Interval-based Pencil Beam Pattern Synthesis in Linear Antenna Arrays

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Abstract

An innovative strategy for the robust design of linear antenna arrays is analyzed in this report. Being the array elements characterised by tolerance errors, the synthesis is aimed at determining the intervals of values fitting the user-defined mask constraints on the radiated power pattern. With reference to the upper and lower bounds of the power pattern analytically determined for given tolerances through interval analysis, the nominal excitations of the array elements are then optimised by means of a global stochastic optimiser suitably customised to deal with interval numbers.

1 Pencil Beam Patterns

In the following test cases the robustness of a PSO based synthesis is investigated. The SLL and the LM-Height are fixed. Every time the number of elements is raised, UMBW is reduced. The LMBW is chosen as half of the UMBW value.

Fixed Parameters:

- SLL = -20 dB
- LM-Height = -5 dB
- $LMBW = \frac{UMBW}{2}$

Simulations:

1. $N = 10$, UMBW= 0.46 u, LMBW = 0.23 u
   - $\delta a_n = 1\%$
   - $\delta a_n = 5\%$
   - $\delta a_n = 10\%$
2. $N = 20$, UMBW = 0.38 u, LMBW= 0.19 u
   - $\delta a_n = 1\%$
   - $\delta a_n = 5\%$
   - $\delta a_n = 10\%$
3. $N = 40$, UMBW = 0.25 u, LMBW= 0.125 u
   - $\delta a_n = 1\%$
   - $\delta a_n = 5\%$
   - $\delta a_n = 10\%$
4. $N = 100$, UMBW = 0.11 u, LMBW= 0.055 u
   - $\delta a_n = 1\%$
   - $\delta a_n = 5\%$
   - $\delta a_n = 10\%$

1.1 Test Case #2 - Pencil Beam - $N = 10$ - $\delta a_n = 5\%$

Array Geometry:

- Number of Elements: $N = 10$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 5%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.46 u
- LMBW: 0.23 u
- SLL: -20 dB
- LM-Height: -5 dB

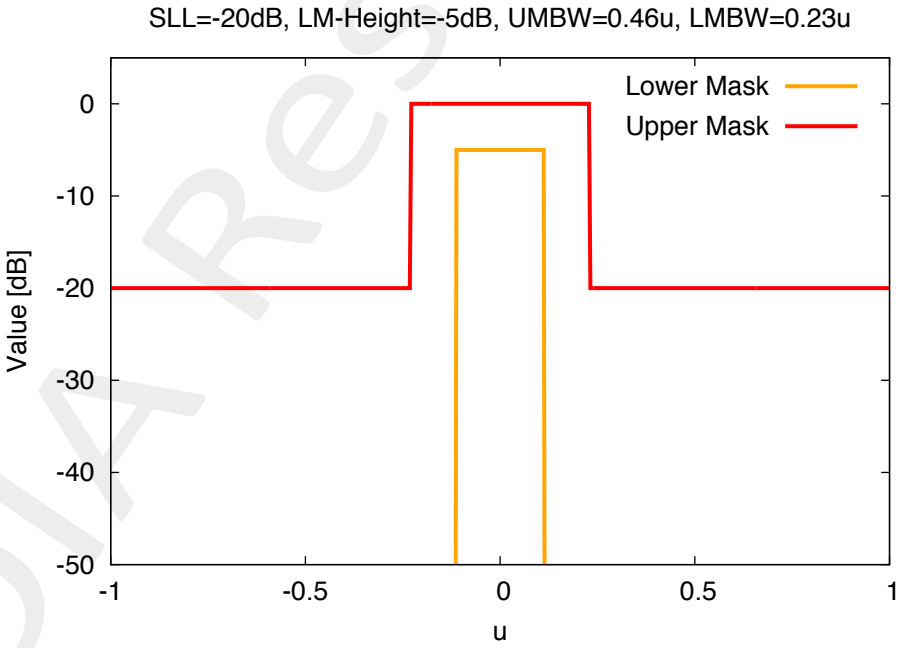

Figure 6.2.1. Power Synthesis Mask

1.1.1 PSO Parameters:

- Unknown Number: 10
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 200
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.2.1.1. Max and Min excitations amplitudes values, for the PSO

1.1.2 Excitations:

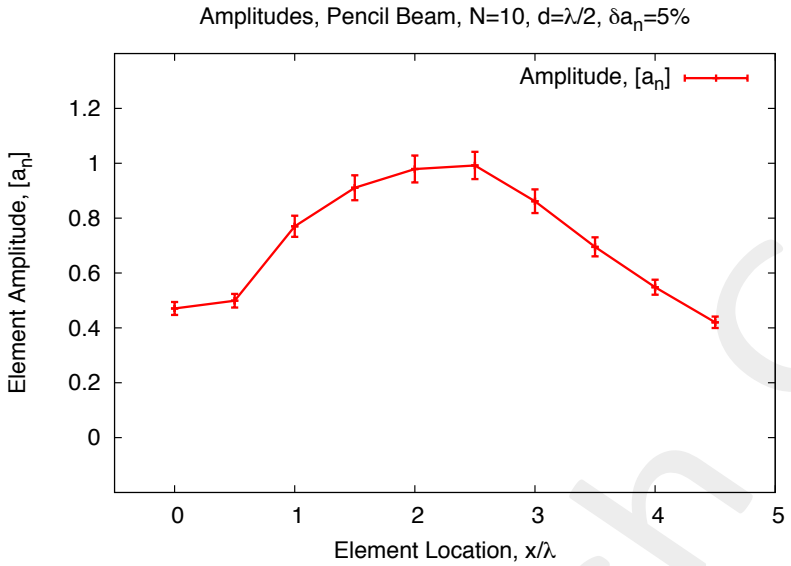

Figure 6.2.2.1. Optimal particle's excitations amplitudes

1.1.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

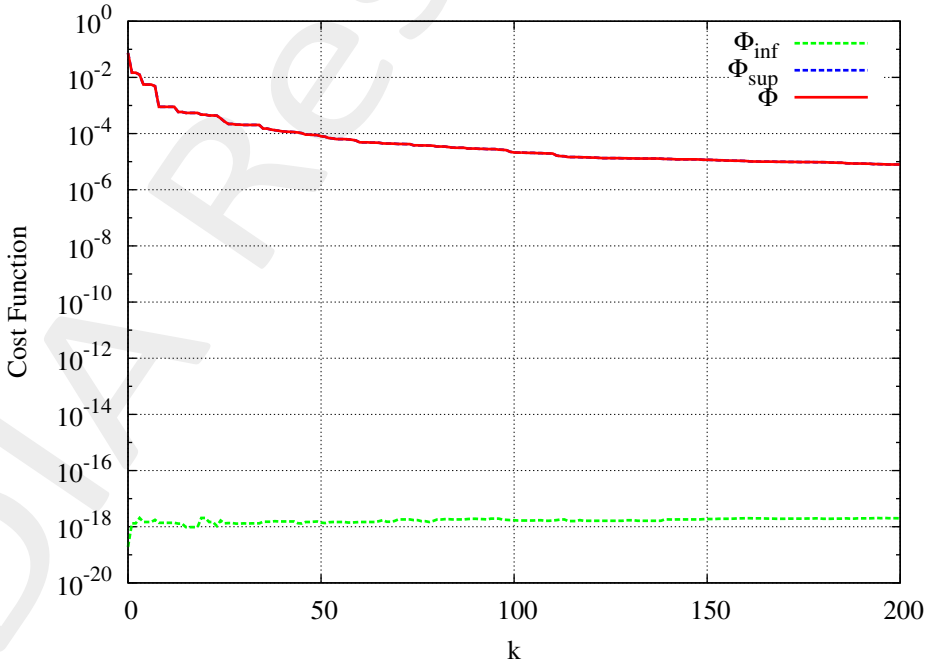

Figure 6.2.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
200	7.9×10^{-6}	5 min

1.1.4 Synthesized Interval Pattern:

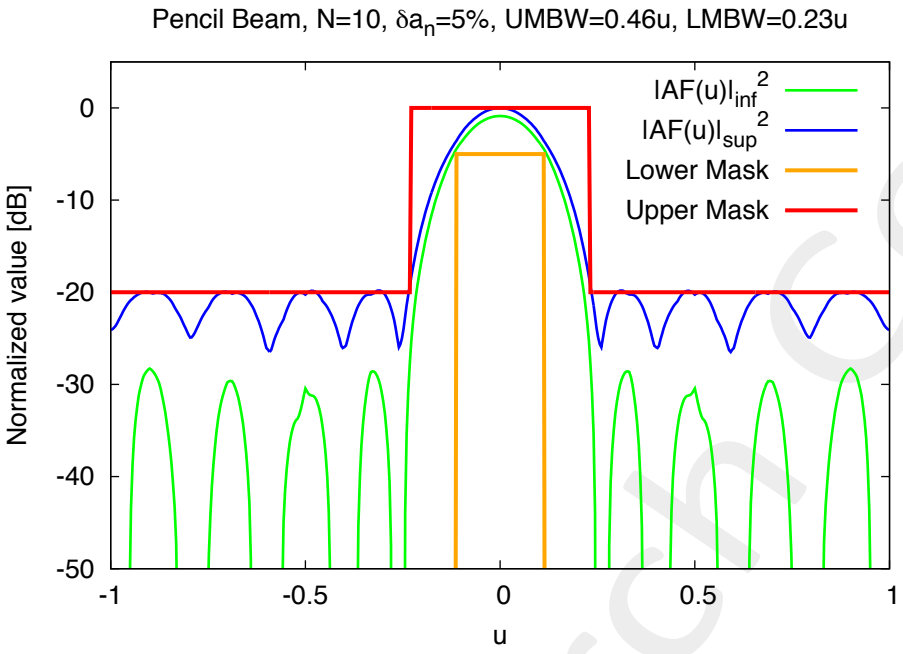

Figure 6.1.5. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	9.6	-23.7	0.208	-0.42	0.055627
Inf.	8.7	-28.2	0.208	-0.87	/
Sup.	10.5	-19.8	0.24	0.0	/

Table 6.1.5. Interval Pattern Parameters

1.2 Test Case #3 - Pencil Beam - $N = 10$ - $\delta a_n = 10\%$

Array Geometry:

- Number of Elements: $N = 10$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 10%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.46 u
- LMBW: 0.23 u
- SLL: -20 dB
- LM-Height: -5 dB

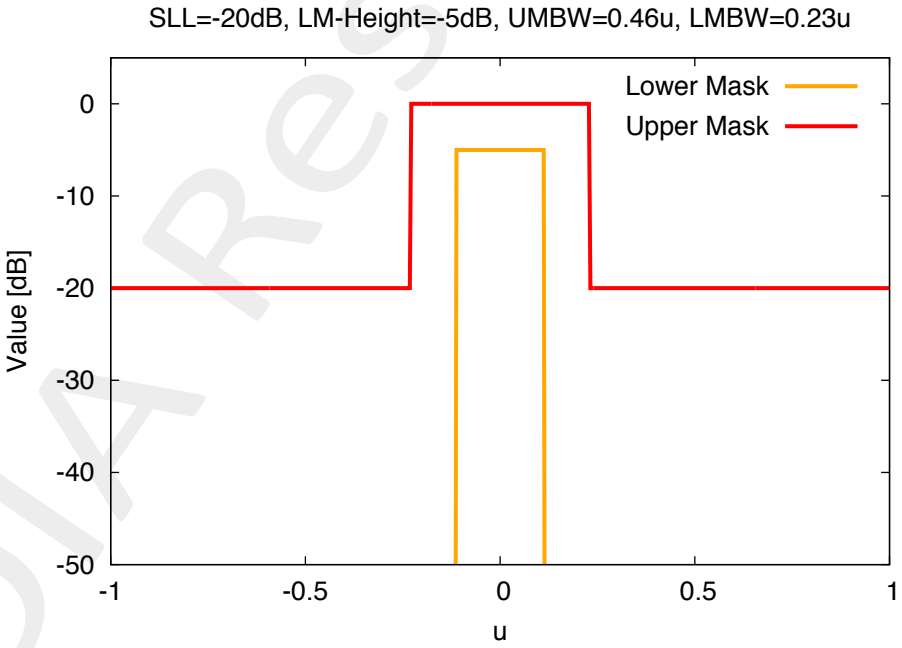

Figure 6.3.1. Power Synthesis Mask

1.2.1 PSO Parameters:

- Unknown Number: 10
- Swarm Dimension: 20
- Random Seed: 11
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 500
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.3.1.1. Max and Min excitations amplitudes values, for the PSO

1.2.2 Excitations:

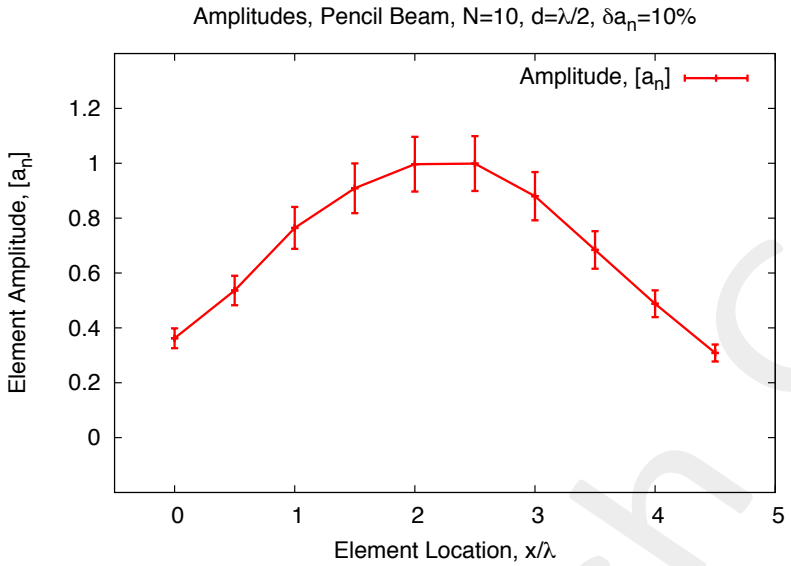

Figure 6.3.2.1. Optimal particle's excitations amplitudes

1.2.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

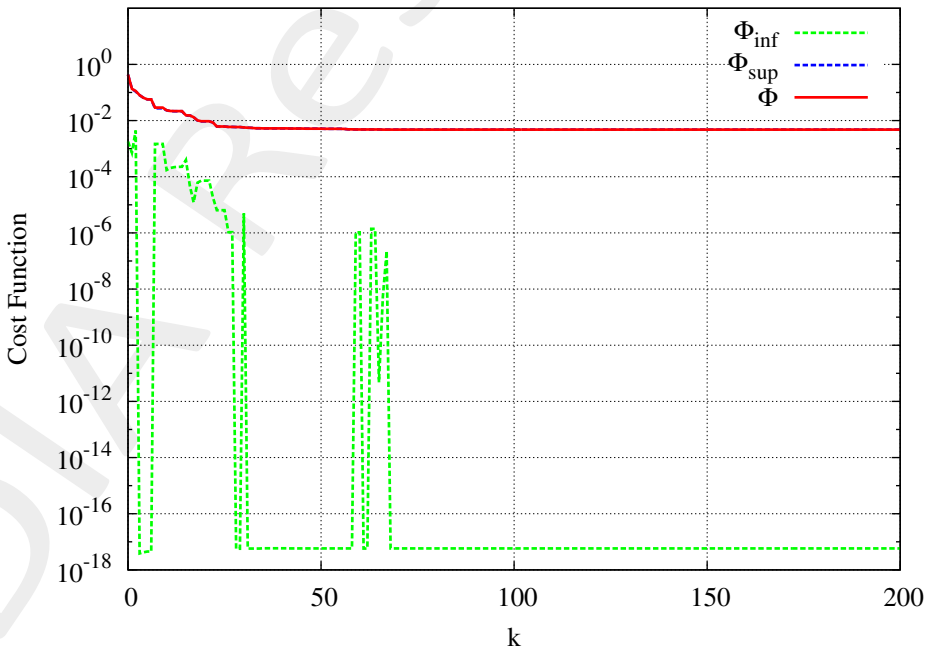

Figure 6.3.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
500	4.7×10^{-3}	11 min. 37 sec.

