Robust Synthesis of Shaped Beams exploiting Interval Analysis and Particle Swarm Optimization

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Abstract

The potentialities and the intrinsic robustness of an IA-based synthesis method in defining the intervals of values for the array excitations still guaranteeing the generation of shaped beam patterns laying within the desired mask-constraint are investigated in this report. A stochastic global optimiser is employed to determine the amplitude excitations defined in terms of intervals of width corresponding to the maximum manufacturing tolerance around the nominal values that allow to match the power pattern constraints.

1 Flat Top Patterns

1.1 Test Case #10 - Flat Top - $\delta a_n = 1\%$

In this test case we try to synthesize a linear array with a Flat Top beam shape. The error on the amplitudes is 1%

Geometry:

- Number of Elements: $N = 30$
- Element Spacing: $d = \frac{\lambda}{2}$
- Sample Points: 301

Test Case Parameters:

- Amplitude Error: 1%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 8.10.1: Max and Min excitations amplitudes values, for the PSO

PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 50
- Fitness Tolerance: 1×10^{-5}
- Max Iterations Number: 500
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Power Mask Constraints

- Upper Mask Main Beam Width: 1 u (60°)
- Lower Mask Main Beam Width: 0.684 u (40°)

- Side Lobe Level: -20 dB
- Upper Mask Height: 0 dB
- Lower Mask Height: -4 dB

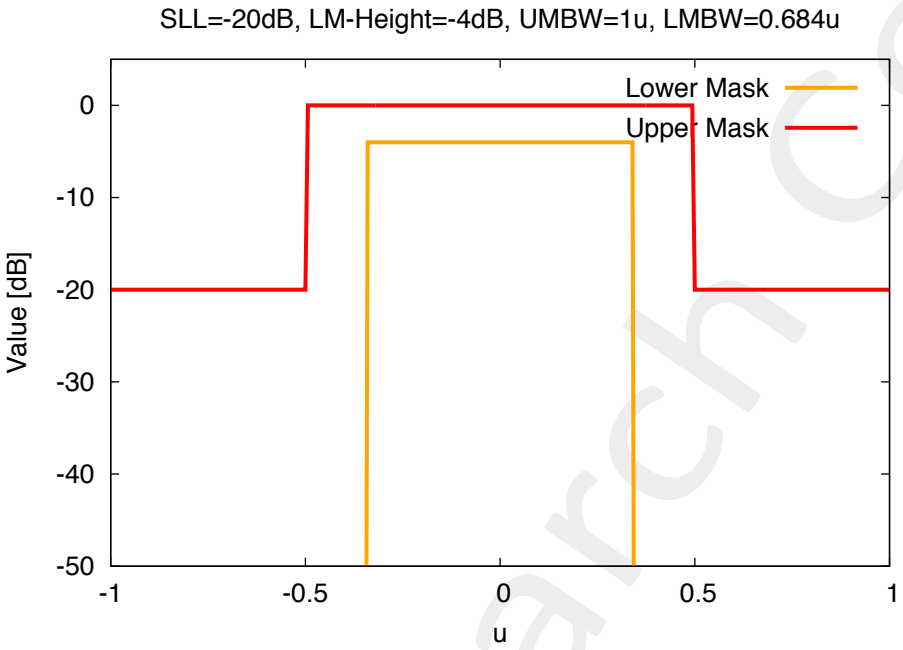

Figure 8.10.1: Power Synthesis Mask

Optimal Particle:

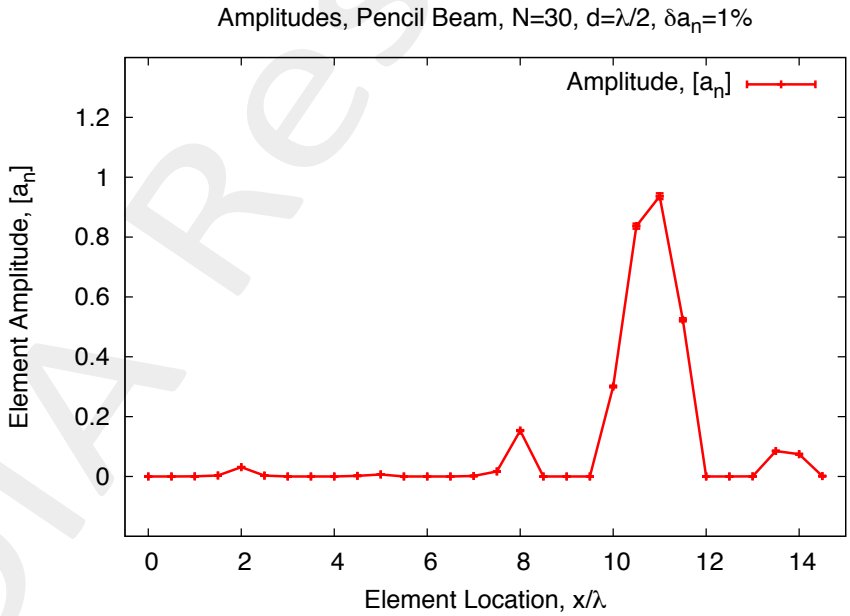

Figure 8.10.2: Optimal particle Amplitudes

Fitness:

- Performed Iterations: 500
- Fitness Finale : 2.7×10^{-3}

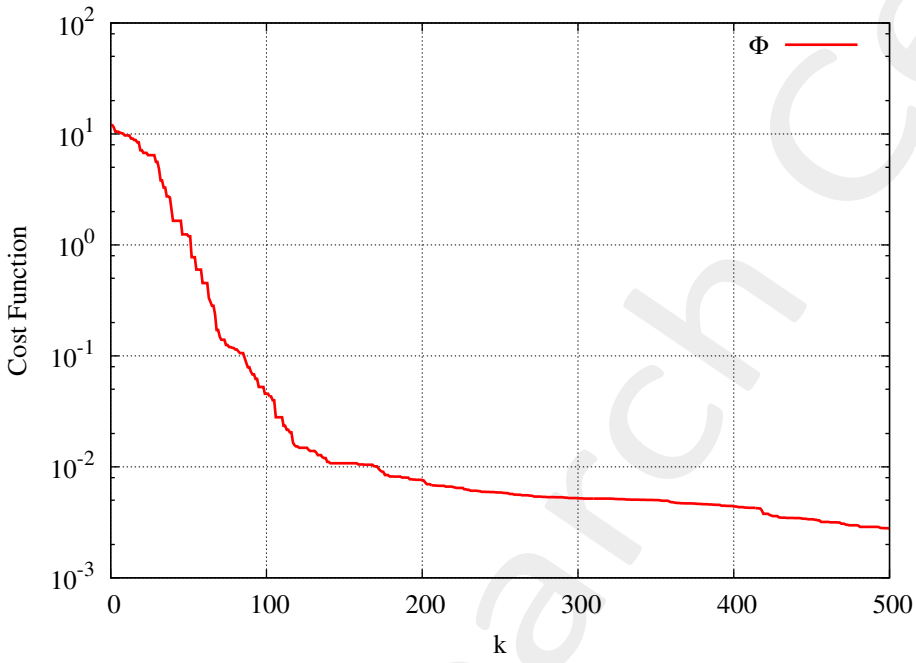

Figure 8.10.3: Fitness

Interval Pattern:

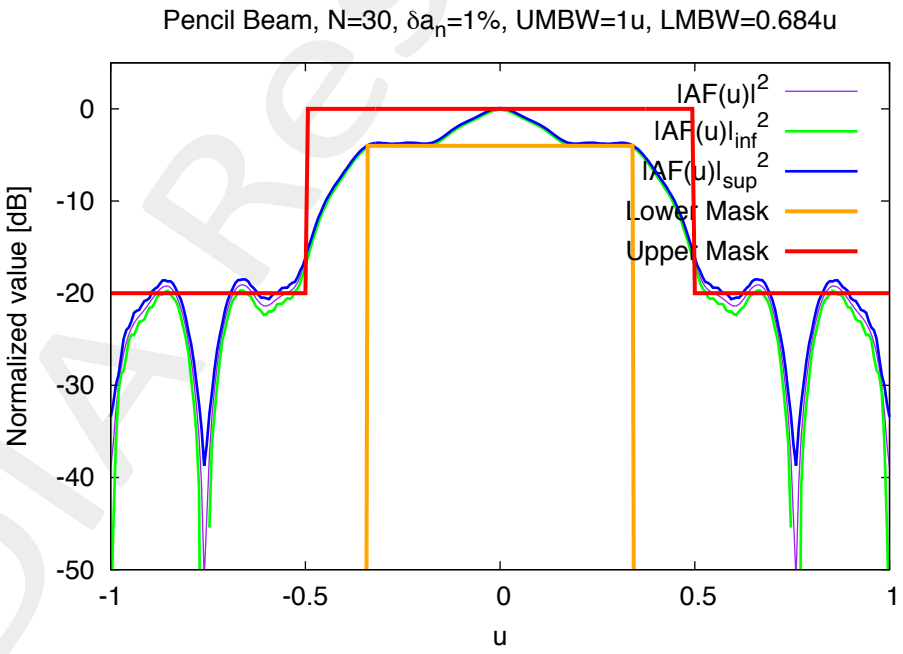

Figure 8.10.4: Optimal Interval Pattern

Simulation Time:

- Time: 20 min, 30 sec

1.2 Test Case #11 - Flat Top - $\delta a_n = 1\%$

In this test case we try to synthesize a linear array with a Flat Top beam. The error on the amplitudes is 1%

Geometry:

- Number of Elements: $N = 30$
- Element Spacing: $d = \frac{\lambda}{2}$
- Sample Points: 301

Test Case Parameters:

- Amplitude Error: 1%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 8.10.1: Max and Min excitations amplitudes values, for the PSO

PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 23
- Fitness Tolerance: 1×10^{-5}
- Max Iterations Number: 500
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Power Mask Constraints

- Upper Mask Main Beam Width: 0.684 u (40°)
- Lower Mask Main Beam Width: 0.517 u (30°)
- Side Lobe Level: -20 dB
- Upper Mask Height: 0 dB

- Lower Mask Height: -5 dB

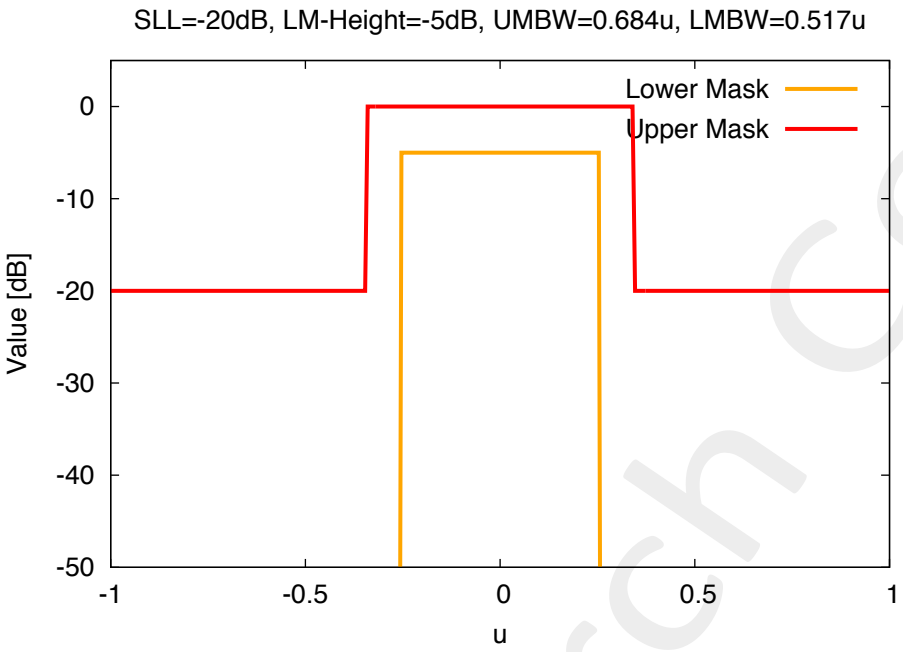

Figure 8.10.1: Power Synthesis Mask

Optimal Particle:

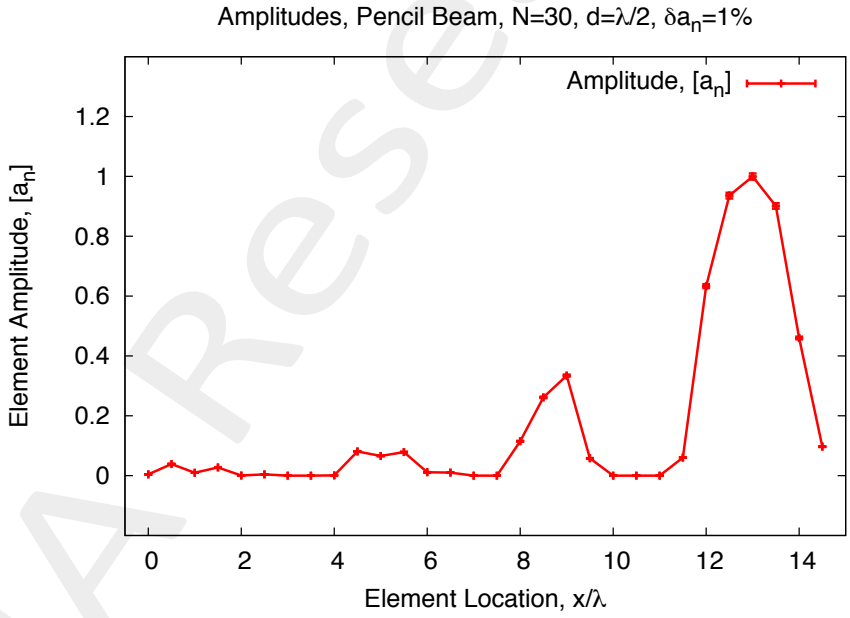

Figure 8.10.2: Optimal particle Amplitudes

Fitness:

- Performed Iterations: 300
- Fitness Finale : 6.9×10^{-4}

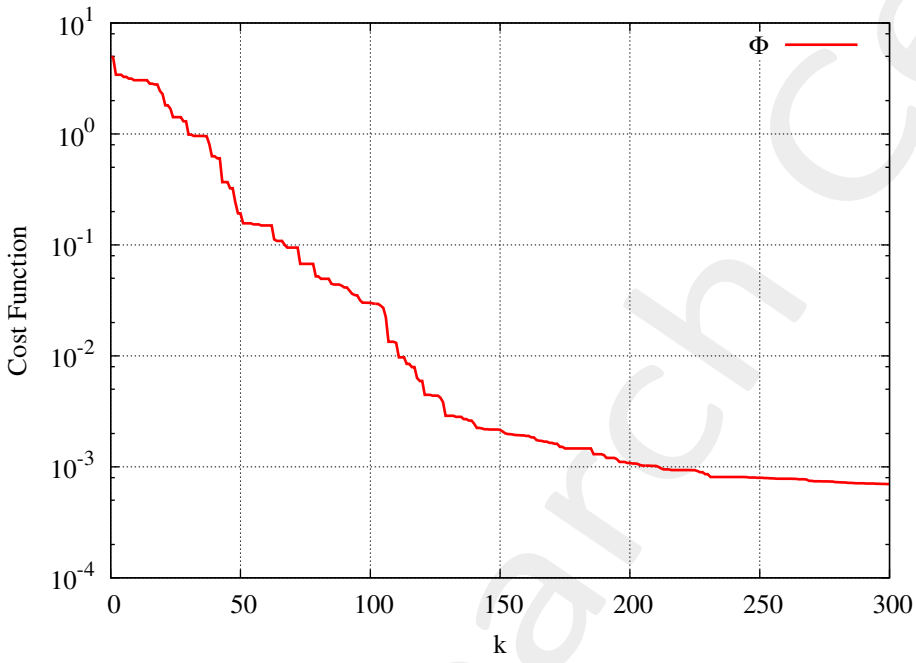

Figure 8.10.3: Fitness

Interval Pattern:

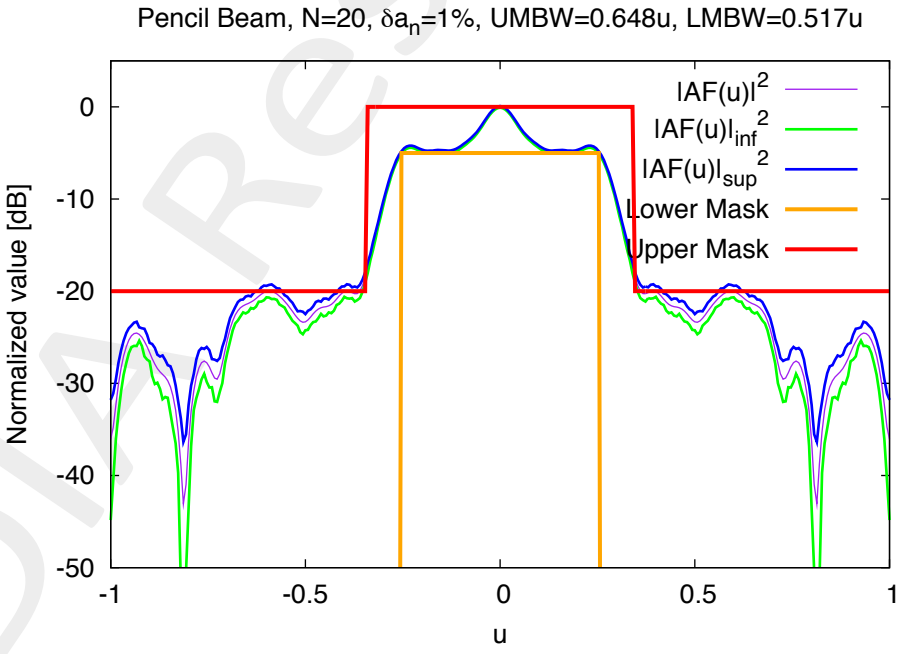

Figure 8.10.4: Optimal Interval Pattern

Simulation Time:

- Time: 7 min, 12 sec

1.3 Test Case #12 - Flat Top Beam - $\delta a_n = \%1$

Due to the fact that a good flat top beam is hard to achieve using the PSO, an already synthesized flat top beam is considered here. An error on the amplitudes of 1% is then inserted into the analysis. The already synthesized solution is suggested to the optimizer which tries to find a more robust radiation pattern with respect to the amplitude error.

