: Interval Pattern cut on the plane $u = 0$ (a) cut on the plane $v = 0$ (b) cut on the plane $u = 0.255$ (c) and cut on the plane $v = 0.255$ (d), 1 - $\delta\beta_n = 0$ $[deg]$, 2 - $\delta\beta_n = \pm 1$ $[deg]$, 3 - $\delta\beta_n = \pm 3$ $[deg]$, 4 - $\delta\beta_n = \pm 5$ $[deg]$

1.2 $SLL = 20\,dB$

Nominal Pattern

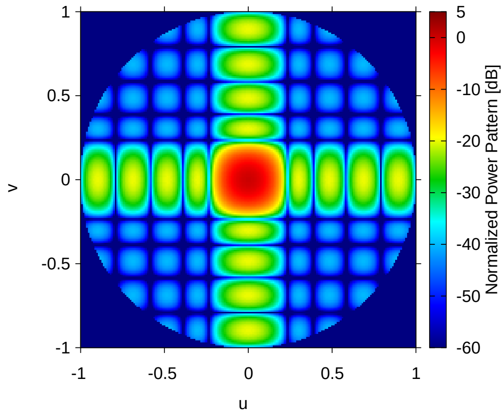


Figure 88:

Nominal Pattern Features

$BW\ [v] - u = 0$	$BW\ [u] - v = 0$	$SLL\ [dB] - u = 0$	$SLL\ [dB] - v = 0$	$PP\ [dB]$
0.196	0.196	-20.0	-20.0	35.84

Table $XXIV$:

1.2.1 Amplitude Tolerance $\delta\alpha_n = 0\,\%$

3D Pattern

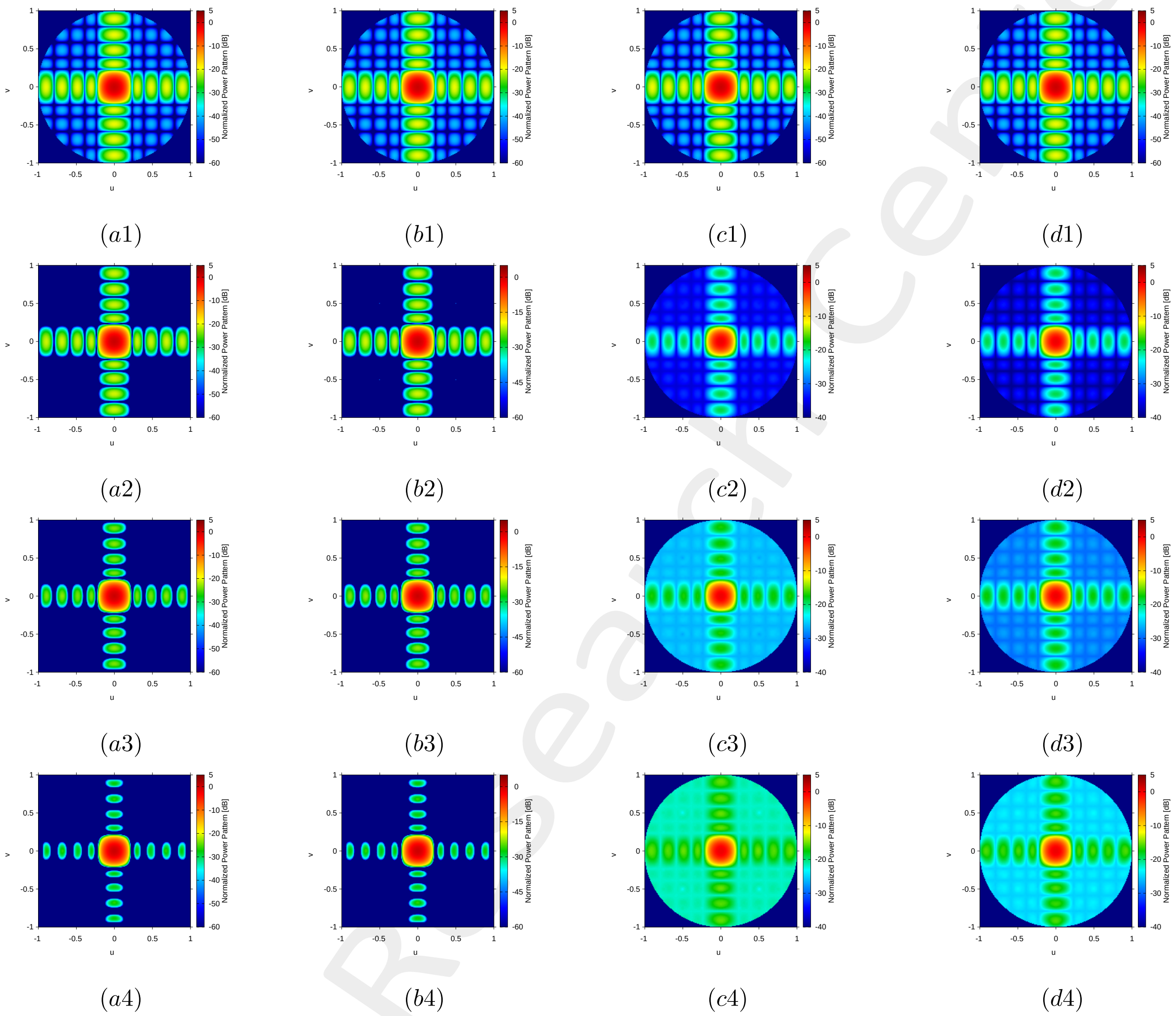


Figure 89: Infimum of the power Cartesian (a) and Minkowski sum (b), Supremum of the power pattern Cartesian (c) and Minkoski sum (d), 1- $\delta\beta_n = 0$ $[deg]$, 2 - $\delta\beta_n = \pm 1$ $[deg]$, 3 $\delta\beta = 3$ $[deg]$, 4 $\delta\beta = 5$ $[deg]$

Pattern: Cuts on the plane

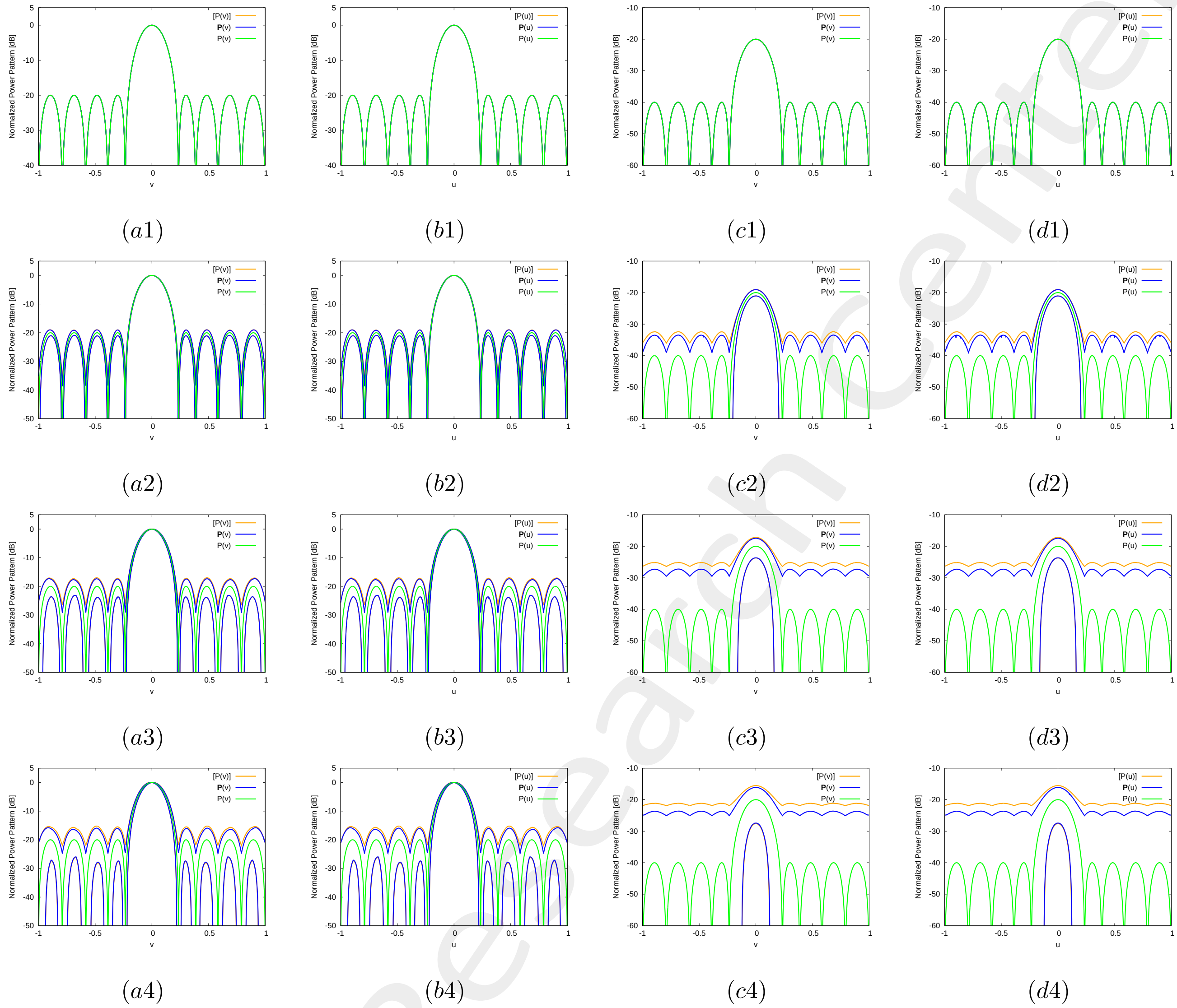
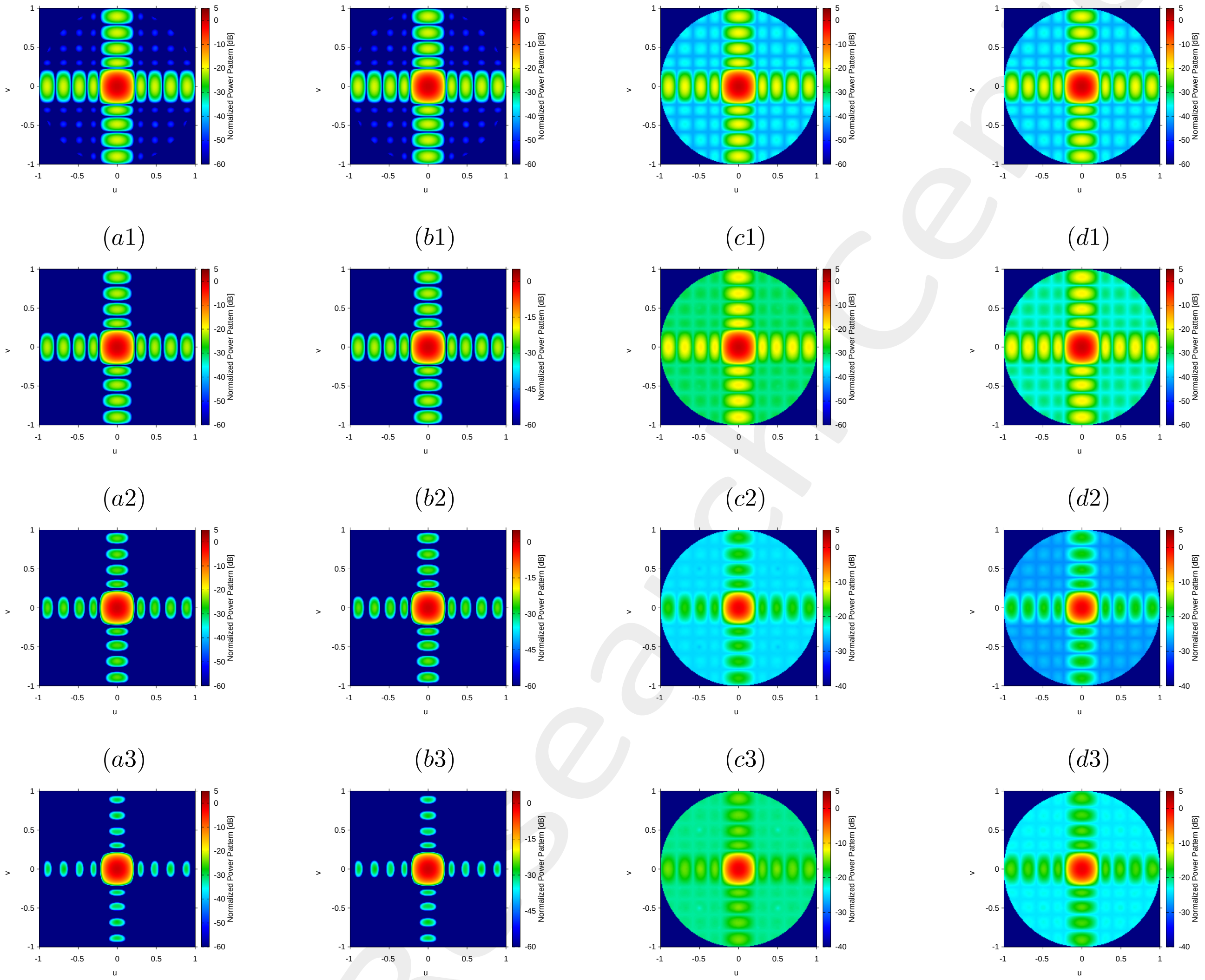


