Reconfigurable Thinning For Pattern Nulling In Circular Ring Arrays

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Abstract

The pattern nulling performances of reconfigurable ring arrays are investigated in this report. By controlling the on/off status of the array elements with a binary genetic algorithm, the SINR will be maximized in order to generate deep nulls along the unknown direction of arrival of an interference impinging on the array. Representative numerical results will be proposed in order to show the trade-off existing between the nulling capability and the number of elements.

Mathematical Formulation

Multiple Interferences at the Central Frequency - SINR Maximization

Consider a linear array of N isotropic elements equally spaced along the x axis: the desired signal received by the n-th element of the antenna array can be defined as

$$S_n^d(t) = p_d(t)e^{j\beta_n^d} \quad n = 1, ..., N \tag{1}$$

where $\beta_n^d = (2\pi/\lambda)(u_d x_n)$, $u_d = \sin\theta_d \cos\phi_d$, x_n is the distance between the n-th element and the center of the array e θ_d, ϕ_d are the polar coordinates defining the direction of arrival (DOA) of the desired signal characterized by envelope $p_d(t)$. Assuming that one or more (I) interfering signals can be received by the antenna at the same angular frequency ω_d of the desired signal, it is possible to evaluate the contribution of each interference at the n-th element:

$$S_n^i(t) = p_i(t)e^{j\beta_n^i} \quad \begin{cases} n = 1, ..., N \\ i = 1, ..., I \end{cases} \tag{2}$$

where $\beta_n^i = (2\pi/\lambda)(u_i x_n)$, $u_i = \sin\theta_i \cos\phi_i$ e θ_i, ϕ_i are the polar coordinates defining the direction of arrival (DOA) of the i-th interfering signal characterized by envelope $p_i(t)$. Moreover, let assume the presence of the noise modelled with an additive gaussian process with power $\wp_n$.

Hence, the coefficients of the covariance matrix $(N \times N)$ of the desired signal Φ_d are

$$\Phi_d^{mn} = E\left\{S_m^{d*}(t)S_n^d(t)\right\} \quad m, n = 1, ..., N \tag{3}$$

Similarly, it is possible to write the coefficients of the covariance matrix Φ_i of the i-th interfering signal $(i = 1, ..., I)$ as

$$\Phi_i^{mn} = E\left\{S_m^{i*}(t)S_n^i(t)\right\} \quad m, n = 1, ..., N \tag{4}$$

while the covariance matrix of the noise is defined

$$\Phi_n = p_n 1^N \tag{5}$$

where 1^N is an identity matrix with dimension $N \times N$.

Let us write the covariance matrix of the undesired signal with the form

$$\Phi_u = \sum_{i=1}^{I} \Phi_i + \Phi_n \tag{6}$$

The power of the undesired signal received at the central frequency is

$$\wp_u = \frac{1}{2}\underline{W}^{T*}\Phi_u\underline{W} \tag{7}$$

where $\underline{W}$ is defined as

$$\underline{W} = \left\{\alpha_n e^{j\gamma_n}, \;\; n = 1, ..., N\right\} \tag{8}$$

where α_n amplitude excitation coefficients of the n-th element and γ_n is the phase excitation coefficient of the n-th element of the array. Using a thinning technique, the possible solutions of α_n are just two values: $\alpha_n \in \Upsilon$, $n = 1, ..., N$, where $\Upsilon = [\{0\}, \{1\}]$. We consider $\gamma_n = 0$, $n = 1, ..., N$.

The power contribution of the desired signal at the receiver is

$$\wp_d = \frac{1}{2} p_d^2(t) \left| W^T \underline{U}(\theta_d, \phi_d) \right|^2 \tag{9}$$

where

$$\underline{U}(\theta_d, \phi_d) = \left\{ e^{j\beta_n^d}, \;\; n = 1, ..., N \right\} \tag{10}$$

Considering (7) and (9) the SINR (*Signal to Interferience plus Noise Ratio)* can be defined as:

$$\Psi(\underline{G}) \triangleq \frac{\wp_d}{\wp_u} = \frac{p_d^2(t) \left| W^T \underline{U}(\theta_d, \phi_d) \right|^2}{\underline{W}^{T*} \Phi_u \underline{W}} \tag{11}$$

Since Φ_u and $p_d^2(t)$ are not directly misurable, (11) is not useful. However, it is possible to reformulate the SINR maximization problem through the following cost function

$$f(\underline{G}) = \frac{\left| W^T \underline{U}(\theta_d, \phi_d) \right|^2}{\underline{W}^{T*} \Phi_t \underline{W}} \tag{12}$$

where $\Phi_t = \Phi_d + \sum_{i=1}^{I} \Phi_i + \Phi_n$ is a quantity that can be measured at the receiver.