Geometry:

- Number of Elements: $N = 30$
- Element Spacing: $d = \frac{\lambda}{2}$
- Sample Points: 501

Test Case Parameters:

- Amplitude Error: 1%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 8.12.1: Max and Min excitations amplitudes values, for the PSO

Max Phase Value	4.0
Min Phase Value	-4.0

Table 8.12.2: Max and Min excitations phases values, for the PSO

PSO Parameters:

- Unknown Number: 60
- Swarm Dimension: 20
- Random Seed: 43
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 2000
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Power Mask Constraints

- Upper Mask Main Beam Width: 0.829 u (49°)
- Lower Mask Main Beam Width: 0.716 u (42°)
- Side Lobe Level: −20 dB
- Lower Mask Height: −4 dB

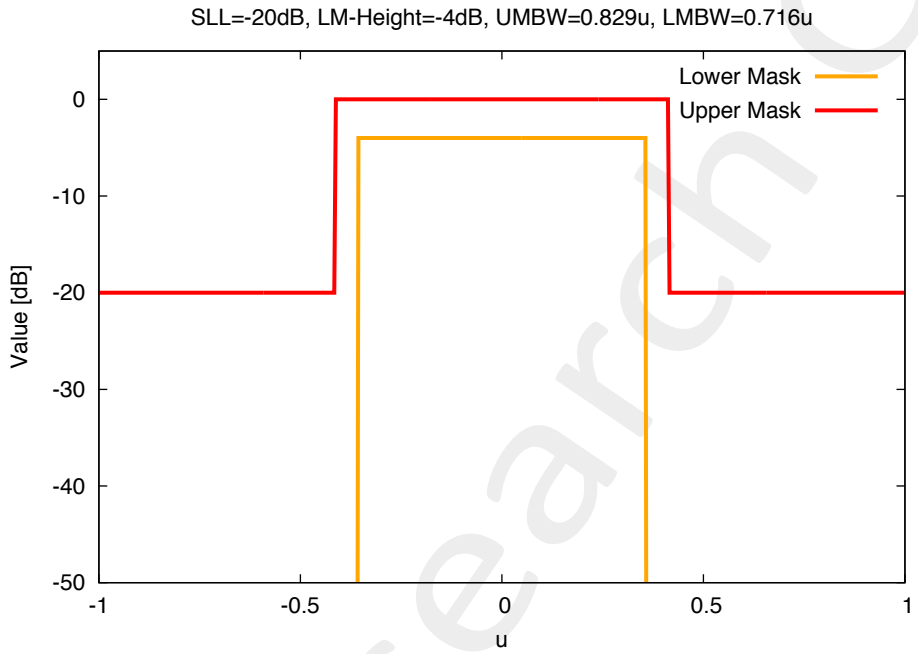

Figure 8.12.1: Power Synthesis Mask

Optimal Particle:

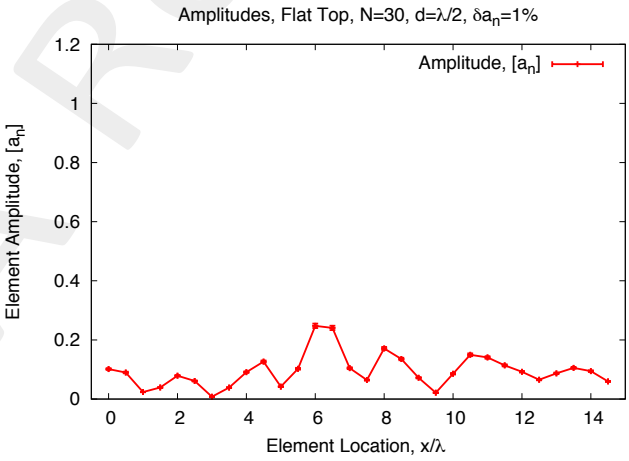

Figure 8.12.2: Optimal particle Amplitudes

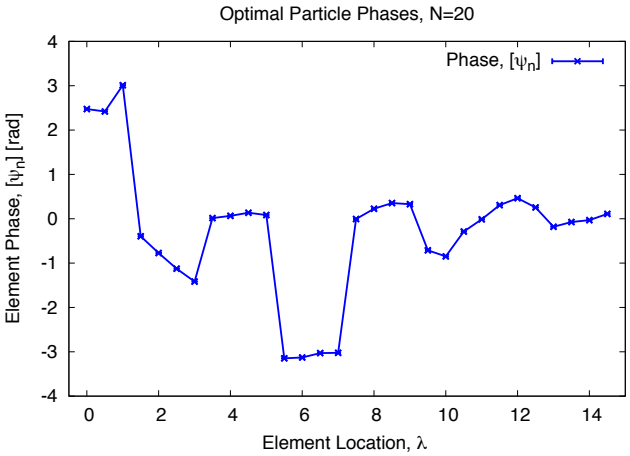

Figure 8.12.3: Optimal particle phases

Fitness:

- Performed Iterations: 2500
- Fitness Finale : 5×10^{-3}

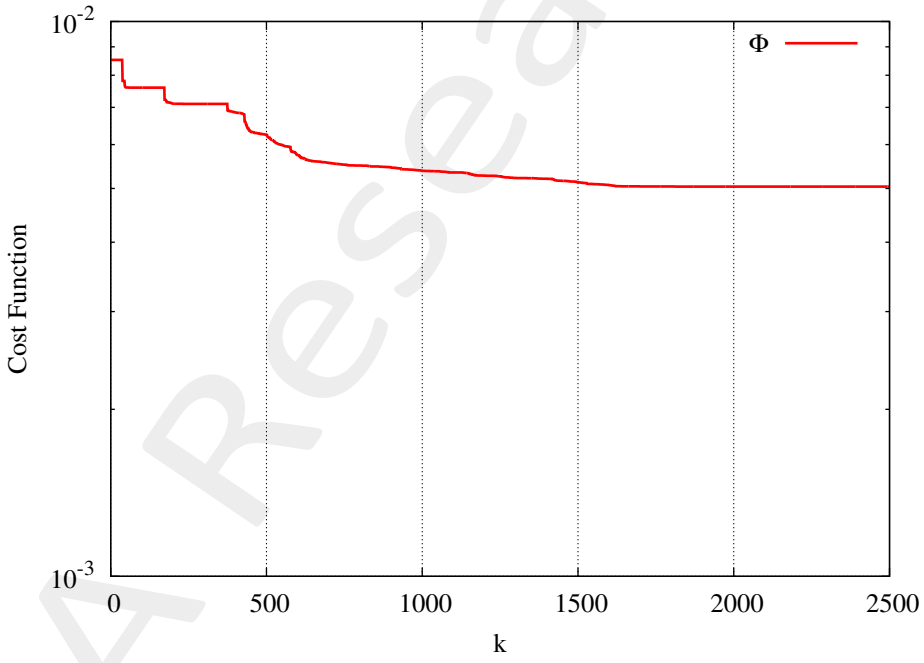

Figure 8.12.4: Fitness

Analytical suggested solution:

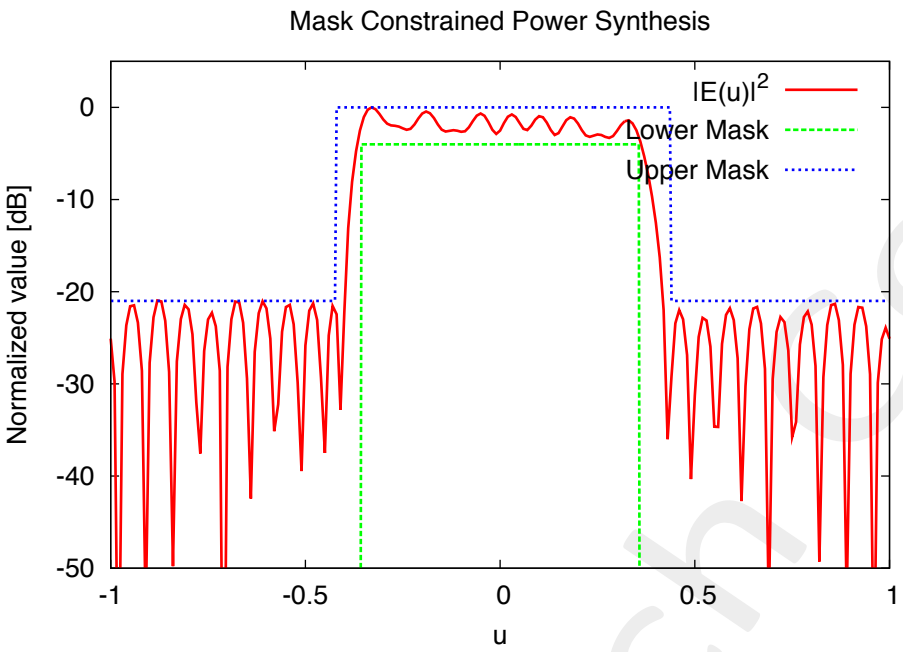

Figure 8.12.5: Flat Top Beam

Interval Pattern:

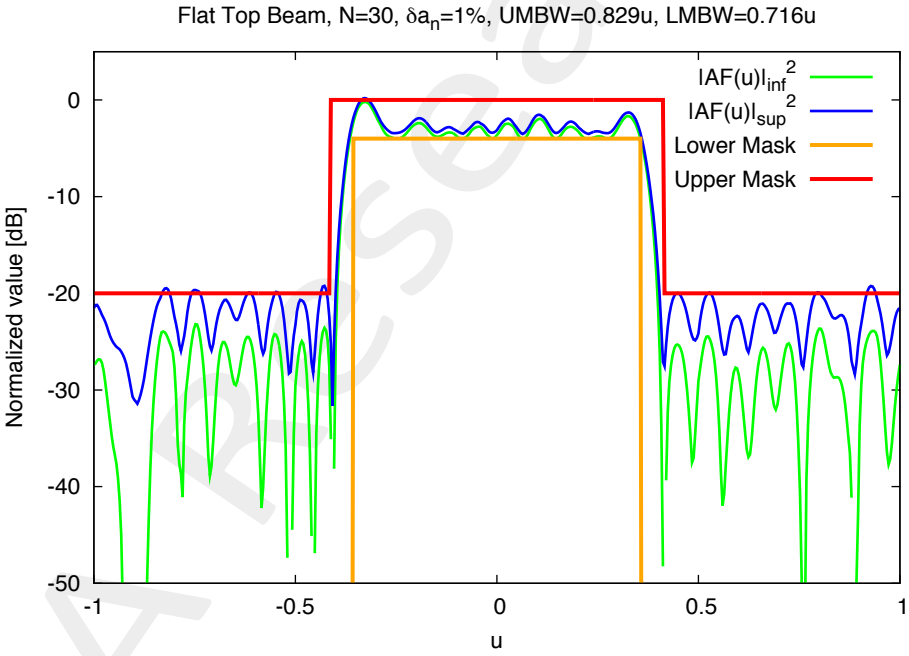

Figure 8.12.6: Optimal Interval Pattern

Parameter	inf	sup	nominal
$[D_{max}]$ [dB]	3.1	3.7	3.4
$[SLL]$ [dB]	-23.3	-19	-23
$[HPBW]$ [u]	0.08	0.092	0.088
$[\|E(u_{max})\|^2]$ [dB]	-0.2	0.19	0
Pattern Matching	//	//	0.051731

Table 8.12.3: Interval Pattern Parameters

Simulation Time:

- Time: 52 min, 40 sec

2 Amplifier Complexity vs Number of Elements

Considering a certain tolerance on the excitations amplitudes can leads the PSO optimizer to find a solution that doesnt't respect the constraints.

In this section is shown how raising the number of the array's elements, helps the optimizer to find a suitable solution.

The following cases are considered:

- $N = 16$; $\delta a_n = 1\%$, $\delta a_n = 5\%$, $\delta a_n = 8\%$
- $N = 18$; $\delta a_n = 1\%$, $\delta a_n = 5\%$, $\delta a_n = 8\%$
- $N = 20$; $\delta a_n = 1\%$, $\delta a_n = 5\%$, $\delta a_n = 8\%$
- $N = 26$; $\delta a_n = 1\%$, $\delta a_n = 5\%$, $\delta a_n = 8\%$

2.1 Test Case #1 - Pencil Beam - $N = 16$ - $\delta a_n = 1\%$

Array Geometry:

- Number of Elements: $N = 16$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 1%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.22 u
- LMBW: 0.1 u
- SLL: -20 dB
- LM-Height: -5 dB

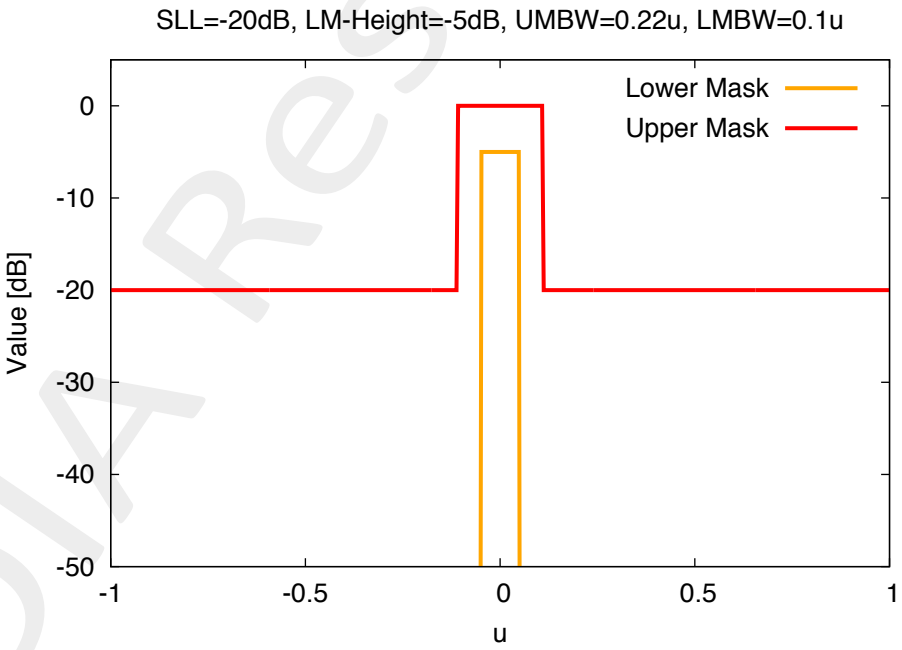

Figure 6.1.1. Power Synthesis Mask

2.1.1 PSO Parameters:

- Unknown Number: 16
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 400
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.1.1. Max and Min excitations amplitudes values, for the PSO