1.2.4 Synthesized Interval Pattern:

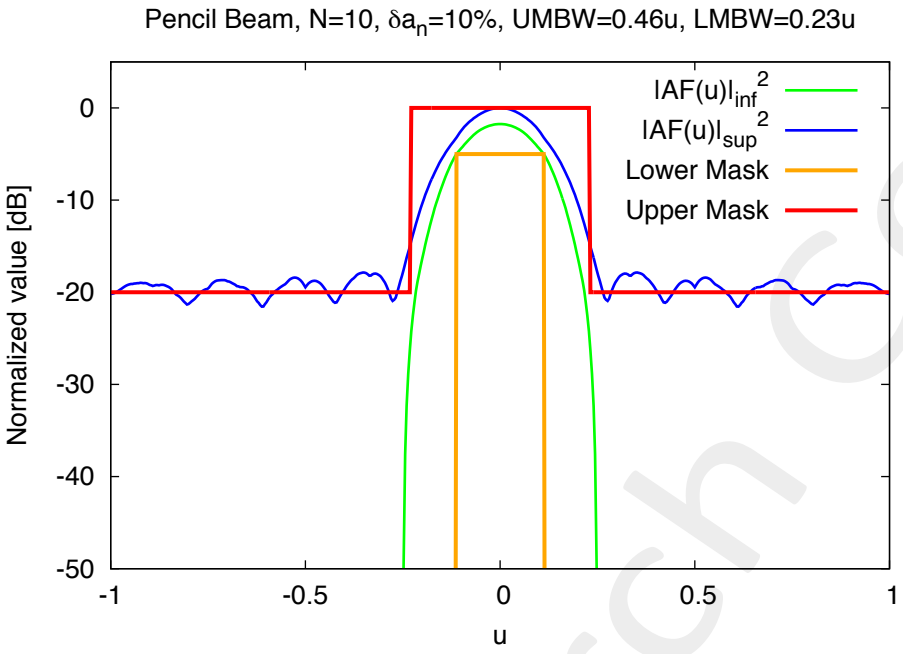

Figure 6.3.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	9.5	24.5	0.216	-0.83	0.10417
Inf.	7.7	$-\infty$	0.216	-1.74	/
Sup.	11.2	-16.11	0.28	0.0	/

Table 6.3.4.1. Interval Pattern Parameters

1.3 Test Case #4 - Pencil Beam - $N = 20$ - $\delta a_n = 1\%$

Array Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 1%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.38 u
- LMBW: 0.19 u
- SLL: -20 dB
- LM-Height: -5 dB

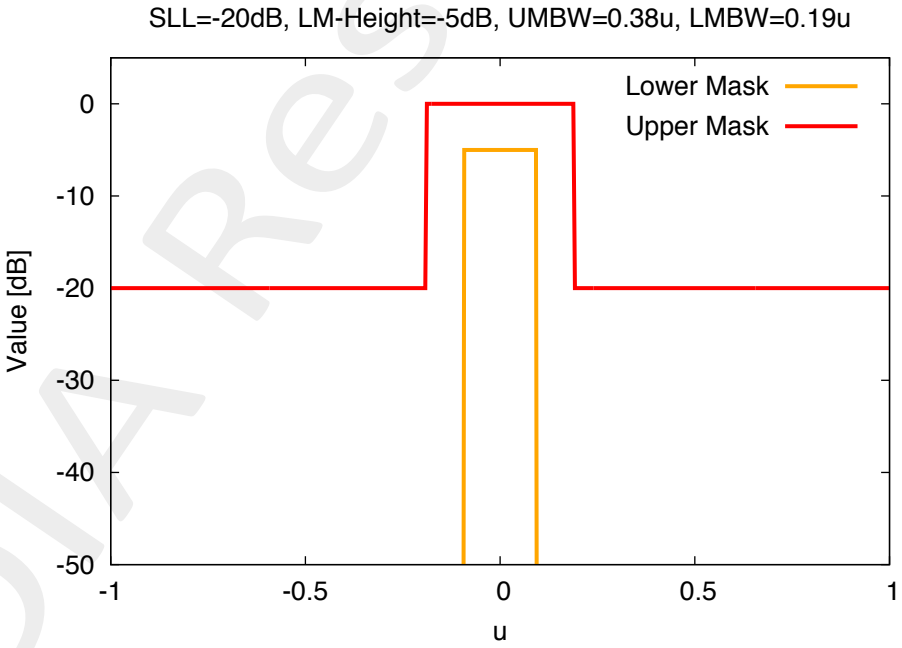

Figure 6.4.1. Power Synthesis Mask

1.3.1 PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 45
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 200
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.4.1.1. Max and Min excitations amplitudes values, for the PSO

1.3.2 Excitations:

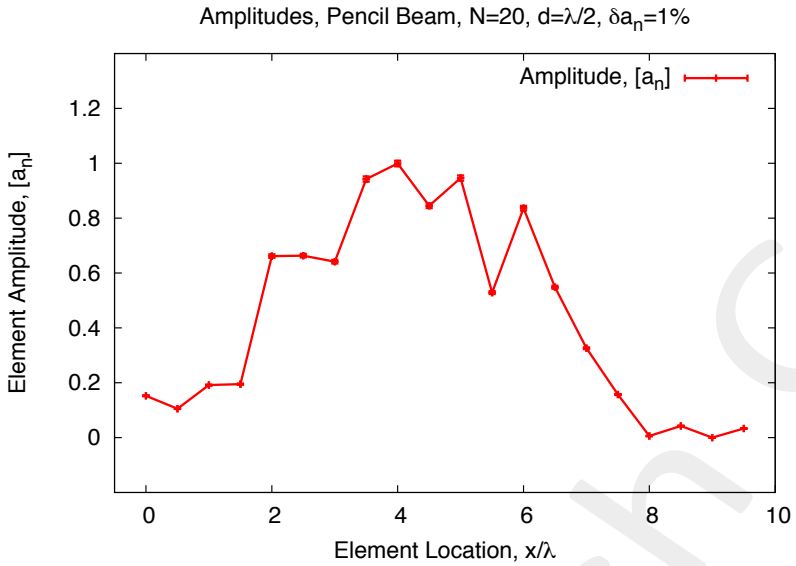

Figure 6.4.2.1. Optimal particle's excitations amplitudes

1.3.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{inf}}$$

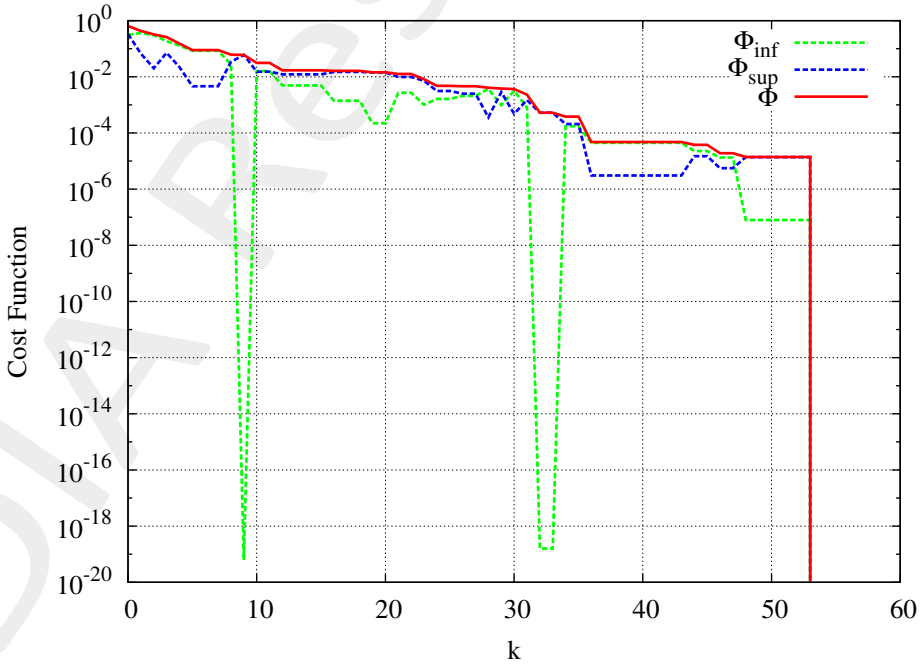

Figure 6.4.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
53	0.0	40 sec.

1.3.4 Synthesized Interval Pattern:

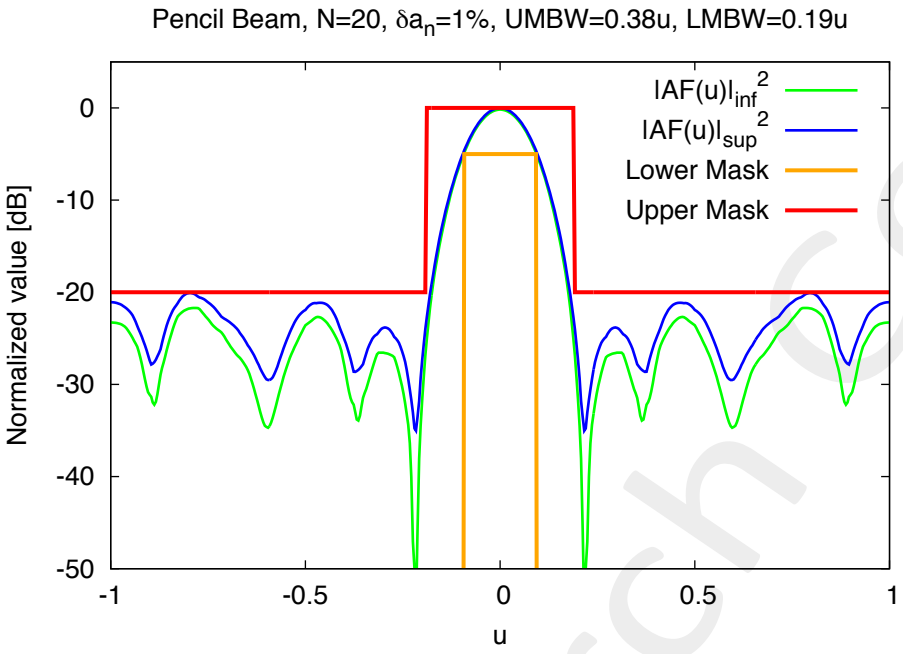

Figure 6.4.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	10.91	-20.9	0.152	0.08	0.01035
Inf.	10.74	-21.7	0.152	-0.17	/
Sup.	11.08	-20	0.16	0.0	/

Table 6.4.4.1. Interval Pattern Parameters

1.4 Test Case #5 - Pencil Beam - $N = 20$ - $\delta a_n = 5\%$

Array Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 5%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.38 u
- LMBW: 0.19 u
- SLL: -20 dB
- LM-Height: -5 dB

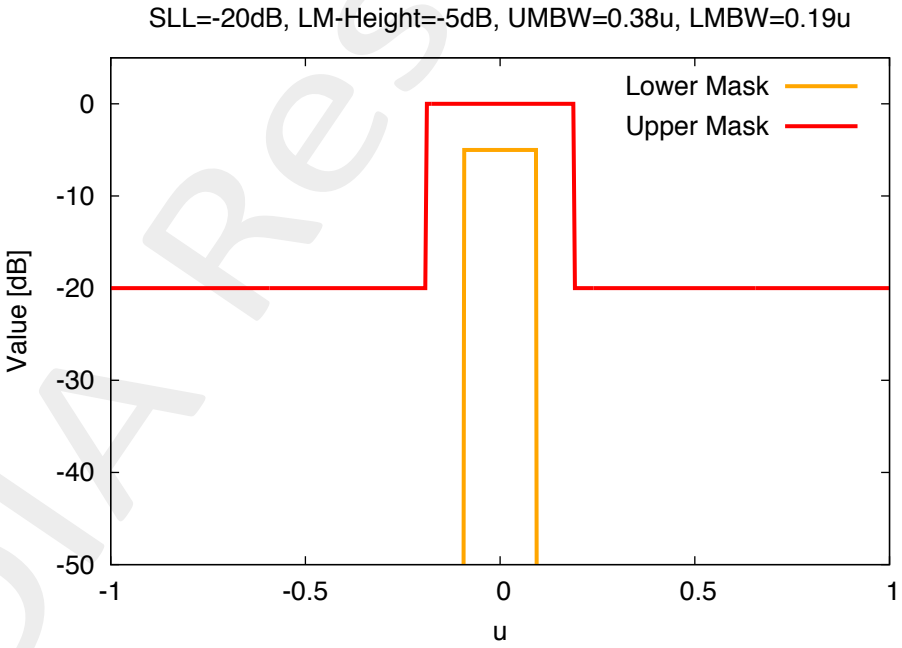

Figure 6.5.1. Power Synthesis Mask

1.4.1 PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 82
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 200
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.5.1.1. Max and Min excitations amplitudes values, for the PSO

1.4.2 Excitations:

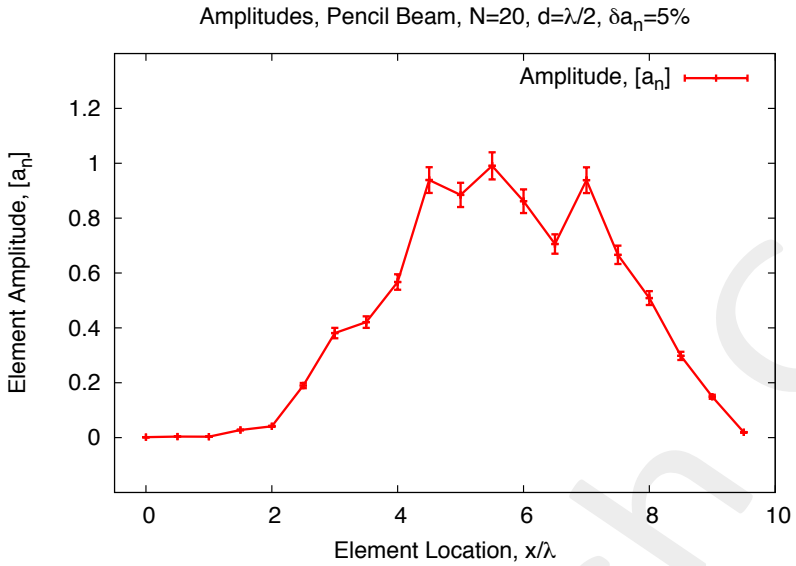

Figure 6.5.2.1. Optimal particle's excitations amplitudes

1.4.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

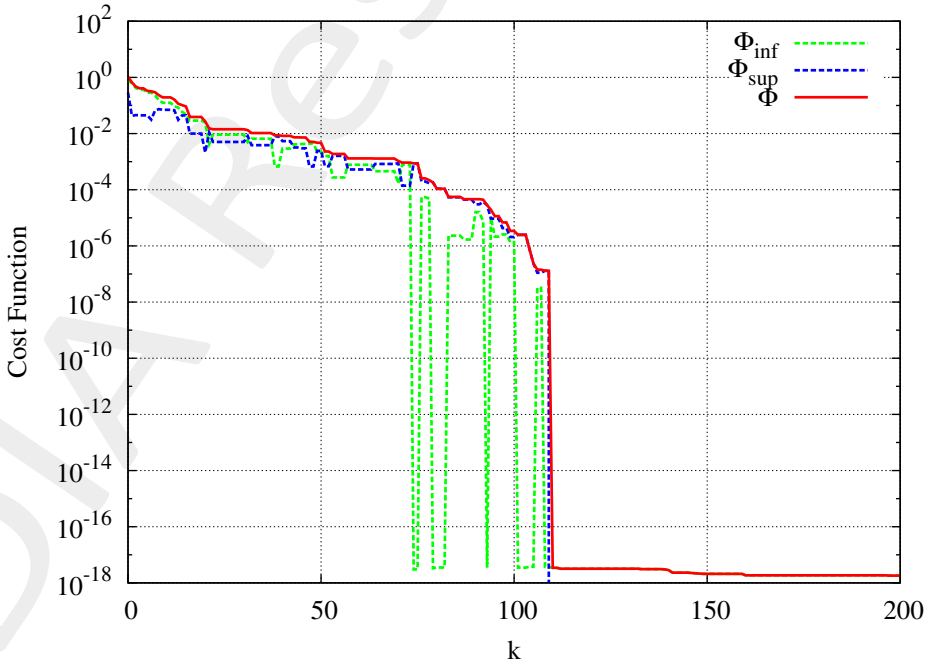

Figure 6.5.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
200	1.8×10^{-18}	6 min. 10 sec.