Figure 90: Interval Pattern cut on the plane $u = 0$ (a) cut on the plane $v = 0$ (b) cut on the plane $u = 0.300$ (c) and cut on the plane $v = 0.300$ (d), 1- $\delta\beta = 0$ $[deg]$, 2 - $\delta\beta_n = \pm 1$ $[deg]$, 3 - $\delta\beta_n = \pm 3$ $[deg]$, 4 - $\delta\beta_n = 5$ $[deg]$

1.2.2 Amplitude Tolerance $\delta\alpha_n = 1\,\%$

3D Pattern



(a1) (b1) (c1) (d1)

(a2) (b2) (c2) (d2)

(a3) (b3) (c3) (d3)

(a4) (b4) (c4) (d4)

Figure 91: Infimum of the power Cartesian (a) and Minkowski sum (b), Supremum of the power pattern Cartesian (c) and Minkoski sum (d), 1- $\delta\beta_n = 0$ $[deg]$, 2 - $\delta\beta_n = \pm 1$ $[deg]$, 3 - $\delta\beta_n = \pm 3$ $[deg]$, 4 - $\delta\beta_n = \pm 5$ $[deg]$

Pattern: Cuts on the plane

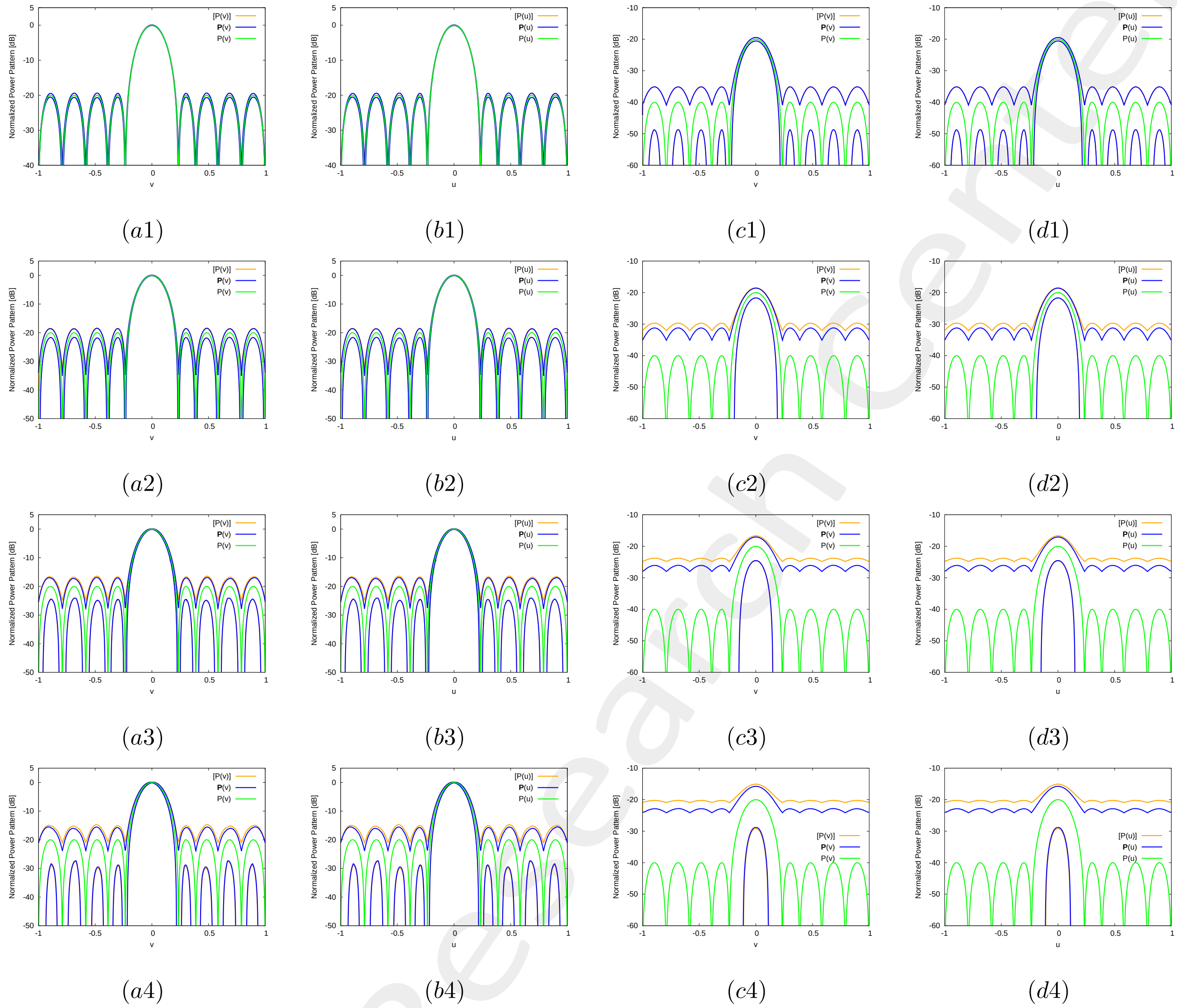


Figure 92: Interval Pattern cut on the plane $u = 0$ (a) cut on the plane $v = 0$ (b) cut on the plane $u = 0.300$ (c) and cut on the plane $v = 0300$ (d), 1- $\delta\beta_n = 0$ $[deg]$, 2 - $\delta\beta_n = \pm 1$ $[deg]$, 3 - $\delta\beta_n = \pm 3$ $[deg]$, 4 - $\delta\beta_n = \pm 5$ $[deg]$

1.2.3 Amplitude Tolerance $\delta\alpha_n = 3\,\%$

3D Pattern

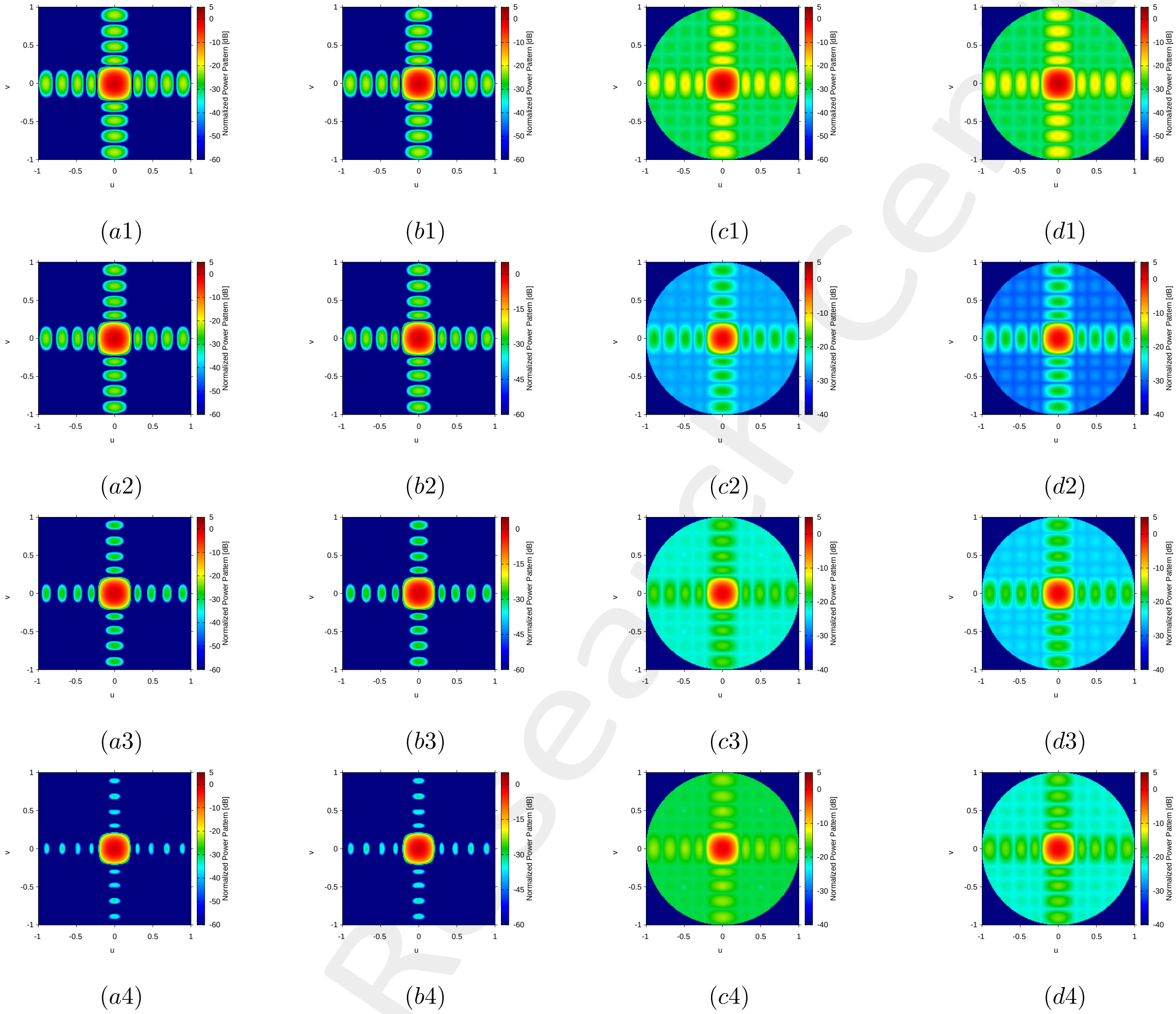


Figure 93: Infimum of the power Cartesian (a) and Minkowski sum (b), Supremum of the power pattern Cartesian (c) and Minkoski sum (d), 1- $\delta\beta_n = 0$ $[deg]$, 2 - $\delta\beta_n = \pm 1$ $[deg]$, 3 - $\delta\beta_n = \pm 3$ $[deg]$, 4 - $\delta\beta_n = \pm 5$ $[deg]$