2.1.2 Excitations:

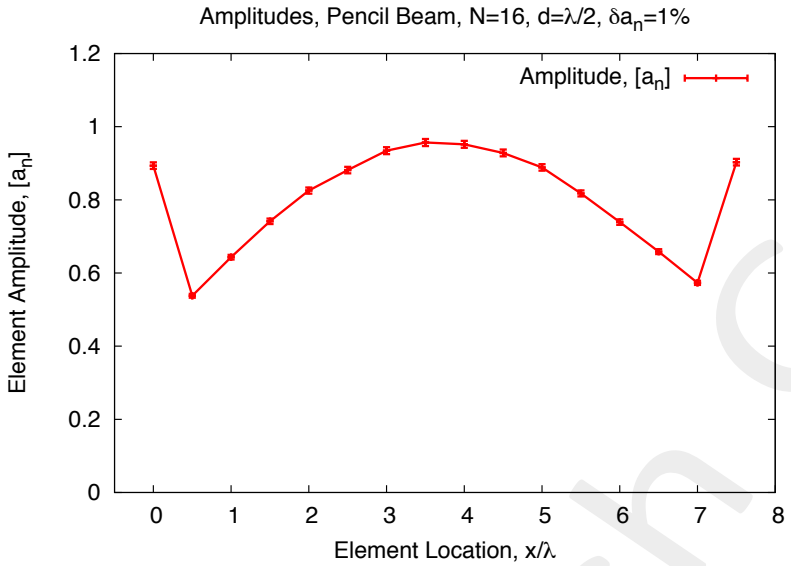

Figure 6.1.2.1. Optimal particle's excitations amplitudes

2.1.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

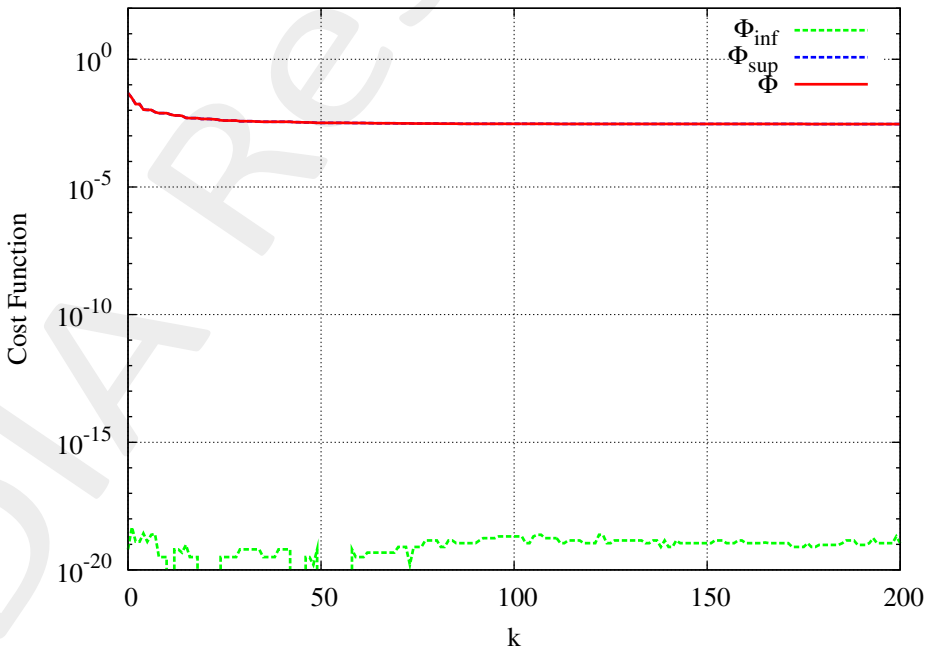

Figure 6.1.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
400	2.8×10^{-3}	5 min. 58 sec.

2.1.4 Synthesized Interval Pattern:

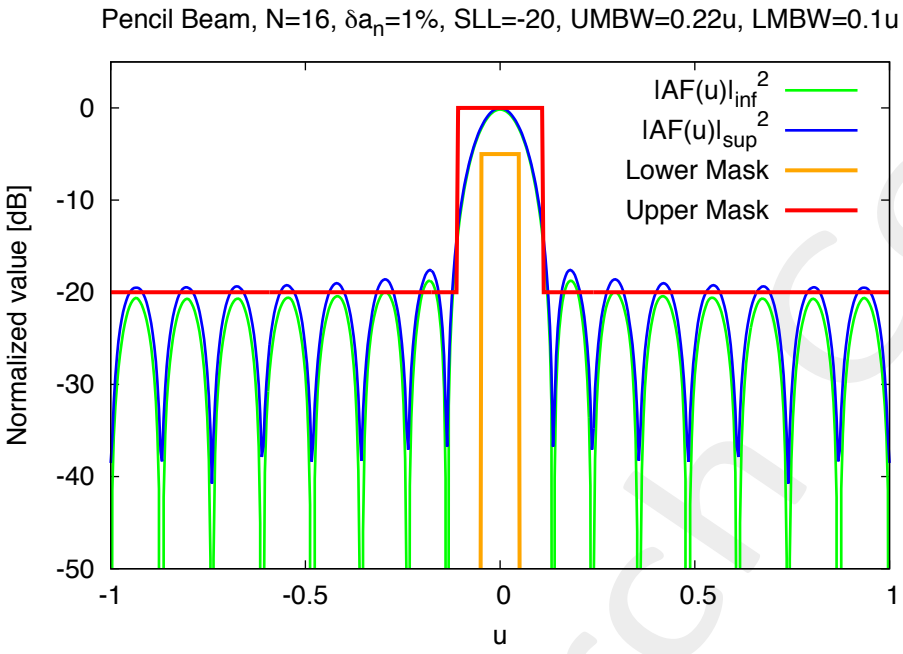

Figure 6.1.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	11.9	-19.2	0.12	-0.086	0.009074
Inf.	11.7	-18.7	0.12	-0.17	/
Sup.	12.1	-17.4	0.12	-0.0	/

Table 6.1.4.1. Interval Pattern Parameters

2.2 Test Case #2 - Pencil Beam - $N = 16$ - $\delta a_n = 5\%$

Array Geometry:

- Number of Elements: $N = 16$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 5%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.22 u
- LMBW: 0.1 u
- SLL: -20 dB
- LM-Height: -5 dB

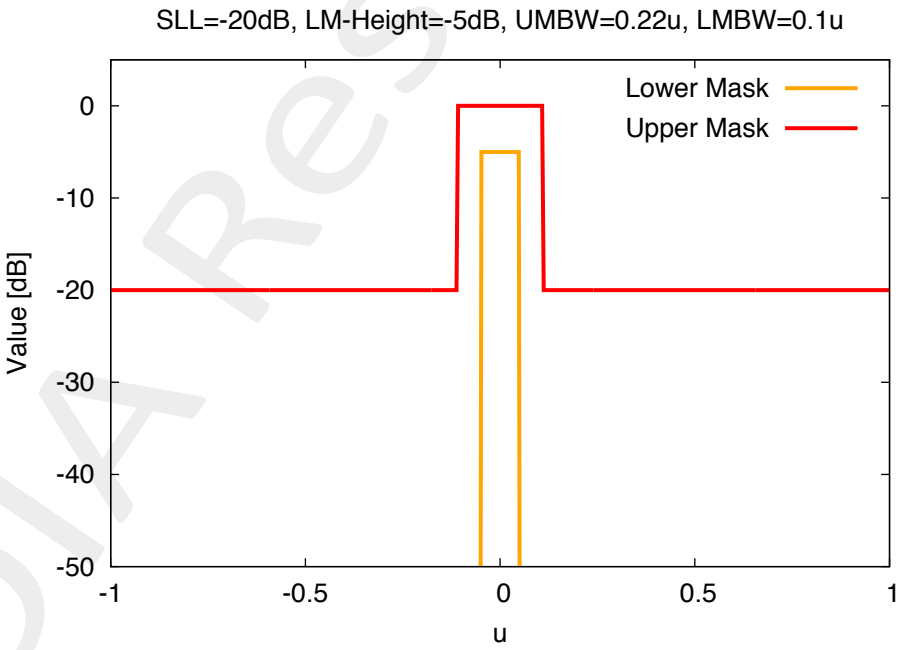

Figure 6.1.1. Power Synthesis Mask

2.2.1 PSO Parameters:

- Unknown Number: 16
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 400
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.1.1. Max and Min excitations amplitudes values, for the PSO

2.2.2 Excitations:

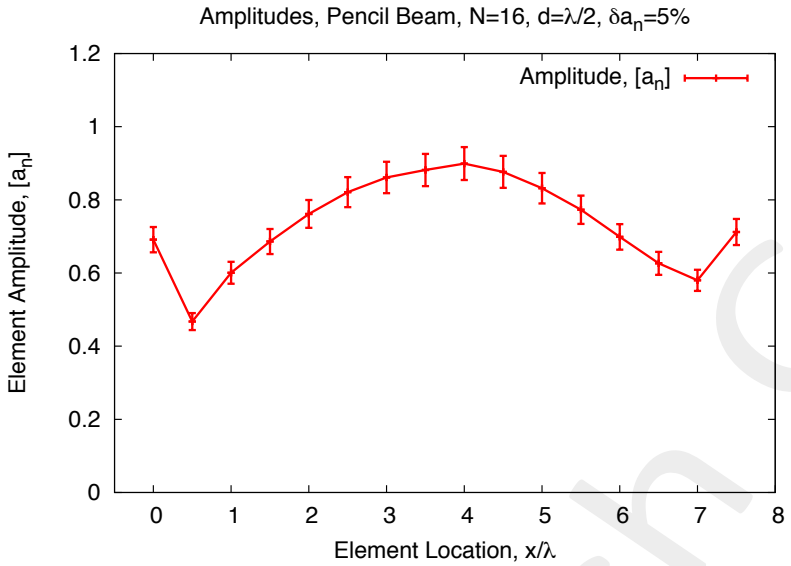

Figure 6.1.2.1. Optimal particle's excitations amplitudes

2.2.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

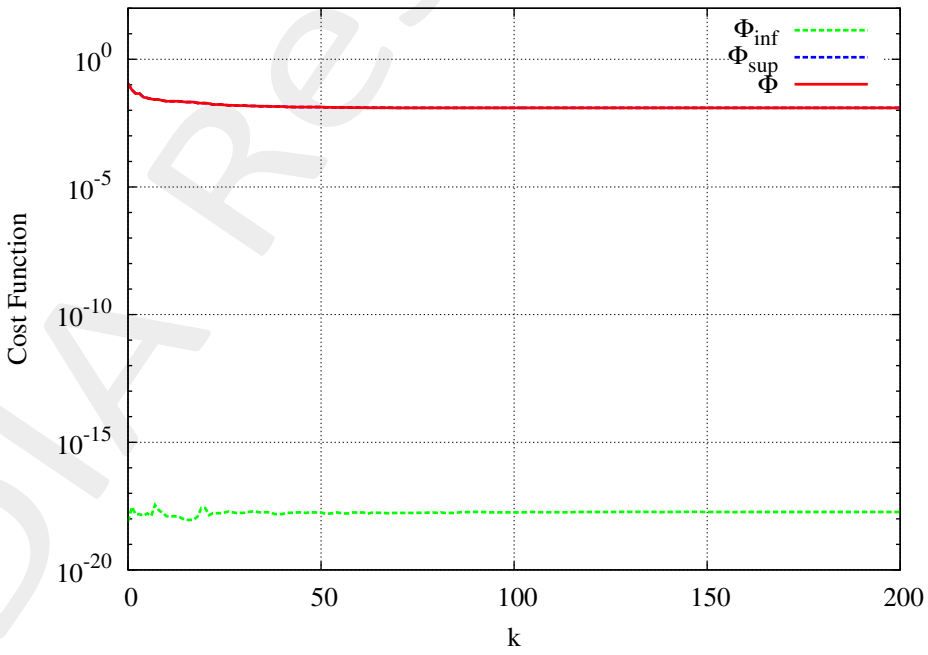

Figure 6.1.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
400	1.2×10^{-2}	6 min. 20 sec.

2.2.4 Synthesized Interval Pattern:

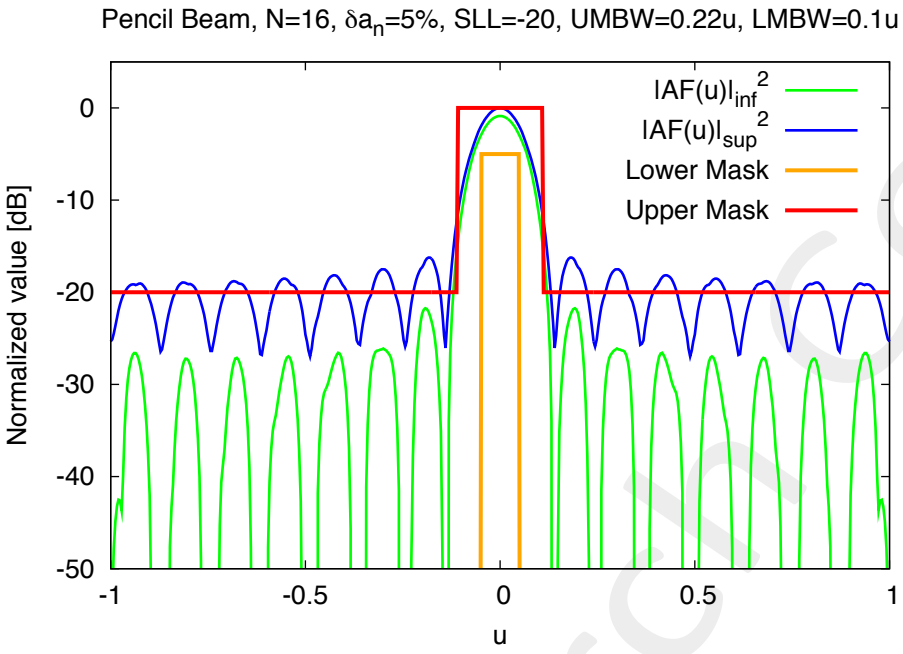

Figure 6.1.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	11.9	-18.8	0.12	-0.42	0.040477
Inf.	11.0	-21.7	0.104	-0.87	/
Sup.	12.8	-15.3	0.12	-0.0	/

Table 6.1.4.1. Interval Pattern Parameters

2.3 Test Case #3 - Pencil Beam - $N = 16$ - $\delta a_n = 8\%$

Array Geometry:

- Number of Elements: $N = 16$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 8%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.22 u
- LMBW: 0.1 u
- SLL: -20 dB
- LM-Height: -5 dB

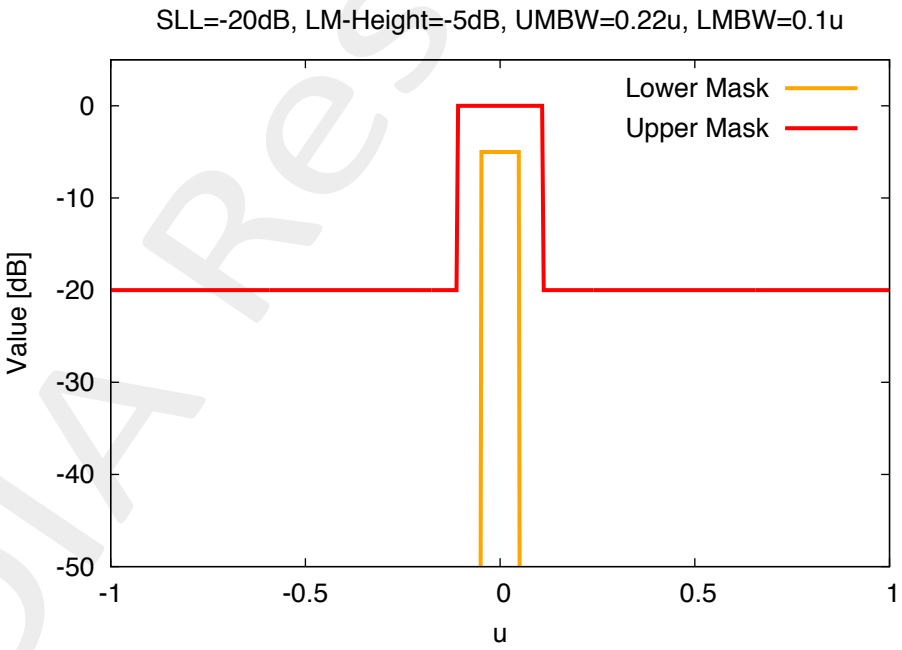

Figure 6.1.1. Power Synthesis Mask

2.3.1 PSO Parameters:

- Unknown Number: 16
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 400
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.1.1. Max and Min excitations amplitudes values, for the PSO