1.4.4 Synthesized Interval Pattern:

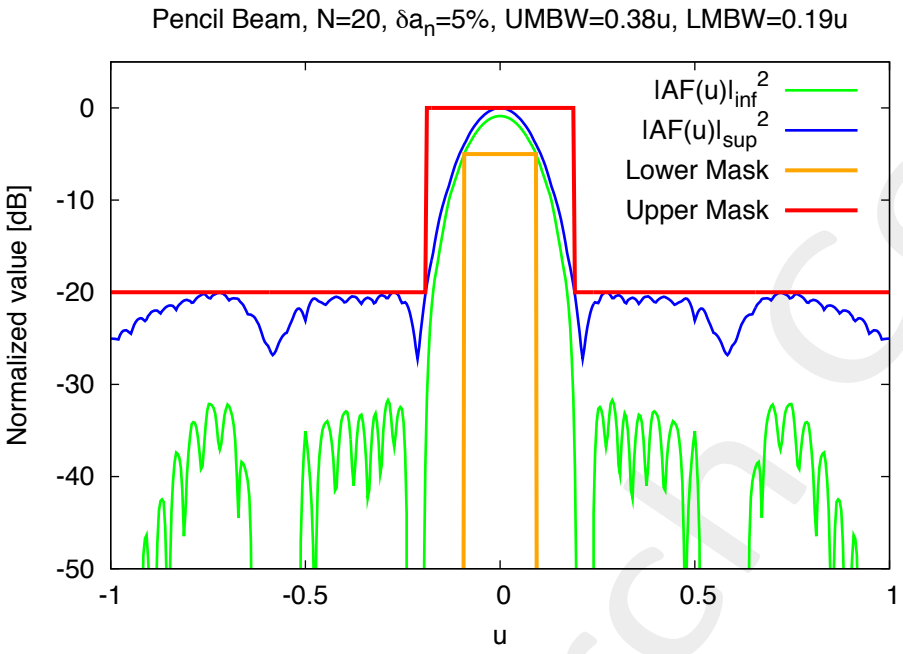

Figure 6.5.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	10.7	-24.8	0.16	-0.42	0.04671
Inf.	9.8	-20	0.16	-0.87	/
Sup.	11.6	-31.7	0.18	0.0	/

Table 6.5.4.1. Interval Pattern Parameters

1.5 Test Case #6 - Pencil Beam - $N = 20$ - $\delta a_n = 10\%$

Array Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 10%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.38 u
- LMBW: 0.19 u
- SLL: -20 dB
- LM-Height: -5 dB

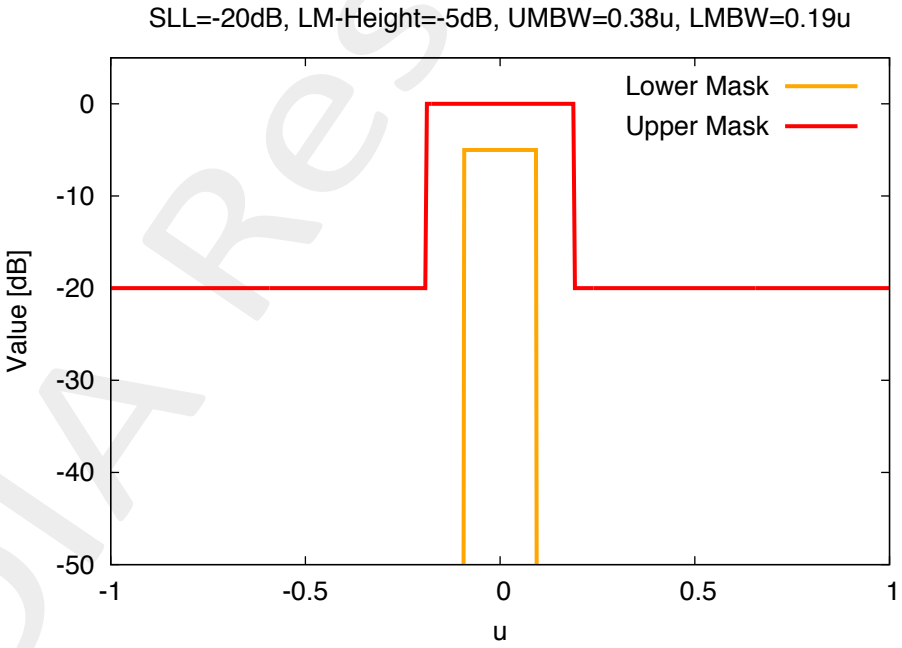

Figure 6.6.1. Power Synthesis Mask

1.5.1 PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 4
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 200
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.6.1.1. Max and Min excitations amplitudes values, for the PSO

1.5.2 Excitations:

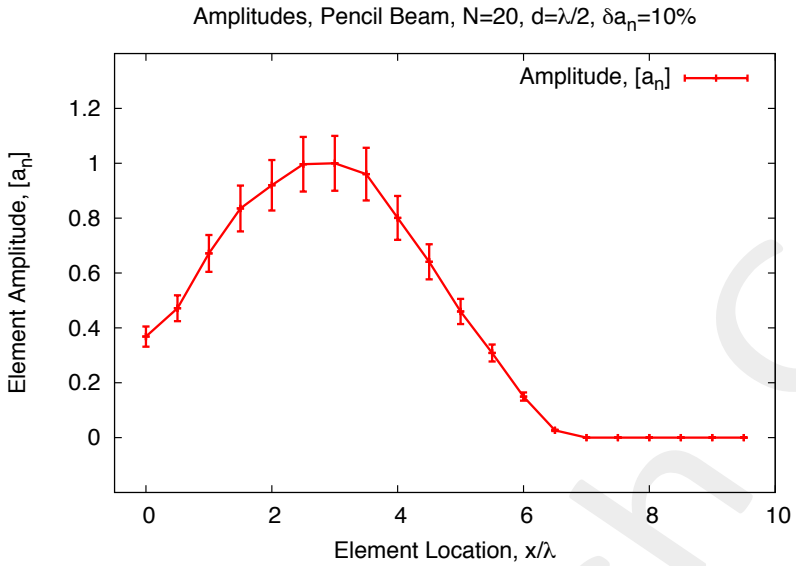

Figure 6.6.2.1. Optimal particle's excitations amplitudes

1.5.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

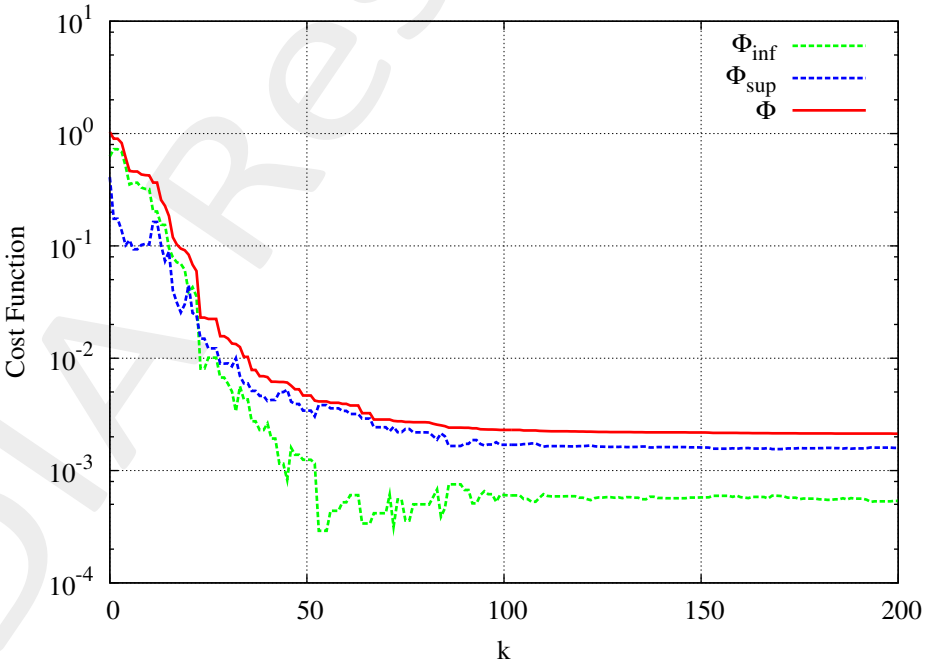

Figure 6.6.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
200	2.1×10^{-3}	6 min. 4 sec.

1.5.4 Synthesized Interval Pattern:

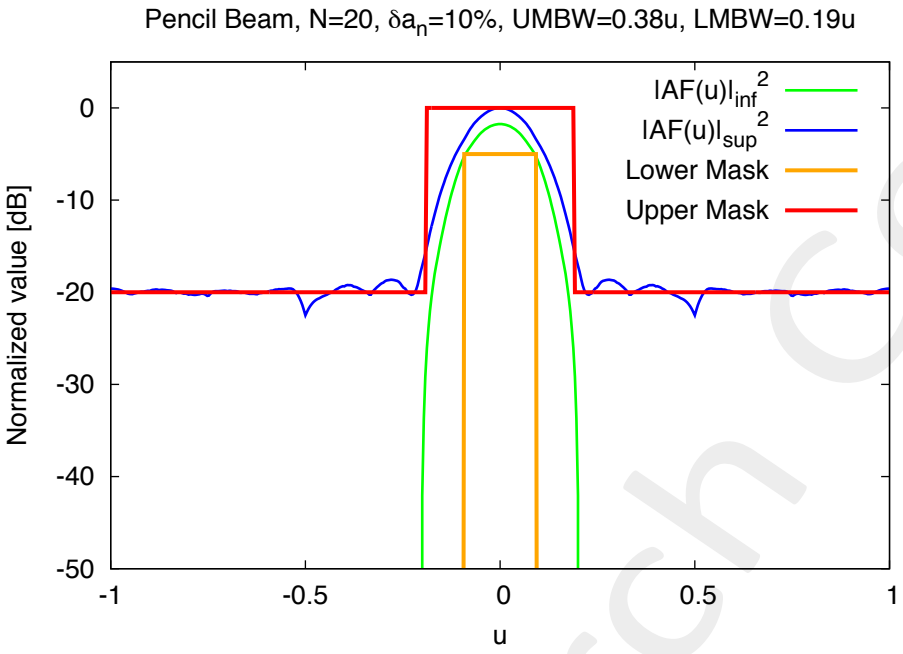

Figure 6.6.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	10.5	-27.4	0.176	-0.82	0.086339
Inf.	8.7	$-\infty$	0.168	-1.7	/
Sup.	12.2	-18.5	0.224	0.0	/

Table 6.6.4.1. Interval Pattern Parameters

1.6 Test Case #7 - Pencil Beam - $N = 40$ - $\delta a_n = 1\%$

Array Geometry:

- Number of Elements: $N = 40$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 1%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.25 u
- LMBW: 0.125 u
- SLL: -20 dB
- LM-Height: -5 dB

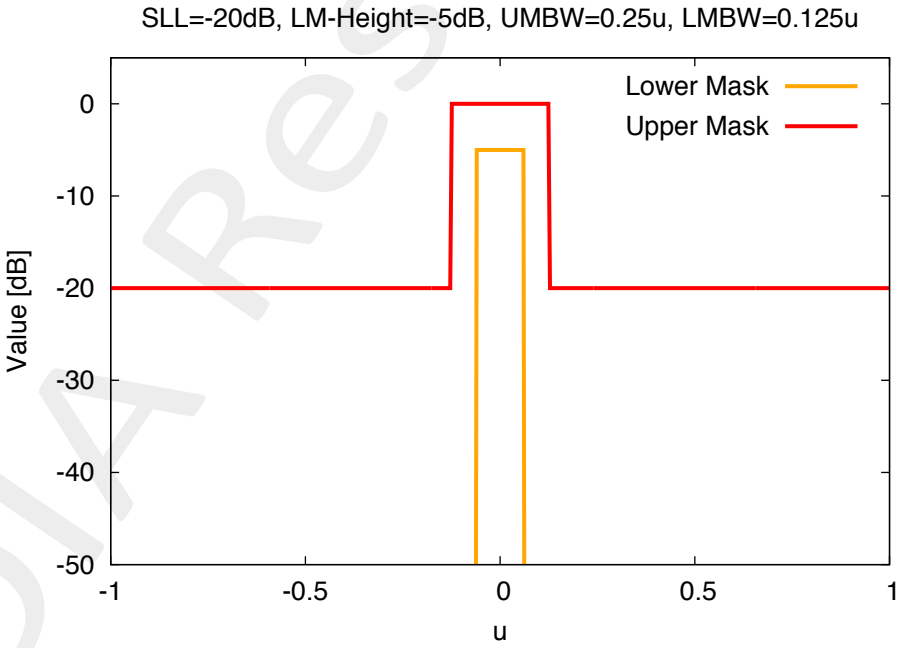

Figure 6.7.1. Power Synthesis Mask

1.6.1 PSO Parameters:

- Unknown Number: 40
- Swarm Dimension: 20
- Random Seed: 25
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 200
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.7.1.1. Max and Min excitations amplitudes values, for the PSO

1.6.2 Excitations:

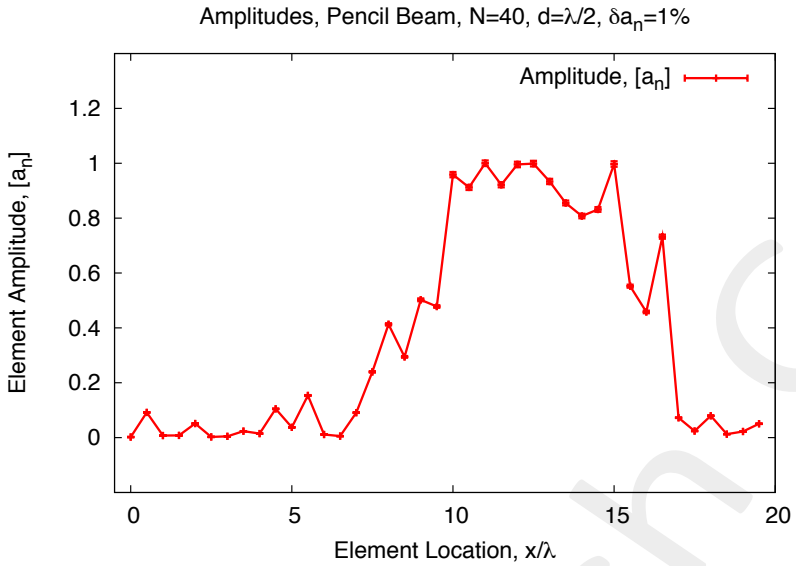

Figure 6.7.2.1. Optimal particle's excitations amplitudes

1.6.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

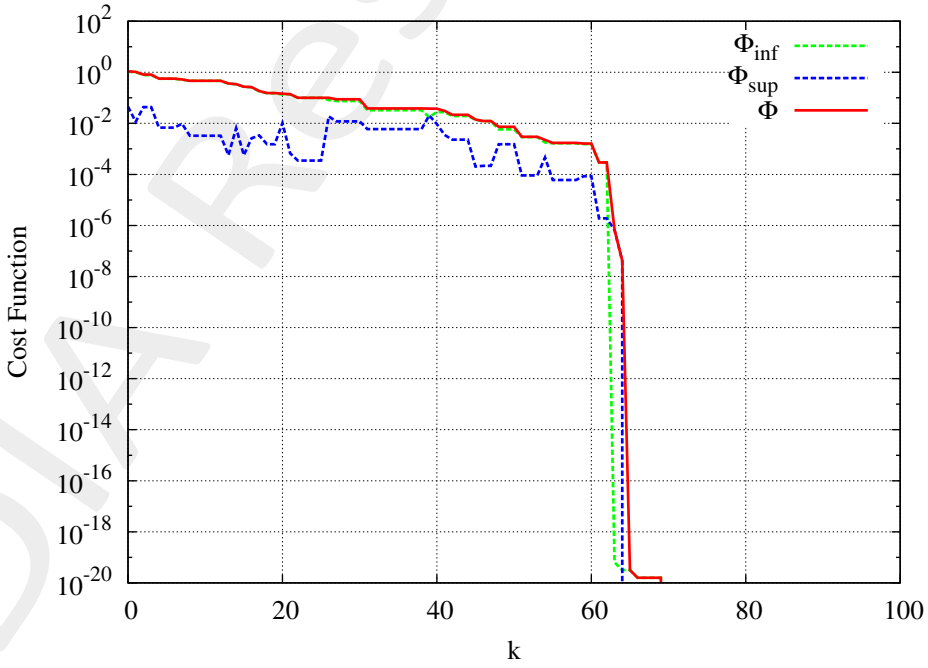

Figure 6.7.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
69	0	3 min. 25 sec.

1.6.4 Synthesized Interval Pattern:

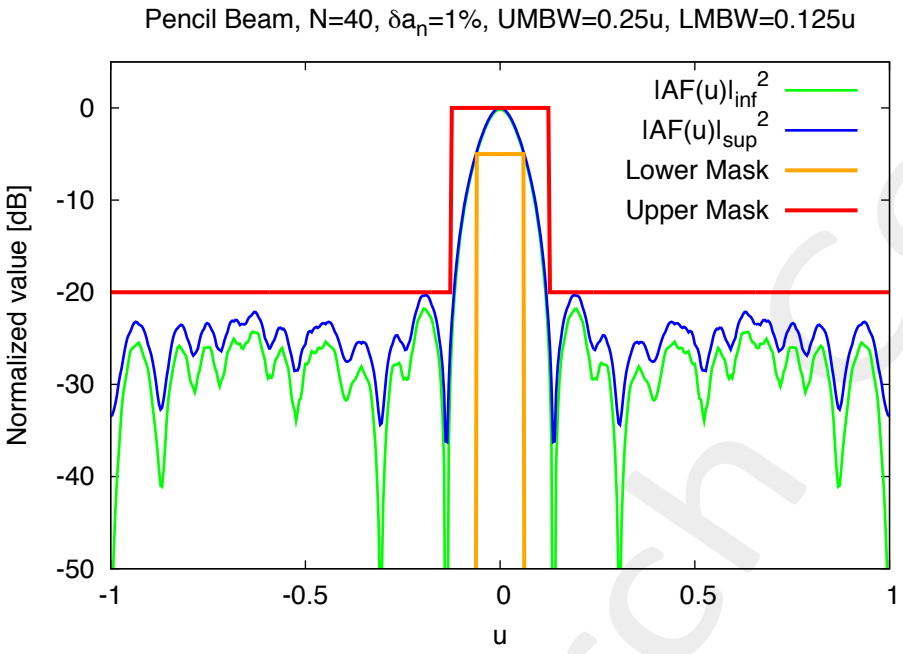

Figure 6.7.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	12.8	-21.0	0.096	0.086	0.007516
Inf.	12.6	-21.8	0.096	-0.17	/
Sup.	12.9	-20.3	0.104	0.0	/

Table 6.7.4.1. Interval Pattern Parameters

1.7 Test Case #8 - Pencil Beam - $N = 40$ - $\delta a_n = 5\%$

Array Geometry:

- Number of Elements: $N = 40$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 5%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.25
- LMBW: 0.125
- SLL: -20 dB
- LM-Height: -5 dB

