A Sensitivity Analysis Approach for Linear Arrays with Uncertain Excitations

L. Tenuti, N. Anselmi, P. Rocca, M. Salucci, and A. Massa

Abstract

When dealing with the design of linear phased arrays, the amplitudes and phases of the array control points are properly synthesized to yield desired radiation features (e.g., high directivity, low sidelobe level, etc.). Unfortunately, the excitation values can deviate from the nominal/expected ones because of unavoidable fabrication tolerances and/or environmental variations. As a consequence, the radiated pattern can differ from the desired one, causing the overall system to fail in meeting the expected performances. In this document, an innovative interval analysis (IA) technique, based on the Minkowski sum, is proposed to analyze the effects of the linear array excitation tolerances on the radiated field. A comparison of the obtained bounds with those obtainable through the standard Cartesian IA is provided, as well.

1 Numerical Assessment - Method Validation - Linear Array

Array geometry:

- Uniform linear array: $N = 10$.
- Inter-element spacing: $d = 0.5$ [λ].

Nominal control points:

- Taylor pattern - $SLL = 20$ [dB] - $\overline{n} = 2$.

Tolerances on the control points:

- Amplitude tolerance: $\delta\alpha_n = 0\%$, $\pm 1\,\%$.
- Phase tolerance: $\delta\beta_n = \pm 1$, ± 5 [deg].

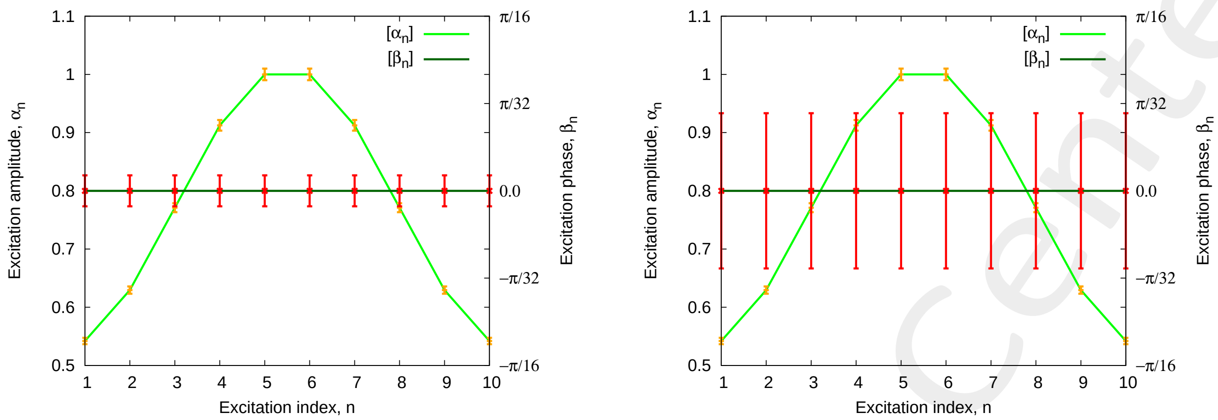
Minkowski sum parameters:

- Number of sides including polygon: $L = 720$

Number of Random Patterns: 10^4 trials

1.1 Amplitude Tolerance: $\delta\alpha_n = \pm 1\,\%$ - Phase Tolerance: $\delta\beta_n = \pm 1,\ \pm 5\ [deg]$

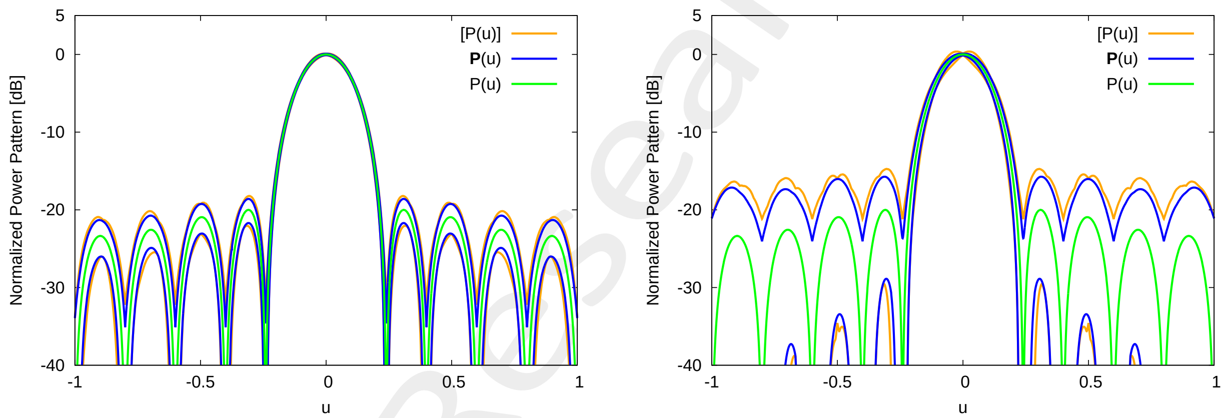
Interval Excitations



(a) (b)

Figure 13

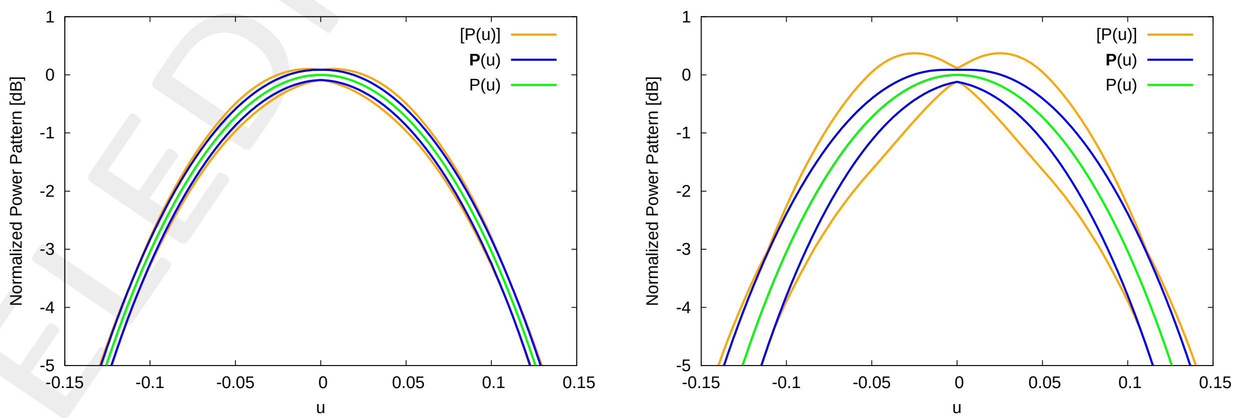
Interval Patterns - $u \in [-1,\ 1]$



(a) (b)

Figure 14

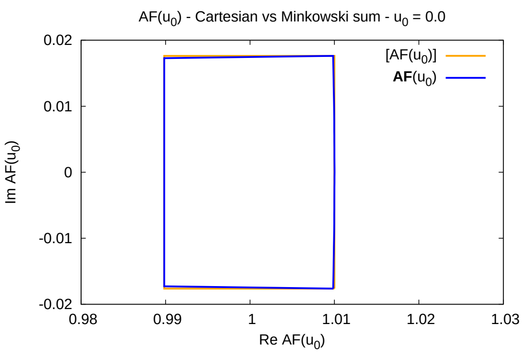
Interval Patterns - Main Lobe Region



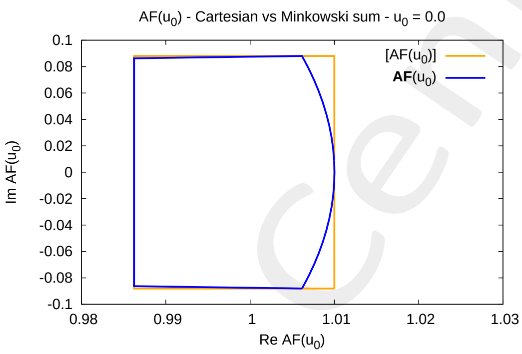
(a) (b)