2.3.2 Excitations:

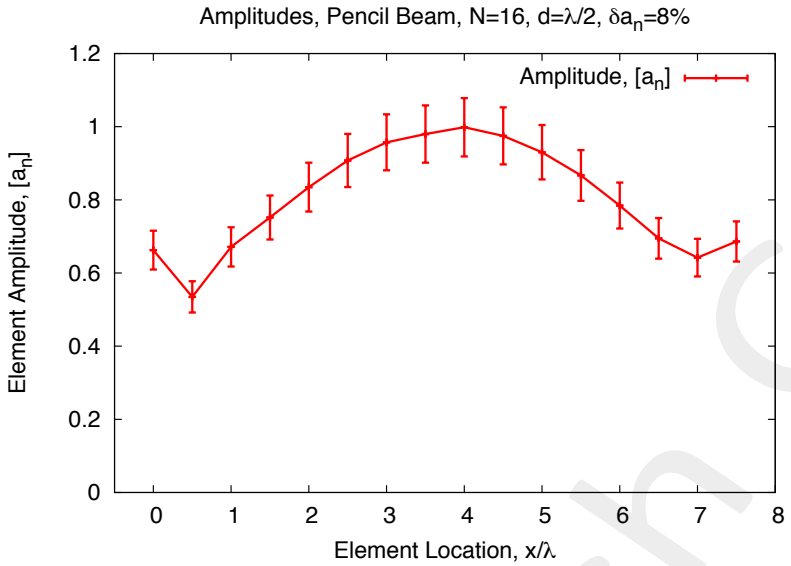

Figure 6.1.2.1. Optimal particle's excitations amplitudes

2.3.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

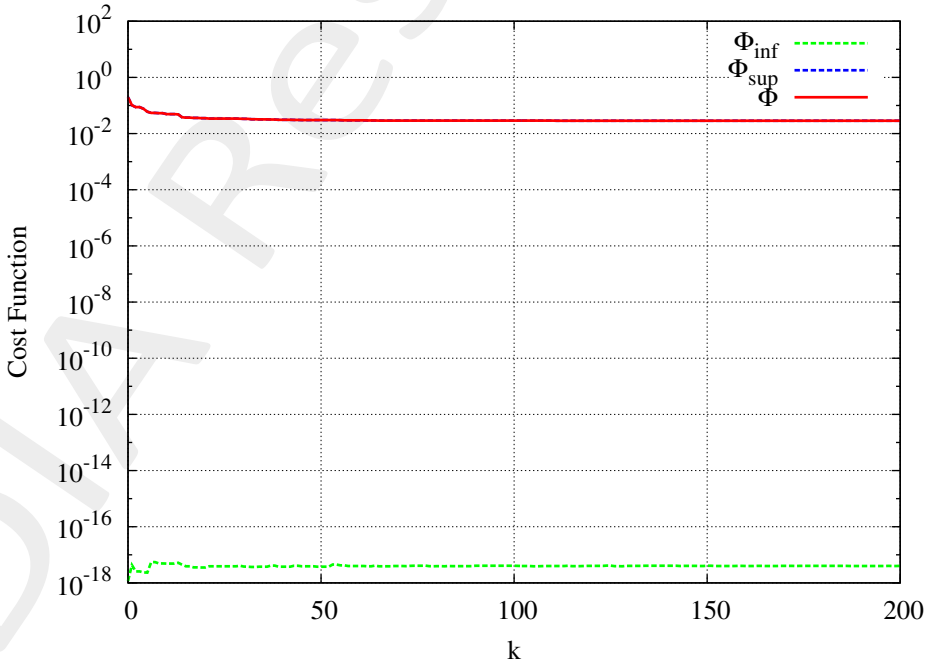

Figure 6.1.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
400	2.8×10^{-3}	6 min. 59 sec.

2.3.4 Synthesized Interval Pattern:

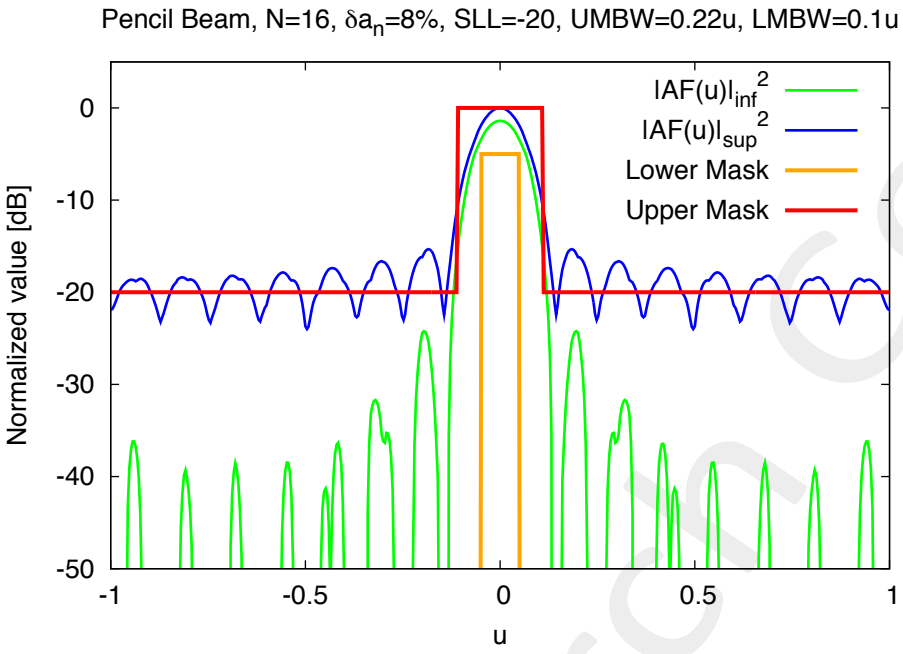

Figure 6.1.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	11.9	-19.2	0.128	-0.67	0.060572
Inf.	10.5	-24.2	0.088	-1.39	/
Sup.	13.3	-13.9	0.152	-0.0	/

Table 6.1.4.1. Interval Pattern Parameters

2.4 Test Case #4 - Pencil Beam - $N = 18$ - $\delta a_n = 1\%$

Array Geometry:

- Number of Elements: $N = 18$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 1%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.22 u
- LMBW: 0.1 u
- SLL: -20 dB
- LM-Height: -5 dB

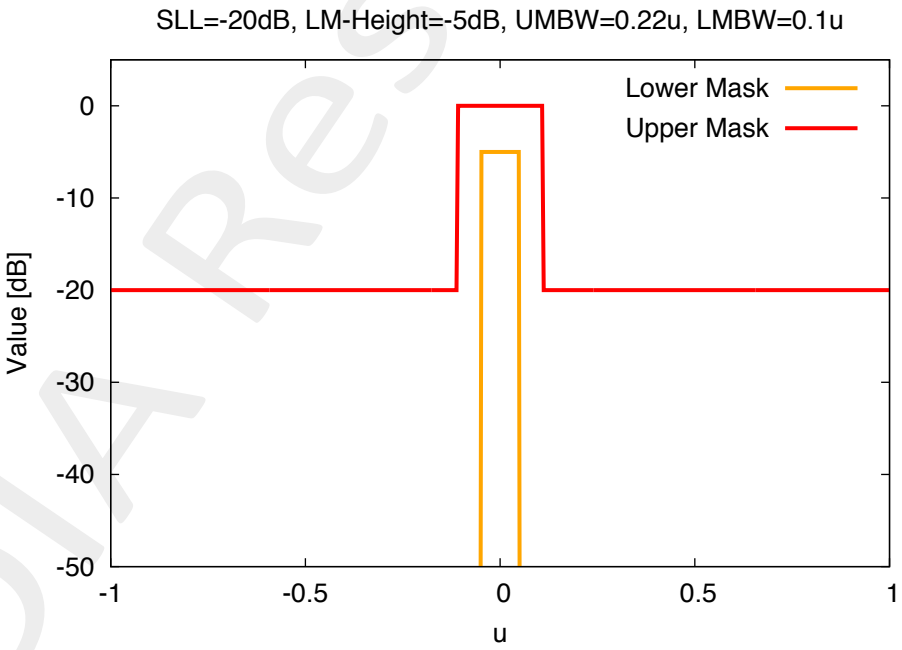

Figure 6.1.1. Power Synthesis Mask

2.4.1 PSO Parameters:

- Unknown Number: 18
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 400
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.1.1. Max and Min excitations amplitudes values, for the PSO

2.4.2 Excitations:

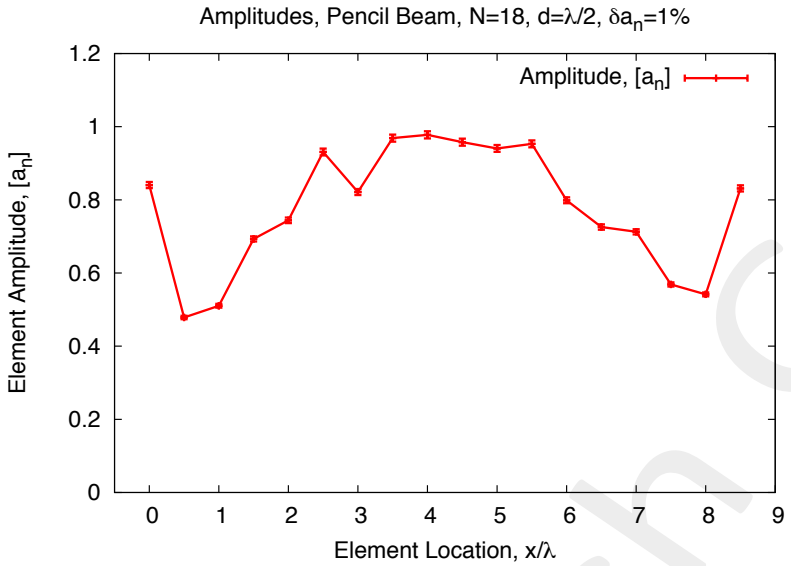

Figure 6.1.2.1. Optimal particle's excitations amplitudes

2.4.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

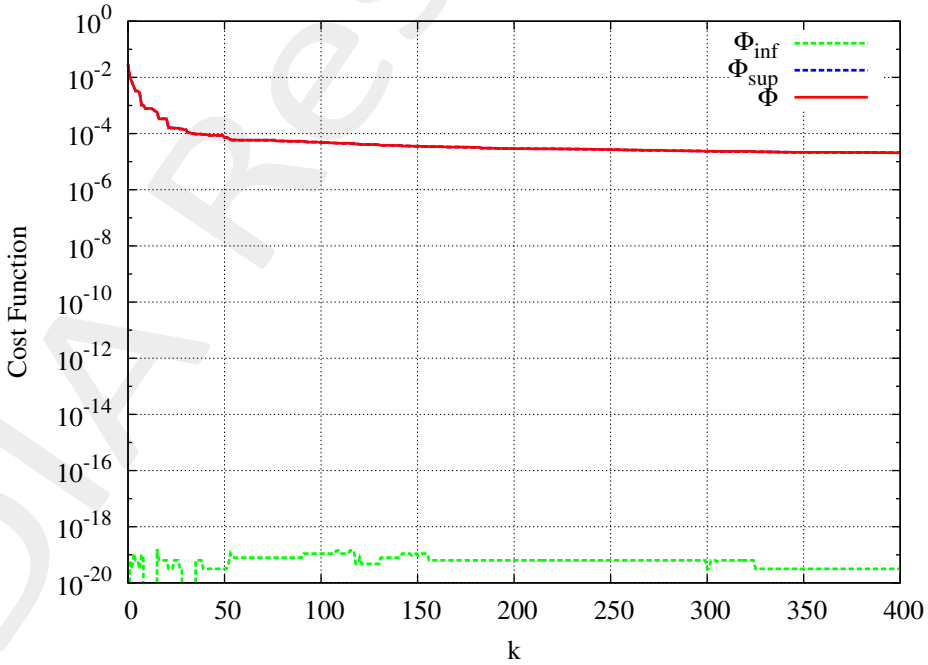

Figure 6.1.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
400	2.1×10^{-5}	6 min. 20 sec.

2.4.4 Synthesized Interval Pattern:

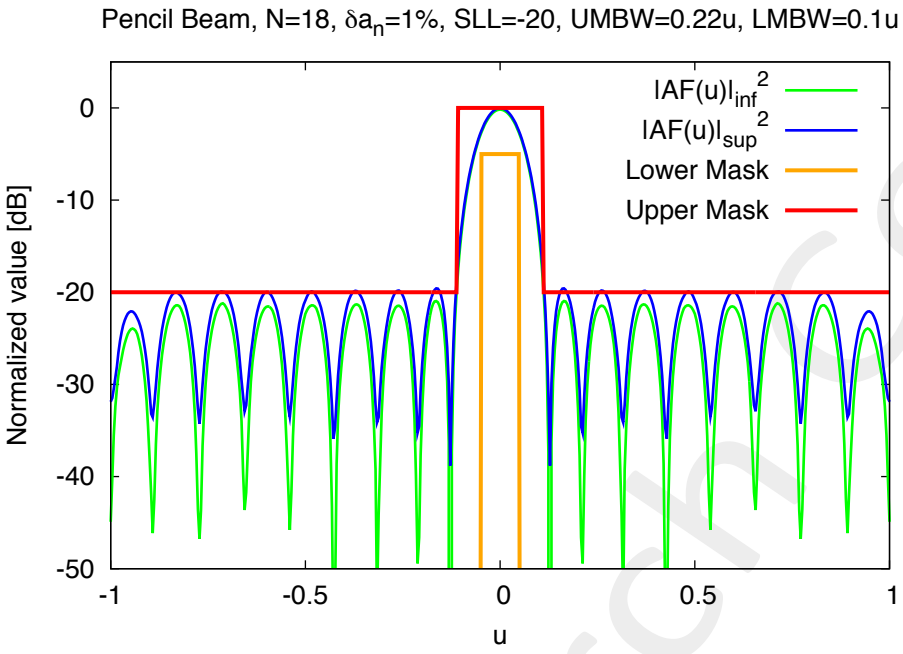

Figure 6.1.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	12.3	-20.2	0.112	-0.086	0.008258
Inf.	12.2	-20.9	0.104	-0.17	/
Sup.	12.5	-19.4	0.112	0.0	/

Table 6.1.4.1. Interval Pattern Parameters

2.5 Test Case #5 - Pencil Beam - $N = 18$ - $\delta a_n = 5\%$

Array Geometry:

- Number of Elements: $N = 18$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 5%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.22 u
- LMBW: 0.1 u
- SLL: -20 dB
- LM-Height: -5 dB

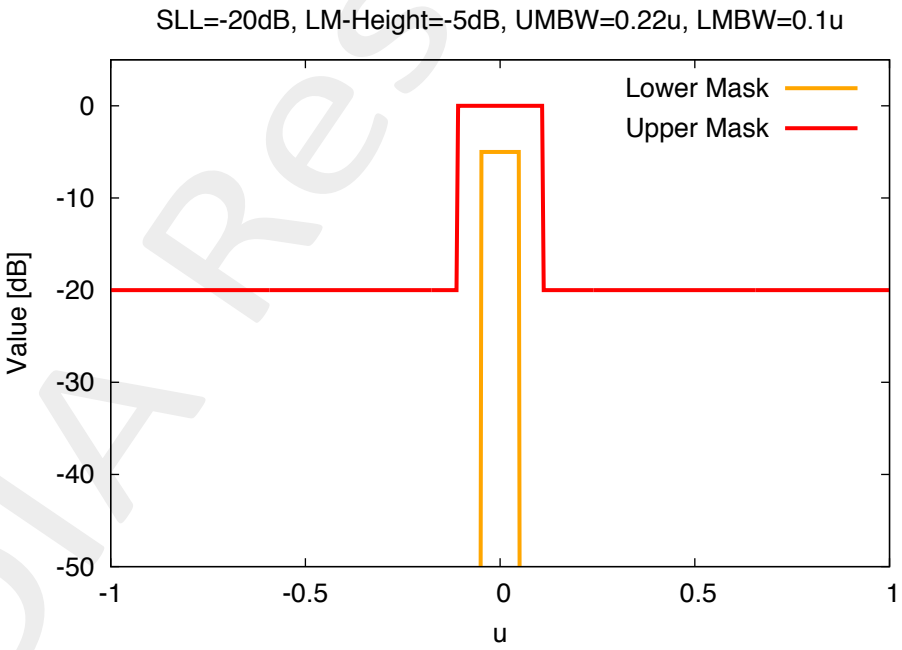

Figure 6.1.1. Power Synthesis Mask

2.5.1 PSO Parameters:

- Unknown Number: 18
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 400
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.1.1. Max and Min excitations amplitudes values, for the PSO