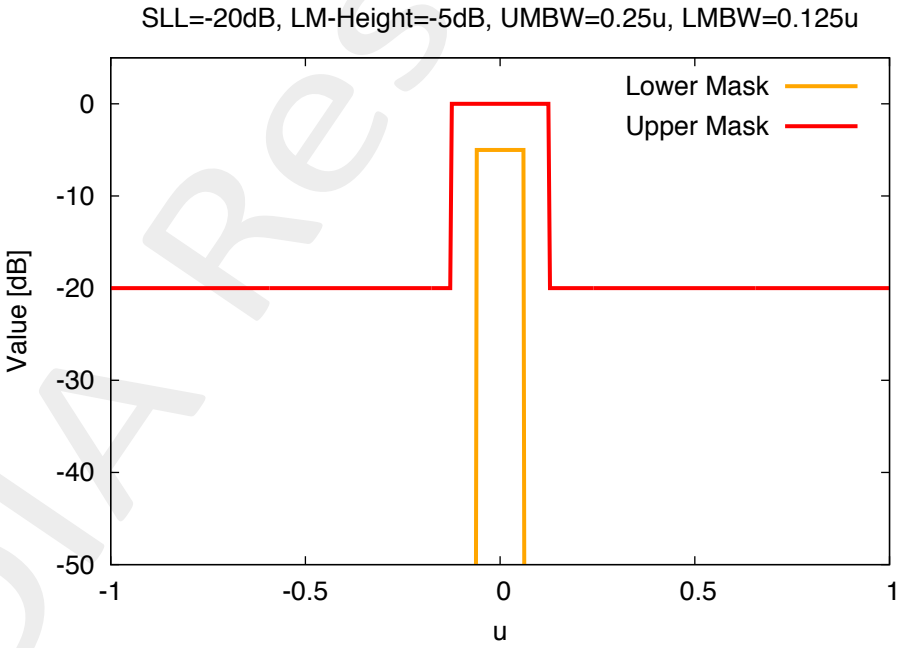

Figure 6.8.1. Power Synthesis Mask

1.7.1 PSO Parameters:

- Unknown Number: 40
- Swarm Dimension: 20
- Random Seed: 3
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 200
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.8.1.1. Max and Min excitations amplitudes values, for the PSO

1.7.2 Excitations:

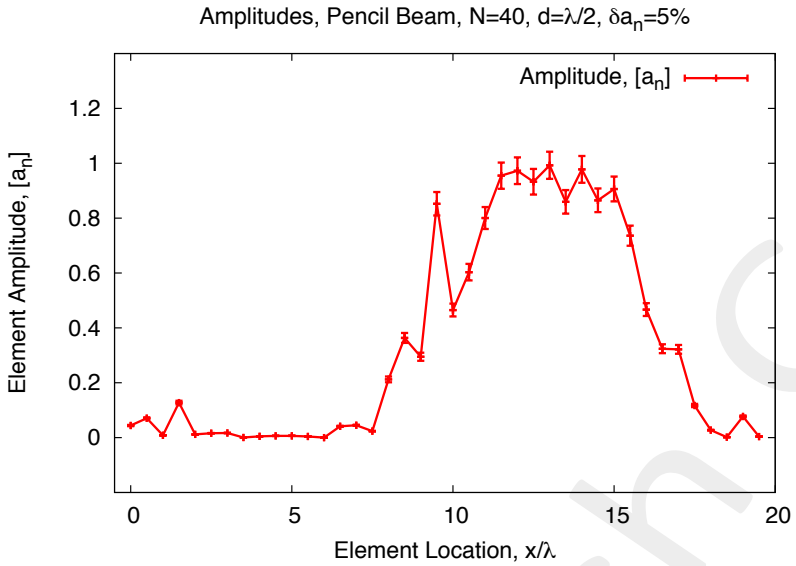

Figure 6.8.2.1. Optimal particle's excitations amplitudes

1.7.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

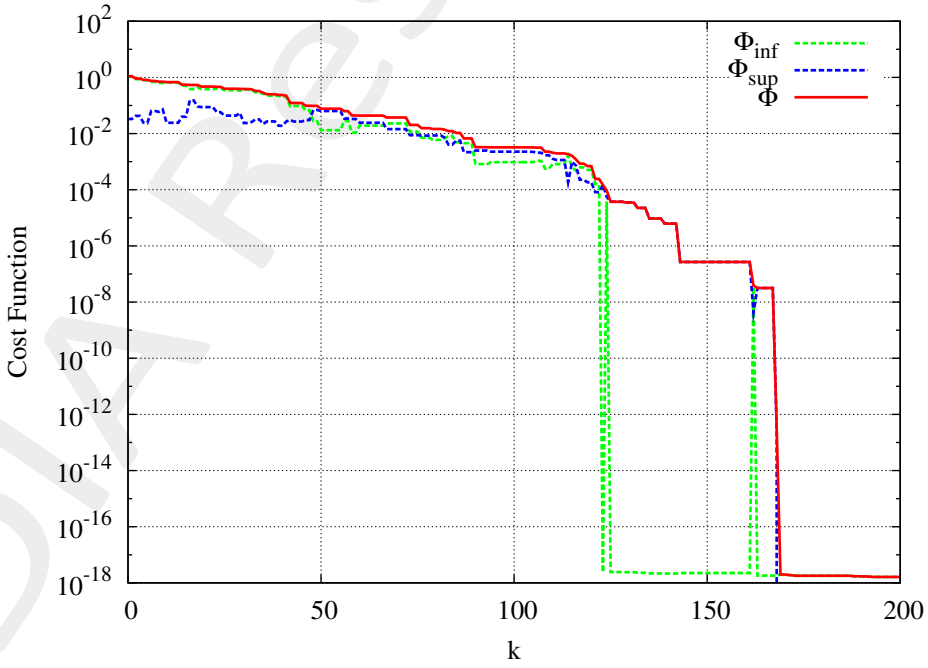

Figure 6.8.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
200	1.6×10^{-18}	9 min. 24 sec.

1.7.4 Synthesized Interval Pattern:

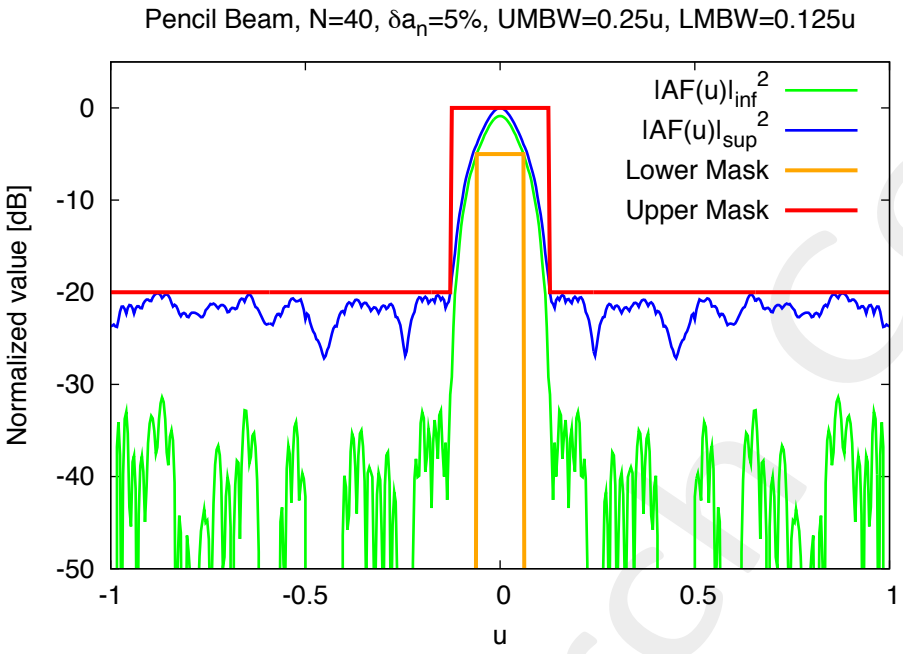

Figure 6.8.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	12.5	-24.9	0.104	-0.42	0.0351406
Inf.	11.7	-31.4	0.104	-0.87	/
Sup.	13.4	-20.1	0.12	0.0	/

Table 6.8.4.1. Interval Pattern Parameters

1.8 Test Case #9 - Pencil Beam - $N = 40$ - $\delta a_n = 10\%$

Array Geometry:

- Number of Elements: $N = 40$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 10%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.25 u
- LMBW: 0.125 u
- SLL: -20 dB
- LM-Height: -5 dB

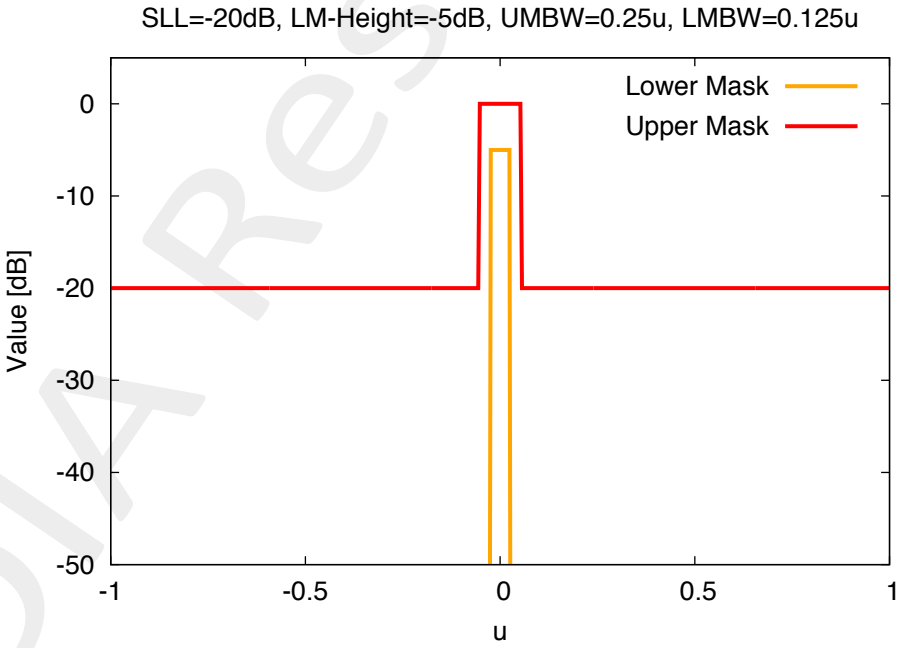

Figure 6.9.1. Power Synthesis Mask

1.8.1 PSO Parameters:

- Unknown Number: 40
- Swarm Dimension: 20
- Random Seed: 8
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 500
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.9.1.1. Max and Min excitations amplitudes values, for the PSO

1.8.2 Excitations:

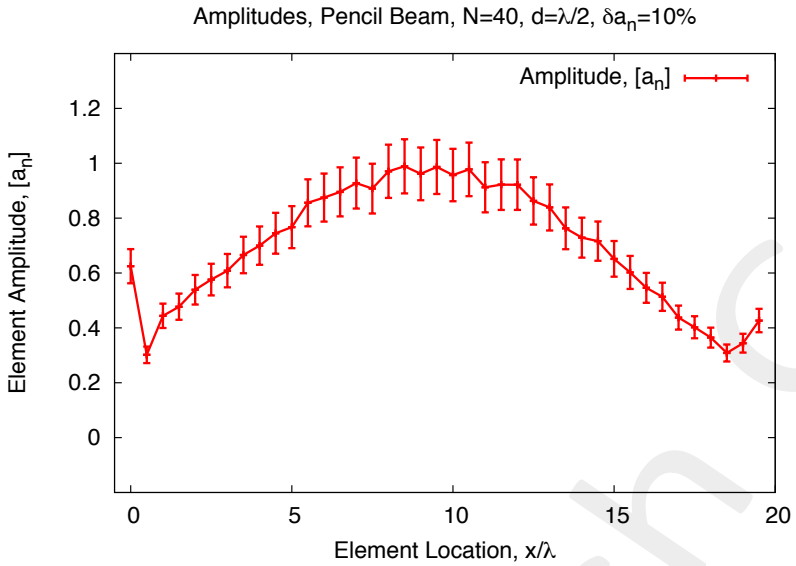

Figure 6.9.2.1. Optimal particle's excitations amplitudes

1.8.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

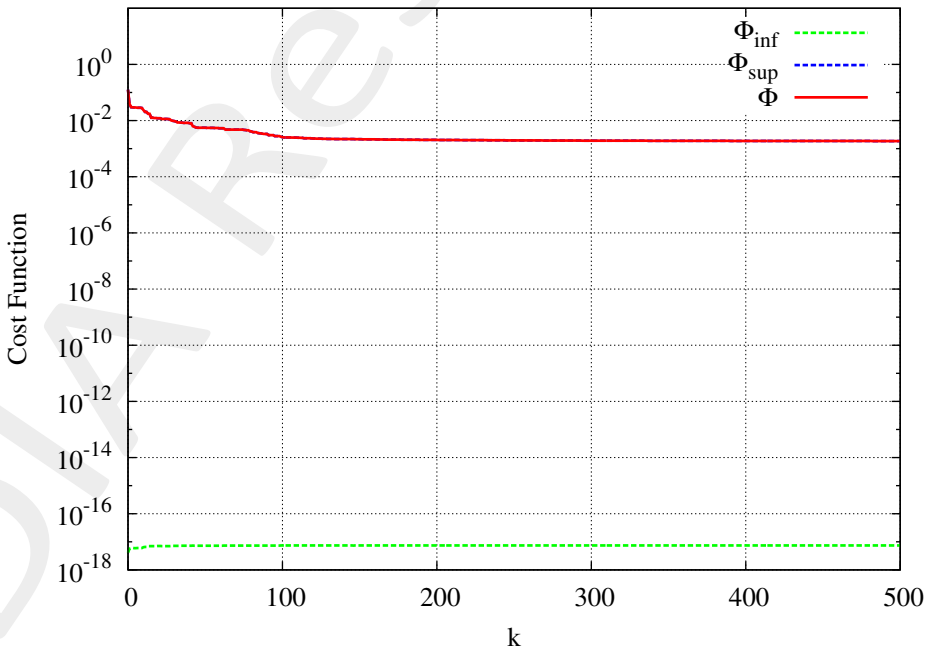

Figure 6.9.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
500	1.86×10^{-3}	13 min. 50 sec.

1.8.4 Synthesized Interval Pattern:

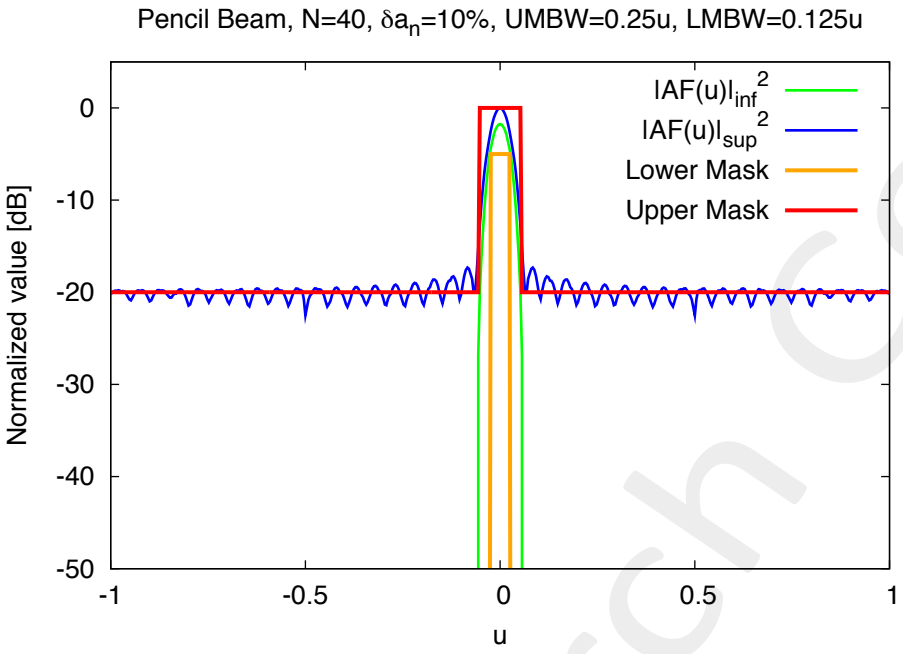

Figure 6.9.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	15.6	-24.5	0.056	-0.82	0.039751
Inf.	13.8	$-\infty$	0.056	-1.7	/
Sup.	17.3	-15.5	0.072	0.0	/

Table 6.9.4.1. Interval Pattern Parameters

1.9 Test Case #9 - Pencil Beam - $N = 100$ - $\delta a_n = 1\%$

Array Geometry:

- Number of Elements: $N = 100$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 1%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.11 u
- LMBW: 0.055 u
- SLL: -20 dB
- LM-Height: -5 dB