Figure 15

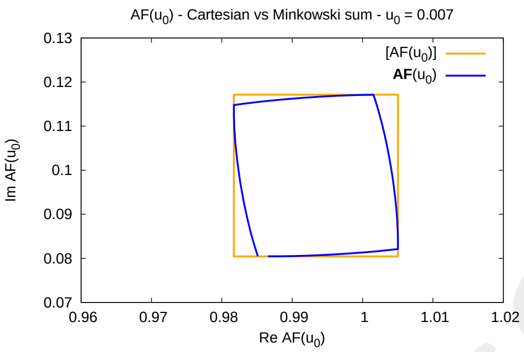
Interval Array Factor samples



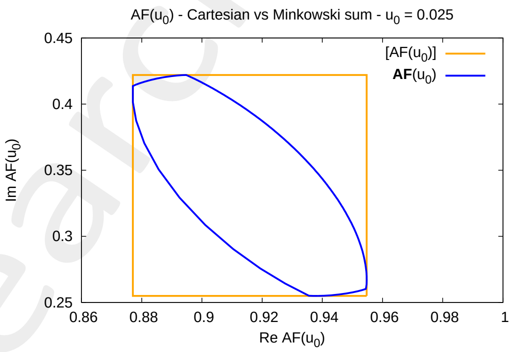
(a)



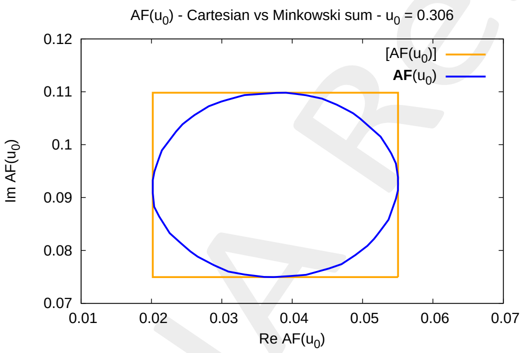
(b)



(c)



(d)



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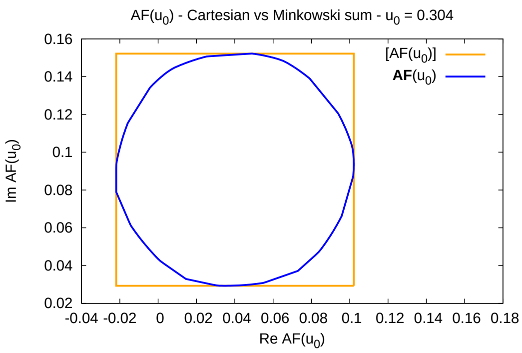

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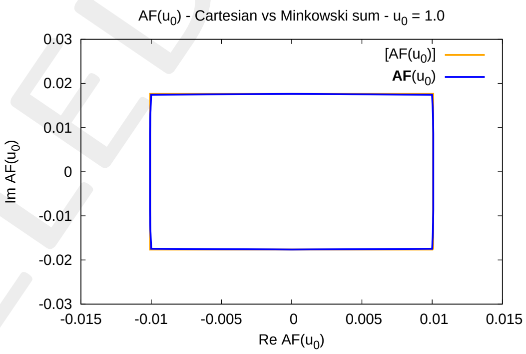
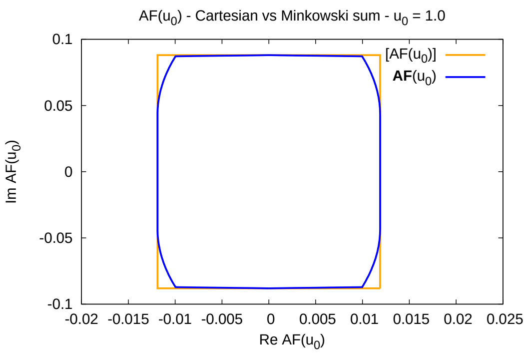