2.5.2 Excitations:

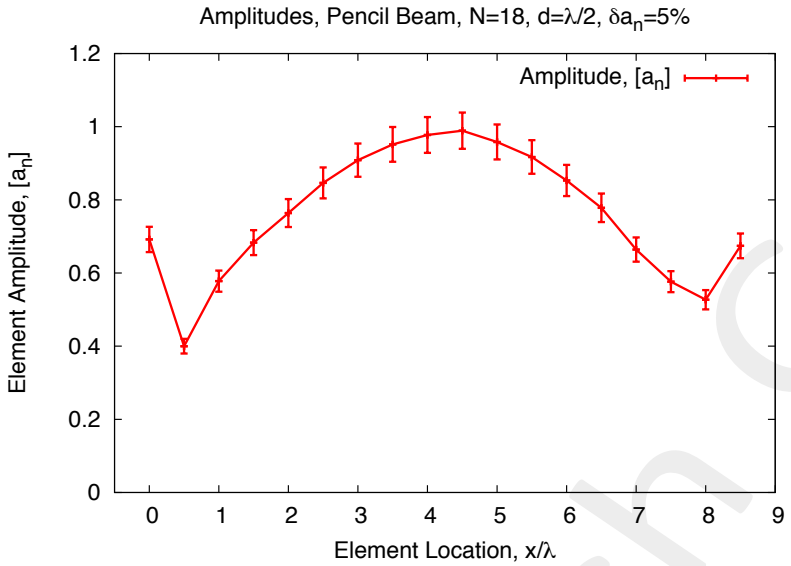

Figure 6.1.2.1. Optimal particle's excitations amplitudes

2.5.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

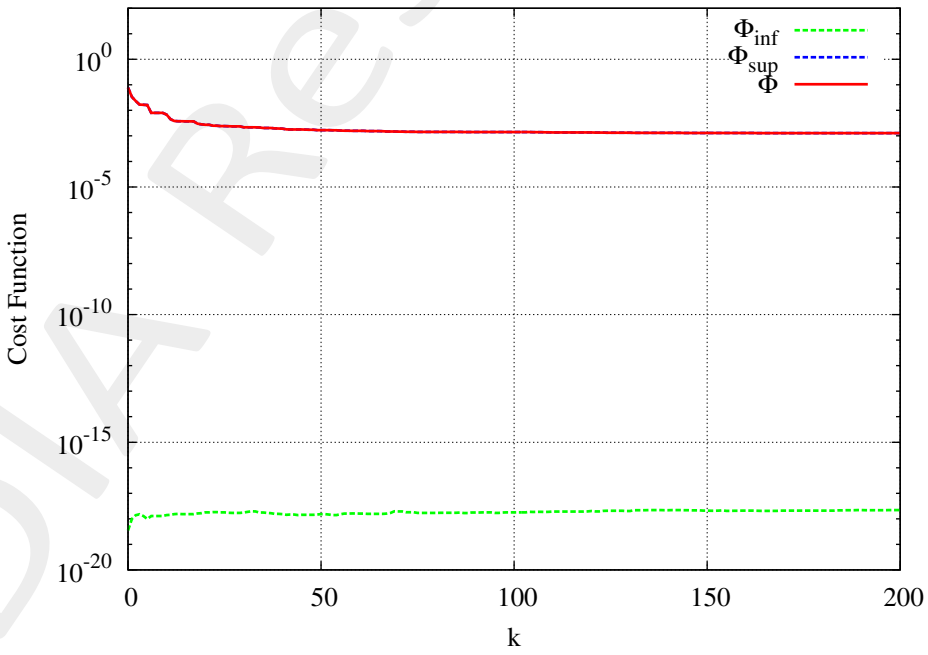

Figure 6.1.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
400	1.2×10^{-3}	6 min. 15 sec.

2.5.4 Synthesized Interval Pattern:

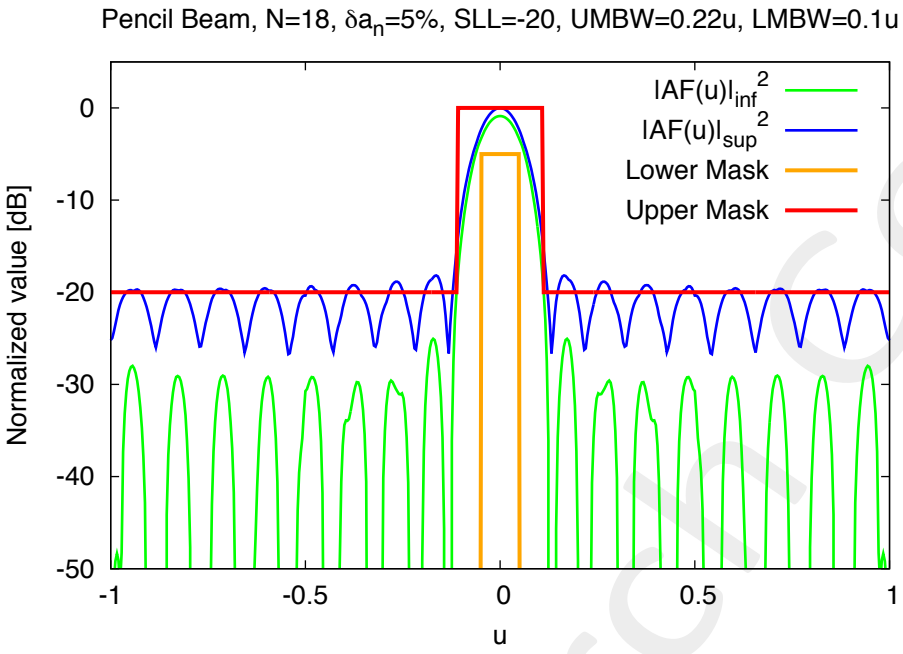

Figure 6.1.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	12.3	-21.3	0.112	-0.42	0.036427
Inf.	11.4	-25	0.096	-0.87	/
Sup.	13.2	-17.3	0.128	0.0	/

Table 6.1.4.1. Interval Pattern Parameters

2.6 Test Case #6 - Pencil Beam - $N = 18$ - $\delta a_n = 8\%$

Array Geometry:

- Number of Elements: $N = 18$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 8%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.22 u
- LMBW: 0.1 u
- SLL: -20 dB
- LM-Height: -5 dB

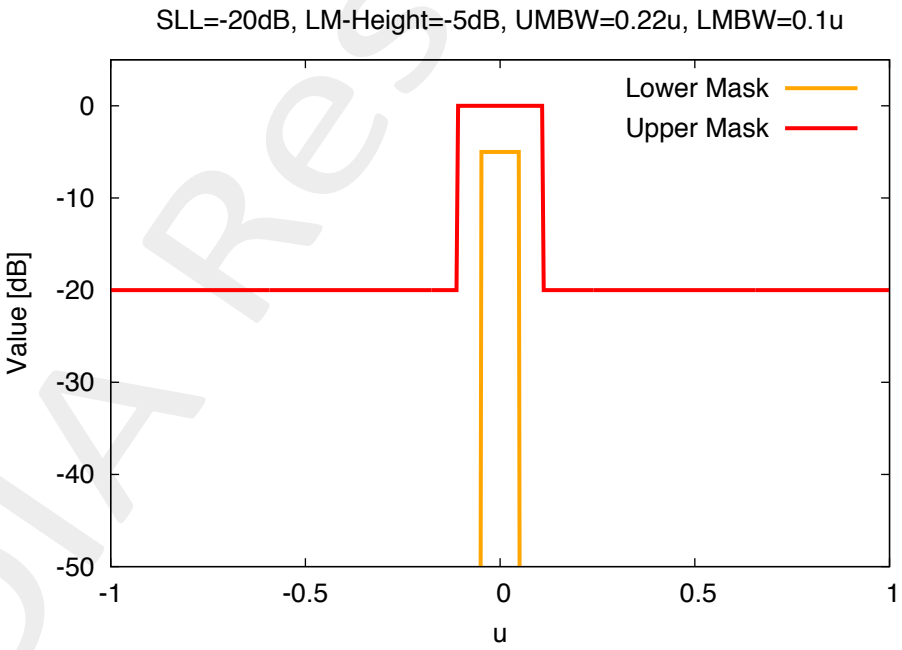

Figure 6.1.1. Power Synthesis Mask

2.6.1 PSO Parameters:

- Unknown Number: 18
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 400
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.1.1. Max and Min excitations amplitudes values, for the PSO

2.6.2 Excitations:

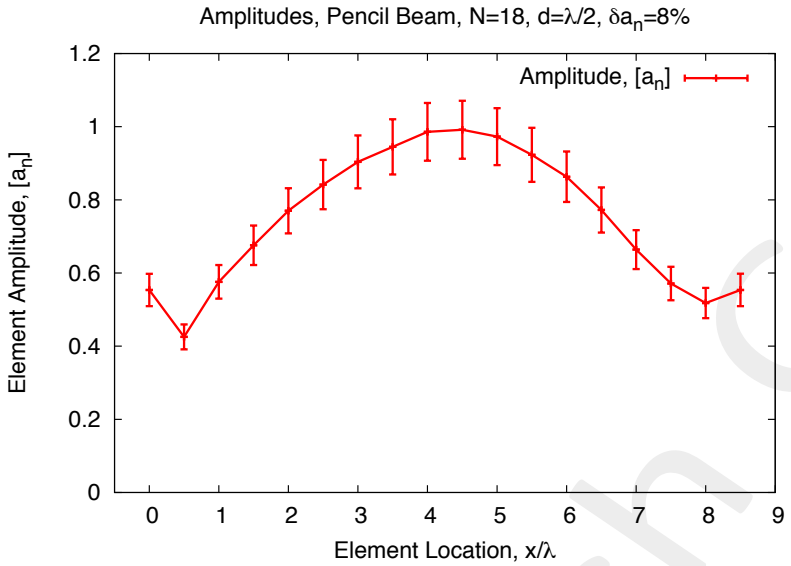

Figure 6.1.2.1. Optimal particle's excitations amplitudes

2.6.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

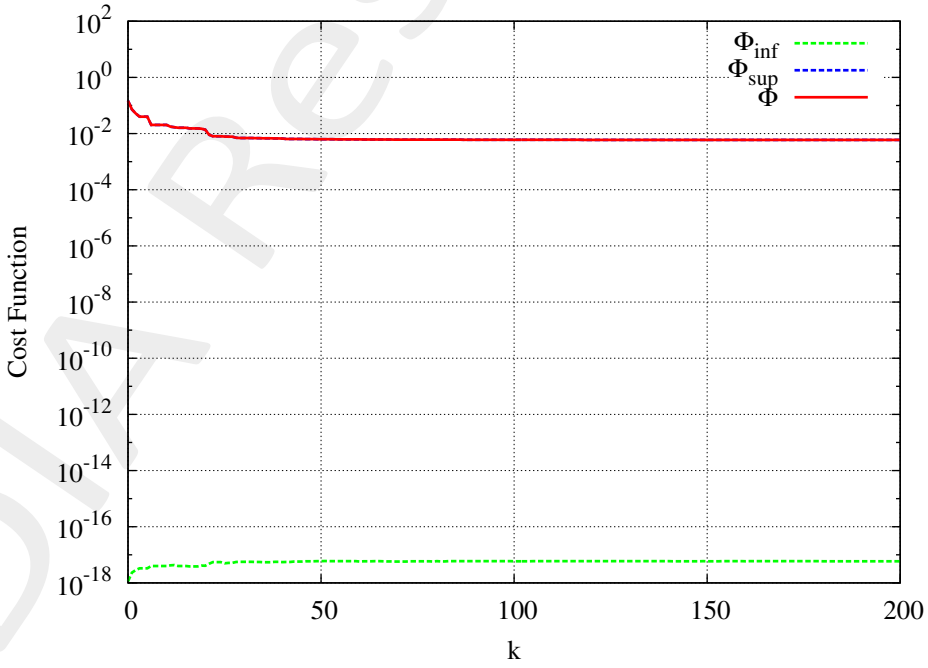

Figure 6.1.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
400	5.9×10^{-3}	6 min. 21 sec.

2.6.4 Synthesized Interval Pattern:

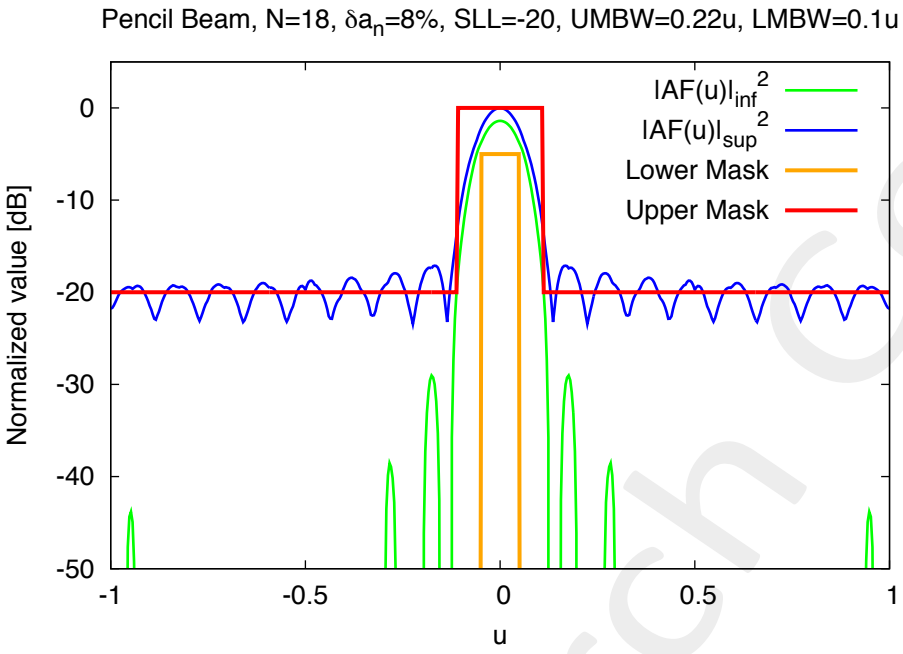

Figure 6.1.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	12.3	-21.8	0.112	-0.67	0.054379
Inf.	10.9	-29	0.088	-1.39	/
Sup.	13.7	-15.7	0.136	0.0	/

Table 6.1.4.1. Interval Pattern Parameters

2.7 Test Case #7 - Pencil Beam - $N = 20$ - $\delta a_n = 1\%$

Array Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 1%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.22 u
- LMBW: 0.1 u
- SLL: -20 dB
- LM-Height: -5 dB

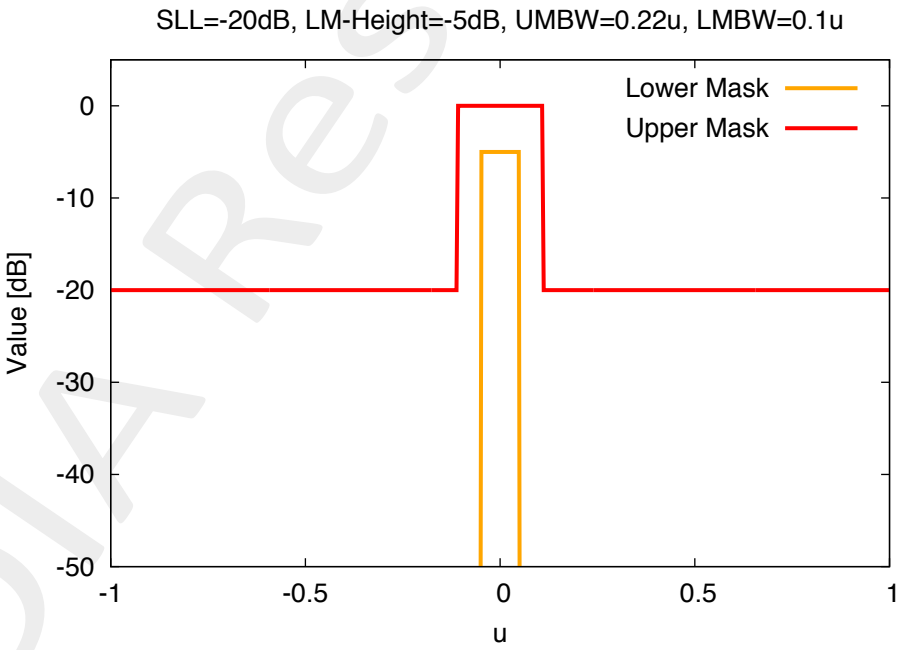

Figure 6.1.1. Power Synthesis Mask

2.7.1 PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 400
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.1.1. Max and Min excitations amplitudes values, for the PSO