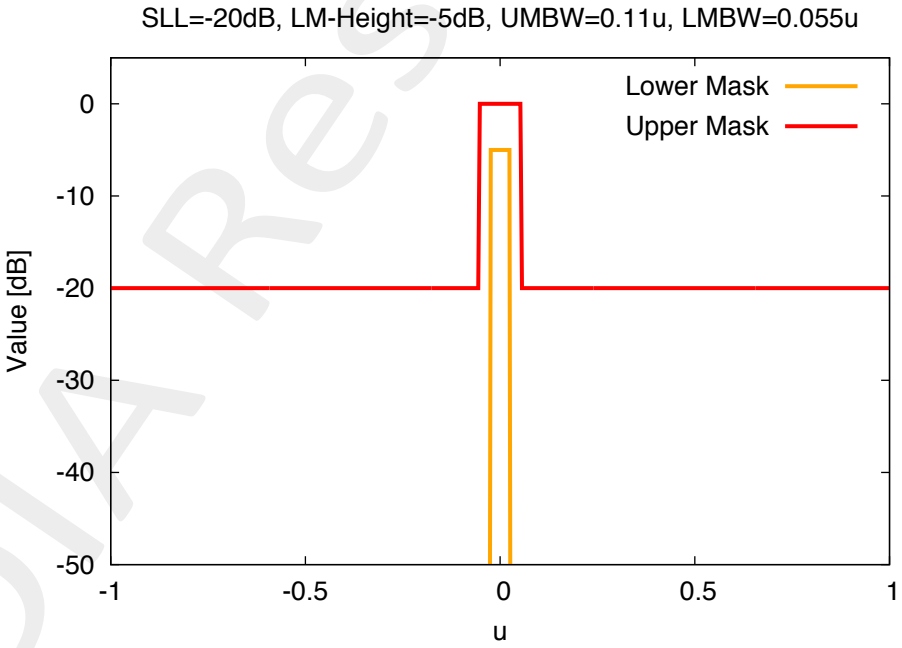

Figure 6.10.1. Power Synthesis Mask

1.9.1 PSO Parameters:

- Unknown Number: 100
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 200
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.10.1.1. Max and Min excitations amplitudes values, for the PSO

1.9.2 Excitations:

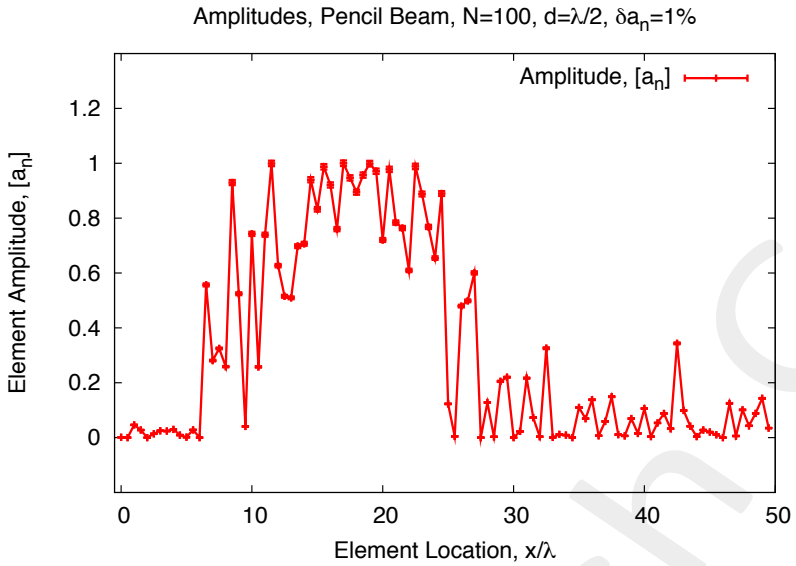

Figure 6.10.2.1. Optimal particle's excitations amplitudes

1.9.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{inf}}$$

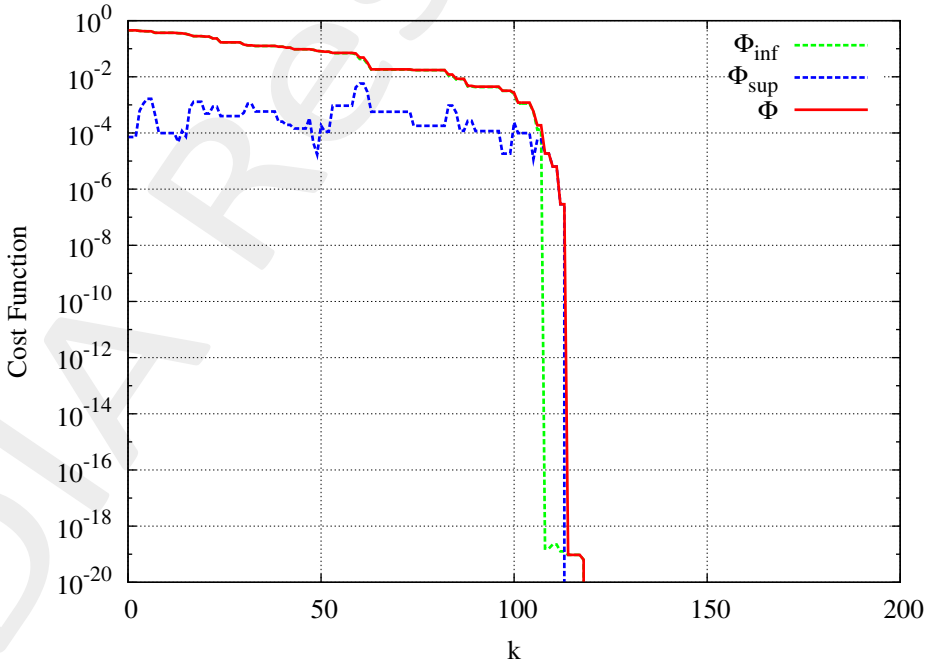

Figure 6.10.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
119	0.0	12 min. 48 sec.

1.9.4 Synthesized Interval Pattern:

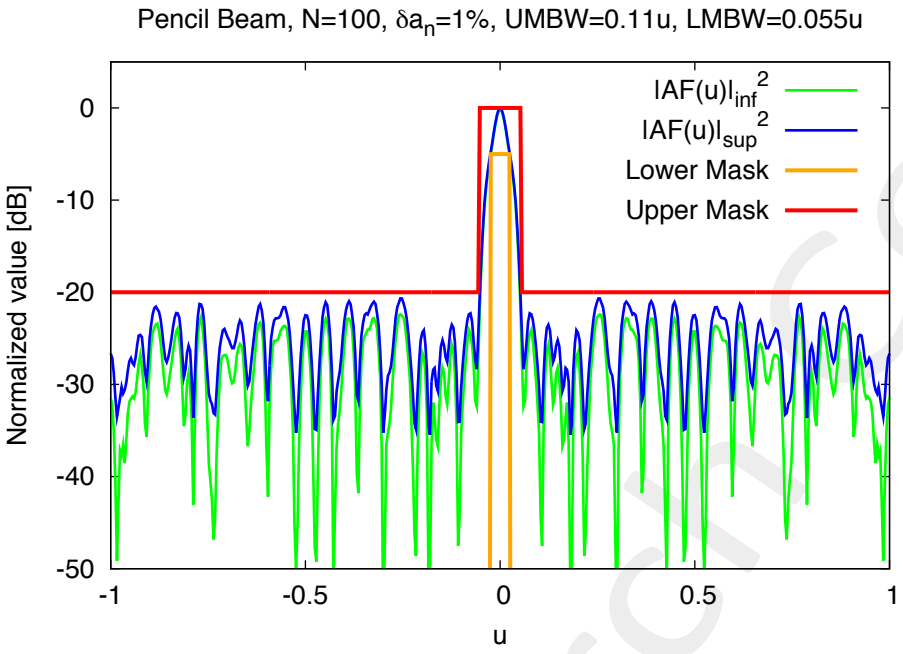

Figure 6.10.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	16.4	-21.5	0.04	0.086	0.004777
Inf.	16.3	-22.4	0.04	-0.17	/
Sup.	16.6	-20.6	0.04	0.0	/

Table 6.10.4.1. Interval Pattern Parameters

1.10 Test Case #9 - Pencil Beam - $N = 100$ - $\delta a_n = 5\%$

Array Geometry:

- Number of Elements: $N = 100$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 5%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.11 u
- LMBW: 0.055 u
- SLL: -20 dB
- LM-Height: -5 dB

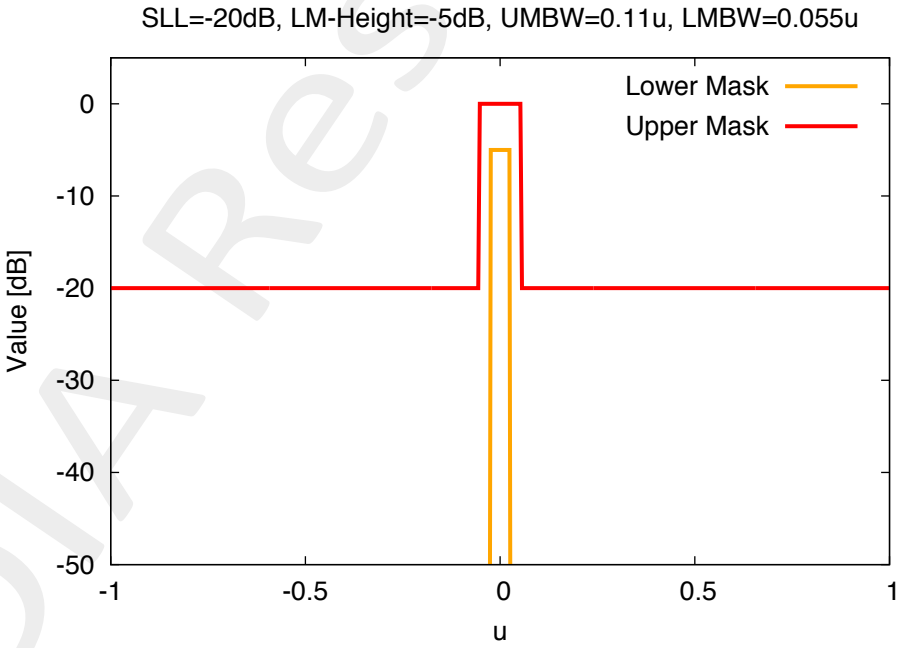

Figure 6.11.1. Power Synthesis Mask

1.10.1 PSO Parameters:

- Unknown Number: 100
- Swarm Dimension: 20
- Random Seed: 62
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 500
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.11.1.1. Max and Min excitations amplitudes values, for the PSO

1.10.2 Excitations:

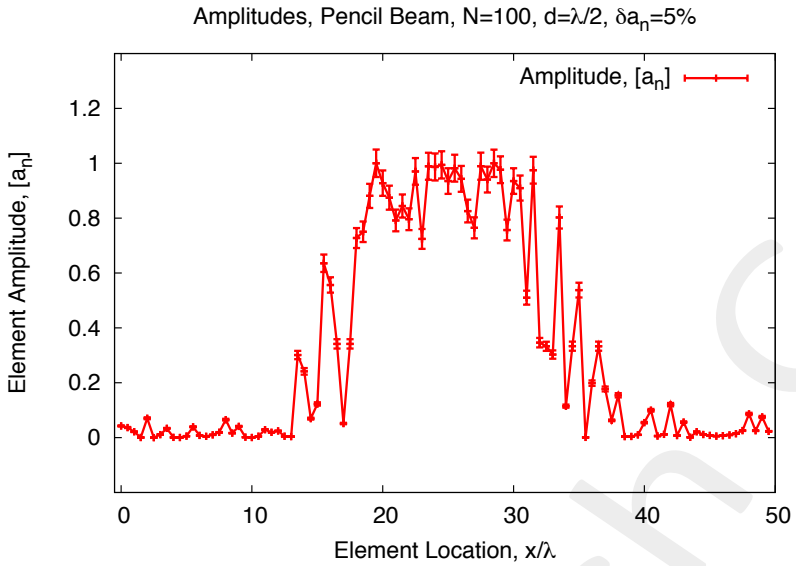

Figure 6.11.2.1. Optimal particle's excitations amplitudes

1.10.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

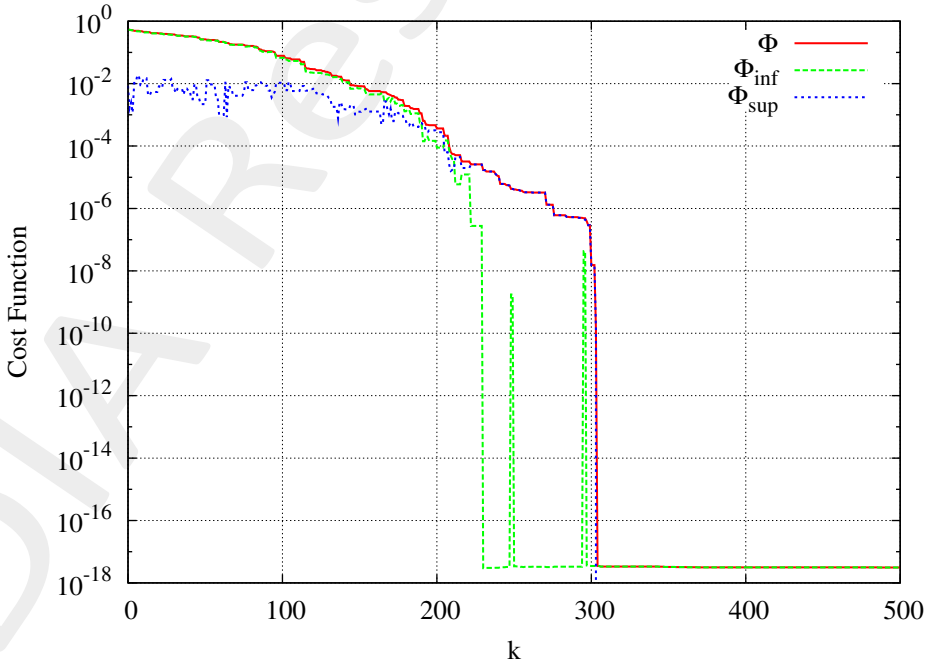

Figure 6.11.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
500	3.1×10^{-18}	49 min.

1.10.4 Synthesized Interval Pattern:

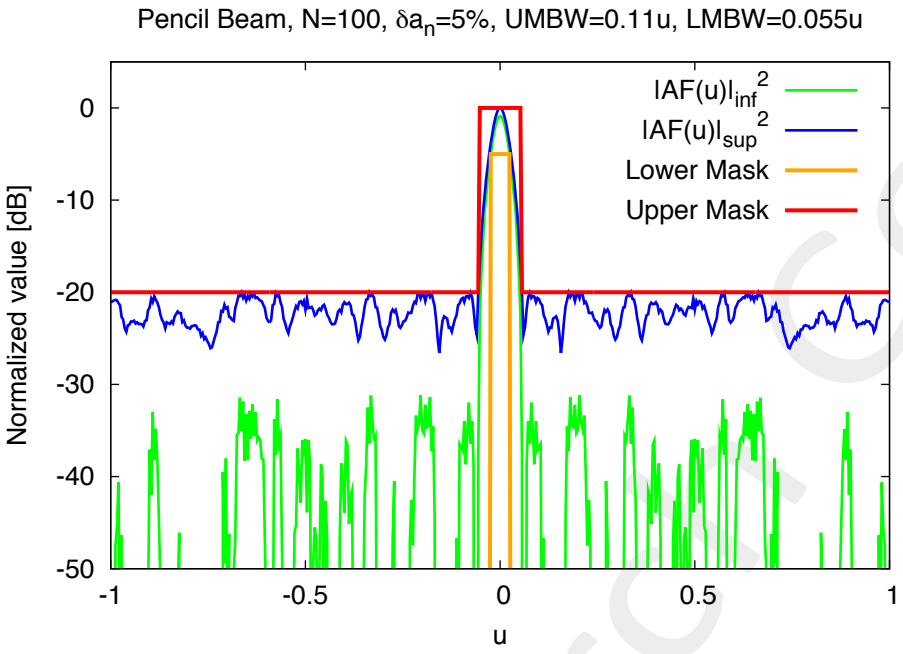

Figure 6.11.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	16.2	-24.9	0.048	-0.42	0.021466
Inf.	15.3	-31.2	0.048	-0.87	/
Sup.	17	-19.1	0.056	0.0	/

Table 6.11.4.1. Interval Pattern Parameters

1.11 Test Case #9 - Pencil Beam - $N = 100$ - $\delta a_n = 10\%$

Array Geometry:

- Number of Elements: $N = 100$
- Element Spacing: $d = \frac{\lambda}{2}$

Test Case Parameters:

- Sample Points: 501
- Amplitude Error: 10%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Power Mask:

- UMBW: 0.11 u
- LMBW: 0.055 u
- SLL: -20 dB
- LM-Height: -5 dB

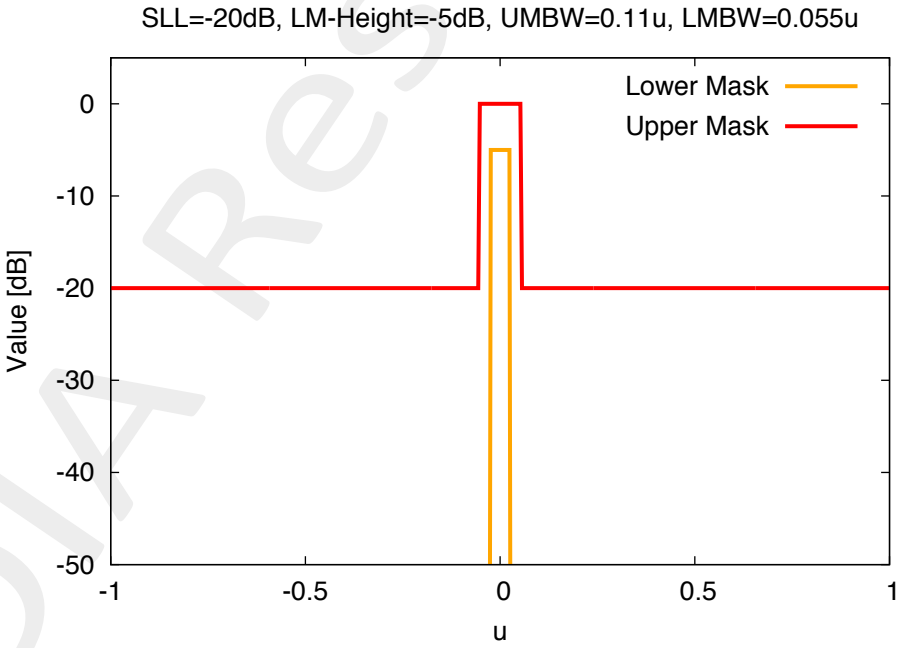

Figure 6.12.1. Power Synthesis Mask

1.11.1 PSO Parameters:

- Unknown Number: 100
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 500
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 6.12.1.1. Max and Min excitations amplitudes values, for the PSO