Pattern: Cuts on the plane

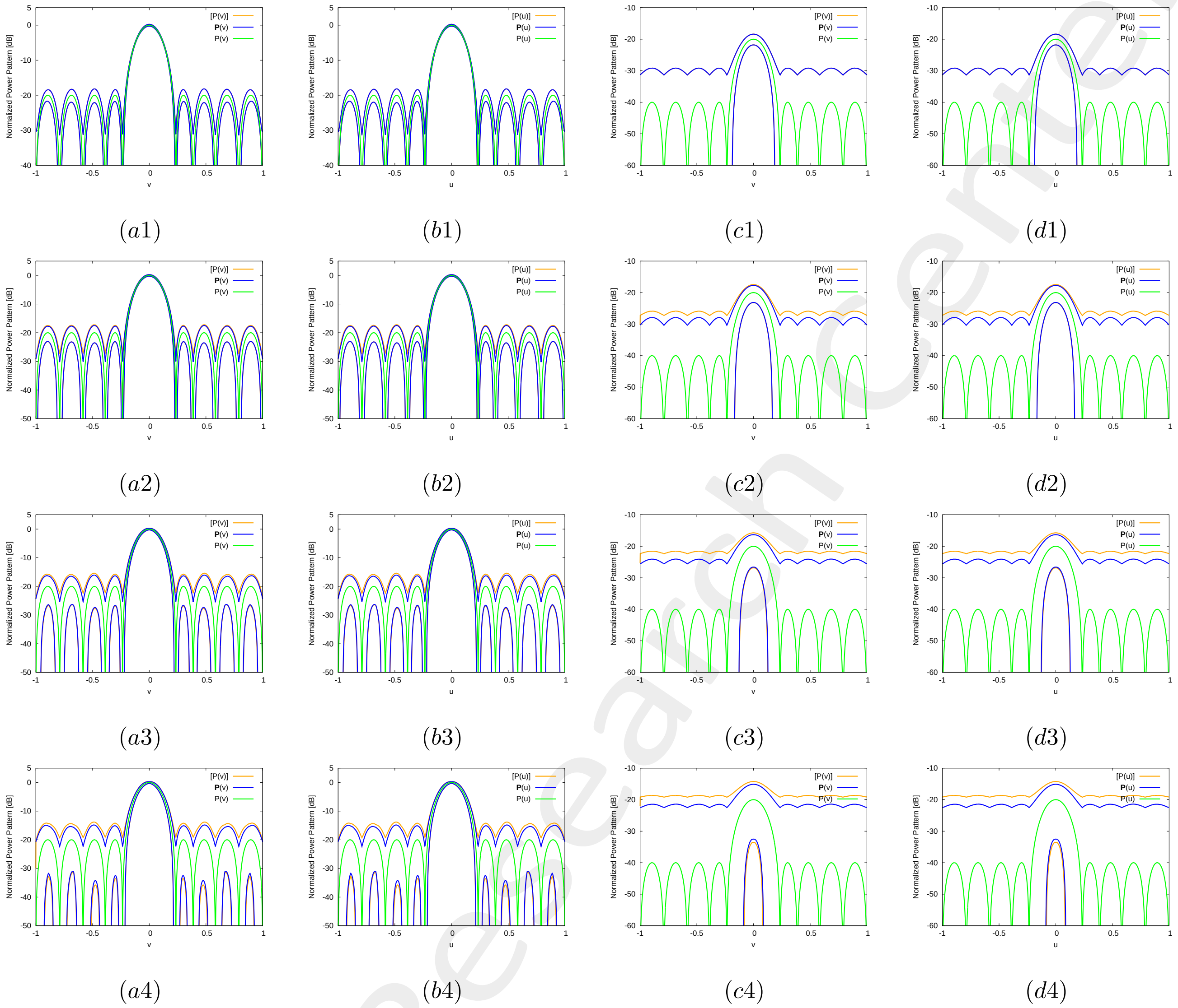


Figure 94: Interval Pattern cut on the plane $u = 0$ (a) cut on the plane $v = 0$ (b) cut on the plane $u = 0.300$ (c) and cut on the plane $v = 0.300$ (d), 1- $\delta\beta_n = 0$ $[deg]$, 2 - $\delta\beta_n = \pm 1$ $[deg]$, 3 - $\delta\beta_n = \pm 3$ $[deg]$, 4 - $\delta\beta_n = \pm 5$ $[deg]$

1.2.4 Amplitude Tolerance $\delta\alpha_n = 5\,\%$

3D Pattern

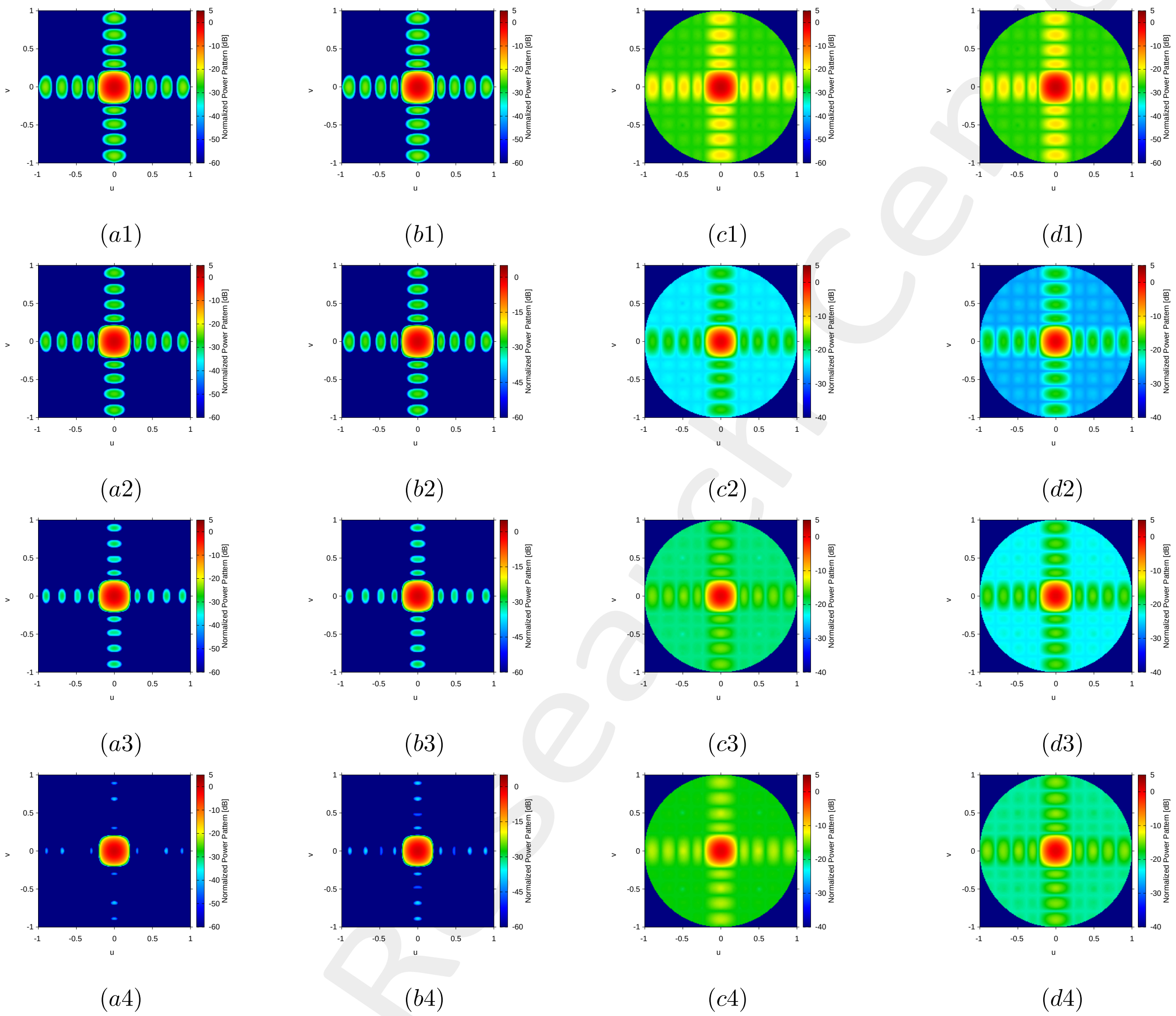


Figure 95: Infimum of the power Cartesian (a) and Minkowski sum (b), Supremum of the power pattern Cartesian (c) and Minkoski sum (d), 1- $\delta\beta_n = 0$ $[deg]$, 2 - $\delta\beta_n = \pm 1$ $[deg]$, 3 - $\delta\beta_n = \pm 3$ $[deg]$, 4 - $\delta\beta_n = \pm 5$ $[deg]$