Numerical Assessment

TEST CASE - $N = 37$ -$Configuration = 3rings$ -$\eta \in [0.0, 1.0]$ - $N_I = 1$

Goal

Maximization of the SINR using genetic algorithms (GA) to determine the optimal thinned ring array configuration, considering a time-varying scenario with a single interference.

Test Case Description

- Number of Elements $N = 37$
- Elements Spacing: $d = 0.5\lambda$
- Max Gain Pattern Direction : $\theta^d = 90°$, $\phi^d = 90°$
- Desired Signal Power: $0\,dB$
- Interference Power: $30\,dB$
- Noise Power: $-30\,dB$
- Number of Interferences: $N_I = 1$
- Interference Direction Of Arrival: $\theta_1^i = 33°$, $\phi_1^i = 151°$

Optimization Approach: GA

- Number of Variables: $X = 37$ (α_n, $n = 1, ..., N$)
- Population: 18
- Crossover Probability: 0.9
- Mutation Probability: 0.01
- Number of Generations: 200
- Minimum Thinning Coefficient: 0.0
- Maximum Thinning Coefficient: 1.0

GA - Single Interference: $\theta_1^i = 33^\circ$, $\phi_1^i = 151^\circ$

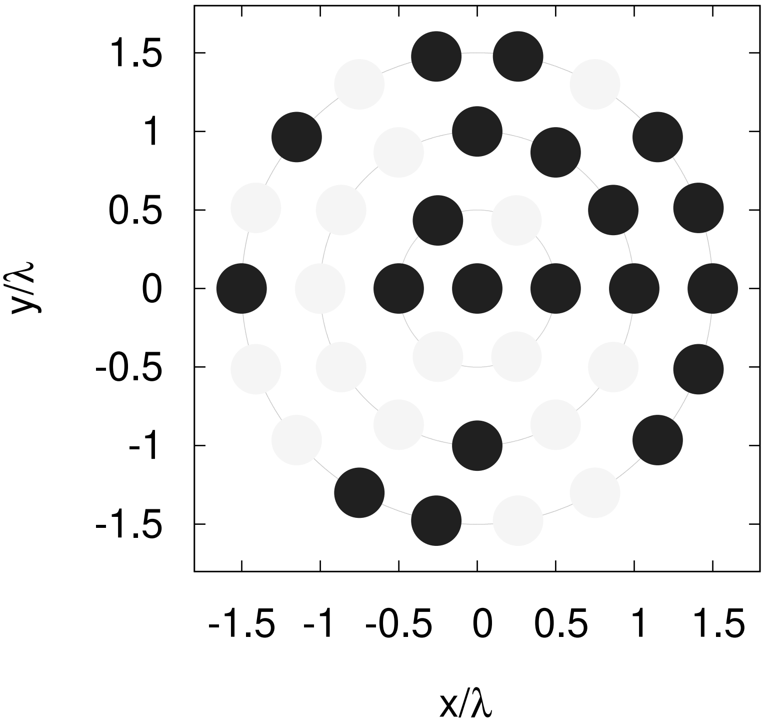

Fig.1 - Thinning Configuration

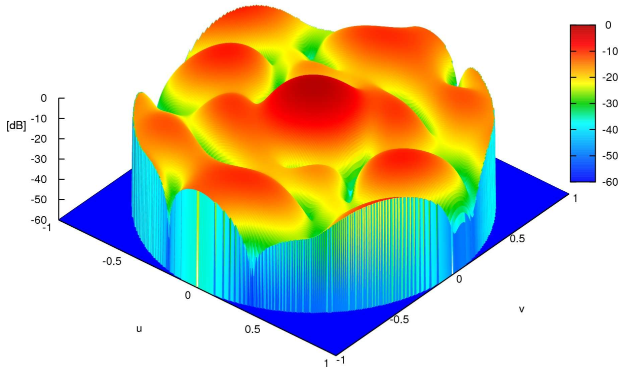

Fig.2 - Pattern

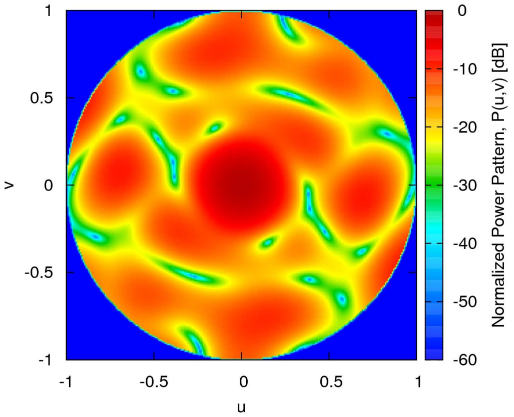

Fig.3 - Pattern projection

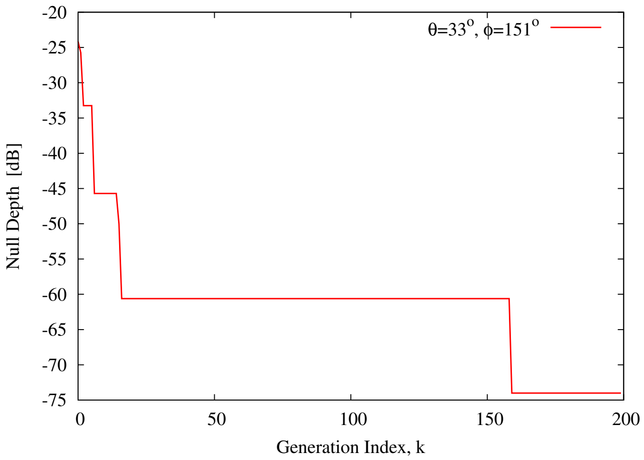

Fig.4 - Nulls Depth

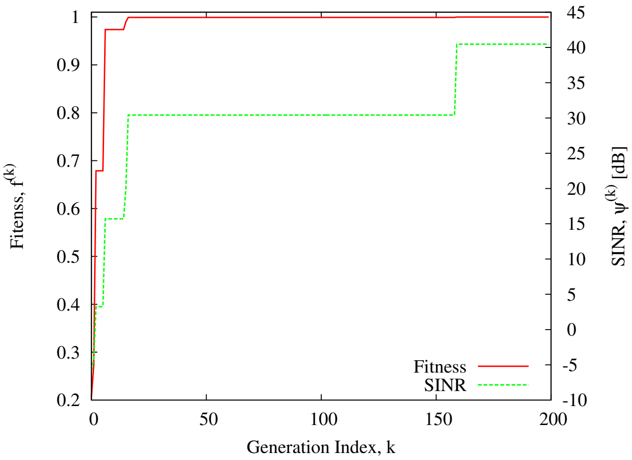

Fig.5 - Fitness - SINR

SINR[dB]: 40.47

Null Depths[dB]: -74

Number of Active Elements: 20

TEST CASE - $N = 91$ -$Configuration = 5rings$ -$\eta \in [0.0, 1.0]$ - $N_I = 1$

Goal

Maximization of the SINR using genetic algorithms (GA) to determine the optimal thinned ring array configuration, considering a time-varying scenario with a single interference.