(g)



(h)

Figure 16

Interval Array Factor Samples - $\delta\alpha_n = \pm 1\,\%$ - $\delta\beta_n = \pm 1$ $[deg]$

	$u_0 = 0.0$	$u_0 = 0.007$	$u_0 = 0.306$	$u_0 = 1.0$
$[\lvert AF(u_0)\rvert]$	[0.9898, 1.0102]	[0.9850, 1.0118]	[0.0770, 0.1227]	[0.0, 0.0203]
$\lvert \mathbf{AF}(u_0)\rvert$	[0.9898, 1.0100]	[0.9874, 1.0091]	[0.0818, 0.1170]	[0.0, 0.0201]

Table IV

Interval Array Factor Samples - $\delta\alpha_n = \pm 1\,\%$ - $\delta\beta_n = \pm 5$ $[deg]$

	$u_0 = 0.0$	$u_0 = 0.025$	$u_0 = 0.304$	$u_0 = 1.0$
$[\lvert AF(u_0)\rvert]$	[0.9862, 1.0138]	[0.9143, 1.0438]	[0.0293 0.1832]	[0.0, 0.0888]
$\lvert \mathbf{AF}(u_0)\rvert$	[0.9862, 1.0100]	[0.9513, 1.0080]	[0.0360, 0.1620]	[0.0, 0.0880]

Table V

Interval Pattern Features

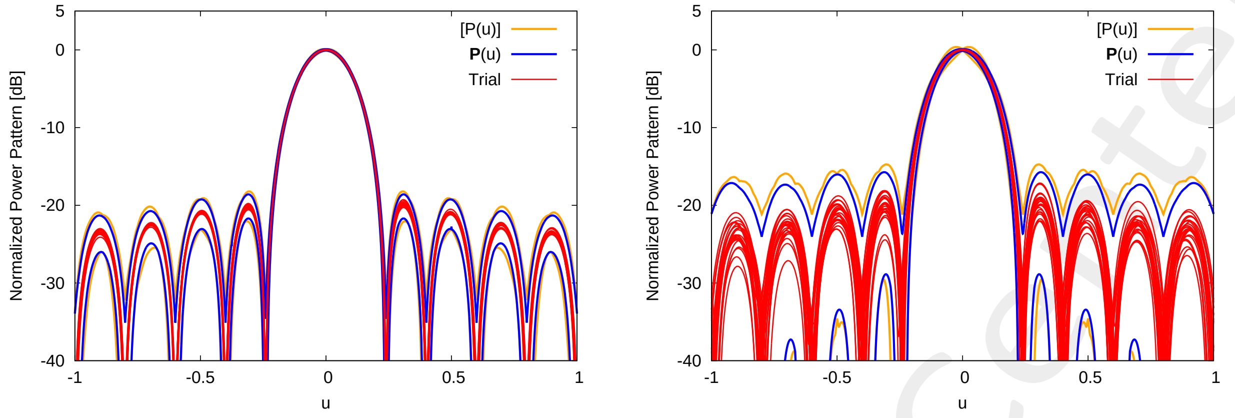
$Feature$	$Nominal$	$Cartesian\,Sum$		$Minkowski\,Sum$	
		$\delta\beta_n = \pm 1$ $[deg]$	$\delta\beta_n = \pm 5$ $[deg]$	$\delta\beta_n = \pm 1$ $[deg]$	$\delta\beta_n = \pm 5$ $[deg]$
BW $[u]$	0.200	[0.190, 0.210]	[0.180, 0.236]	[0.190, 0.208]	[0.182, 0.218]
SLL $[dB]$	-20.0	[-22.16, -18.13]	[-29.94, -14.62]	[-21.78, -18.51]	[-28.96, -15.61]
$P_{\max}$ $[dB]$	0.0	[-0.089, 0.102]	[-0.12, 0.375]	[-0.089, 0.086]	[-0.12, 0.086]
Δ	–	0.0301	0.106	0.0223	0.0698
Δ_{norm}	–	0.0860	0.3032	0.0639	0.1996

Table VI

Comments and Observations:

Also when dealing with both amplitude and phase tolerances on the control points of the array the Minkowski interval are always included in the Cartesian interval. Accordingly, the same conclusions hold true for the only phase tolerance case.