2.7.2 Excitations:

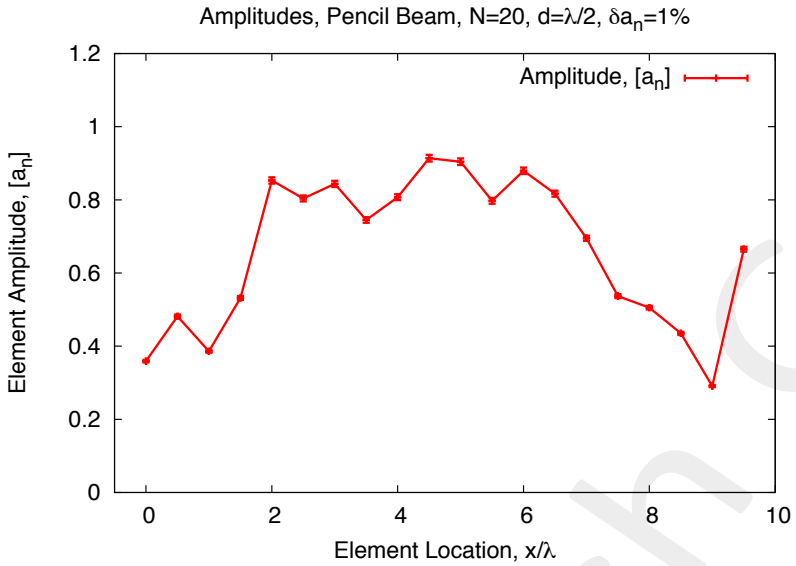

Figure 6.1.2.1. Optimal particle's excitations amplitudes

2.7.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

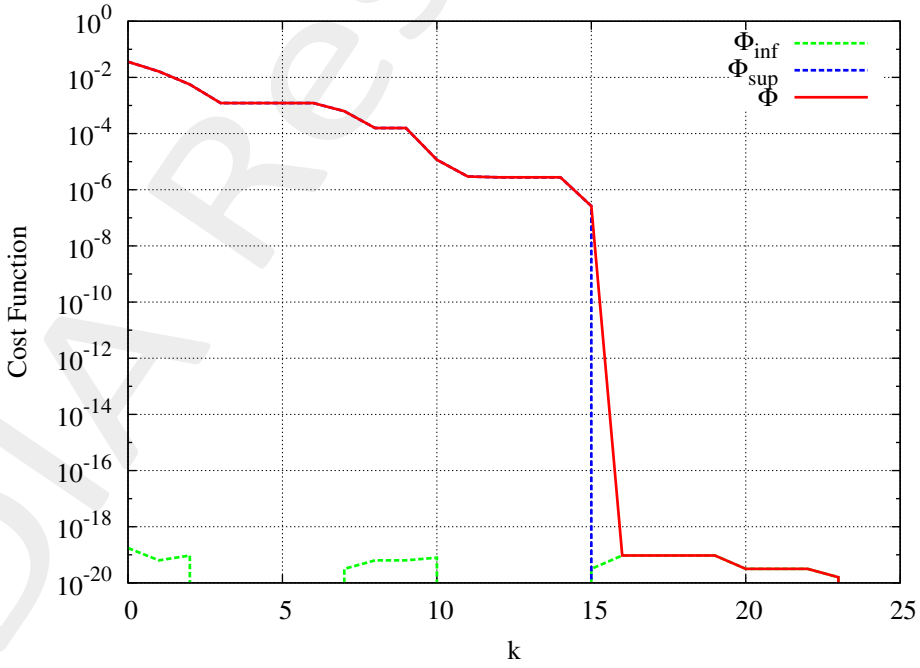

Figure 6.1.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
24	0.0	27 sec.

2.7.4 Synthesized Interval Pattern:

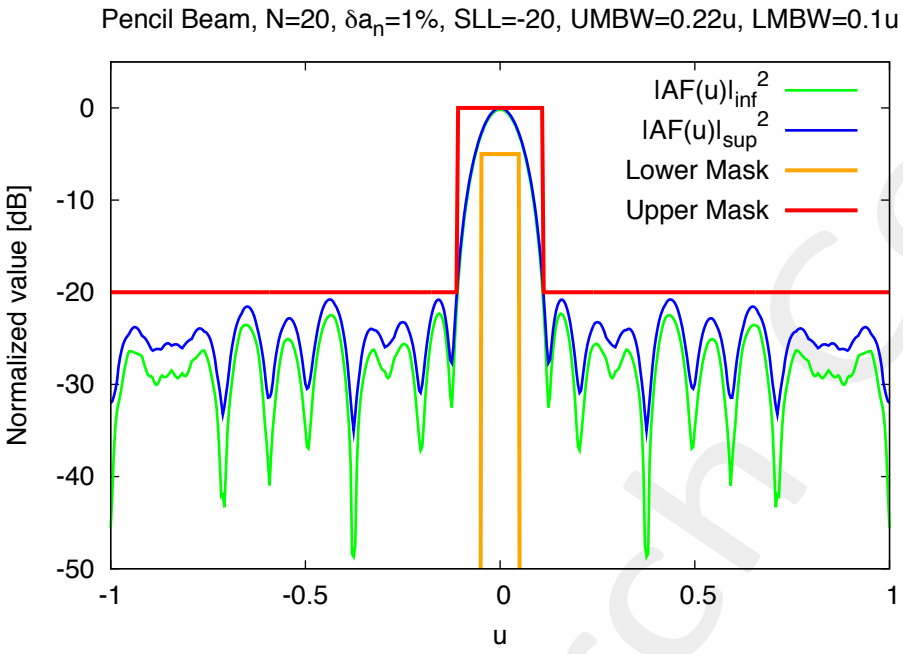

Figure 6.1.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	12.6	-24.3	0.104	-0.086	0.007511
Inf.	12.4	-22.3	0.104	-0.17	/
Sup.	12.8	-20.6	0.104	0.0	/

Table 6.1.4.1. Interval Pattern Parameters

2.8 Test Case #8 - Pencil Beam - $N = 20$ - $\delta a_n = 5\%$

Array Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 5%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.22 u
- LMBW: 0.1 u
- SLL: -20 dB
- LM-Height: -5 dB

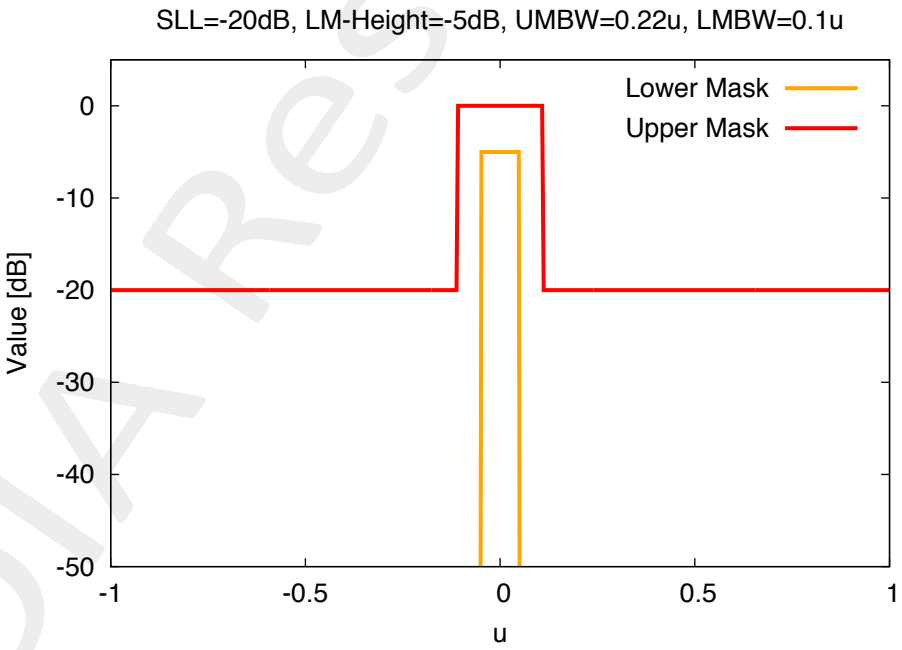

Figure 6.1.1. Power Synthesis Mask

2.8.1 PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 400
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.1.1. Max and Min excitations amplitudes values, for the PSO

2.8.2 Excitations:

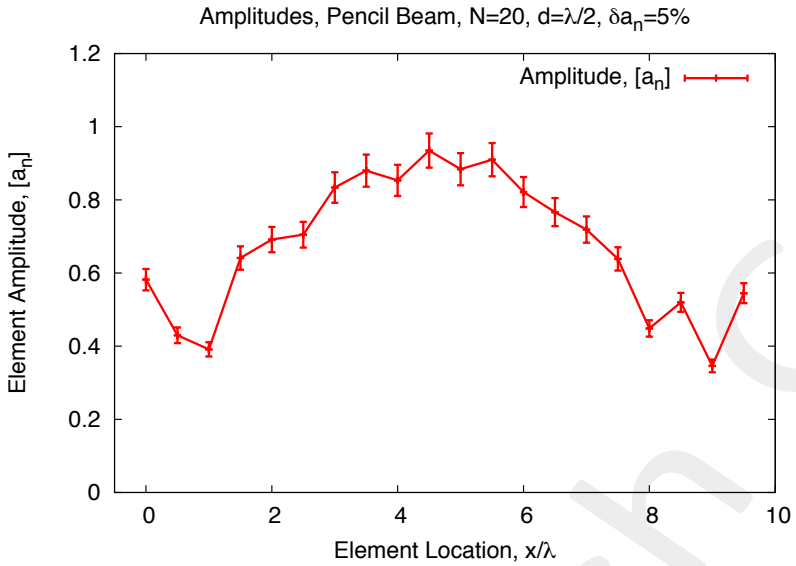

Figure 6.1.2.1. Optimal particle's excitations amplitudes

2.8.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

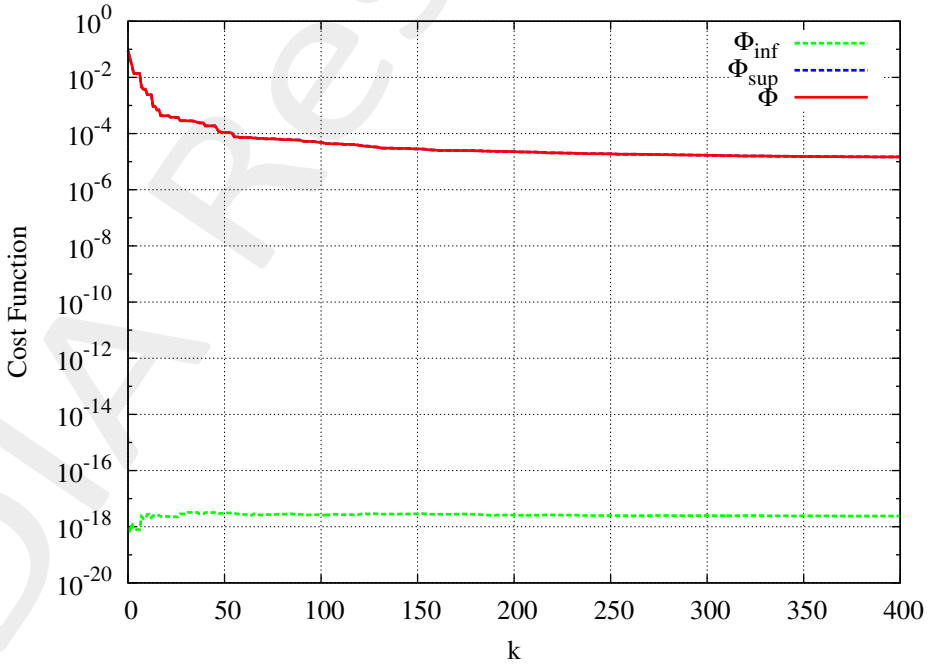

Figure 6.1.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
400	1.4×10^{-5}	7 min. 10 sec.

2.8.4 Synthesized Interval Pattern:

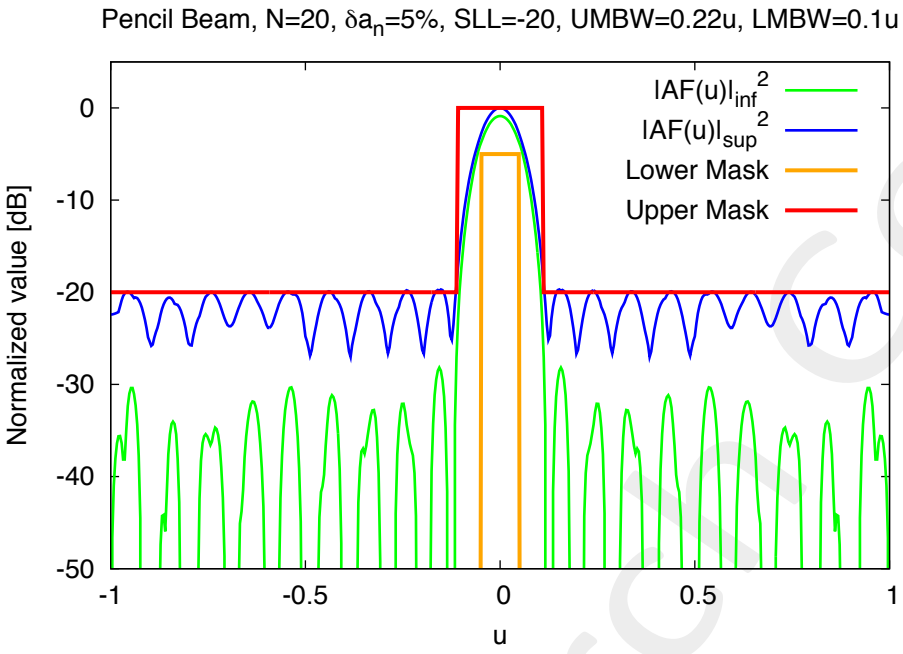

Figure 6.1.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	12.7	-24.5	0.104	-0.42	0.033615
Inf.	15.3	-28.1	0.088	-0.86	/
Sup.	22.8	-18.8	0.12	0.0	/

Table 6.1.4.1. Interval Pattern Parameters

2.9 Test Case #9 - Pencil Beam - $N = 20$ - $\delta a_n = 8\%$

Array Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 8%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.22 u
- LMBW: 0.1 u
- SLL: -20 dB
- LM-Height: -5 dB

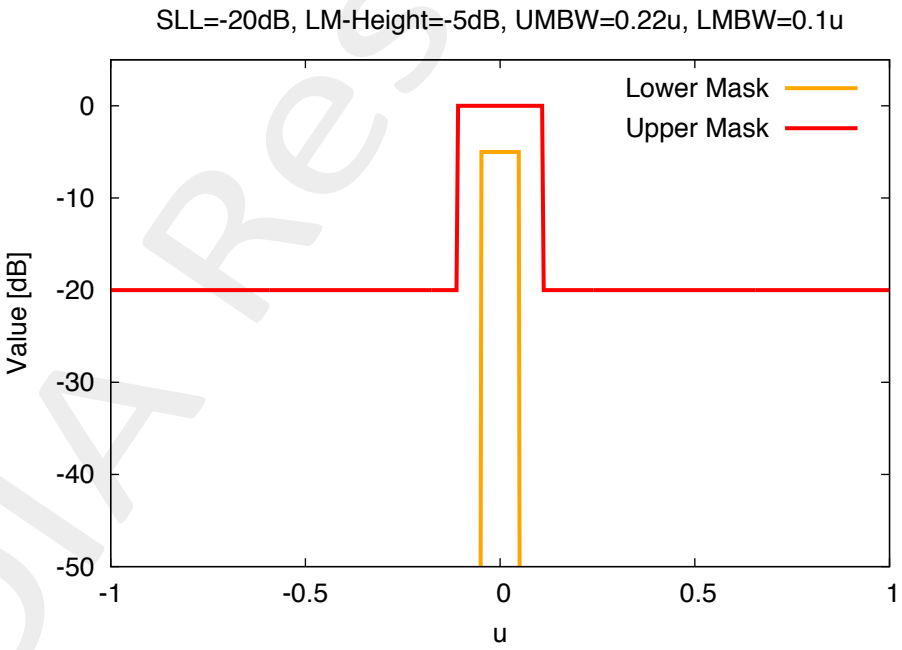

Figure 6.1.1. Power Synthesis Mask

2.9.1 PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 400
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.1.1. Max and Min excitations amplitudes values, for the PSO