Pattern: Cuts on the plane

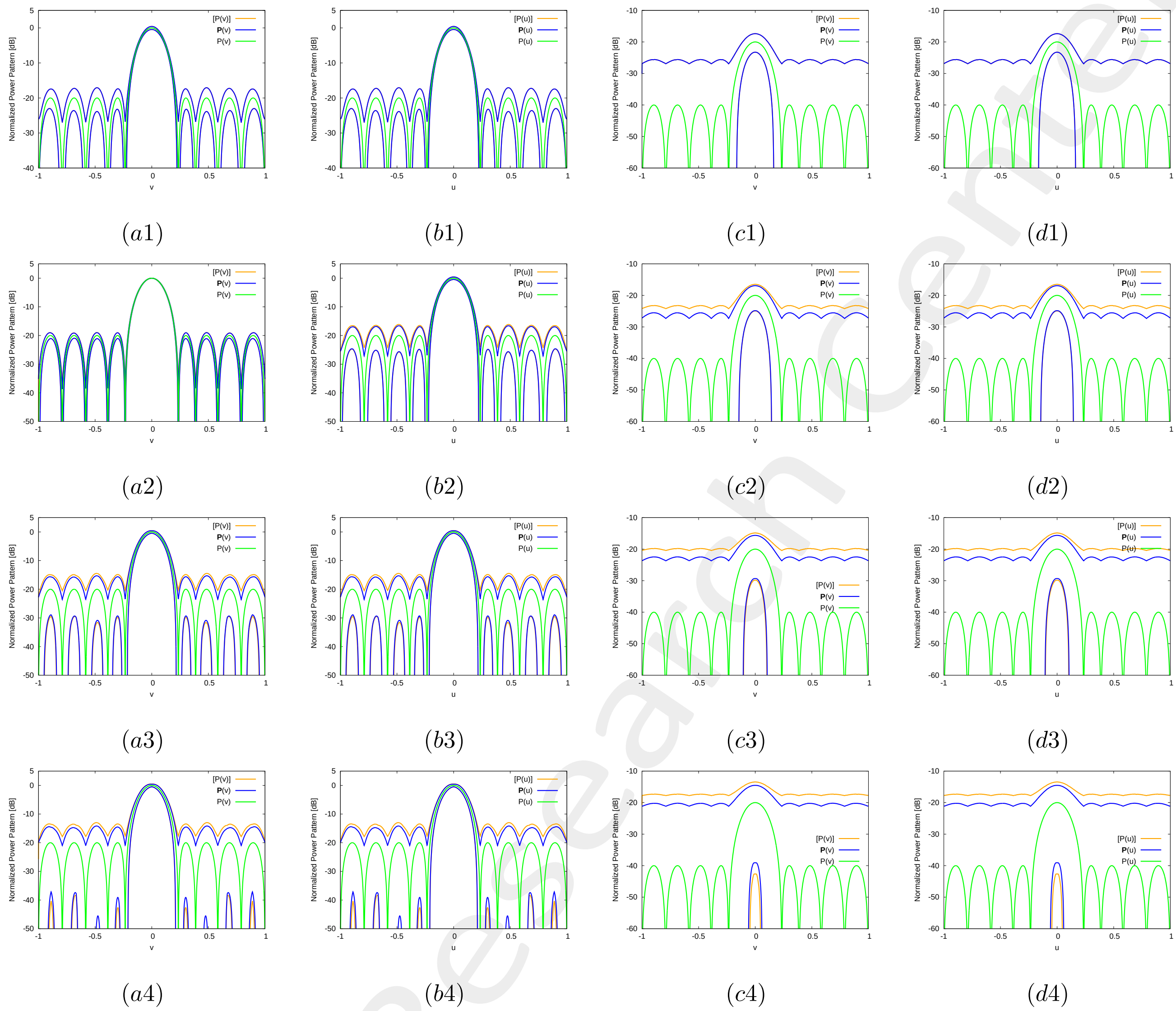


Figure 96: Interval Pattern cut on the plane $u = 0$ (a) cut on the plane $v = 0$ (b) cut on the plane $u = 0.300$ (c) and cut on the plane $v = 0.300$ (d), 1- $\delta\beta_n = 0$ $[deg]$, 2 - $\delta\beta_n = \pm 1$ $[deg]$, 3 - $\delta\beta_n = \pm 3$ $[deg]$, 4 - $\delta\beta_n = \pm 5$ $[deg]$

Note: the pattern figure for patterns with SLL equal to -30, -40, -50 and -60 $[dB]$ are available but not reported here for sake of space.

1.3 Analysis vs SLL

1.3.1 Pattern Features - Interval Beamwidth - plane $v = 0$

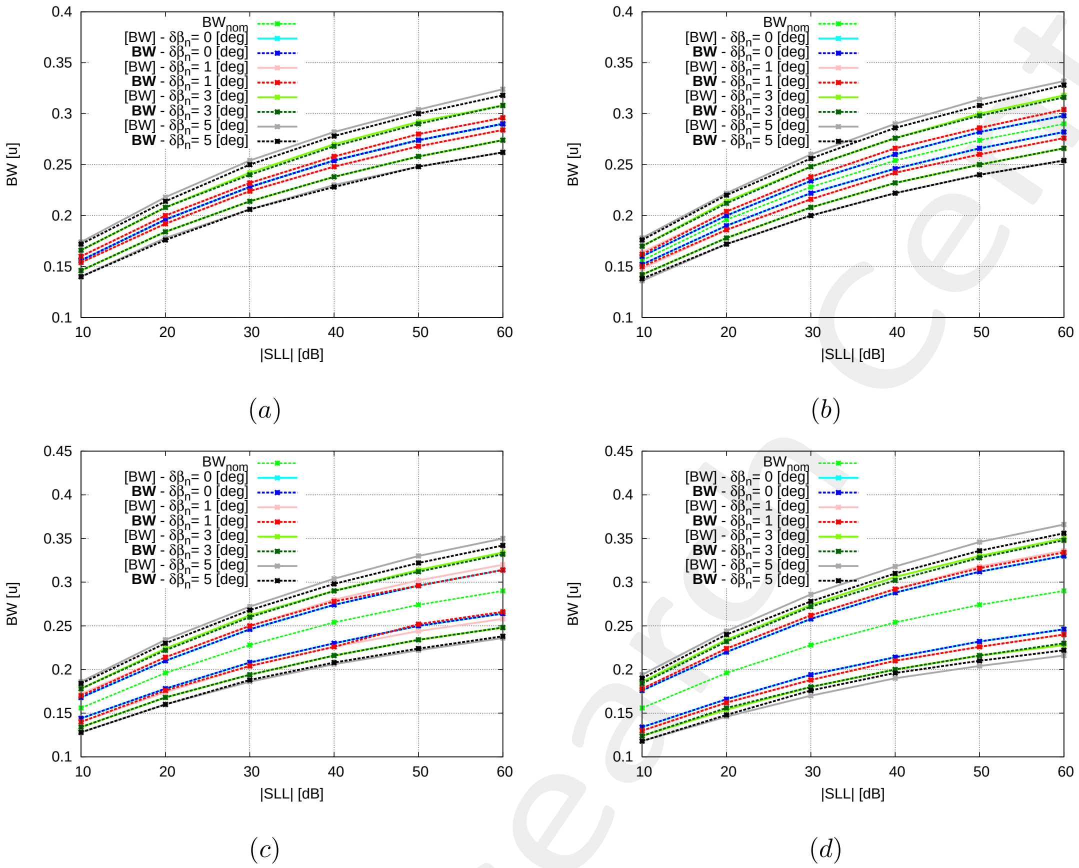


Figure 97: Interval Beamwidth comparison Cartesian vs Minkowski sum

$(a)\ \delta\alpha_n = 0\%$, $(b)\delta\alpha_n = 1\%$, $(c)\delta\alpha_n = 3\%$, $(d)\delta\alpha_n = 5\%$

Amplitude Error: $\delta\alpha_n = 0\%$ **- Cartesian Sum**

	Cartesian			
$SLL\ [dB]\ /\delta\beta_n\ [deg]$	0	1	3	5
−10.00	[0.156, 0.156]	[0.154, 0.160]	[0.146, 0.166]	[0.140, 0.174]
−20.00	[0.196, 0.196]	[0.192, 0.200]	[0.184, 0.208]	[0.178, 0.218]
−30.00	[0.228, 0.228]	[0.224, 0.232]	[0.214, 0.242]	[0.206, 0.254]
−40.00	[0.254, 0.254]	[0.248, 0.258]	[0.238, 0.270]	[0.230, 0.282]
−50.00	[0.274, 0.274]	[0.268, 0.280]	[0.258, 0.292]	[0.248, 0.304]
−60.00	[0.290, 0.290]	[0.284, 0.296]	[0.274, 0.308]	[0.262, 0.324]

Table: Interval Beamwidth vs SLL - $\delta\alpha_n = 0\%$ - Cartesian sum

Amplitude Error: $\delta\alpha_n = 0\%$ **- Minkowski Sum**

	Minkowski			
$SLL\ [dB] / \delta\beta_n\ [deg]$	0	1	3	5
-10.00	[0.156, 0.156]	[0.154, 0.160]	[0.146, 0.166]	[0.140, 0.172]
-20.00	[0.196, 0.196]	[0.192, 0.200]	[0.184, 0.208]	[0.176, 0.214]
-30.00	[0.228, 0.228]	[0.224, 0.232]	[0.214, 0.240]	[0.206, 0.250]
-40.00	[0.254, 0.254]	[0.248, 0.258]	[0.238, 0.268]	[0.228, 0.278]
-50.00	[0.274, 0.274]	[0.268, 0.280]	[0.258, 0.290]	[0.248, 0.300]
-60.00	[0.290, 0.290]	[0.284, 0.296]	[0.274, 0.308]	[0.262, 0.318]

Table: Interval Beamwidth vs SLL - $\delta\alpha_n = 0\%$ - Minkowski sum

Amplitude Error: $\delta\alpha_n = 1\%$ **- Cartesian Sum**

	Cartesian			
$SLL\ [dB] / \delta\beta_n\ [deg]$	0	1	3	5
-10.00	[0.152, 0.160]	[0.148, 0.164]	[0.142, 0.170]	[0.136, 0.178]
-20.00	[0.190, 0.200]	[0.186, 0.204]	[0.178, 0.214]	[0.172, 0.222]
-30.00	[0.222, 0.234]	[0.216, 0.238]	[0.208, 0.248]	[0.200, 0.260]
-40.00	[0.246, 0.260]	[0.242, 0.266]	[0.232, 0.276]	[0.222, 0.290]
-50.00	[0.266, 0.282]	[0.260, 0.286]	[0.250, 0.300]	[0.240, 0.314]
-60.00	[0.282, 0.298]	[0.276, 0.304]	[0.266, 0.318]	[0.254, 0.332]

Table: Interval Beamwidth vs SLL - $\delta\alpha_n = 1\%$ - Cartesian sum

Amplitude Error: $\delta\alpha_n = 1\%$ **- Minkowski Sum**

	Minkowski			
$SLL\ [dB] / \delta\beta_n\ [deg]$	0	1	3	5
-10.00	[0.152, 0.160]	[0.150, 0.162]	[0.142, 0.170]	[0.138, 0.176]
-20.00	[0.190, 0.200]	[0.186, 0.204]	[0.178, 0.212]	[0.172, 0.220]
-30.00	[0.222, 0.234]	[0.216, 0.238]	[0.208, 0.248]	[0.200, 0.256]
-40.00	[0.246, 0.260]	[0.242, 0.266]	[0.232, 0.276]	[0.222, 0.286]
-50.00	[0.266, 0.282]	[0.260, 0.286]	[0.250, 0.298]	[0.240, 0.308]
-60.00	[0.282, 0.298]	[0.276, 0.304]	[0.266, 0.316]	[0.254, 0.328]

Table: Interval Beamwidth vs SLL - $\delta\alpha_n = 1\%$ - Minkowski sum

Amplitude Error: $\delta\alpha_n = 3\%$ **- Cartesian Sum**

	Cartesian			
SLL $[dB]$ / $\delta\beta_n$ $[deg]$	0	1	3	5
-10.00	[0.144, 0.168]	[0.140, 0.172]	[0.134, 0.178]	[0.128, 0.186]
-20.00	[0.178, 0.210]	[0.174, 0.214]	[0.168, 0.224]	[0.160, 0.234]
-30.00	[0.208, 0.246]	[0.204, 0.250]	[0.194, 0.262]	[0.186, 0.272]
-40.00	[0.230, 0.274]	[0.226, 0.280]	[0.216, 0.290]	[0.206, 0.304]
-50.00	[0.250, 0.296]	[0.244, 0.302]	[0.234, 0.314]	[0.222, 0.330]
-60.00	[0.264, 0.314]	[0.258, 0.320]	[0.248, 0.334]	[0.236, 0.350]

Table: Interval Beamwidth vs SLL - $\delta\alpha_n = 3\%$ - Cartesian sum

Amplitude Error: $\delta\alpha_n = 3\%$ **- Minkowski Sum**

	Minkowski			
SLL $[dB]$ / $\delta\beta_n$ $[deg]$	0	1	3	5
-10.00	[0.144, 0.168]	[0.140, 0.170]	[0.134, 0.178]	[0.128, 0.184]
-20.00	[0.178, 0.210]	[0.176, 0.214]	[0.168, 0.222]	[0.160, 0.230]
-30.00	[0.208, 0.246]	[0.204, 0.250]	[0.194, 0.260]	[0.188, 0.268]
-40.00	[0.230, 0.274]	[0.226, 0.278]	[0.216, 0.290]	[0.208, 0.298]
-50.00	[0.250, 0.296]	[0.252, 0.296]	[0.234, 0.312]	[0.224, 0.322]
-60.00	[0.264, 0.314]	[0.266, 0.314]	[0.248, 0.332]	[0.238, 0.342]