Test Case Description

- Number of Elements $N = 91$
- Elements Spacing: $d = 0.5\lambda$
- Max Gain Pattern Direction : $\theta^d = 90°$, $\phi^d = 90°$
- Desired Signal Power: $0\,dB$
- Interference Power: $30\,dB$
- Noise Power: $-30\,dB$
- Number of Interferences: $N_I = 1$
- Interference Direction Of Arrival: $\theta_1^i = 136°$, $\phi_1^i = 116°$

Optimization Approach: GA

- Number of Variables: $X = 91$ (α_n, $n = 1, ..., N$)
- Population: 46
- Crossover Probability: 0.9
- Mutation Probability: 0.01
- Number of Generations: 200
- Minimum Thinning Coefficient: 0.0
- Maximum Thinning Coefficient: 1.0

GA - Single Interference: $\theta_1^i = 136^\circ$, $\phi_1^i = 116^\circ$

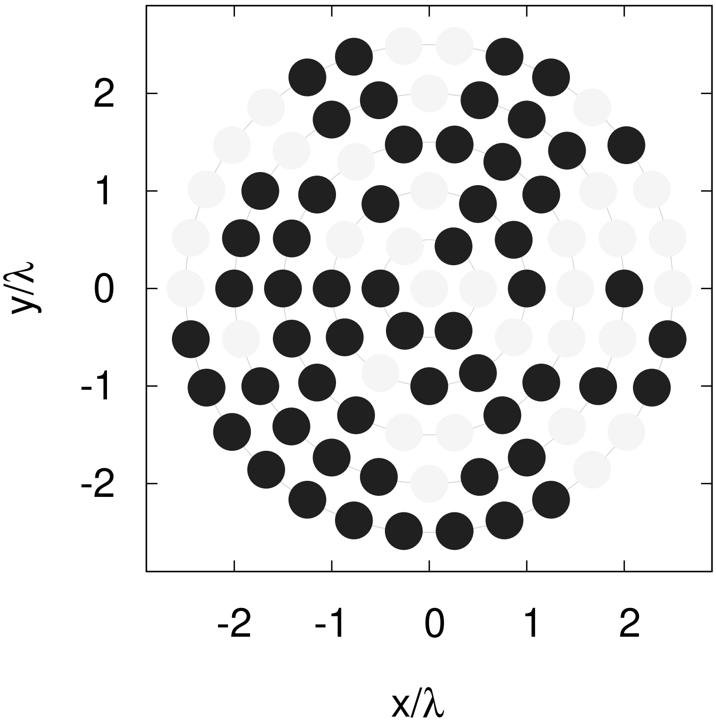

Fig.1 - Thinning Configuration

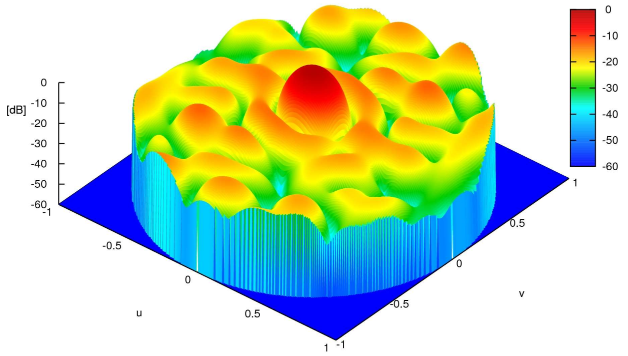

Fig.2 - Pattern

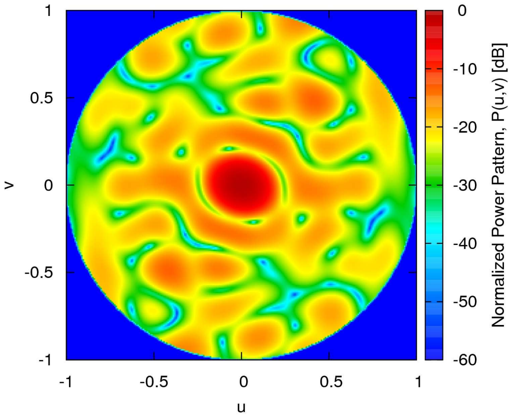

Fig.3 - Pattern projection

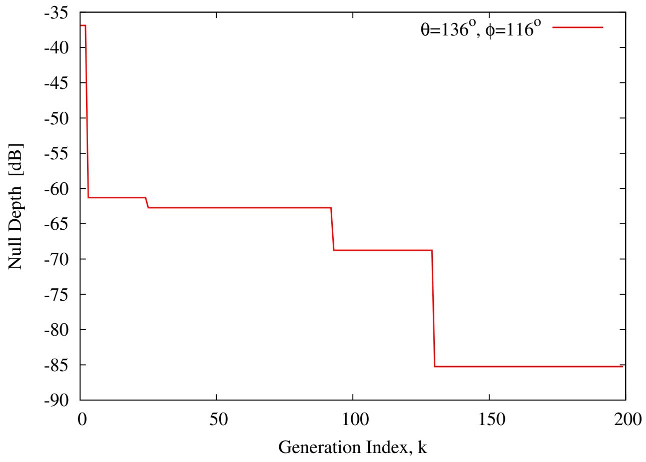

Fig.4 - Nulls Depth

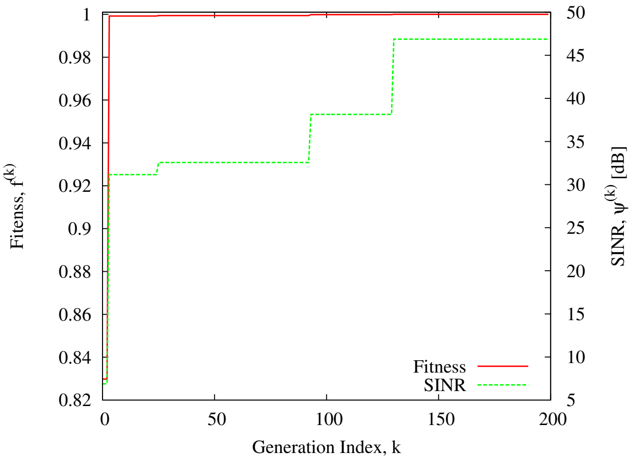

Fig.5 - Fitness - SINR

SINR[dB]: 46.87

Null Depths[dB]: -85.25

Number of Active Elements: 57

TEST CASE - $N = 172$ -$Configuration = 7rings$ -$\eta \in [0.0, 1.0]$ - $N_I = 1$

Goal

Maximization of the SINR using genetic algorithms (GA) to determine the optimal thinned ring array configuration, considering a time-varying scenario with a single interference.

Test Case Description

- Number of Elements $N = 172$
- Elements Spacing: $d = 0.5\lambda$
- Max Gain Pattern Direction : $\theta^d = 90°$, $\phi^d = 90°$
- Desired Signal Power: $0\,dB$
- Interference Power: $30\,dB$
- Noise Power: $-30\,dB$
- Number of Interferences: $N_I = 1$
- Interference Direction Of Arrival: $\theta_1^i = 152°$, $\phi_1^i = 154°$

Optimization Approach: GA

- Number of Variables: $X = 172$ (α_n, $n = 1, ..., N$)
- Population: 86
- Crossover Probability: 0.9
- Mutation Probability: 0.01
- Number of Generations: 200
- Minimum Thinning Coefficient: 0.0
- Maximum Thinning Coefficient: 1.0