2.9.2 Excitations:

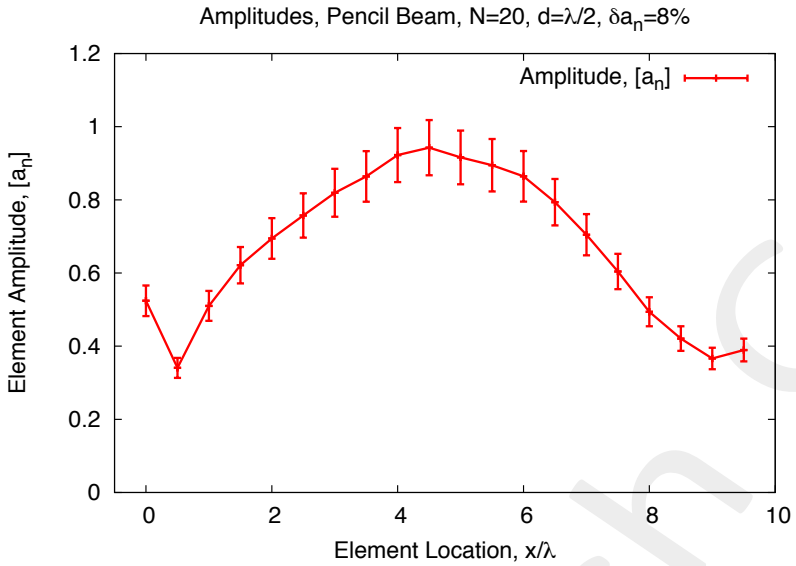

Figure 6.1.2.1. Optimal particle's excitations amplitudes

2.9.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

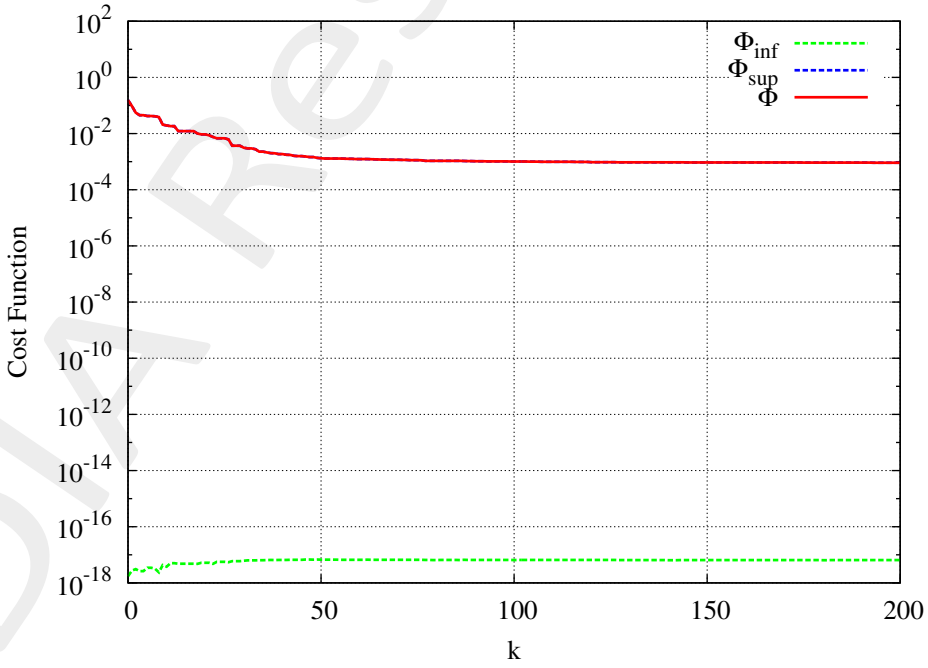

Figure 6.1.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
400	8.9×10^{-4}	6 min. 58 sec.

2.9.4 Synthesized Interval Pattern:

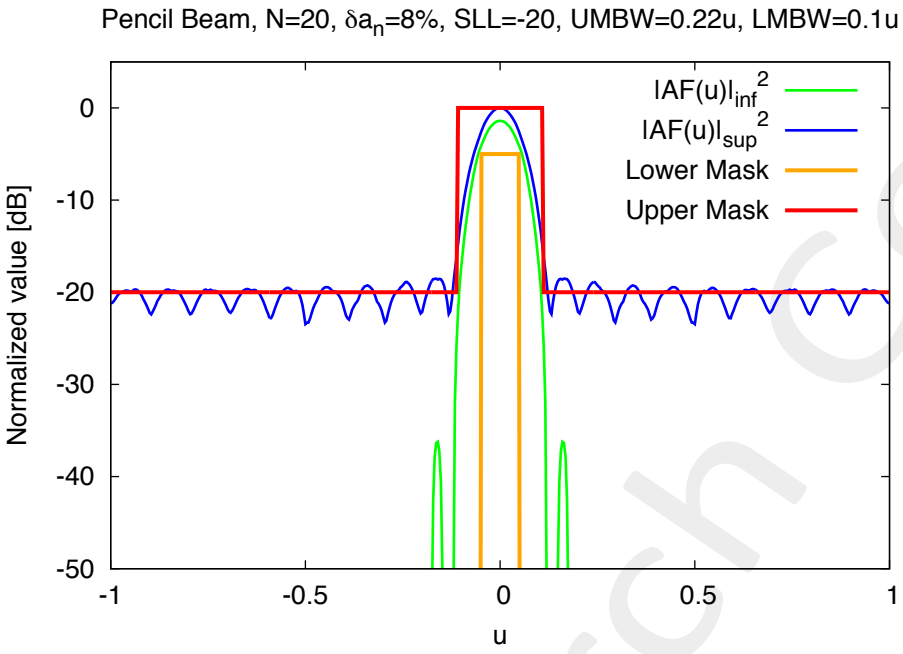

Figure 6.1.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	12.6	-24.3	0.104	-0.66	0.050377
Inf.	13.3	-36.2	0.08	-1.39	/
Sup.	25.3	-17.13	0.104	0.0	/

Table 6.1.4.1. Interval Pattern Parameters

2.10 Test Case #10 - Pencil Beam - $N = 26$ - $\delta a_n = 1\%$

Array Geometry:

- Number of Elements: $N = 26$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 1%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.22 u
- LMBW: 0.1 u
- SLL: -20 dB
- LM-Height: -5 dB

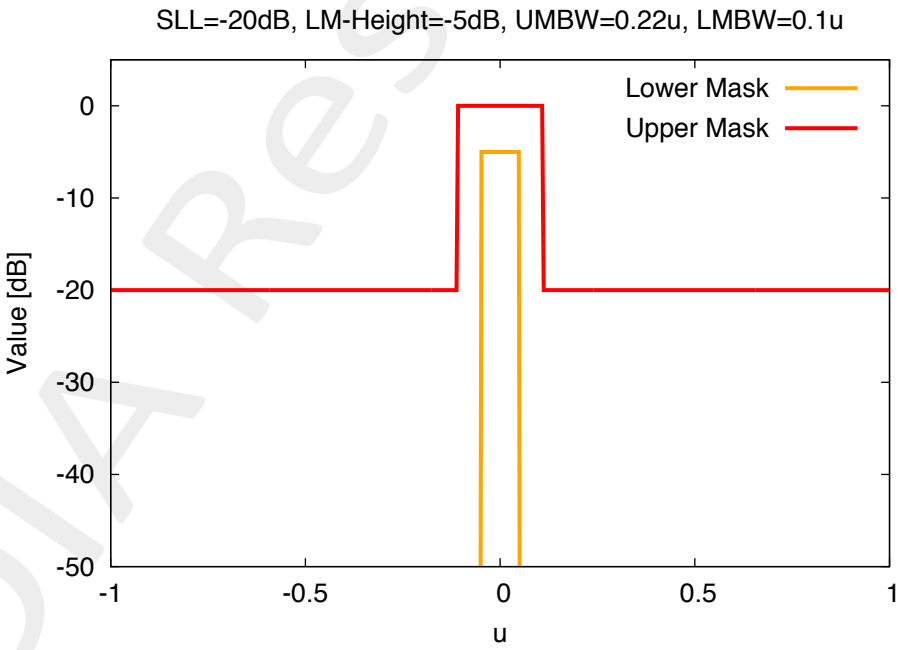

Figure 6.1.1. Power Synthesis Mask

2.10.1 PSO Parameters:

- Unknown Number: 26
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 400
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.1.1. Max and Min excitations amplitudes values, for the PSO

2.10.2 Excitations:

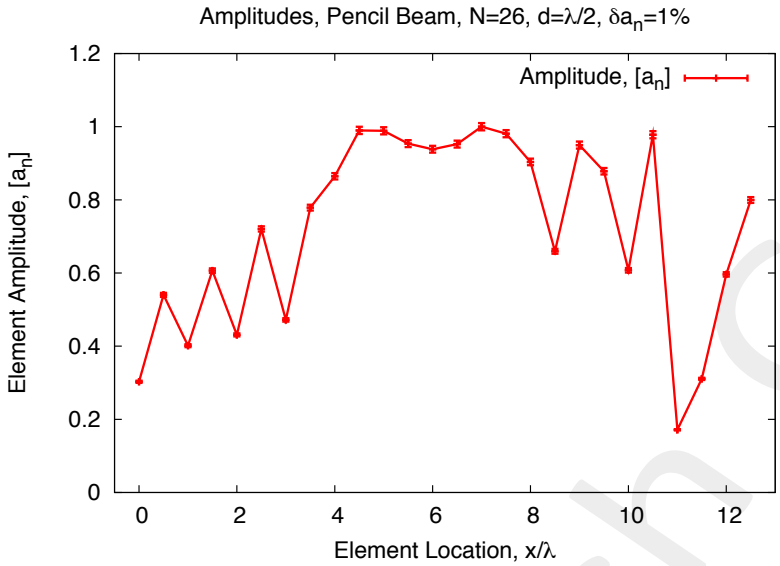

Figure 6.1.2.1. Optimal particle's excitations amplitudes

2.10.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

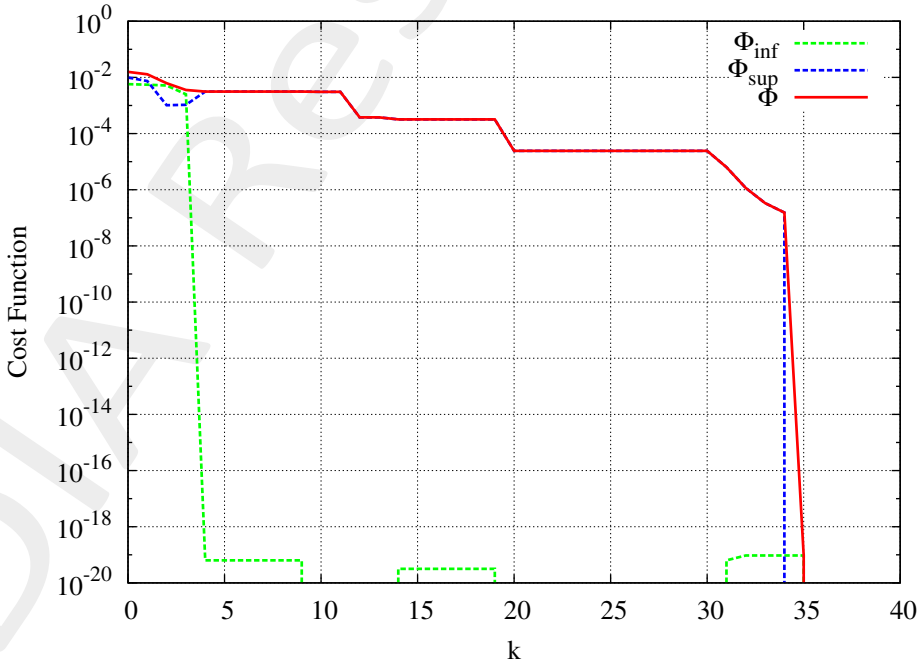

Figure 6.1.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
36	0.0	45 sec.

2.10.4 Synthesized Interval Pattern:

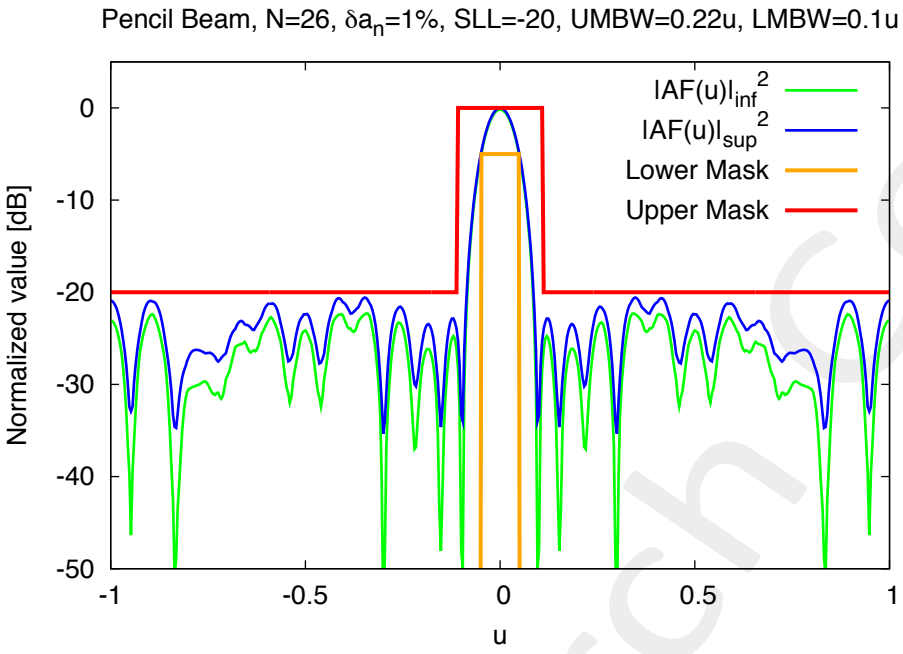

Figure 6.1.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	13.6	-21.5	0.08	-0.086	0.006775
Inf.	13.4	-22.3	0.08	-0.17	/
Sup.	13.8	-20.4	0.8	0.0	/

Table 6.1.4.1. Interval Pattern Parameters

2.11 Test Case #11 - Pencil Beam - $N = 26$ - $\delta a_n = 5\%$

Array Geometry:

- Number of Elements: $N = 26$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 5%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.22 u
- LMBW: 0.1 u
- SLL: -20 dB
- LM-Height: -5 dB

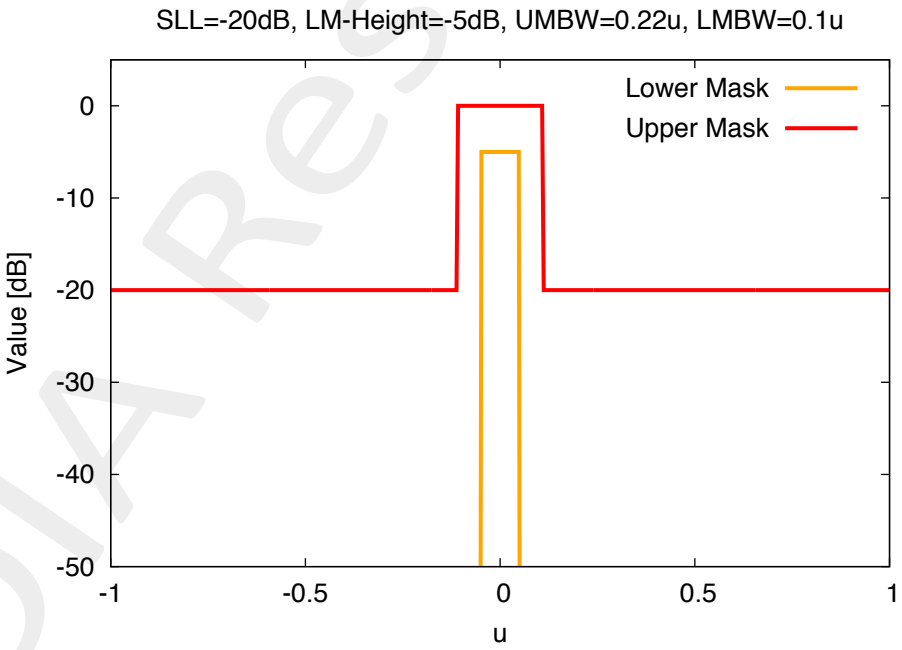

Figure 6.1.1. Power Synthesis Mask

2.11.1 PSO Parameters:

- Unknown Number: 26
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 400
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.1.1. Max and Min excitations amplitudes values, for the PSO