Table: Interval Beamwidth vs SLL - $\delta\alpha_n = 3\%$ - Minkowski sum

Amplitude Error: $\delta\alpha_n = 5\%$ **- Cartesian Sum**

	Cartesian			
SLL $[dB]$ / $\delta\beta_n$ $[deg]$	0	1	3	5
-10.00	[0.134, 0.176]	[0.130, 0.178]	[0.124, 0.186]	[0.118, 0.194]
-20.00	[0.166, 0.220]	[0.162, 0.224]	[0.154, 0.234]	[0.146, 0.244]
-30.00	[0.194, 0.258]	[0.188, 0.262]	[0.180, 0.274]	[0.170, 0.286]
-40.00	[0.214, 0.288]	[0.210, 0.292]	[0.200, 0.306]	[0.190, 0.318]
-50.00	[0.232, 0.312]	[0.226, 0.318]	[0.216, 0.330]	[0.204, 0.346]
-60.00	[0.246, 0.330]	[0.240, 0.336]	[0.228, 0.350]	[0.216, 0.366]

Table: Interval Beamwidth vs SLL - $\delta\alpha_n = 5\%$ - Cartesian sum

Amplitude Error: $\delta\alpha_n = 5\%$ **- Minkowski Sum**

	Minkowski			
$SLL\ [dB]\,/\delta\beta_n\ [deg]$	0	1	3	5
-10.00	[0.134, 0.176]	[0.130, 0.178]	[0.124, 0.184]	[0.118, 0.190]
-20.00	[0.166, 0.220]	[0.162, 0.224]	[0.156, 0.232]	[0.148, 0.240]
-30.00	[0.194, 0.258]	[0.188, 0.262]	[0.180, 0.272]	[0.176, 0.278]
-40.00	[0.214, 0.288]	[0.210, 0.292]	[0.200, 0.302]	[0.196, 0.310]
-50.00	[0.232, 0.312]	[0.226, 0.316]	[0.216, 0.328]	[0.210, 0.336]
-60.00	[0.246, 0.330]	[0.240, 0.334]	[0.230, 0.348]	[0.222, 0.356]

Table: Interval Beamwidth vs SLL - $\delta\alpha_n = 5\%$ - Minkowski sum

1.3.2 Pattern Features - Interval SLL - plane $v = 0$

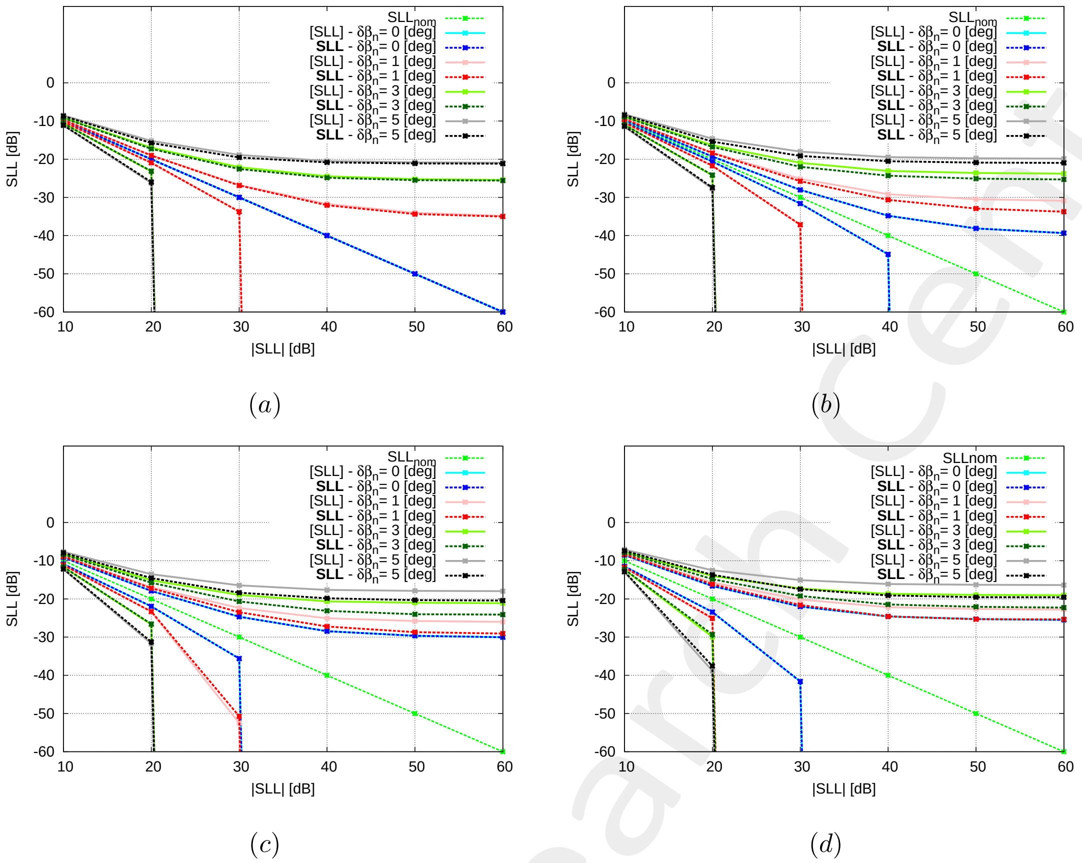


Figure 98: Interval SLL comparison Cartesian vs Minkowski sum

$(a)\ \delta\alpha_n = 0\%$, $(b)\delta\alpha_n = 1\%$, $(c)\delta\alpha_n = 3\%$, $(d)\delta\alpha_n = 5\%$

Amplitude Error: $\delta\alpha_n = 0\%$ - Cartesian Sum

	Cartesian			
$SLL\ [dB]\ /\delta\beta_n\ [deg]$	0	1	3	5
-10.00	$[-10.00, -10.00]$	$[-10.21, -9.72]$	$[-10.68, -9.13]$	$[-11.18, -8.53]$
-20.00	$[-20.00, -20.00]$	$[-20.94, -18.96]$	$[-23.20, -16.97]$	$[-26.24, -15.19]$
-30.00	$[-30.00, -30.00]$	$[-33.75, -26.72]$	$[-\infty, -22.09]$	$[-\infty, -18.87]$
-40.00	$[-40.00, -40.00]$	$[-\infty, -31.72]$	$[-\infty, -24.46]$	$[-\infty, -20.47]$
-50.00	$[-50.00, -50.00]$	$[-\infty, -34.03]$	$[-\infty, -25.22]$	$[-\infty, -20.82]$
-60.00	$[-60.00, -60.00]$	$[-\infty, -34.82]$	$[-\infty, -25.41]$	$[-\infty, -20.93]$

Table: Interval SLL vs SLL - $\delta\alpha_n = 0\%$ - Cartesian sum

Amplitude Error: $\delta\alpha_n = 0\%$ **- Minkowski Sum**

	Minkowski			
SLL $[dB]$ / $\delta\beta_n$ $[deg]$	0	1	3	5
-10.00	$[-10.00, -10.00]$	$[-10.21, -9.73]$	$[-10.64, -9.21]$	$[-11.10, -8.71]$
-20.00	$[-20.00, -20.00]$	$[-20.93, -19.02]$	$[-23.13, -17.29]$	$[-25.98, -15.76]$
-30.00	$[-30.00, -30.00]$	$[-33.73, -26.87]$	$[-\infty, -22.56]$	$[-\infty, -19.55]$
-40.00	$[-40.00, -40.00]$	$[-\infty, -32.05]$	$[-\infty, -24.86]$	$[-\infty, -20.84]$
-50.00	$[-50.00, -50.00]$	$[-\infty, -34.35]$	$[-\infty, -25.48]$	$[-\infty, -21.11]$
-60.00	$[-60.00, -60.00]$	$[-\infty, -35.01]$	$[-\infty, -25.59]$	$[-\infty, -21.15]$

Table: Interval SLL vs SLL - $\delta\alpha_n = 0\%$ - Minkowski sum

Amplitude Error: $\delta\alpha_n = 1\%$ **- Cartesian Sum**

	Cartesian			
SLL $[dB]$ / $\delta\beta_n$ $[deg]$	0	1	3	5
-10.00	$[-10.28, -9.70]$	$[-10.52, -9.43]$	$[-11.00, -8.84]$	$[-11.52, -8.24]$
-20.00	$[-20.62, -19.29]$	$[-21.72, -18.26]$	$[-24.24, -16.33]$	$[-27.74, -14.62]$
-30.00	$[-31.60, -28.03]$	$[-37.18, -25.14]$	$[-\infty, -20.95]$	$[-\infty, -18.01]$
-40.00	$[-44.90, -34.79]$	$[-\infty, -29.16]$	$[-\infty, -23.09]$	$[-\infty, -19.44]$
-50.00	$[-\infty, -38.12]$	$[-\infty, -30.47]$	$[-\infty, -23.61]$	$[-\infty, -19.75]$
-60.00	$[-\infty, -39.34]$	$[-\infty, -30.88]$	$[-\infty, -23.77]$	$[-\infty, -19.85]$

Table: Interval SLL vs SLL - $\delta\alpha_n = 1\%$ - Cartesian sum

Amplitude Error: $\delta\alpha_n = 1\%$ **- Minkowski Sum**

	Minkowski			
SLL $[dB]$ / $\delta\beta_n$ $[deg]$	0	1	3	5
-10.00	$[-10.28, -9.70]$	$[-10.41, -9.50]$	$[-10.97, -8.93]$	$[-11.34, -8.46]$
-20.00	$[-20.62, -19.29]$	$[-21.71, -18.39]$	$[-24.15, -16.79]$	$[-27.44, -15.38]$
-30.00	$[-31.60, -28.03]$	$[-37.14, -25.76]$	$[-\infty, -21.98]$	$[-\infty, -19.15]$
-40.00	$[-44.90, -34.79]$	$[-\infty, -30.65]$	$[-\infty, -24.32]$	$[-\infty, -20.53]$
-50.00	$[-\infty, -38.12]$	$[-\infty, -32.91]$	$[-\infty, -25.09]$	$[-\infty, -20.89]$
-60.00	$[-\infty, -39.34]$	$[-\infty, -33.75]$	$[-\infty, -25.30]$	$[-\infty, -20.98]$

Table: Interval SLL vs SLL - $\delta\alpha_n = 1\%$ - Minkowski sum

Amplitude Error: $\delta\alpha_n = 3\%$ **- Cartesian Sum**

	Cartesian			
SLL $[dB]$ $/\delta\beta_n$ $[deg]$	0	1	3	5
-10.00	$[-10.87, -9.11]$	$[-11.14, -8.84]$	$[-11.67, -8.26]$	$[-12.23, -7.66]$
-20.00	$[-21.95, -17.92]$	$[-23.35, -16.94]$	$[-26.69, -15.12]$	$[-31.73, -13.52]$
-30.00	$[-35.61, -24.66]$	$[-52.47, -22.39]$	$[-\infty, -18.98]$	$[-\infty, -16.45]$
-40.00	$[-44.90, -28.46]$	$[-\infty, -25.07]$	$[-\infty, -20.65]$	$[-\infty, -17.65]$
-50.00	$[-\infty, -29.65]$	$[-\infty, -25.78]$	$[-\infty, -21.01]$	$[-\infty, -17.89]$
-60.00	$[-\infty, -30.03]$	$[-\infty, -26.00]$	$[-\infty, -21.13]$	$[-\infty, -17.97]$

