# DoA estimation via MT-BCS exploiting multiple-snapshots

M.Carlin, P. Rocca, G. Oliveri, and A. Massa

**Abstract**

In this report, an innovative strategy for the estimation of the directions of arrival of signals impinging on linear arrays of electromagnetic sensors has been assessed. Starting from a sparse representation of the problem solution, the DoA estimation problem has been addressed by means of a methodology based on the BCS paradigm. A customized implementation exploiting the measurements collected at multiple time instants (multiple-snapshots) providing robust and very accurate estimates when correlating the information from multiple snapshots has been validated.

## MT-BCS DoA estimation

**GOAL:** The goal of this section is the analysis of the performances of the MT-BCS method for the DoA estimation with $W > 1$ snapshots. The performances of the method are compared with the standard single-task BCS (ST-BCS) and with the ROOT-MUSIC and ESPRIT algorithms.

$$\underline{\hat{x}}_h^{(ave)} = \frac{1}{W}\sum_{w=1}^{W} |\underline{\hat{x}}_h(t_w)| \quad (1)$$

being $W$ the number of snapshots and $h \in \{ST - BCS, MT - BCS\}$. The main difference between the $ST$ and $MT\ BCS$ furmulations is that in the second case the non-zero elements of the estimated vectors $\underline{\hat{x}}_h(t_w)$ are forced to be in the same locations.

**Analysis vs number of snapshots $W$**

**Simulation Parameters**

- Scenario
  - BPSK signals ($E_l^{inc} \in \{-1, 1\}$)
  - Number of incident signals: $L = 2$
  - Signal directions: $\underline{\theta} = \{0, 7\}$ $[deg]$
  - Signal to noise ratio: $SNR = 7\ dB$ (equivalent to a $SNR = 4\ dB$ if the literature's definition is taken into account)
- Array parameters
  - Elements spacing: $d = 0.5\lambda$
  - Number of elements: $M = 10$
- MT-BCS parameters
  - Number of angular locations: $K = 181$
  - $a = 3.162$
  - $b = 3.981 \times 10^1$
- BCS parameters
  - Number of angular locations: $K = 181$
  - $\sigma_0^2 = 4.642 \times 10^{-1}$
  - **Number of snapshots**: $W \in [1, 25]$

- Simulation
  - Number of independent realizations $Q = 150$ (the noise and the signal amplitudes are random, while the DoAs are fixed)

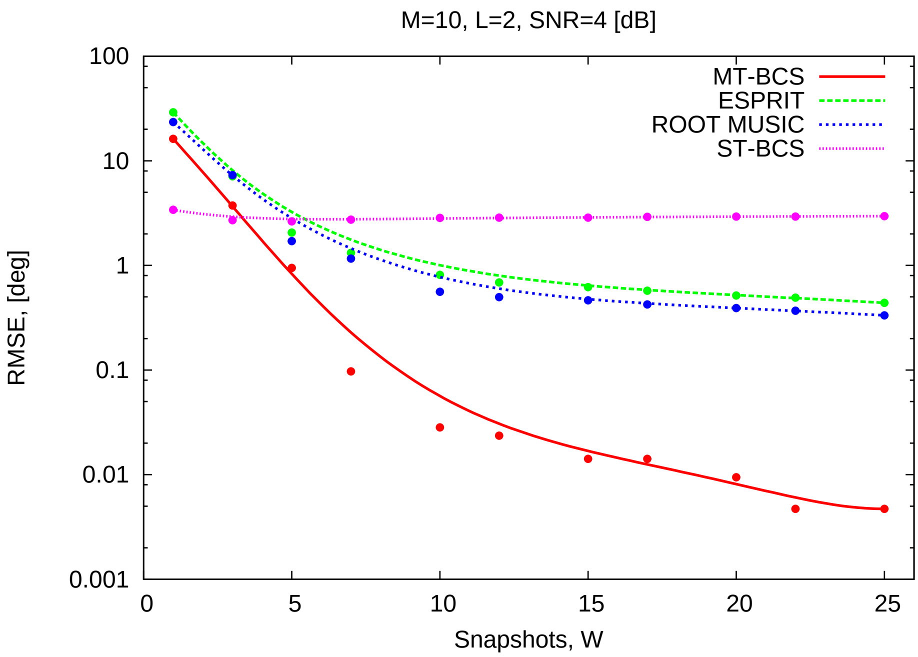


Figure 1: $RMSE$ vs the number of snapshots $W$.