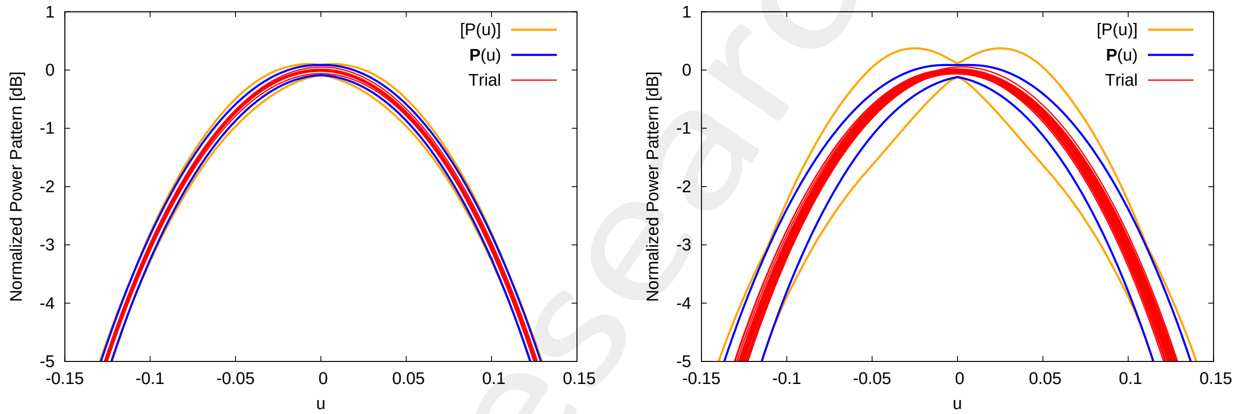
Interval Pattern vs Random Pattern $u \in [-1, 1]$



(a) (b)

Figure 17

Interval Pattern - Main Lobe



(a) (b)