Table: Interval SLL vs SLL - $\delta\alpha_n = 3\%$ - Cartesian sum

Amplitude Error: $\delta\alpha_n = 3\%$ **- Minkowski Sum**

	Minkowski			
SLL $[dB]$ $/\delta\beta_n$ $[deg]$	0	1	3	5
-10.00	$[-10.87, -9.11]$	$[-11.12, -8.89]$	$[-11.63, -8.39]$	$[-12.14, -7.93]$
-20.00	$[-21.95, -17.92]$	$[-23.30, -17.22]$	$[-26.56, -15.78]$	$[-31.25, -14.54]$
-30.00	$[-35.61, -24.66]$	$[-50.71, -23.48]$	$[-\infty, -20.67]$	$[-\infty, -18.36]$
-40.00	$[-44.90, -28.46]$	$[-\infty, -27.22]$	$[-\infty, -23.13]$	$[-\infty, -19.84]$
-50.00	$[-\infty, -29.65]$	$[-\infty, -28.69]$	$[-\infty, -24.00]$	$[-\infty, -20.31]$
-60.00	$[-\infty, -30.03]$	$[-\infty, -29.07]$	$[-\infty, -24.10]$	$[-\infty, -20.44]$

Table: Interval SLL vs SLL - $\delta\alpha_n = 3\%$ - Minkowski sum

Amplitude Error: $\delta\alpha_n = 5\%$ **- Cartesian Sum**

	Cartesian			
SLL $[dB]$ $/\delta\beta_n$ $[deg]$	0	1	3	5
-10.00	$[-11.48, -8.53]$	$[-11.76, -8.26]$	$[-12.35, -7.68]$	$[-12.97, -7.09]$
-20.00	$[-23.43, -16.62]$	$[-25.17, -15.69]$	$[-29.93, -14.00]$	$[-38.84, -12.51]$
-30.00	$[-41.61, -22.00]$	$[-\infty, -20.16]$	$[-\infty, -17.29]$	$[-\infty, -15.08]$
-40.00	$[-\infty, -24.59]$	$[-\infty, -22.16]$	$[-\infty, -18.65]$	$[-\infty, -16.10]$
-50.00	$[-\infty, -25.27]$	$[-\infty, -22.64]$	$[-\infty, -18.94]$	$[-\infty, -16.34]$
-60.00	$[-\infty, -25.48]$	$[-\infty, -22.79]$	$[-\infty, -19.03]$	$[-\infty, -16.37]$

Table: Interval SLL vs SLL - $\delta\alpha_n = 5\%$ - Cartesian sum

Amplitude Error: $\delta\alpha_n = 5\%$ - Minkowski Sum

	Minkowski			
$SLL\ [dB] / \delta\beta_n\ [deg]$	0	1	3	5
-10.00	$[-11.48,\ -8.53]$	$[-11.74,\ -8.33]$	$[-12.27,\ -7.87]$	$[-12.85,\ -7.41]$
-20.00	$[-23.43,\ -16.62]$	$[-25.06,\ -16.15]$	$[-29.32,\ -14.84]$	$[-37.54,\ -13.69]$
-30.00	$[-41.61,\ -22.00]$	$[-\infty,\ -20.60]$	$[-\infty,\ -19.22]$	$[-\infty,\ -17.45]$
-40.00	$[-\infty,\ -24.59]$	$[-\infty,\ -24.60]$	$[-\infty,\ -21.45]$	$[-\infty,\ -19.09]$
-50.00	$[-\infty,\ -25.27]$	$[-\infty,\ -25.32]$	$[-\infty,\ -22.07]$	$[-\infty,\ -19.55]$
-60.00	$[-\infty,\ -25.48]$	$[-\infty,\ -25.33]$	$[-\infty,\ -22.27]$	$[-\infty,\ -19.56]$

Table: Interval SLL vs SLL - $\delta\alpha_n = 5\%$ - Minkowski sum

1.3.3 Pattern Features - Interval Power Peak - $(u, v) = (0, 0)$

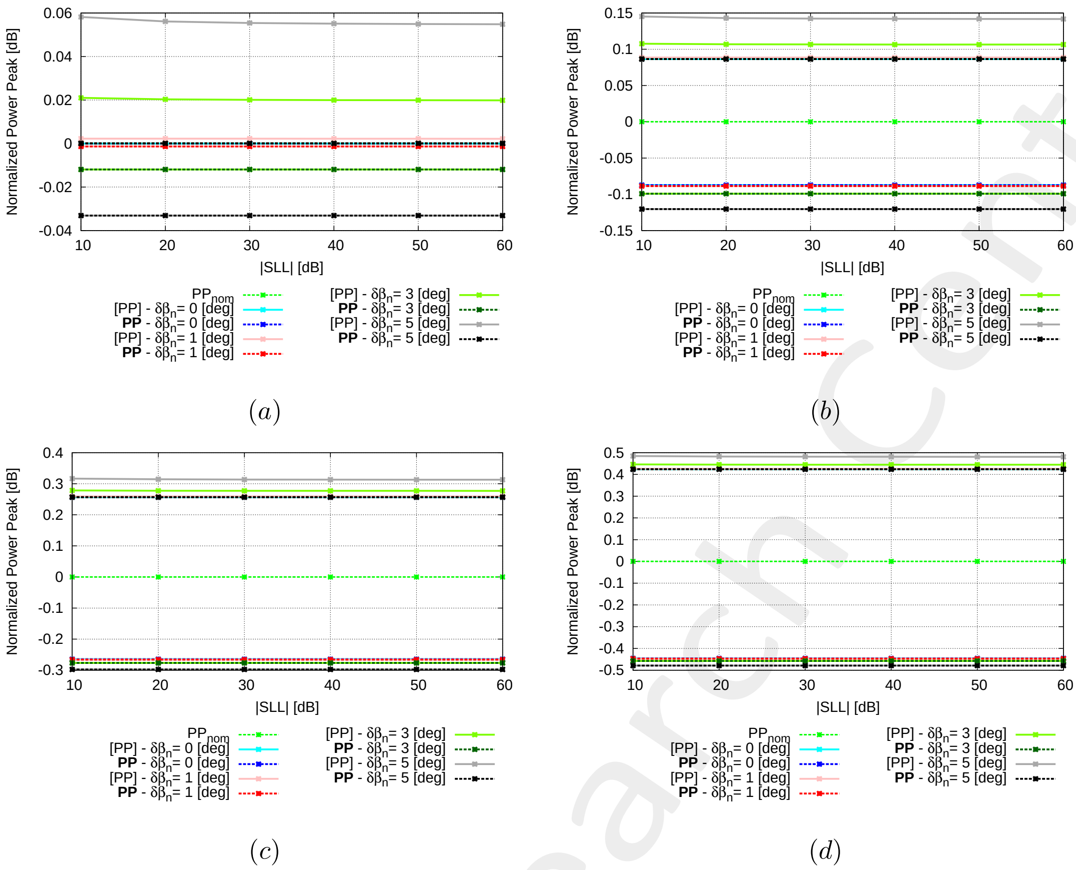


Figure 100: Interval SLL comparison Cartesian vs Minkowski sum

$(a)\ \delta\alpha_n = 0\%,\ (b)\delta\alpha_n = 1\%,\ (c)\delta\alpha_n = 3\%,\ (d)\delta\alpha_n = 5\%$

Amplitude Error: $\delta\alpha_n = 0\%$ **- Cartesian Sum**

	Cartesian			
$SLL\ [dB]\ /\delta\beta_n\ [deg]$	0	1	3	5
-10.00	$[0.0,\ 0.0]$	$[-1.3,\ 2.2] \times 10^{-3}$	$[-1.1,\ 2.1] \times 10^{-2}$	$[-3.3,\ 5.8] \times 10^{-2}$
-20.00	$[0.0,\ 0.0]$	$[-1.3,\ 2.2] \times 10^{-3}$	$[-1.1,\ 2.1] \times 10^{-2}$	$[-3.3,\ 5.8] \times 10^{-2}$
-30.00	$[0.0,\ 0.0]$	$[-1.3,\ 2.2] \times 10^{-3}$	$[-1.1,\ 2.1] \times 10^{-2}$	$[-3.3,\ 5.8] \times 10^{-2}$
-40.00	$[0.0,\ 0.0]$	$[-1.3,\ 2.2] \times 10^{-3}$	$[-1.1,\ 2.1] \times 10^{-2}$	$[-3.3,\ 5.8] \times 10^{-2}$
-50.00	$[0.0,\ 0.0]$	$[-1.3,\ 2.2] \times 10^{-3}$	$[-1.1,\ 2.1] \times 10^{-2}$	$[-3.3,\ 5.8] \times 10^{-2}$
-60.00	$[0.0,\ 0.0]$	$[-1.3,\ 2.2] \times 10^{-3}$	$[-1.1,\ 2.1] \times 10^{-2}$	$[-3.3,\ 5.8] \times 10^{-2}$

Table: Interval PP vs SLL - $\delta\alpha_n = 0\%$ - Cartesian sum

Amplitude Error: $\delta\alpha_n = 0\%$ **- Minkowski Sum**

	Minkowski			
$SLL\ [dB]\ /\delta\beta_n\ [deg]$	0	1	3	5
-10.00	$[0.0, 0.0]$	$[-1.3, 0.0] \times 10^{-3}$	$[-1.1, 0.0] \times 10^{-2}$	$[-3.3, 0.0] \times 10^{-2}$
-20.00	$[0.0, 0.0]$	$[-1.3, 0.0] \times 10^{-3}$	$[-1.1, 0.0] \times 10^{-2}$	$[-3.3, 0.0] \times 10^{-2}$
-30.00	$[0.0, 0.0]$	$[-1.3, 0.0] \times 10^{-3}$	$[-1.1, 0.0] \times 10^{-2}$	$[-3.3, 0.0] \times 10^{-2}$
-40.00	$[0.0, 0.0]$	$[-1.3, 0.0] \times 10^{-3}$	$[-1.1, 0.0] \times 10^{-2}$	$[-3.3, 0.0] \times 10^{-2}$
-50.00	$[0.0, 0.0]$	$[-1.3, 0.0] \times 10^{-3}$	$[-1.1, 0.0] \times 10^{-2}$	$[-3.3, 0.0] \times 10^{-2}$
-60.00	$[0.0, 0.0]$	$[-1.3, 0.0] \times 10^{-3}$	$[-1.1, 0.0] \times 10^{-2}$	$[-3.3, 0.0] \times 10^{-2}$

Table: Interval PP vs SLL - $\delta\alpha_n = 0\%$ - Minkowski sum

Amplitude Error: $\delta\alpha_n = 1\%$ **- Cartesian Sum**

	Cartesian			
$SLL\ [dB]\ /\delta\beta_n\ [deg]$	0	1	3	5
-10.00	$[-0.087, 0.086]$	$[-0.087, 0.089]$	$[-0.099, 0.107]$	$[-0.120, 0.145]$
-20.00	$[-0.087, 0.086]$	$[-0.087, 0.089]$	$[-0.099, 0.107]$	$[-0.120, 0.143]$
-30.00	$[-0.087, 0.086]$	$[-0.087, 0.089]$	$[-0.099, 0.107]$	$[-0.120, 0.142]$
-40.00	$[-0.087, 0.086]$	$[-0.087, 0.089]$	$[-0.099, 0.107]$	$[-0.120, 0.142]$
-50.00	$[-0.087, 0.086]$	$[-0.087, 0.089]$	$[-0.099, 0.107]$	$[-0.120, 0.142]$
-60.00	$[-0.087, 0.086]$	$[-0.087, 0.089]$	$[-0.099, 0.107]$	$[-0.120, 0.142]$