**Analysis vs $SNR$**

**Simulation Parameters**

- Scenario
  - BPSK signals ($E_l^{inc} \in \{-1, 1\}$)
  - Number of incident signals: $L = 2$
  - Signal directions: $\underline{\theta} = \{0, 7\}$ $[deg]$
  - **Signal to noise ratio:** $SNR \in [-5, 20]$ $dB$ ($SNR \in [-8, 17]$ $dB$ if the literature's definition is taken into account)
- Array parameters
  - Elements spacing: $d = 0.5\lambda$
  - Number of elements: $M = 10$
- MT-BCS parameters
  - Number of angular locations: $K = 181$
  - $a = 3.162$
  - $b = 3.981 \times 10^1$
- BCS parameters
  - Number of angular locations: $K = 181$
  - $\sigma_0^2 = 4.642 \times 10^{-1}$
  - Number of snapshots**:** $W = 20$
- Simulation
  - Number of independent realizations $Q = 150$ (the noise and the signal amplitudes are random, while the DoAs are fixed)

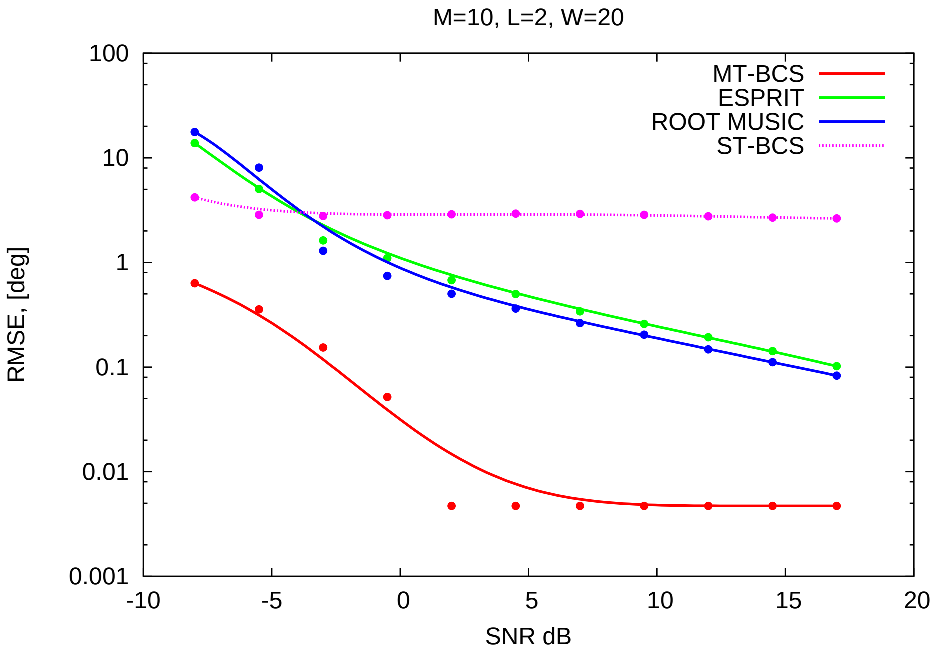


Figure 2: $RMSE$ vs the $SNR$.

**Analysis vs $\Delta\theta^{l(l+1)}$**

**Simulation Parameters**

- Scenario
  - BPSK signals ($E_l^{inc} \in \{-1, 1\}$)
  - Number of incident signals: $L = 2$
  - **Signals spacing**: $\Delta\theta^{l(l+1)} \in [2, 20]\ \ deg$
  - **Signals directions**: $\underline{\theta} = \left\{-\frac{\Delta\theta^{l(l+1)}}{2}, \frac{\Delta\theta^{l(l+1)}}{2}\right\}\ \ [deg]$
  - Signal to noise ratio: $SNR = 7\ dB$ (equivalent to a $SNR = 4\ dB$ if the literature's definition is taken into account)
- Array parameters
  - Elements spacing:
  - Number of elements: $M = 10$
- MT-BCS parameters
  - Number of angular locations: $K = 181$
  - $a = 3.162$
  - $b = 3.981 \times 10^1$
- BCS parameters
  - Number of angular locations: $K = 181$
  - $\sigma_0^2 = 4.642 \times 10^{-1}$
  - Number of snapshots**:** $W = 20$
- Simulation
  - Number of independent realizations $Q = 150$ (the noise and the signal amplitudes are random, while the DoAs are fixed)