Figure 18

Array Factor sample

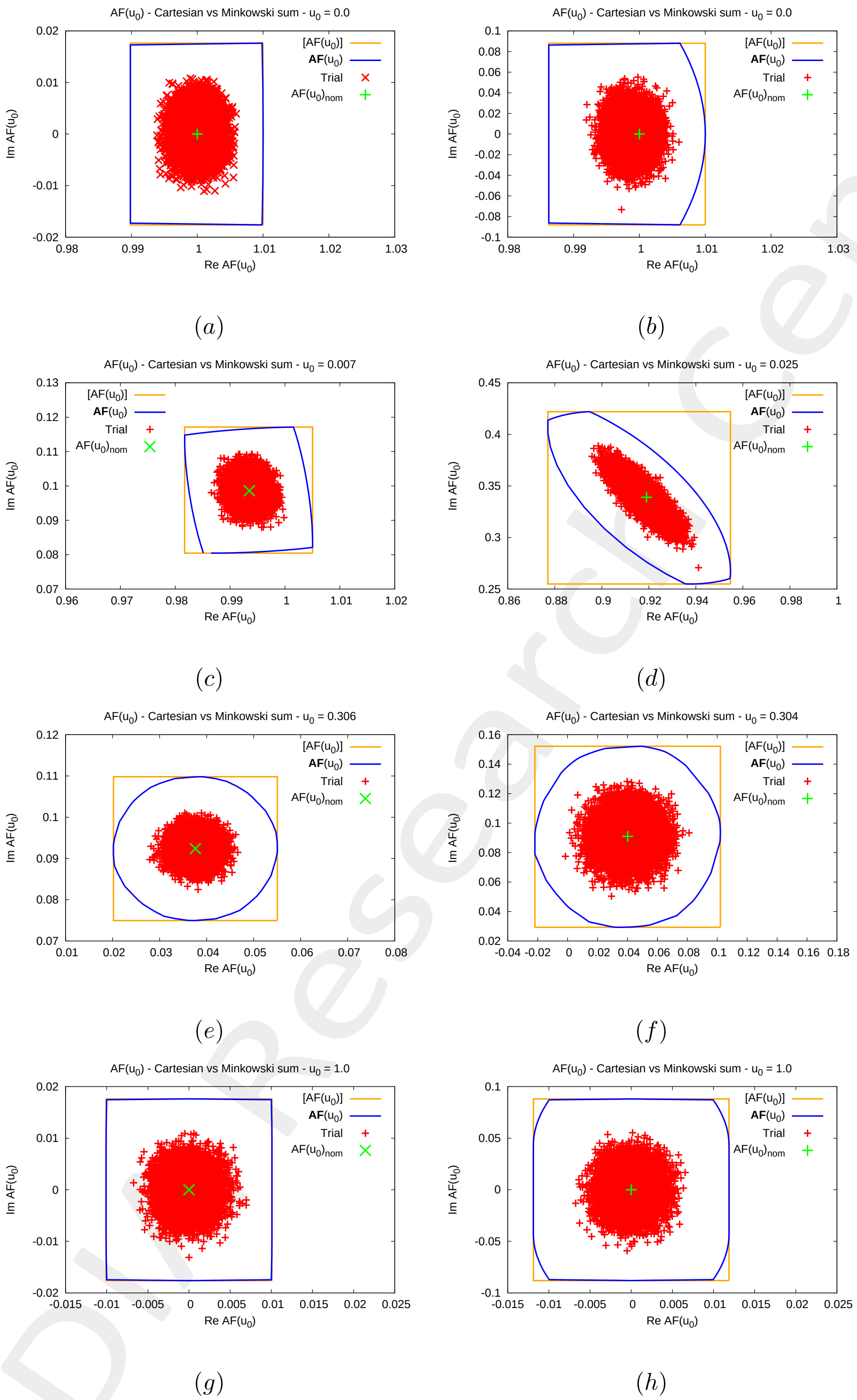
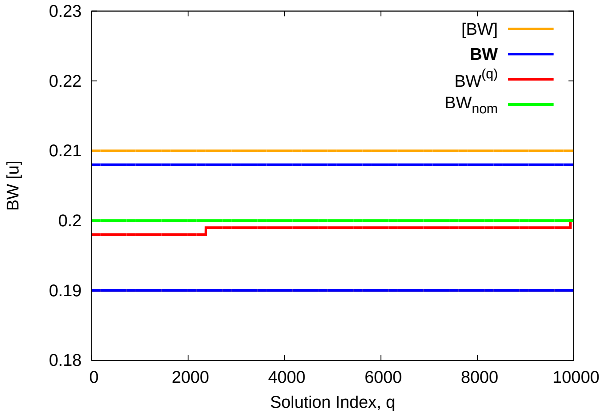
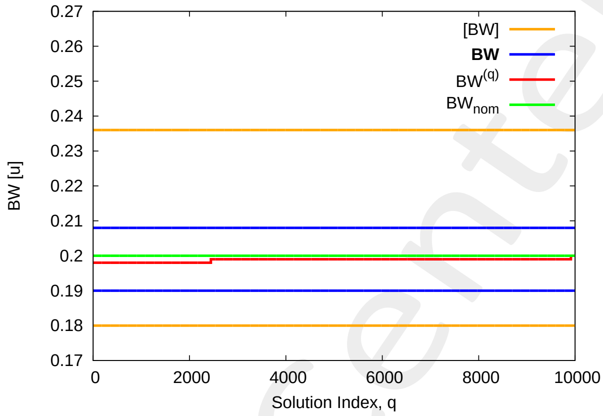