Table: Interval PP vs SLL - $\delta\alpha_n = 1\%$ - Cartesian sum

Amplitude Error: $\delta\alpha_n = 1\%$ **- Minkowski Sum**

	Minkowski			
$SLL\ [dB]\ /\delta\beta_n\ [deg]$	0	1	3	5
-10.00	$[-0.087, 0.086]$	$[-0.089, 0.086]$	$[-0.099, 0.086]$	$[-0.120, 0.086]$
-20.00	$[-0.087, 0.086]$	$[-0.089, 0.086]$	$[-0.099, 0.086]$	$[-0.120, 0.086]$
-30.00	$[-0.087, 0.086]$	$[-0.089, 0.086]$	$[-0.099, 0.086]$	$[-0.120, 0.086]$
-40.00	$[-0.087, 0.086]$	$[-0.089, 0.086]$	$[-0.099, 0.086]$	$[-0.120, 0.086]$
-50.00	$[-0.087, 0.086]$	$[-0.089, 0.086]$	$[-0.099, 0.086]$	$[-0.120, 0.086]$
-60.00	$[-0.087, 0.086]$	$[-0.089, 0.086]$	$[-0.099, 0.086]$	$[-0.120, 0.086]$

Table: Interval PP vs SLL - $\delta\alpha_n = 1\%$ - Minkowski sum

Amplitude Error: $\delta\alpha_n = 3\%$ **- Cartesian Sum**

	Cartesian			
$SLL\ [dB] / \delta\beta_n\ [deg]$	0	1	3	5
-10.00	$[-0.265, 0.256]$	$[-0.266, 0.259]$	$[-0.276, 0.278]$	$[-0.297, 0.316]$
-20.00	$[-0.265, 0.256]$	$[-0.266, 0.259]$	$[-0.276, 0.277]$	$[-0.297, 0.314]$
-30.00	$[-0.265, 0.256]$	$[-0.266, 0.259]$	$[-0.276, 0.277]$	$[-0.297, 0.313]$
-40.00	$[-0.265, 0.256]$	$[-0.266, 0.259]$	$[-0.276, 0.277]$	$[-0.297, 0.313]$
-50.00	$[-0.265, 0.256]$	$[-0.266, 0.259]$	$[-0.276, 0.277]$	$[-0.297, 0.313]$
-60.00	$[-0.265, 0.256]$	$[-0.266, 0.259]$	$[-0.276, 0.277]$	$[-0.297, 0.313]$

Table: Interval PP vs SLL - $\delta\alpha_n = 3\%$ - Cartesian sum

Amplitude Error: $\delta\alpha_n = 3\%$ **- Minkowski Sum**

	Minkowski			
$SLL\ [dB] / \delta\beta_n\ [deg]$	0	1	3	5
-10.00	$[-0.265, 0.256]$	$[-0.266, 0.256]$	$[-0.276, 0.256]$	$[-0.297, 0.256]$
-20.00	$[-0.265, 0.256]$	$[-0.266, 0.256]$	$[-0.276, 0.256]$	$[-0.297, 0.256]$
-30.00	$[-0.265, 0.256]$	$[-0.266, 0.256]$	$[-0.276, 0.256]$	$[-0.297, 0.256]$
-40.00	$[-0.265, 0.256]$	$[-0.266, 0.256]$	$[-0.276, 0.256]$	$[-0.297, 0.256]$
-50.00	$[-0.265, 0.256]$	$[-0.266, 0.256]$	$[-0.276, 0.256]$	$[-0.297, 0.256]$
-60.00	$[-0.265, 0.256]$	$[-0.266, 0.256]$	$[-0.276, 0.256]$	$[-0.297, 0.256]$

Table: Interval PP vs SLL - $\delta\alpha_n = 3\%$ - Minkowski sum

Amplitude Error: $\delta\alpha_n = 5\%$ **- Cartesian Sum**

	Cartesian			
$SLL\ [dB] / \delta\beta_n\ [deg]$	0	1	3	5
-10.00	$[-0.446, 0.424]$	$[-0.447, 0.426]$	$[-0.457, 0.445]$	$[-0.478, 0.485]$
-20.00	$[-0.446, 0.424]$	$[-0.447, 0.426]$	$[-0.457, 0.445]$	$[-0.478, 0.482]$
-30.00	$[-0.446, 0.424]$	$[-0.447, 0.426]$	$[-0.457, 0.445]$	$[-0.478, 0.481]$
-40.00	$[-0.446, 0.424]$	$[-0.447, 0.426]$	$[-0.457, 0.445]$	$[-0.478, 0.481]$
-50.00	$[-0.446, 0.424]$	$[-0.447, 0.426]$	$[-0.457, 0.445]$	$[-0.478, 0.480]$
-60.00	$[-0.446, 0.424]$	$[-0.447, 0.426]$	$[-0.457, 0.445]$	$[-0.478, 0.481]$

Table: Interval PP vs SLL - $\delta\alpha_n = 5\%$ - Cartesian sum

Amplitude Error: $\delta\alpha_n = 5\%$ - Minkowski Sum

	Minkowski			
$SLL\ [dB]\,/\delta\beta_n\ [deg]$	0	1	3	5
-10.00	$[-0.446, 0.424]$	$[-0.447, 0.424]$	$[-0.457, 0.424]$	$[-0.478, 0.424]$
-20.00	$[-0.446, 0.424]$	$[-0.447, 0.424]$	$[-0.457, 0.424]$	$[-0.478, 0.424]$
-30.00	$[-0.446, 0.424]$	$[-0.447, 0.424]$	$[-0.457, 0.424]$	$[-0.478, 0.424]$
-40.00	$[-0.446, 0.424]$	$[-0.447, 0.424]$	$[-0.457, 0.424]$	$[-0.478, 0.424]$
-50.00	$[-0.446, 0.424]$	$[-0.447, 0.424]$	$[-0.457, 0.424]$	$[-0.478, 0.424]$
-60.00	$[-0.446, 0.424]$	$[-0.447, 0.424]$	$[-0.457, 0.424]$	$[-0.478, 0.424]$

Table: Interval PP vs SLL - $\delta\alpha_n = 5\%$ - Minkowski sum

1.3.4 Pattern Features - Pattern Tolerance Δ

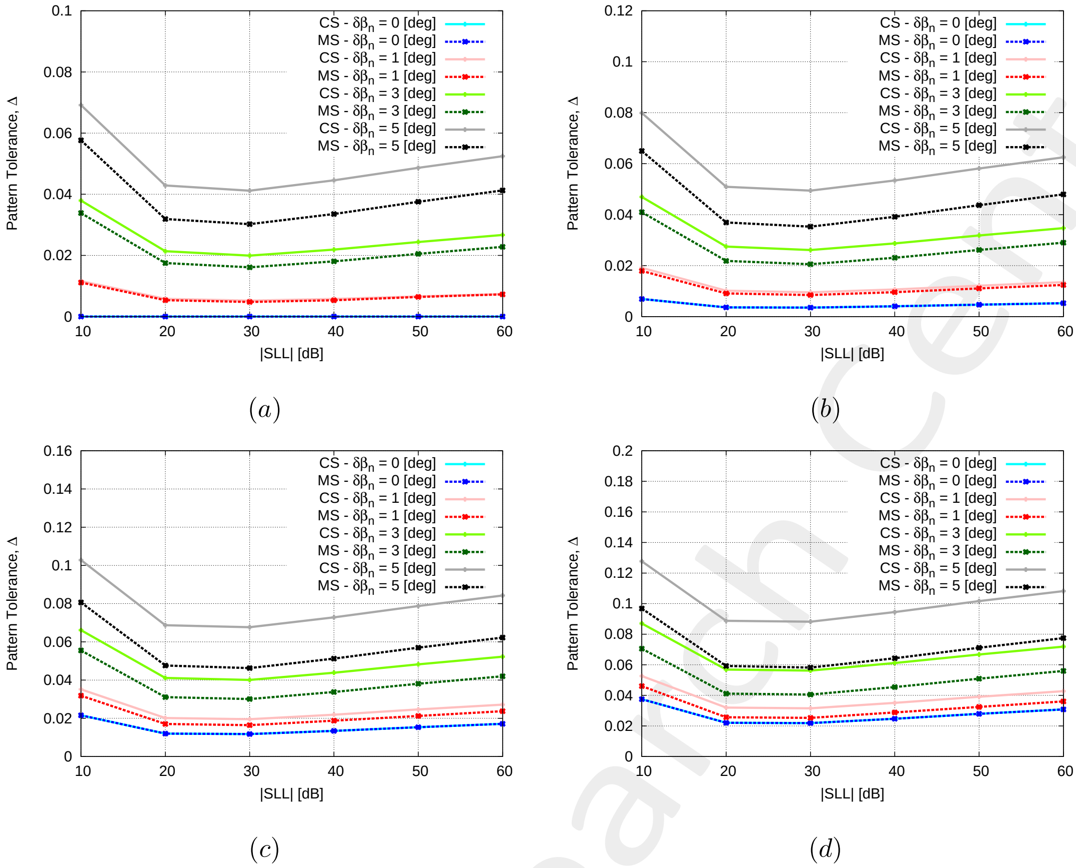


Figure 101: Pattern Matching comparison Cartesian vs Minkowski sum

$(a)\ \delta\alpha_n = 0\%,\ (b)\delta\alpha_n = 1\%,\ (c)\delta\alpha_n = 3\%,\ (d)\delta\alpha_n = 5\%$

Amplitude Error: $\delta\alpha_n = 0\%$

	Cartesian				*Minkowski*			
$SLL\ [dB]\ /\delta\beta_n\ [deg]$	0	1	3	5	0	1	3	5
-10.00	0.0	1.16×10^{-2}	3.79×10^{-2}	6.91×10^{-2}	0.0	1.11×10^{-2}	3.38×10^{-2}	5.76×10^{-2}
-20.00	0.0	0.58×10^{-2}	2.13×10^{-2}	4.28×10^{-2}	0.0	0.53×10^{-2}	1.75×10^{-2}	3.19×10^{-2}
-30.00	0.0	0.52×10^{-2}	1.99×10^{-2}	4.11×10^{-2}	0.0	0.48×10^{-2}	1.61×10^{-2}	3.02×10^{-2}
-40.00	0.0	0.57×10^{-2}	2.19×10^{-2}	4.45×10^{-2}	0.0	0.53×10^{-2}	1.81×10^{-2}	3.35×10^{-2}
-50.00	0.0	0.65×10^{-2}	2.44×10^{-2}	4.86×10^{-2}	0.0	0.64×10^{-2}	2.05×10^{-2}	3.75×10^{-2}
-60.00	0.0	0.73×10^{-2}	2.67×10^{-2}	5.24×10^{-2}	0.0	0.72×10^{-2}	2.27×10^{-2}	4.13×10^{-2}

Table: Δ vs SLL - $\delta\alpha_n = 0\%$

Amplitude Error: $\delta\alpha_n = 1\%$

	Cartesian				*Minkowski*			
$SLL\ [dB] / \delta\beta_n\ [deg]$	0	1	3	5	0	1	3	5
-10.00	6.91×10^{-3}	1.91×10^{-2}	4.69×10^{-2}	7.99×10^{-2}	6.91×10^{-3}	1.79×10^{-2}	4.10×10^{-2}	6.50×10^{-2}
-20.00	3.62×10^{-3}	1.01×10^{-2}	2.75×10^{-2}	5.09×10^{-2}	3.62×10^{-3}	0.91×10^{-2}	2.18×10^{-2}	3.69×10^{-2}
-30.00	3.51×10^{-3}	0.95×10^{-2}	2.61×10^{-2}	4.94×10^{-2}	3.51×10^{-3}	0.85×10^{-2}	2.05×10^{-2}	3.52×10^{-2}
-40.00	4.03×10^{-3}	1.06×10^{-2}	2.87×10^{-2}	5.34×10^{-2}	4.03×10^{-3}	0.96×10^{-2}	2.31×10^{-2}	3.91×10^{-2}
-50.00	4.66×10^{-3}	1.21×10^{-2}	3.18×10^{-2}	5.81×10^{-2}	4.66×10^{-3}	1.11×10^{-2}	2.61×10^{-2}	4.37×10^{-2}
-60.00	5.26×10^{-3}	1.35×10^{-2}	3.47×10^{-2}	6.25×10^{-2}	5.26×10^{-3}	1.24×10^{-2}	2.89×10^{-2}	4.79×10^{-2}