2.11.2 Excitations:

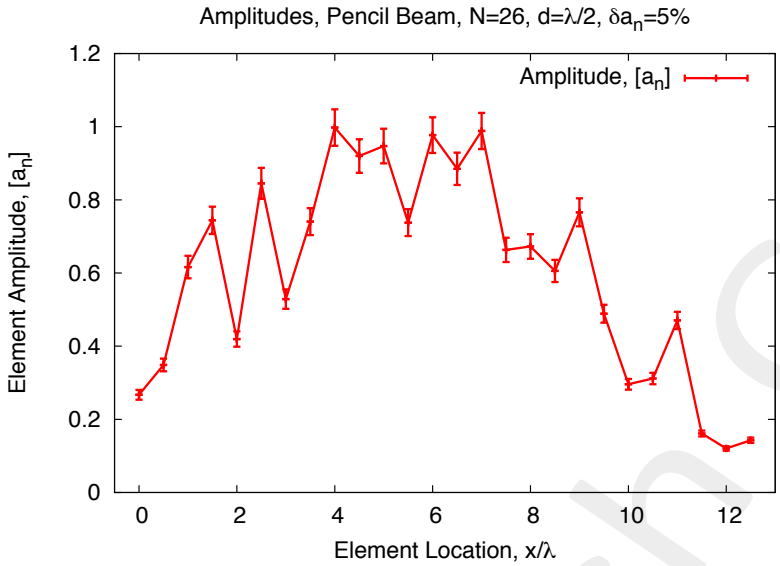

Figure 6.1.2.1. Optimal particle's excitations amplitudes

2.11.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

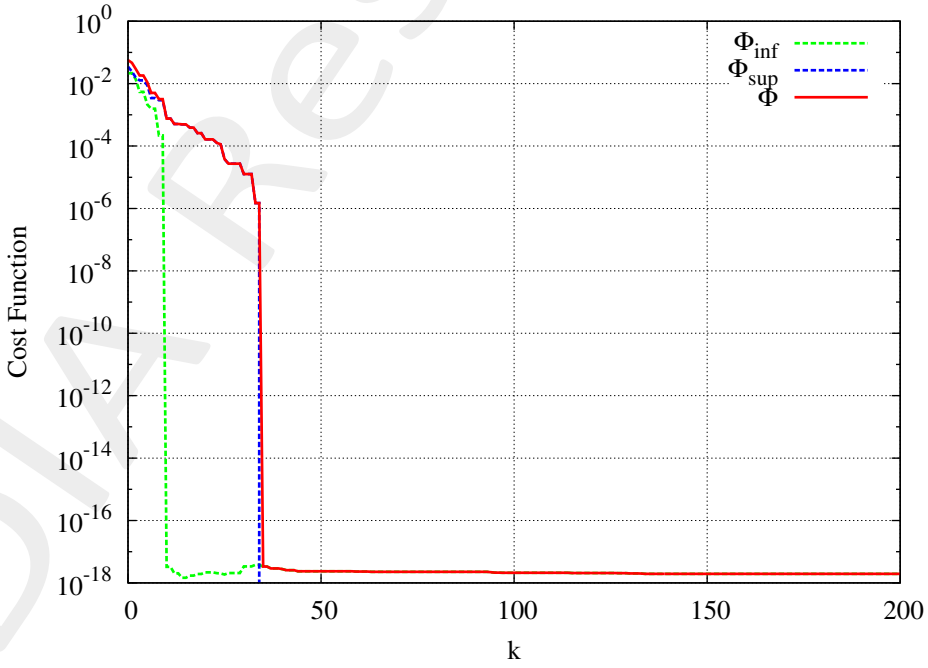

Figure 6.1.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
400	9.9×10^{-19}	8 min. 3 sec.

2.11.4 Synthesized Interval Pattern:

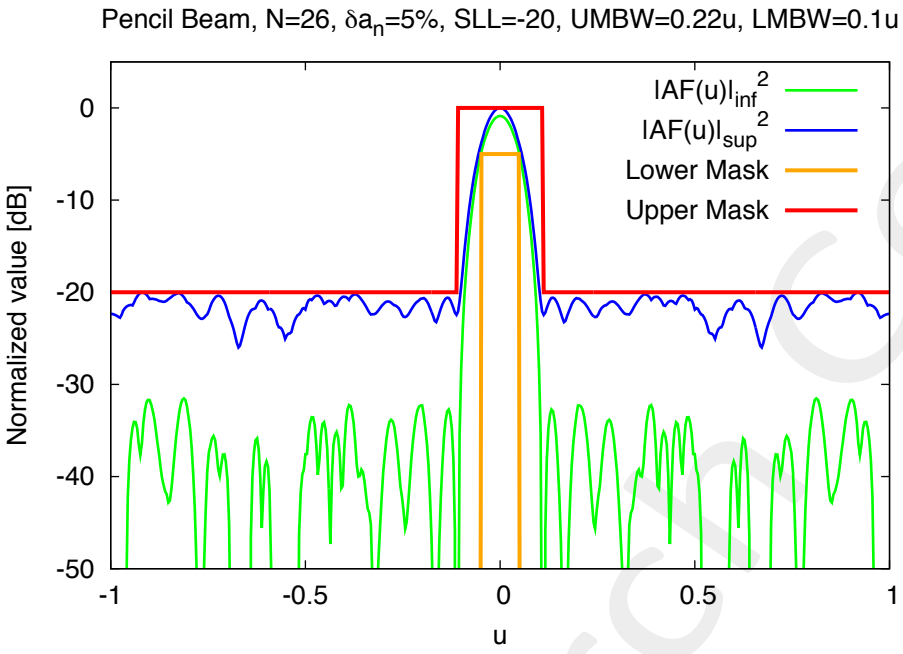

Figure 6.1.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	13.3	-24.8	0.088	-0.42	0.031574
Inf.	12.4	-31.5	0.08	-0.87	/
Sup.	14.2	-19.1	0.104	0.0	/

Table 6.1.4.1. Interval Pattern Parameters

2.12 Test Case #12 - Pencil Beam - $N = 26$ - $\delta a_n = 8\%$

Array Geometry:

- Number of Elements: $N = 26$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 8%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.22 u
- LMBW: 0.1 u
- SLL: -20 dB
- LM-Height: -5 dB

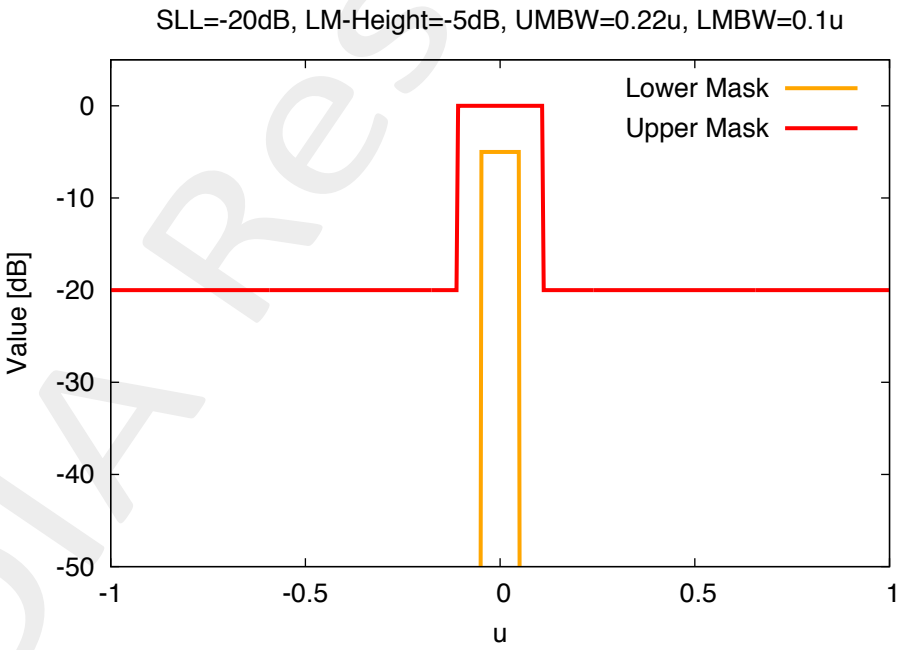

Figure 6.1.1. Power Synthesis Mask

2.12.1 PSO Parameters:

- Unknown Number: 26
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 400
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.1.1. Max and Min excitations amplitudes values, for the PSO

2.12.2 Excitations:

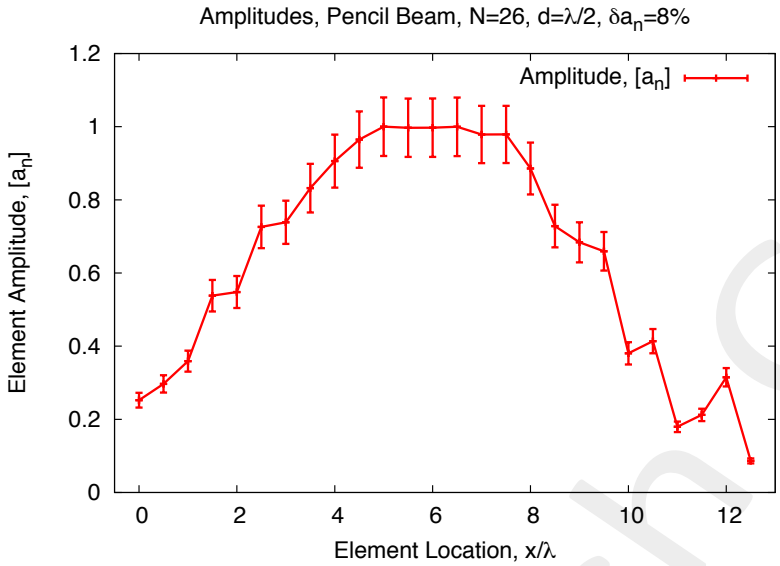

Figure 6.1.2.1. Optimal particle's excitations amplitudes

2.12.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

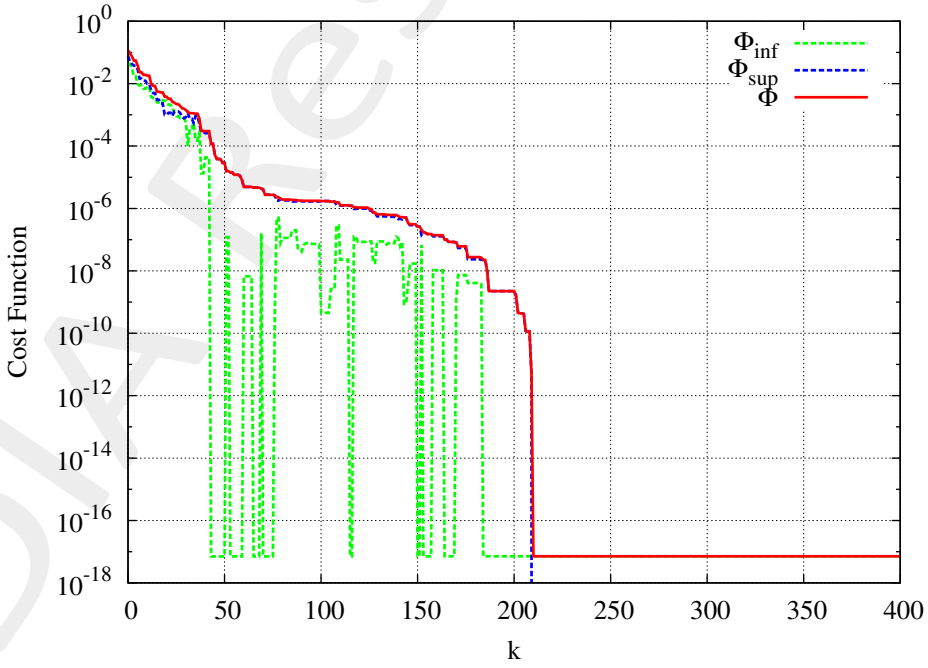

Figure 6.1.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
400	7.8×10^{-18}	8 min.

2.12.4 Synthesized Interval Pattern:

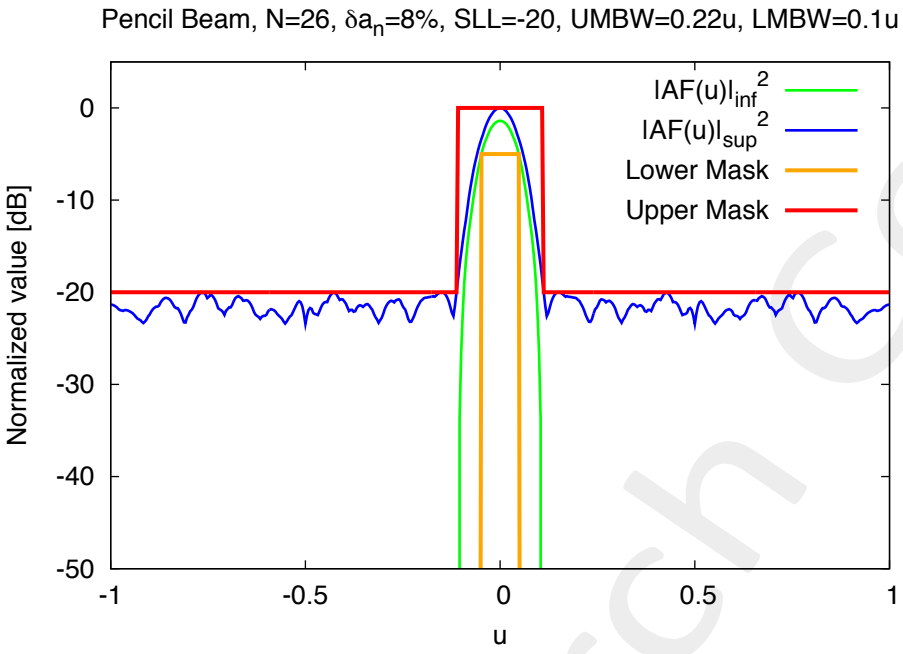

Figure 6.1.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	13.3	-29.1	0.096	-0.66	0.043039
Inf.	11.9	$-\infty$	0.072	-1.39	/
Sup.	14.6	-18.6	0.096	0.0	/

Table 6.1.4.1. Interval Pattern Parameters

3 Simulation Results

Final Fitness value (Φ) for $N = 16, 18, 20, 26$

1. Each test case is here considered with $\delta a_n = 1\%, 5\%, 8\%$

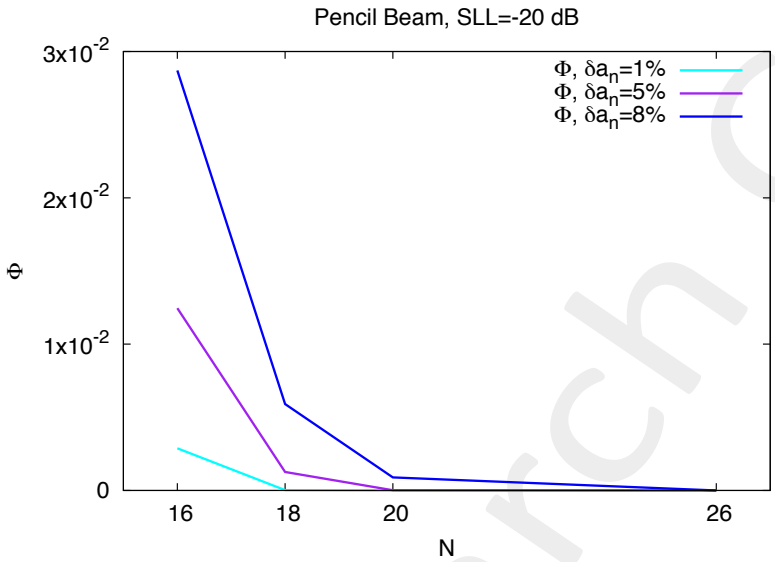

Figure 10.2. Φ vs N for each group of test cases

	$N = 16$	$N = 18$	$N = 20$	$N = 26$
$\delta a_n = 1\%$	2.8×10^{-3}	2.1×10^{-5}	0.0	0.0
$\delta a_n = 5\%$	1.2×10^{-2}	1.3×10^{-3}	1.4×10^{-5}	9.9×10^{-19}
$\delta a_n = 8\%$	2.8×10^{-2}	5.9×10^{-3}	8.9×10^{-4}	7.1×10^{-18}

Table 7.2. Φ values plotted in Fig.10.2

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