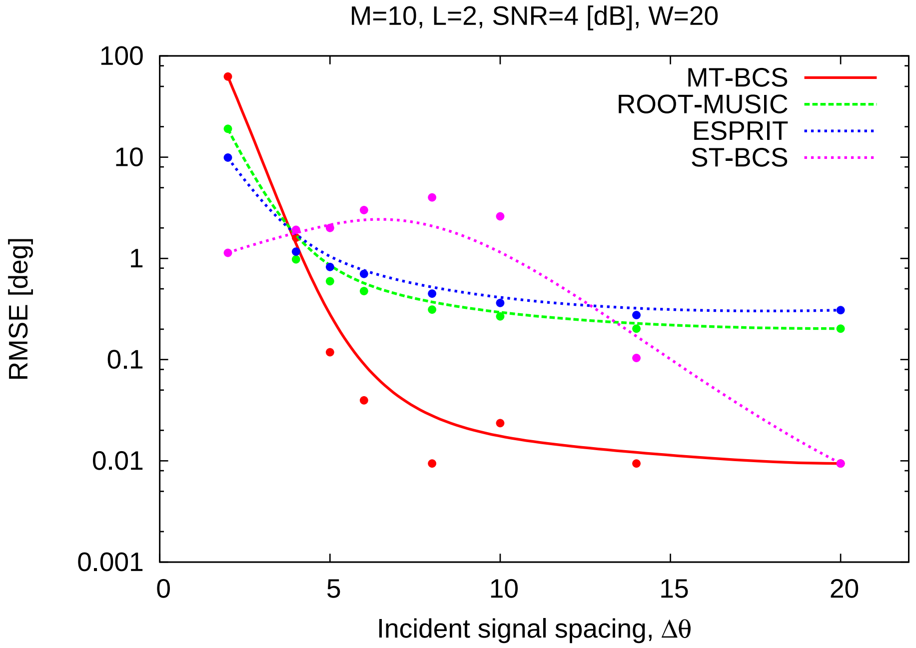


Figure 3: $RMSE$ vs the signal spacing $\Delta\theta^{l(l+1)}$.

## MT-BCS vs ST-BCS comparison: estimation examples

### Simulation Parameters

- Scenario
  - BPSK signals ($E_l^{inc} \in \{-1, 1\}$)
  - **Number of incident signals:** $L \in [1, 9]$
  - **Signal directions:**

| $L$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_5$ | $\theta_6$ | $\theta_7$ | $\theta_8$ | $\theta_9$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | - | - | - | - | - | - | - | - |
| 2 | 0 | 7 | - | - | - | - | - | - | - |
| 4 | 0 | 7 | 35 | | - | - | - | - | - |
| 6 | 0 | 7 | 35 | $-20$ | 22 | $-37$ | - | - | - |
| 8 | 0 | 7 | 35 | $-20$ | 22 | $-37$ | $-9$ | $-67$ | - |
| 9 | 0 | 7 | 35 | $-20$ | 22 | $-37$ | $-9$ | $-67$ | 54 |

Table 1: Signal directions for different numbers of signals.

  - Signal to noise ratio**:** $SNR = 7\ dB$
- Array parameters
  - Elements spacing: $d = 0.5\lambda$
  - Number of elements: $M = 10$
- $ST - BCS$ and $MT - BCS$ parameters
  - Number of angular locations: $K = 181$
  - Number of snapshots**:** $W = 20$

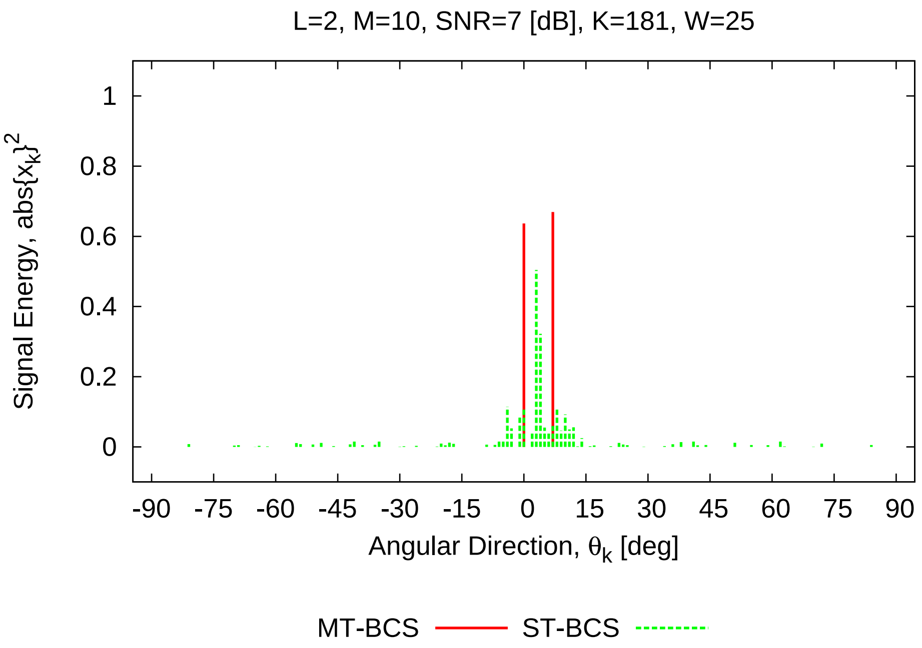


Figure 4: $MT-BCS$ vs $ST-BCS$: esstimated signal amplitudes when $L=2$ signals impinging on the array. The number of snapshots is $W=25$.