Figure 19

Pattern Features vs Random Pattern

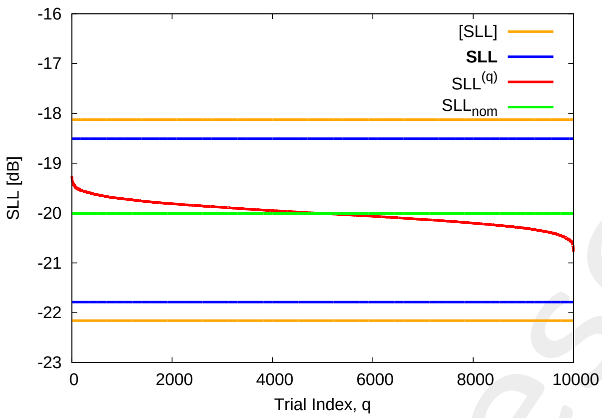


(a)

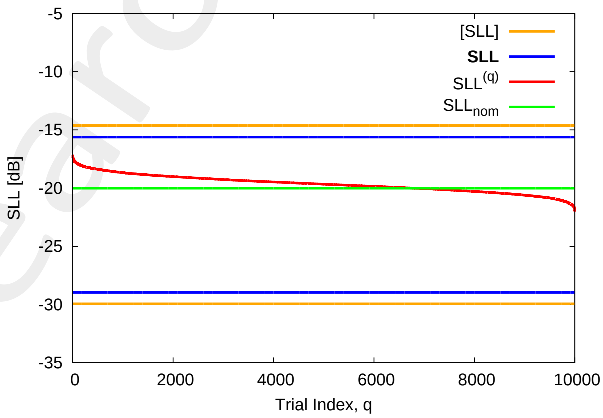


(b)

Figure 20

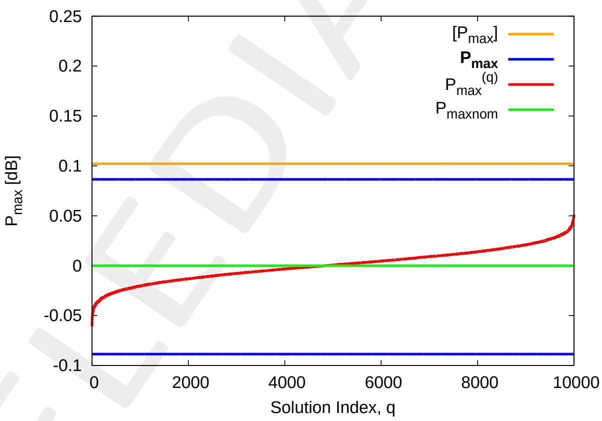


(a)

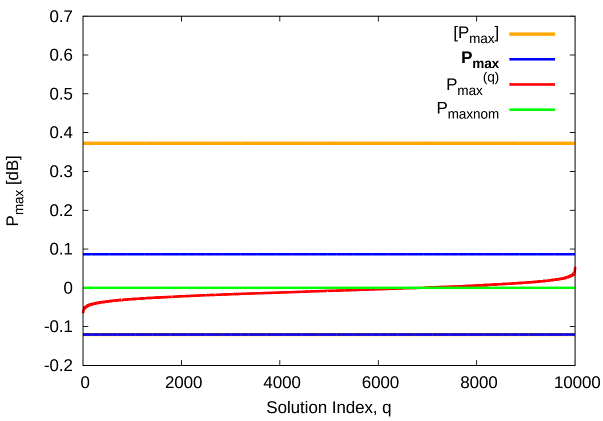


(b)

Figure 21



(a)



(b)

Figure 22

1.1.1 Comments and Observations:

Considering 10000 random patterns, the inclusivity for both cartesian and Minkowski interval seems assured for small (± 1 $[deg]$ and $\pm 1\%$) and big (± 5 $[deg]$ and $\pm 5\%$) tolerances on the control points of a linear array. For small errors the difference between the two including intervals (Cartesian and Minkowski) is very small for particular values of the angular variable u ($u = 0$ and $u = 1$).

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