Table: Δ vs SLL - $\delta\alpha_n = 1\%$

Amplitude Error: $\delta\alpha_n = 3\%$

	Cartesian				*Minkowski*			
$SLL\ [dB] / \delta\beta_n\ [deg]$	0	1	3	5	0	1	3	5
-10.00	2.16×10^{-2}	3.52×10^{-2}	6.61×10^{-2}	10.2×10^{-2}	2.16×10^{-2}	3.18×10^{-2}	5.55×10^{-2}	8.06×10^{-2}
-20.00	1.20×10^{-2}	2.02×10^{-2}	4.11×10^{-2}	6.87×10^{-2}	1.20×10^{-2}	1.70×10^{-2}	3.11×10^{-2}	4.76×10^{-2}
-30.00	1.18×10^{-2}	1.95×10^{-2}	4.01×10^{-2}	6.76×10^{-2}	1.18×10^{-2}	1.64×10^{-2}	3.00×10^{-2}	4.62×10^{-2}
-40.00	1.34×10^{-2}	2.19×10^{-2}	4.39×10^{-2}	7.28×10^{-2}	1.34×10^{-2}	1.87×10^{-2}	3.38×10^{-2}	5.12×10^{-2}
-50.00	1.53×10^{-2}	2.46×10^{-2}	4.82×10^{-2}	7.87×10^{-2}	1.53×10^{-2}	2.12×10^{-2}	3.81×10^{-2}	5.69×10^{-2}
-60.00	1.71×10^{-2}	2.72×10^{-2}	5.22×10^{-2}	8.42×10^{-2}	1.71×10^{-2}	2.37×10^{-2}	4.20×10^{-2}	6.22×10^{-2}

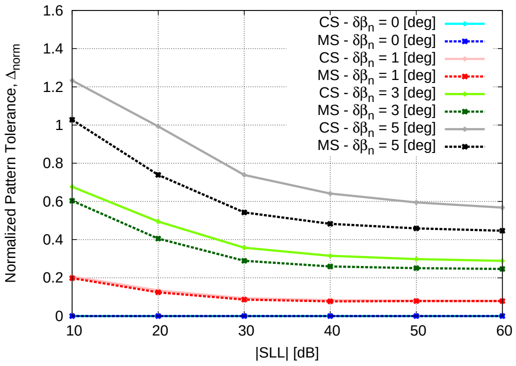
Table: Δ vs SLL - $\delta\alpha_n = 3\%$

Amplitude Error: $\delta\alpha_n = 5\%$

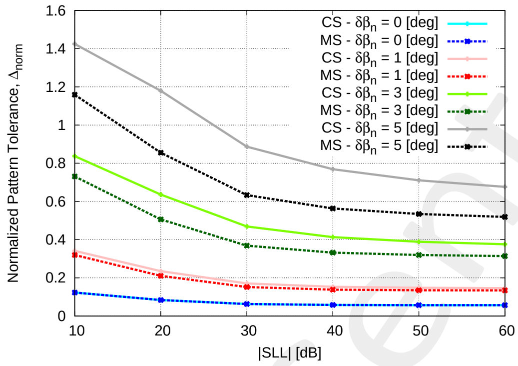
	Cartesian				*Minkowski*			
$SLL\ [dB] / \delta\beta_n\ [deg]$	0	1	3	5	0	1	3	5
-10.00	3.75×10^{-2}	5.26×10^{-2}	8.71×10^{-2}	12.7×10^{-2}	3.75×10^{-2}	4.61×10^{-2}	7.06×10^{-2}	9.68×10^{-2}
-20.00	2.21×10^{-2}	3.20×10^{-2}	5.68×10^{-2}	8.87×10^{-2}	2.21×10^{-2}	2.57×10^{-2}	4.12×10^{-2}	5.93×10^{-2}
-30.00	2.19×10^{-2}	3.15×10^{-2}	5.62×10^{-2}	8.82×10^{-2}	2.19×10^{-2}	2.52×10^{-2}	4.06×10^{-2}	5.82×10^{-2}
-40.00	2.47×10^{-2}	3.51×10^{-2}	6.11×10^{-2}	9.45×10^{-2}	2.47×10^{-2}	2.88×10^{-2}	4.54×10^{-2}	6.43×10^{-2}
-50.00	2.79×10^{-2}	3.91×10^{-2}	6.67×10^{-2}	10.2×10^{-2}	2.79×10^{-2}	3.24×10^{-2}	5.09×10^{-2}	7.11×10^{-2}
-60.00	3.08×10^{-2}	4.27×10^{-2}	7.19×10^{-2}	10.8×10^{-2}	3.08×10^{-2}	3.61×10^{-2}	5.59×10^{-2}	7.74×10^{-2}

Table: Δ vs SLL - $\delta\alpha_n = 5\%$

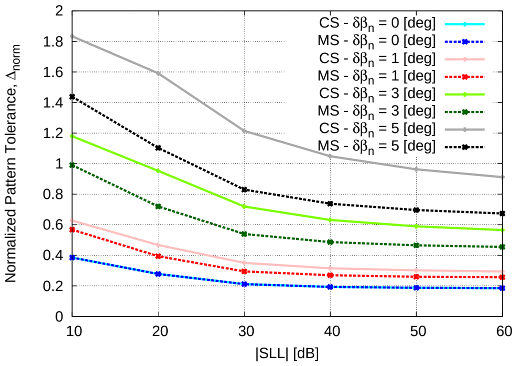
1.3.5 Pattern Features - Normalized Pattern Tolerance Δ_{norm}



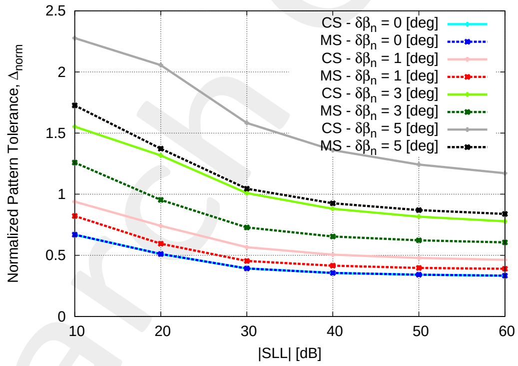
(a)



(b)



(c)



(d)

Figure 102: Pattern Matching comparison Cartesian vs Minkowski sum

$(a)\ \delta\alpha_n = 0\%$, $(b)\delta\alpha_n = 1\%$, $(c)\delta\alpha_n = 3\%$, $(d)\delta\alpha_n = 5\%$

Amplitude Error: $\delta\alpha_n = 0\%$

	Cartesian				*Minkowski*			
$SLL\ [dB]\ /\delta\beta_n\ [deg]$	0	1	3	5	0	1	3	5
-10.00	0.0	2.07×10^{-1}	0.68	1.23	0.0	1.98×10^{-1}	0.60	1.03
-20.00	0.0	1.34×10^{-1}	0.49	0.99	0.0	1.24×10^{-1}	0.41	0.74
-30.00	0.0	0.93×10^{-1}	0.36	0.74	0.0	0.86×10^{-1}	0.29	0.54
-40.00	0.0	0.83×10^{-1}	0.32	0.64	0.0	0.77×10^{-1}	0.26	0.48
-50.00	0.0	0.81×10^{-1}	0.30	0.59	0.0	0.78×10^{-1}	0.25	0.46
-60.00	0.0	0.79×10^{-1}	0.29	0.57	0.0	0.78×10^{-1}	0.24	0.44

Table: Δ_{norm} vs SLL - $\delta\alpha_n = 0\%$

Amplitude Error: $\delta\alpha_n = 1\%$

	Cartesian				*Minkowski*			
$SLL\ [dB]\ /\delta\beta_n\ [deg]$	0	1	3	5	0	1	3	5
-10.00	1.23×10^{-1}	0.341	0.837	1.425	1.23×10^{-1}	0.320	0.731	1.159
-20.00	0.84×10^{-1}	0.235	0.636	1.179	0.84×10^{-1}	0.211	0.506	0.855
-30.00	0.63×10^{-1}	0.171	0.469	0.887	0.63×10^{-1}	0.152	0.368	0.633
-40.00	0.58×10^{-1}	0.153	0.413	0.768	0.58×10^{-1}	0.138	0.332	0.563
-50.00	0.57×10^{-1}	0.148	0.389	0.710	0.57×10^{-1}	0.135	0.319	0.534
-60.00	0.57×10^{-1}	0.146	0.376	0.676	0.57×10^{-1}	0.134	0.314	0.519

Table: Δ_{norm} vs SLL - $\delta\alpha_n = 1\%$

Amplitude Error: $\delta\alpha_n = 3\%$

	Cartesian				*Minkowski*			
$SLL\ [dB]\ /\delta\beta_n\ [deg]$	0	1	3	5	0	1	3	5
-10.00	0.385	0.627	1.179	1.833	0.385	0.568	0.990	1.438
-20.00	0.277	0.467	0.952	1.591	0.277	0.394	0.720	1.103
-30.00	0.211	0.351	0.719	1.214	0.211	0.295	0.540	0.830
-40.00	0.193	0.315	0.631	1.048	0.193	0.270	0.486	0.738
-50.00	0.188	0.301	0.589	0.963	0.188	0.259	0.465	0.696
-60.00	0.185	0.294	0.565	0.912	0.185	0.256	0.454	0.674

Table: Δ_{norm} vs SLL - $\delta\alpha_n = 3\%$

Amplitude Error: $\delta\alpha_n = 5\%$

	Cartesian				*Minkowski*			
$SLL\ [dB]\ /\delta\beta_n\ [deg]$	0	1	3	5	0	1	3	5
-10.00	0.669	0.939	1.552	2.277	0.669	0.822	1.258	1.726
-20.00	0.510	0.741	1.317	2.055	0.510	0.595	0.953	1.372
-30.00	0.392	0.566	1.010	1.583	0.392	0.453	0.727	1.045
-40.00	0.356	0.505	0.880	1.360	0.356	0.415	0.654	0.925
-50.00	0.341	0.478	0.816	1.243	0.341	0.396	0.623	0.870
-60.00	0.333	0.463	0.778	1.171	0.333	0.390	0.606	0.838

Table: Δ_{norm} vs SLL - $\delta\alpha_n = 5\%$

1.3.6 Comments and Observations:

With respect the values of Δ_{norm} , the pattern with higher SLL (-10.0 $[dB]$) seems to be more sensitive to the tolerance on the control points if compared with the pattern with lower SLL. On the other hand, the pattern features (particularly the interval SLL) are more sensitive to the tolerance in pattern with low SLL. Moreover, when increasing the tolerances the supremum of the interval SLL assumes a "flat" beahavior. Such a fact indicates as the tolerance on control points are a limiting factor to obtain very low SLL pattern.

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