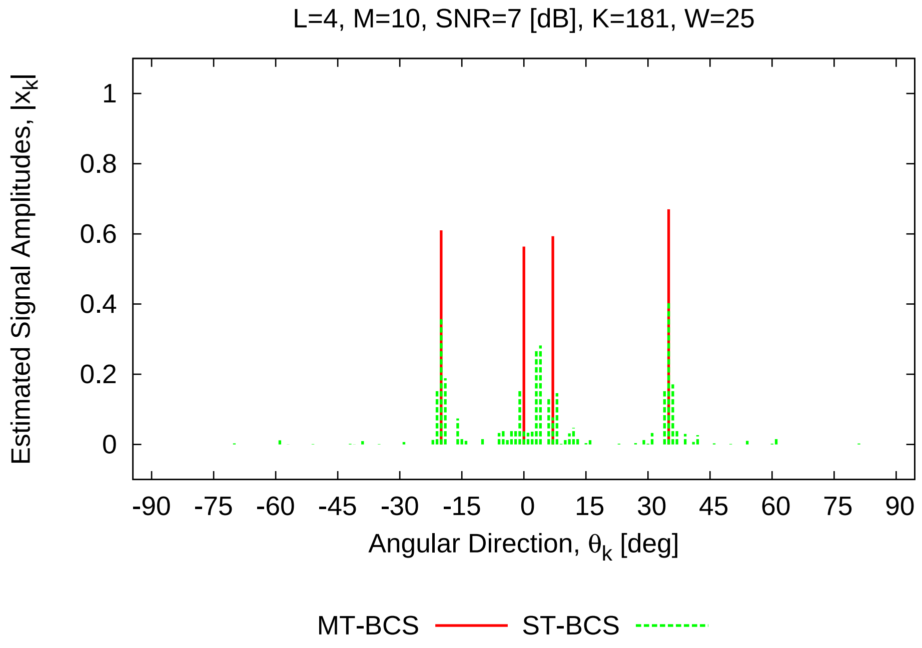


Figure 5: $MT-BCS$ vs $ST-BCS$: esstimated signal amplitudes when $L=4$ signals impinging on the array. The number of snapshots is $W=25$.

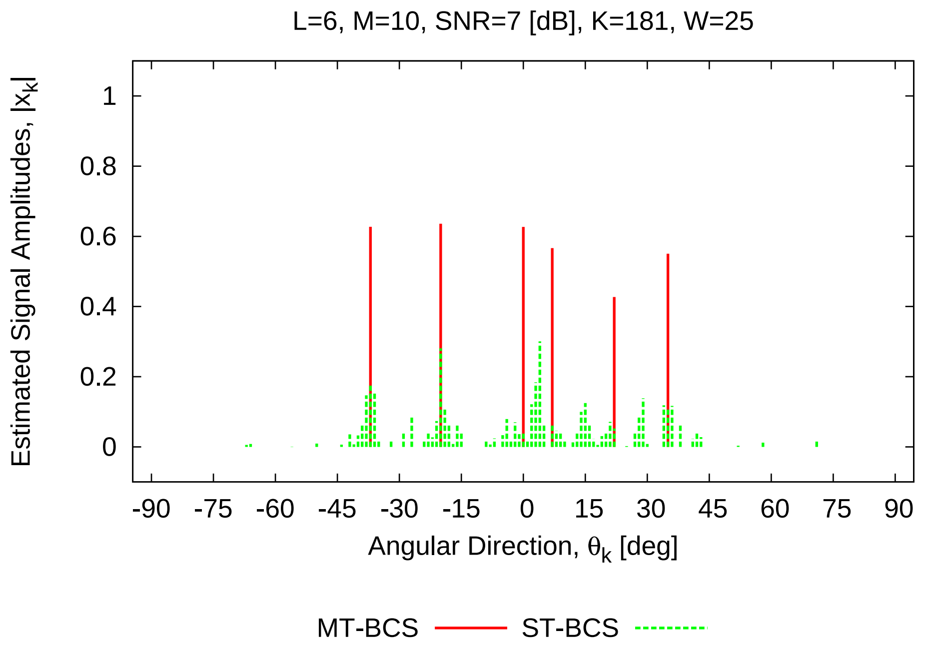


Figure 6: $MT-BCS$ vs $ST-BCS$: esstimated signal amplitudes when $L=6$ signals impinging on the array. The number of snapshots is $W=25$.