GA - Single Interference: $\theta_1^i = 152°$, $\phi_1^i = 154°$

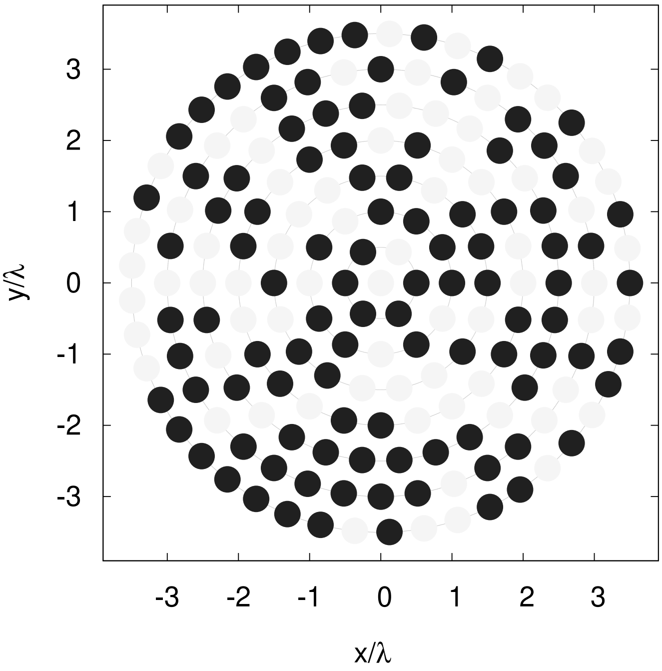

Fig.1 - Thinning Configuration

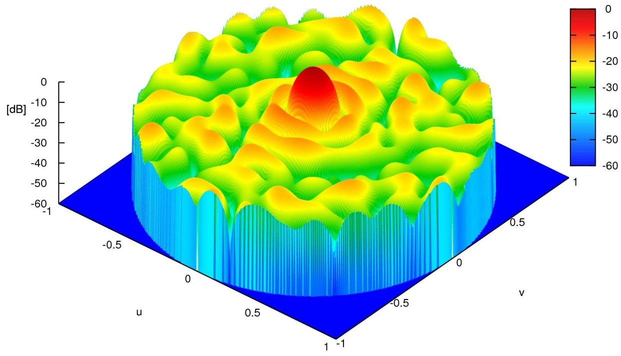

Fig.2 - Pattern

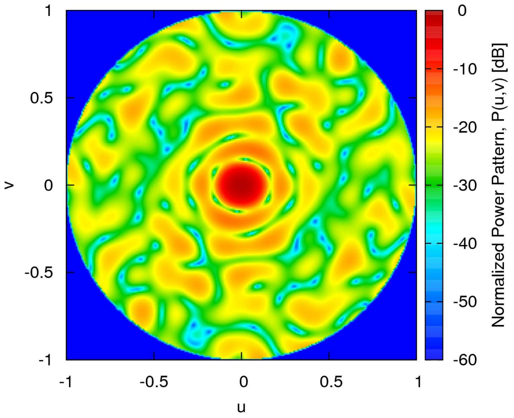

Fig.3 - Pattern projection

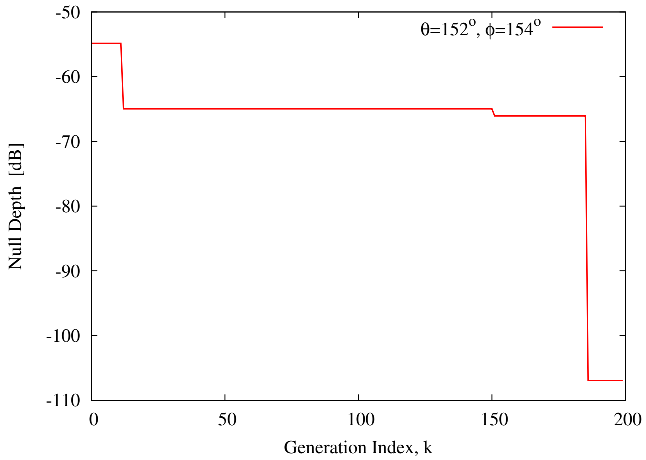

Fig.4 - Nulls Depth

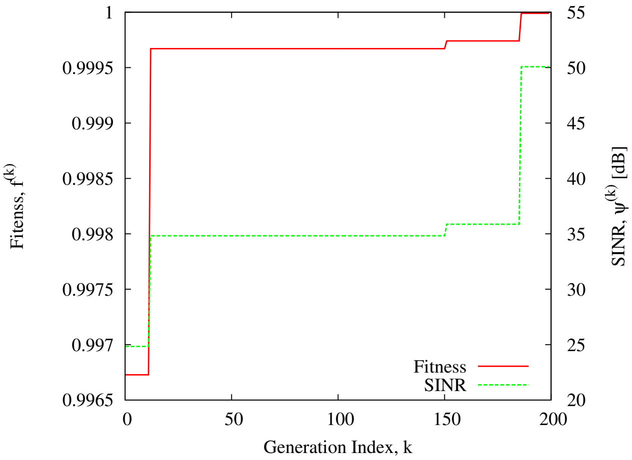

Fig.5 - Fitness - SINR

SINR[dB]: 50.07

Null Depths[dB]: -106.93

Number of Active Elements: 102

More information on the topics of this document can be found in the following list of references.

References

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