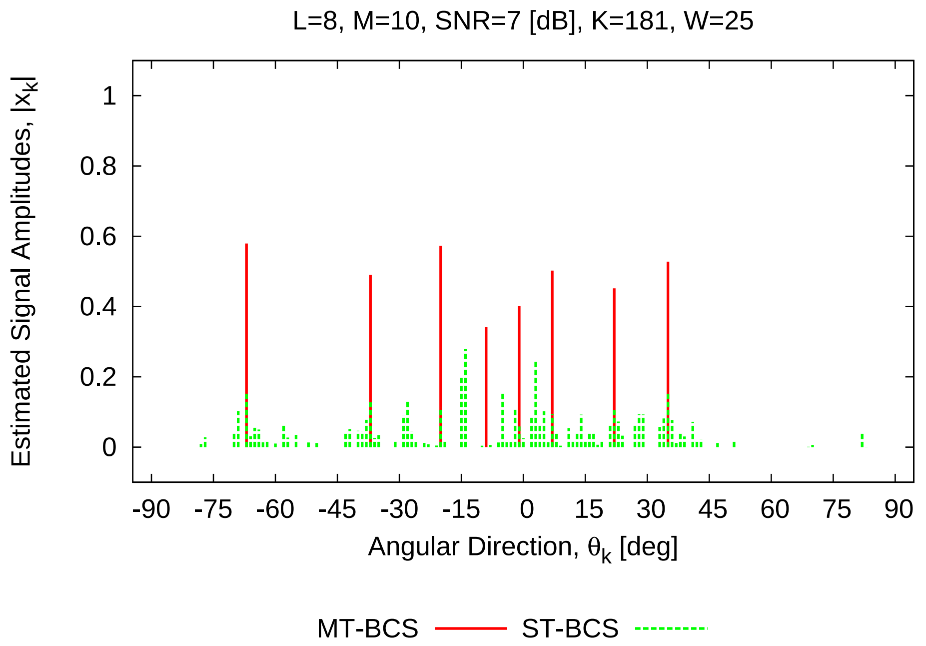


Figure 7: $MT-BCS$ vs $ST-BCS$: esstimated signal amplitudes when $L=8$ signals impinging on the array. The number of snapshots is $W=25$.

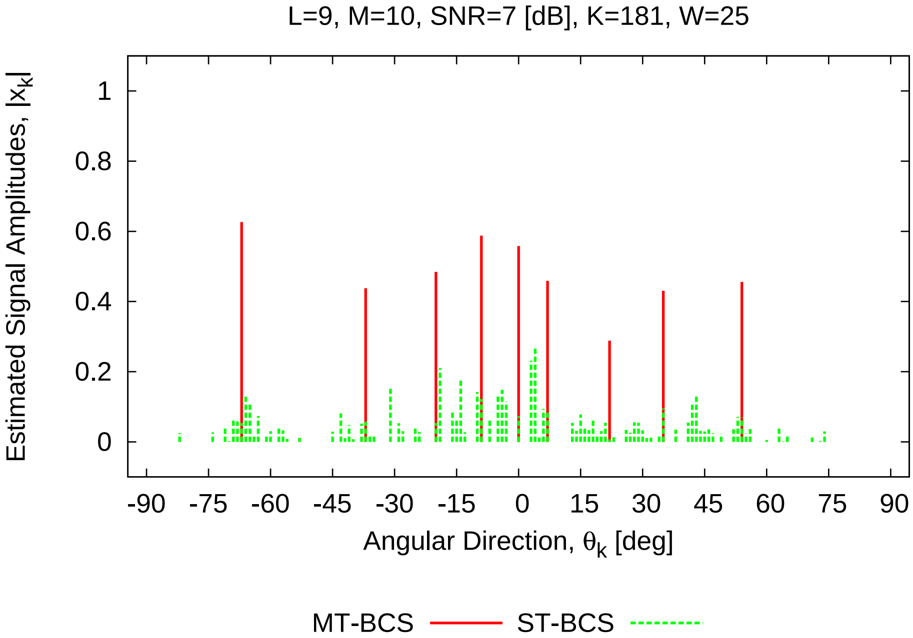


Figure 8: $MT-BCS$ vs $ST-BCS$: esstimated signal amplitudes when $L=9$ signals impinging on the array. The number of snapshots is $W=25$.

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