1.11.2 Excitations:

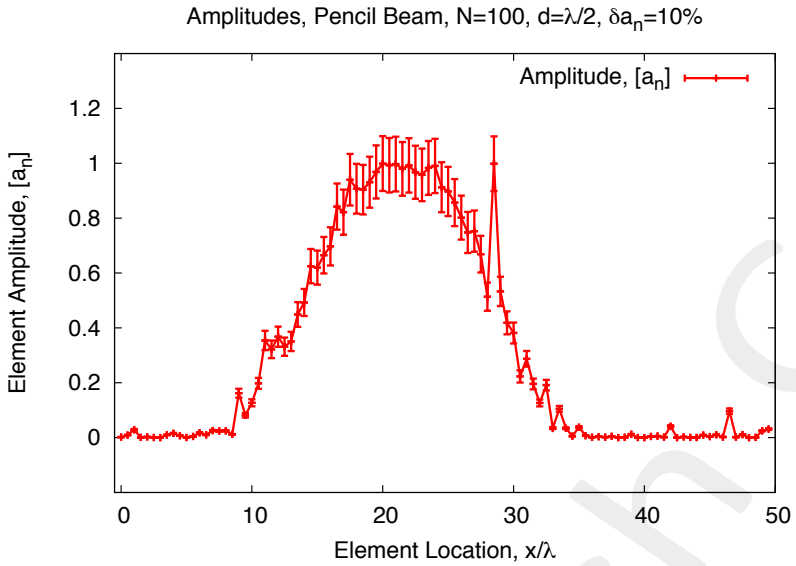

Figure 6.12.2.1. Optimal particle's excitations amplitudes

1.11.3 Fitness:

The following figure shows the variation of the weighted fitness, along with its components (**??**). Remember that:

$$\phi = \phi_{\text{inf}} + \phi_{\text{sup}}$$

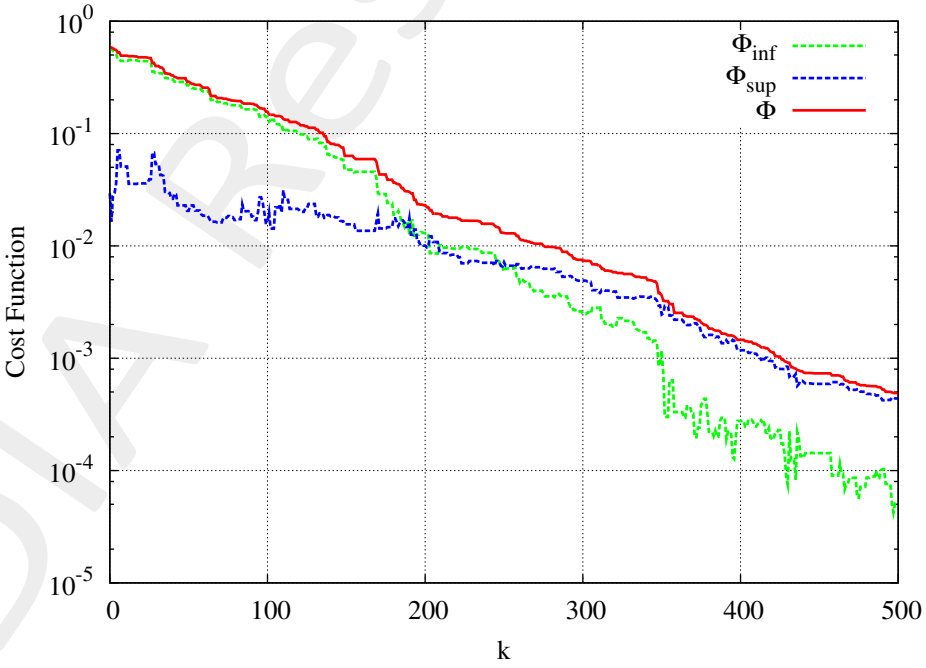

Figure 6.12.3.1. Fitness

Performed Iterations	Final Fitness Value	Simulation time
500	4.9×10^{-4}	46 min. 5 sec.

1.11.4 Synthesized Interval Pattern:

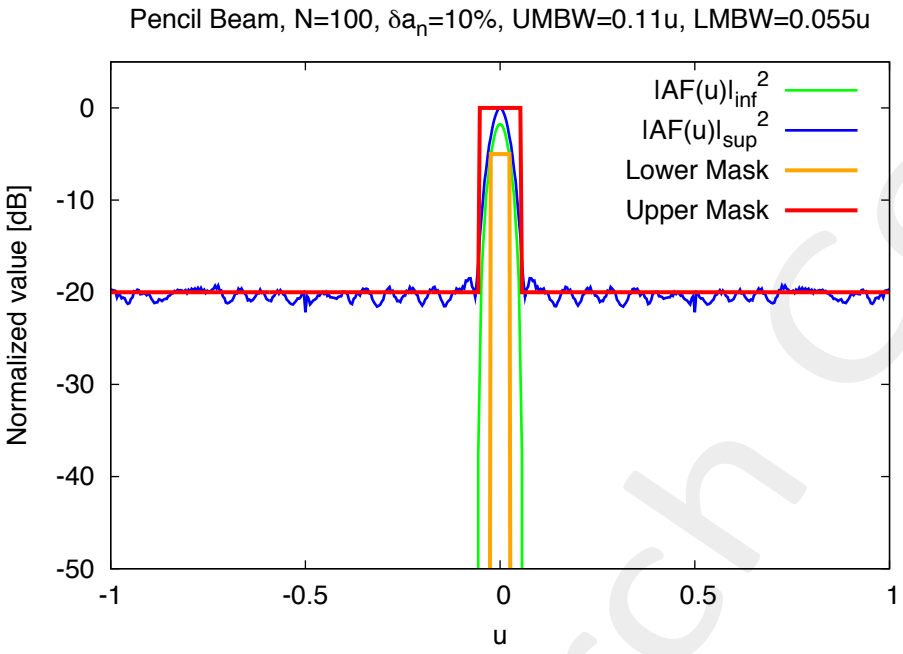

Figure 6.12.4.1. Optimal Interval Pattern

Parameter	$[D_{max}]$ [dB]	$[SLL]$ [dB]	$[HPBW]$ [u]	$[\|AF(u_{max})\|^2]$ [dB]	Δ
Nominal	16.1	-28.1	0.048	-0.83	0.037215
Inf.	14.3	$-\infty$	0.048	-1.74	/
Sup.	17.8	-18.5	0.064	0.0	/

Table 6.11.4.1. Interval Pattern Parameters

2 Summary of the results

In this section the results obtained in the previous test cases are compared.

- Pattern Tolerance (Δ) for $N = 10, 20, 40, 100$

  Each test case is here considered with $\delta a_n = 1\%, 5\%, 10\%$

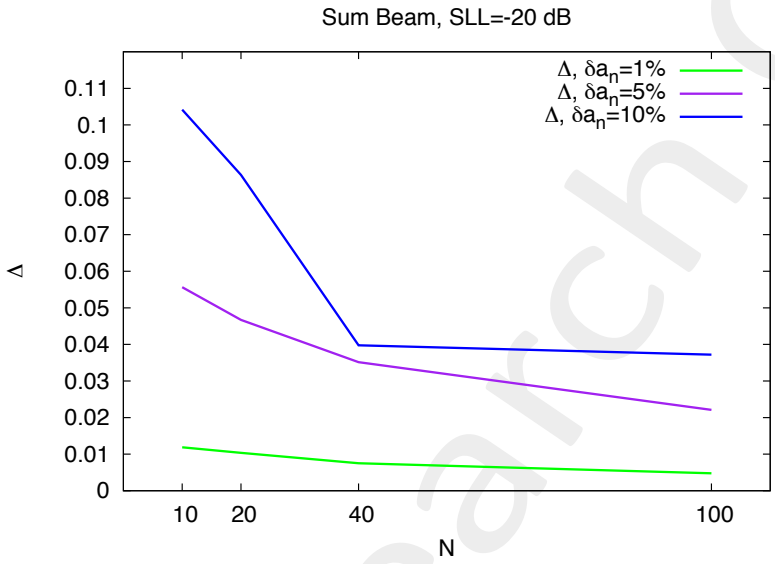

Figure 7.1. Δ vs N for each group of test cases

	$N = 10$	$N = 20$	$N = 40$	$N = 100$
$\delta a_n = 1\%$	0.01186	0.01035	0.00752	0.00477
$\delta a_n = 5\%$	0.05563	0.04671	0.03514	0.02211
$\delta a_n = 10\%$	0.10417	0.08634	0.03975	0.03721

Table 7.1. Δ values plotted in Fig.7.1

Note: In Fig. 7.1 we can see how the Δ parameter decreases, increasing the elements number. This is due to the fact that the main contribution is given by the area between the two bounds in the main lobe u zone.

- Final Fitness value (Φ) for $N = 10, 20, 40, 100$

  1. Each test case is here considered with $\delta a_n = 1\%, 5\%, 10\%$

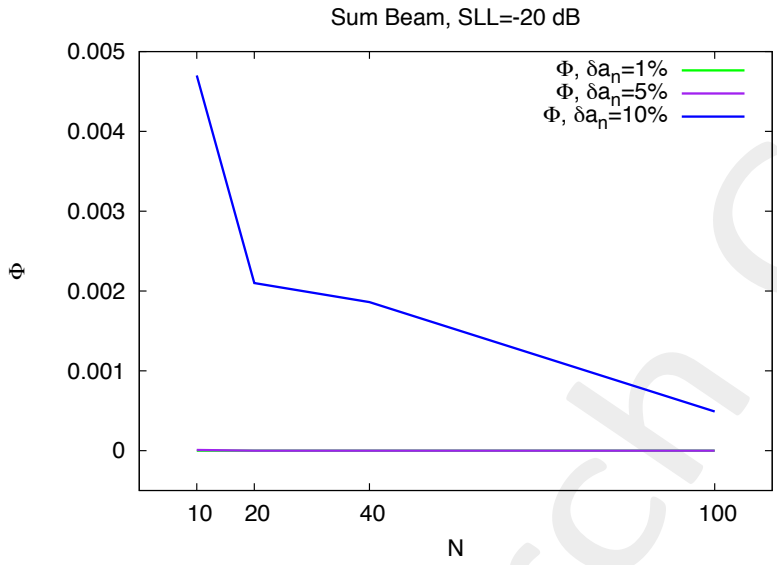

Figure 7.2. Φ vs N for each group of test cases

	$N = 10$	$N = 20$	$N = 40$	$N = 100$
$\delta a_n = 1\%$	0.0	0.0	0.0	0.0
$\delta a_n = 5\%$	7.9×10^{-6}	1.8×10^{-18}	1.6×10^{-18}	3.1×10^{-18}
$\delta a_n = 10\%$	4.7×10^{-3}	2.1×10^{-3}	1.8×10^{-3}	4.9×10^{-4}

Table 7.2. Φ values plotted in Fig.7.2

Note: In Fig. 7.2 the simulations final fitness value is plotted. While in cases $\delta a_n = 1\%$ and $\delta a_n = 5\%$ the mask constraints are almost satified, with $\delta a_n = 10\%$ are not. Moreover increasing N, the PSO is more able to find a suitable solution.

3 PSO Synthesis

3.1 Test Case #1 - Sum Beam -$\delta a_n = 1\%$

In this test case a mask constrained power synthesis is performed using Interval Analysis. The amplitude error is set to 1%.

Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$
- Sample Points: 501

Test Case Parameters:

- Amplitude Error: 1%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 8.1.1: Max and Min excitations amplitudes values, for the PSO

PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 100
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Power Mask Constraints

- Upper Mask Main Beam Width: 0.347 u (20°)
- Lower Mask Main Beam Width: 0.174 u (10°)

- Side Lobe Level: -20 dB
- Lower Mask Height: -5 dB

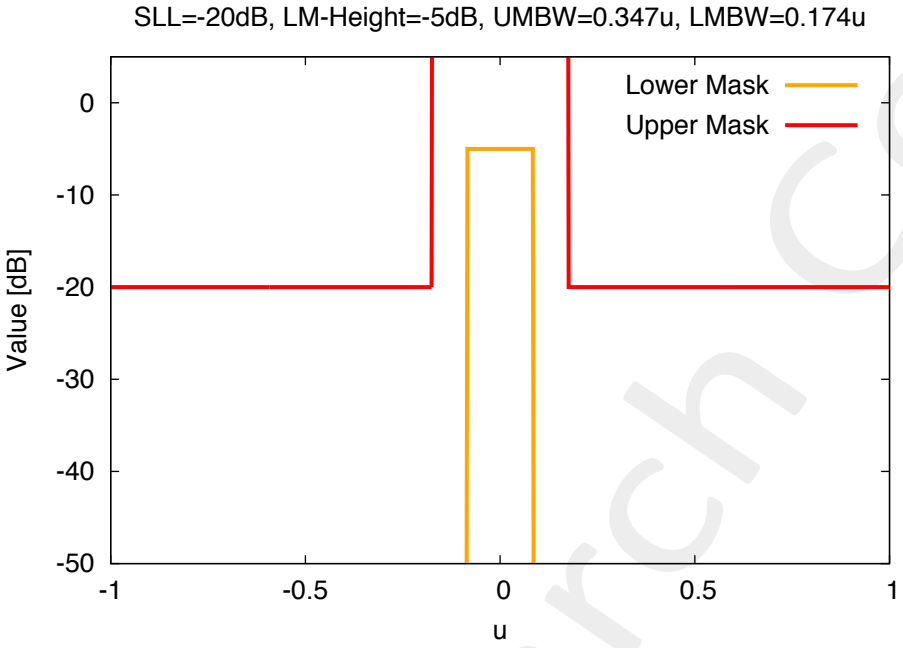

Figure 8.1.1: Power Synthesis Mask

Optimal Particle:

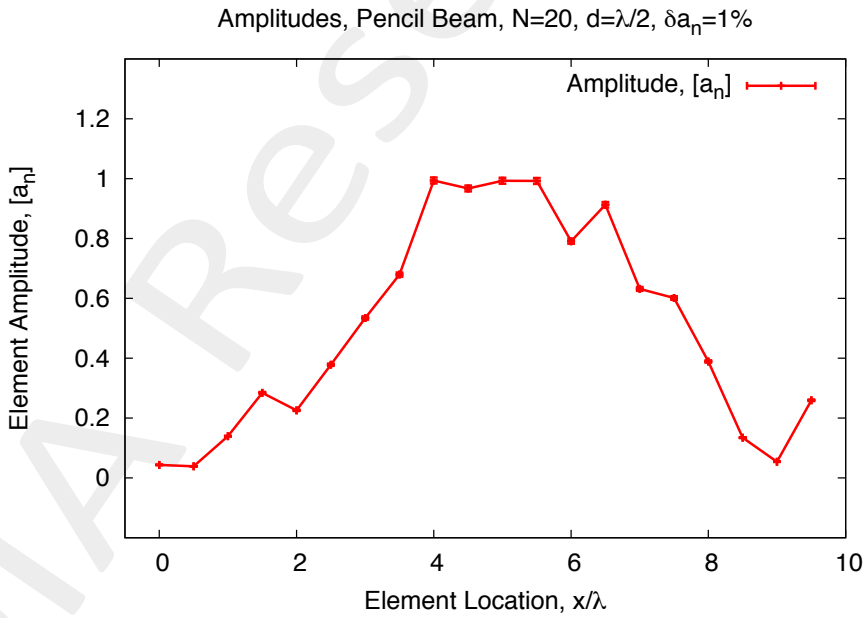

Figure 8.1.2: Optimal particle Amplitudes

Fitness:

- Performed Iterations: 23
- Fitness Finale : 0.0

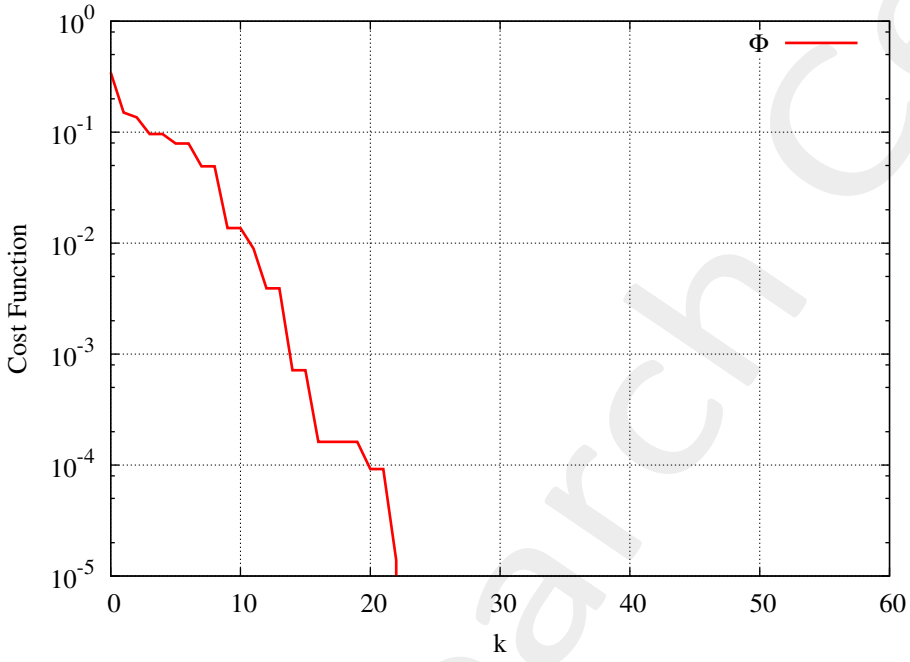

Figure 8.1.3: Fitness

Interval Pattern:

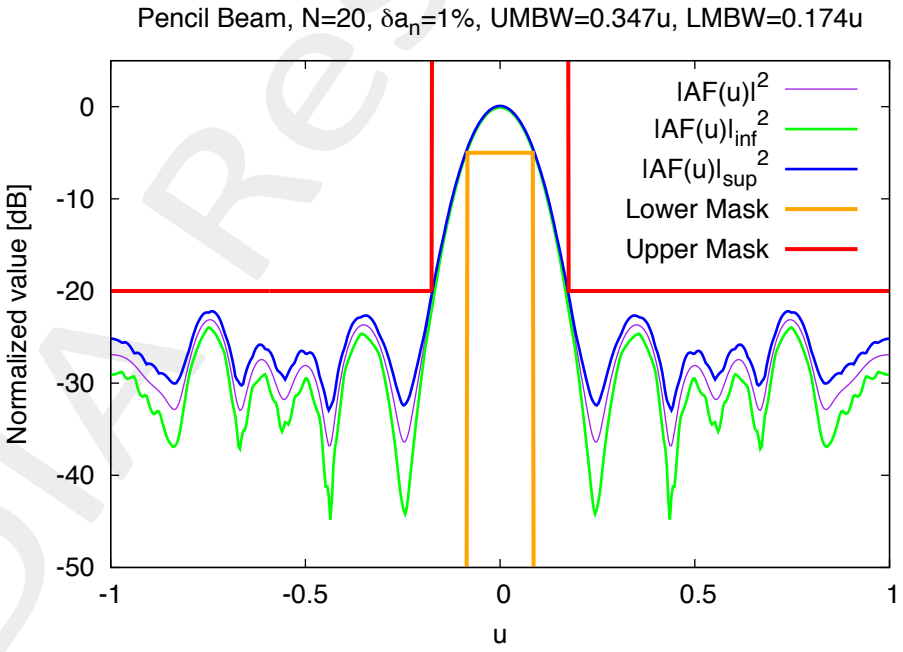

Figure 8.1.2: Optimal Interval Pattern

Parameter	inf	sup	nominal
$[D_{max}]$ [dB]	11.17	11.52	11.34
$[SLL]$ [dB]	-24	-22.1	-23
$[HPBW]$ [deg]	7.79°	8.25°	8.25°
$[\|E(u_{max})\|^2]$ [dB]	-0.087	0.086	0
Pattern Matching	//	//	0.0091

Table 8.1.2: Interval Pattern Parameters

3.2 Test Case #2 - Pencil Beam - $\delta a_n = 5\%$

In this test case a mask constrained power synthesis is performed using Interval Analysis. The amplitude error is set to 5% and the mask is the same of the previous test case (Test Case #1).

Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$
- Sample Points: 501

Test Case Parameters:

- Amplitude Error: 5%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 8.2.1: Max and Min excitations amplitudes values, for the PSO

PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 100
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Power Mask Constraints

- Upper Mask Main Beam Width: 0.347 u (20°)
- Lower Mask Main Beam Width: 0.174 u (10°)
- Side Lobe Level: -20 dB
- Lower Mask Height: -5 dB

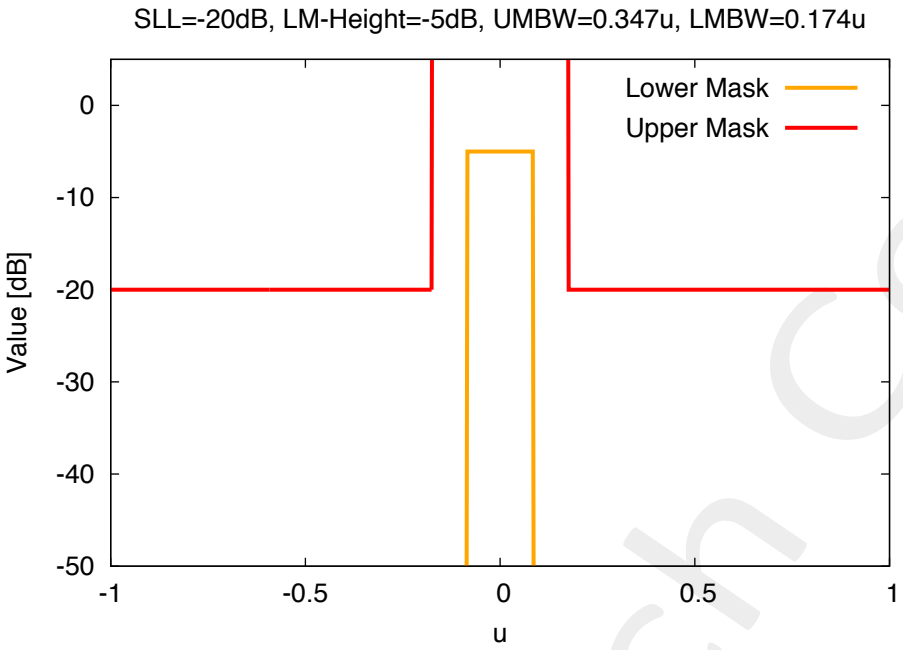

Figure 8.2.1: Power Synthesis Mask

Optimal Particle:

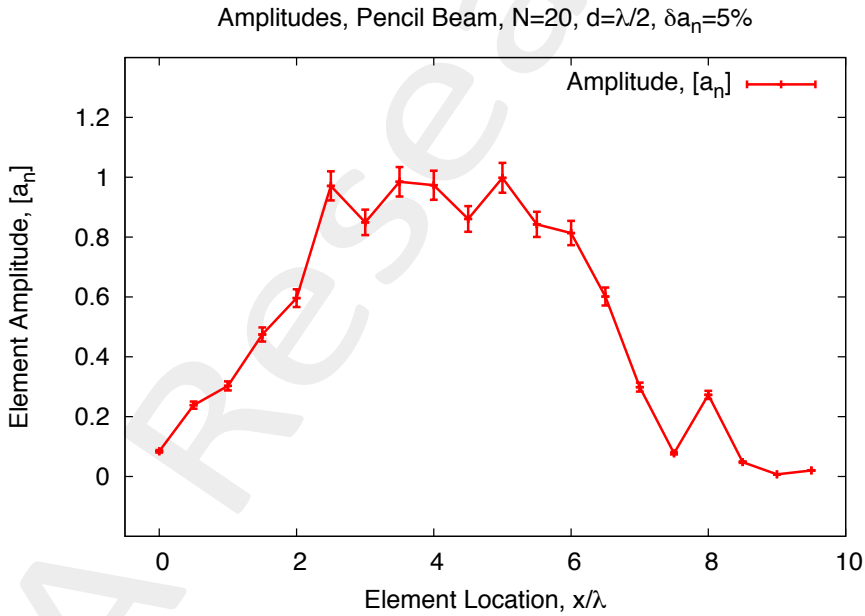

Figure 8.2.2: Optimal particle Amplitudes

Fitness:

- Performed Iterations: 100
- Fitness Finale : 2.3×10^{-18}

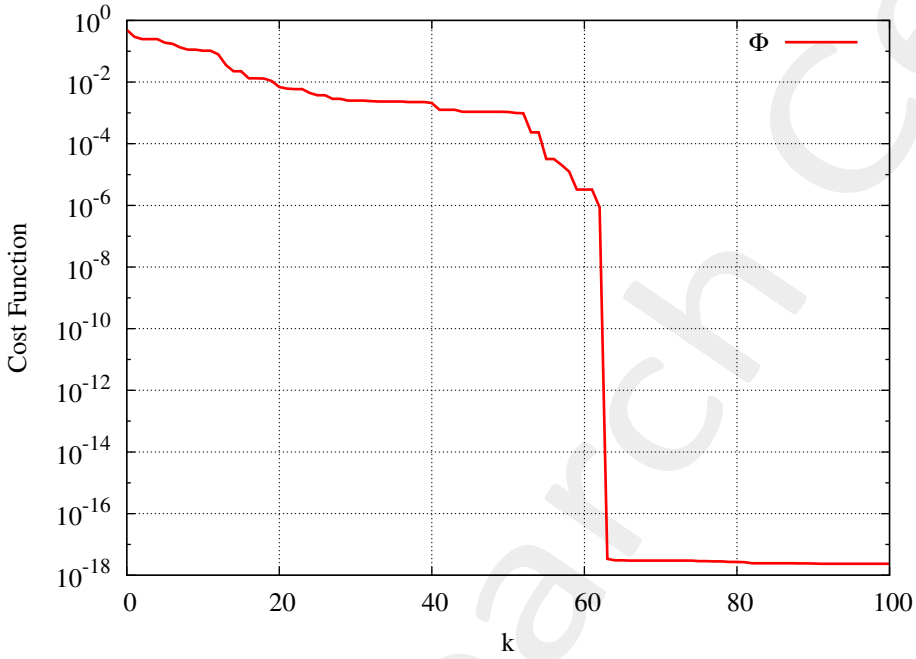

Figure 8.2.3: Fitness

Interval Pattern:

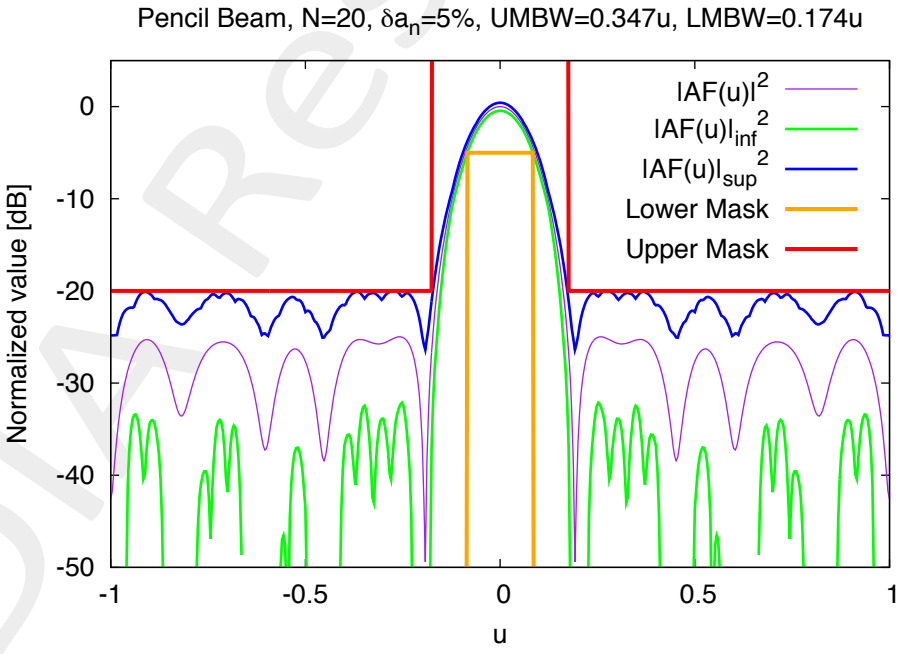

Figure 8.2.2: Optimal Interval Pattern

Parameter	inf	sup	nominal
$[D_{max}]$ [dB]	10.39	12.12	11.26
$[SLL]$ [dB]	-32.5	-19.57	-25
$[HPBW]$ [deg]	8.25°	9.63°	8.25°
$[\|E(u_{max})\|^2]$ [dB]	-0.44	0.42	0
Pattern Matching	//	//	0.046

Table 8.2.2: Interval Pattern Parameters

3.3 Test Case #3 - Pencil Beam - $\delta a_n = 10\%$

In this test case a mask constrained power synthesis is performed using Interval Analysis. The amplitude error is set to 10% and the mask is the same of the previous tests cases (Test Cases #1 and #2).

Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$
- Sample Points: 501

Test Case Parameters:

- Amplitude Error: 10%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 8.3.1: Max and Min excitations amplitudes values, for the PSO

PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 1
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 100
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Power Mask Constraints

- Upper Mask Main Beam Width: 0.347 u (20°)
- Lower Mask Main Beam Width: 0.174 u (10°)
- Side Lobe Level: -20 dB
- Lower Mask Height: -5 dB

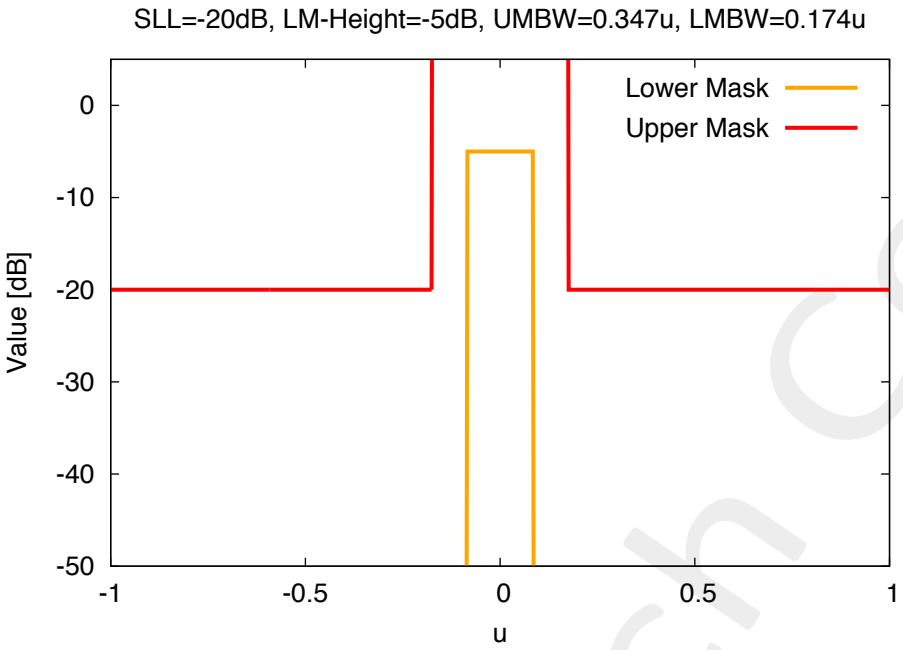

Figure 8.3.1: Power Synthesis Mask

Optimal Particle:

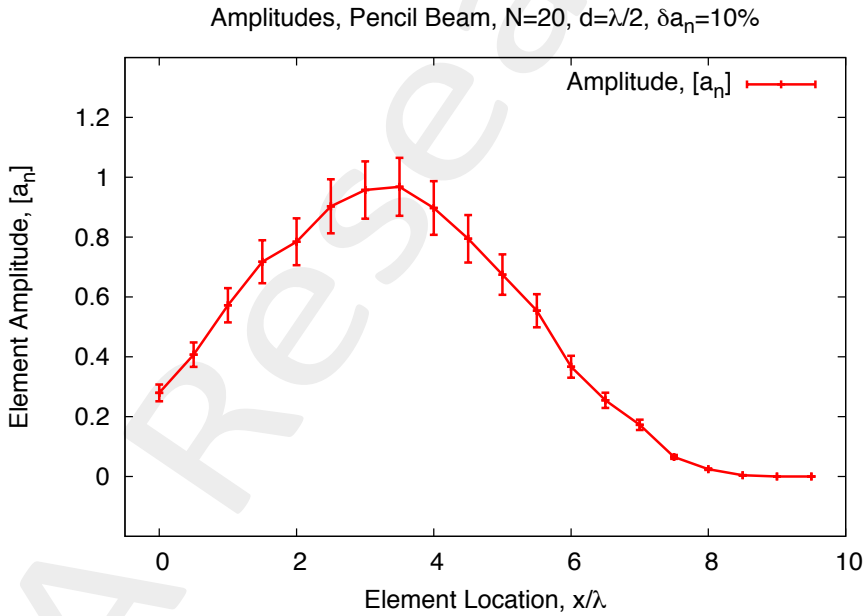

Figure 8.3.2: Optimal particle Amplitudes

Fitness:

- Performed Iterations: 100
- Fitness Finale : 2.1×10^{-3}

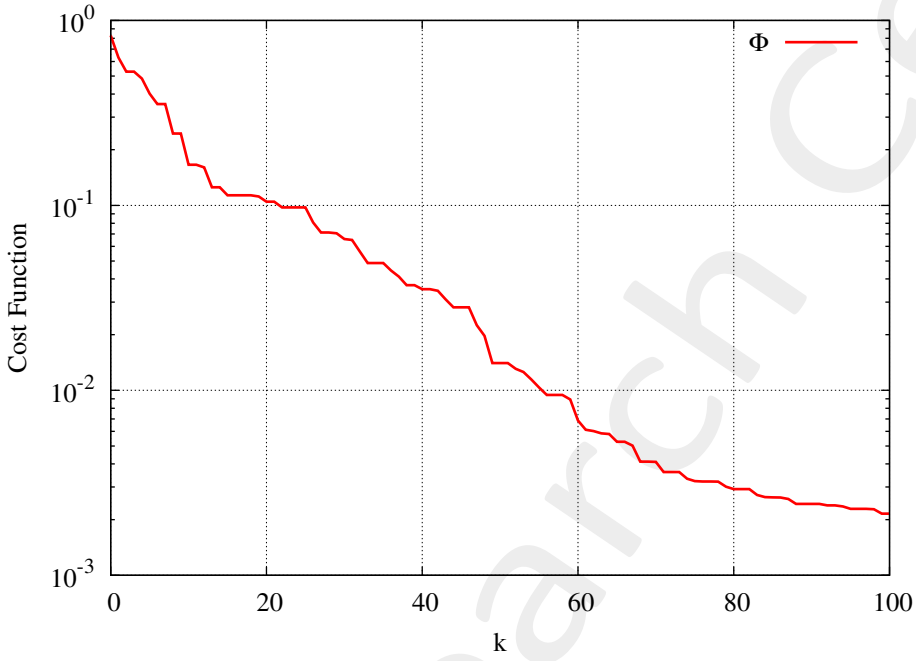

Figure 8.3.3: Fitness

Interval Pattern:

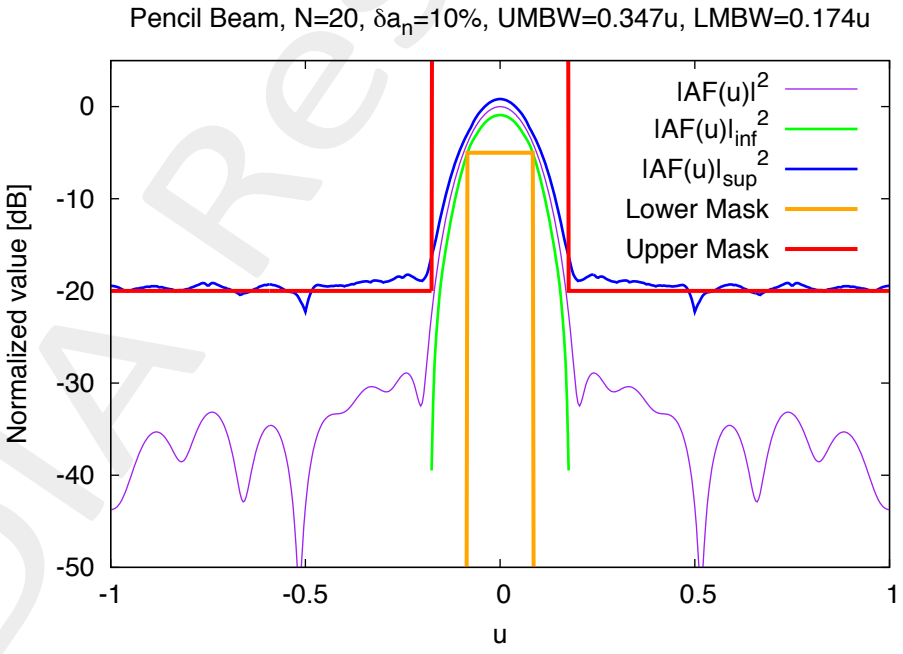

Figure 8.3.2: Optimal Interval Pattern

Parameter	inf	sup	nominal
$[D_{max}]$ [dB]	9.39	12.87	11.13
$[SLL]$ [dB]	$-\infty$	-16.7	-29
$[HPBW]$ [deg]	8.71°	11°	8.71°
$[\|E(u_{max})\|^2]$ [dB]	-0.915	0.827	0
Pattern Matching	//	//	0.091

Table 8.3.2: Interval Pattern Parameters

3.4 Test Case #4 - Pencil Beam - $\delta a_n = 1\%$

In this test case a mask constrained power synthesis is performed using Interval Analysis. The amplitude error is set to 1%. Now the lower mask is 2° larger.

Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$
- Sample Points: 501

Test Case Parameters:

- Amplitude Error: 1%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 8.4.1: Max and Min excitations amplitudes values, for the PSO

PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 2
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 100
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Power Mask Constraints

- Upper Mask Main Beam Width: 0.347 u (20°)
- Lower Mask Main Beam Width: 0.209 u (12°)
- Side Lobe Level: -20 dB
- Lower Mask Height: -5 dB

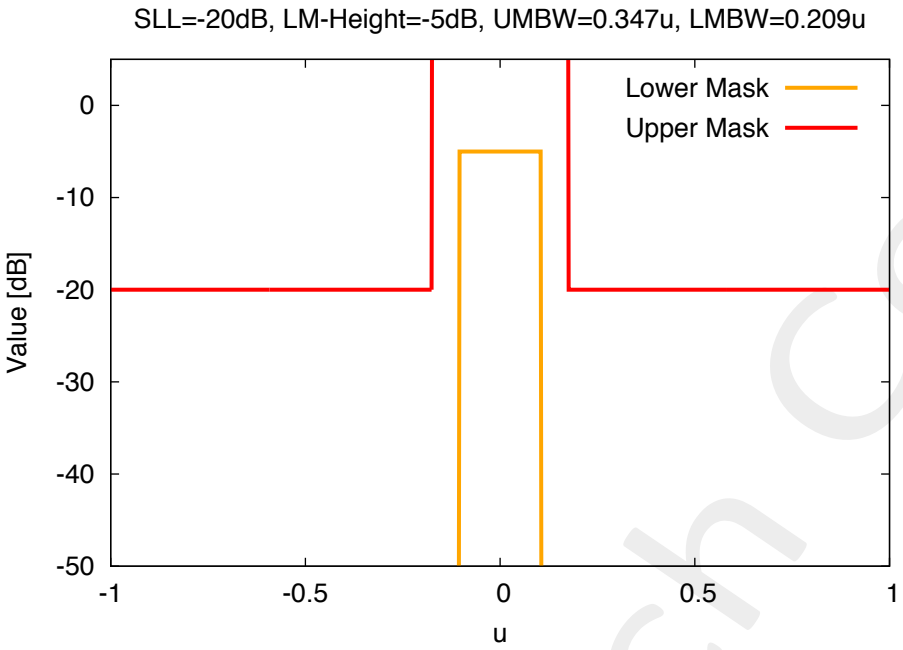

Figure 8.4.1: Power Synthesis Mask

Optimal Particle:

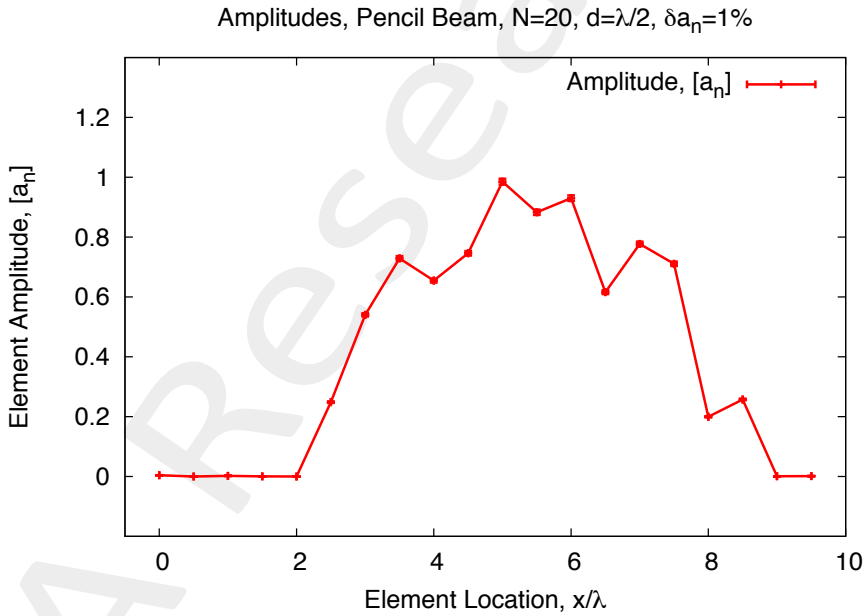

Figure 8.4.2: Optimal particle Amplitudes

Fitness:

- Performed Iterations: 100
- Fitness Finale : 7.4×10^{-5}

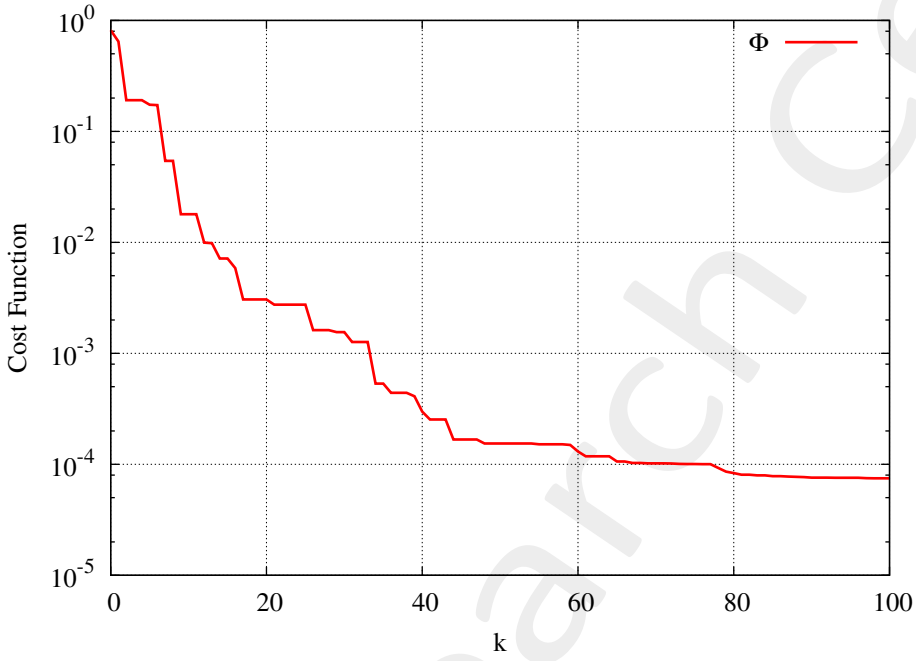

Figure 8.4.3: Fitness

Interval Pattern:

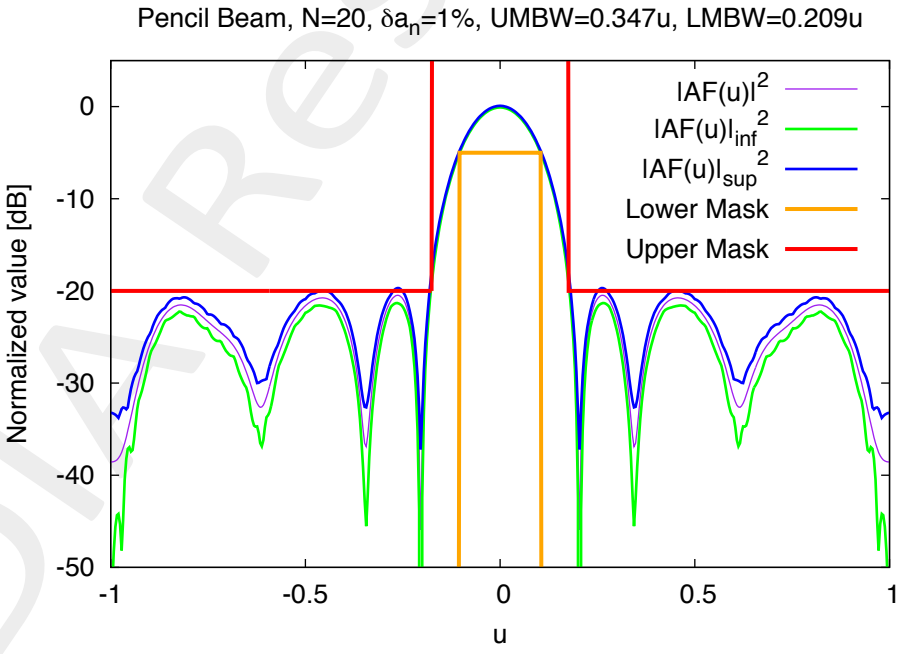

Figure 8.4.2: Optimal Interval Pattern

Parameter	inf	sup	nominal
$[D_{max}]$ [dB]	10.3	10.7	10.5
$[SLL]$ [dB]	-21.4	-19.6	-20
$[HPBW]$ [deg]	9.6°	10.09°	9.6°
$[\|E(u_{max})\|^2]$ [dB]	-0.087	0.086	0
Pattern Matching	//	//	0.011

Table 8.4.2: Interval Pattern Parameters

3.5 Test Case #5 - Pencil Beam -$\delta a_n = 5\%$

In this test case a mask constrained power synthesis is performed using Interval Analysis. The amplitude error is set to 5%. The mask is the same of the previous test case (Test case #4).

Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$
- Sample Points: 501

Test Case Parameters:

- Amplitude Error: 5%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 8.5.1: Max and Min excitations amplitudes values, for the PSO

PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 2
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 100
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Power Mask Constraints

- Upper Mask Main Beam Width: 0.347 u (20°)
- Lower Mask Main Beam Width: 0.209 u (12°)
- Side Lobe Level: -20 dB
- Lower Mask Height: -5 dB

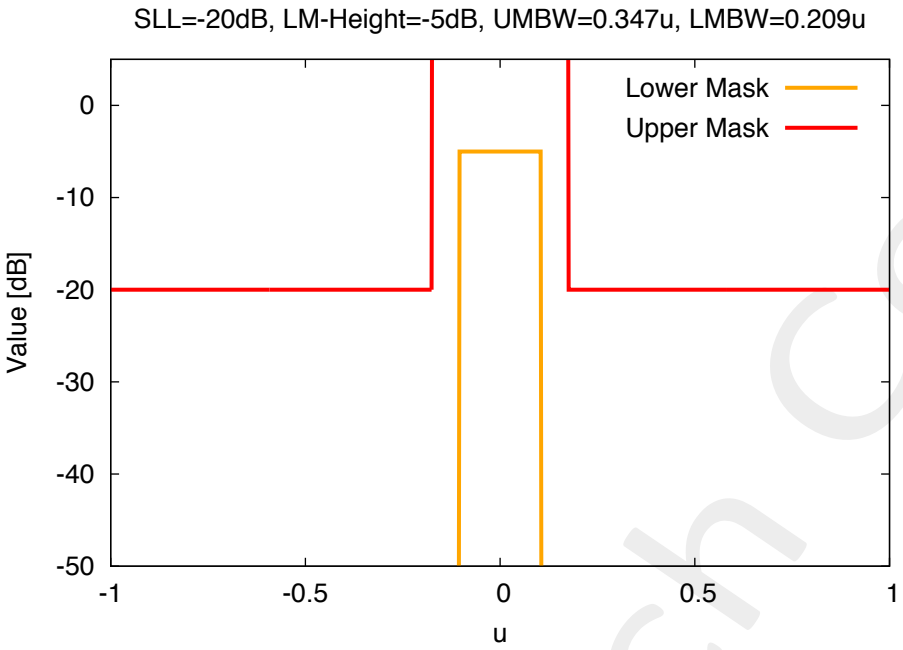

Figure 8.5.1: Power Synthesis Mask

Optimal Particle:

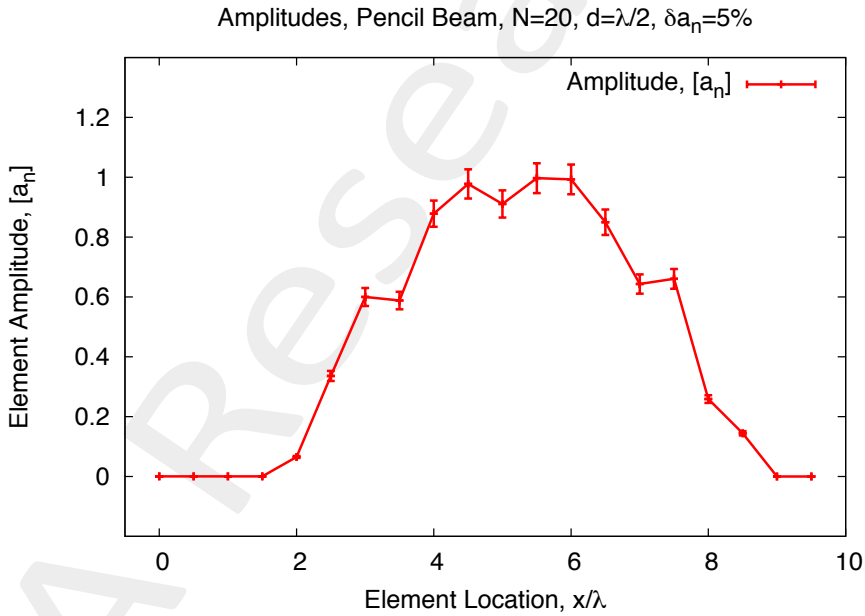

Figure 8.5.2: Optimal particle Amplitudes

Fitness:

- Performed Iterations: 100
- Fitness Finale : 3.1×10^{-3}

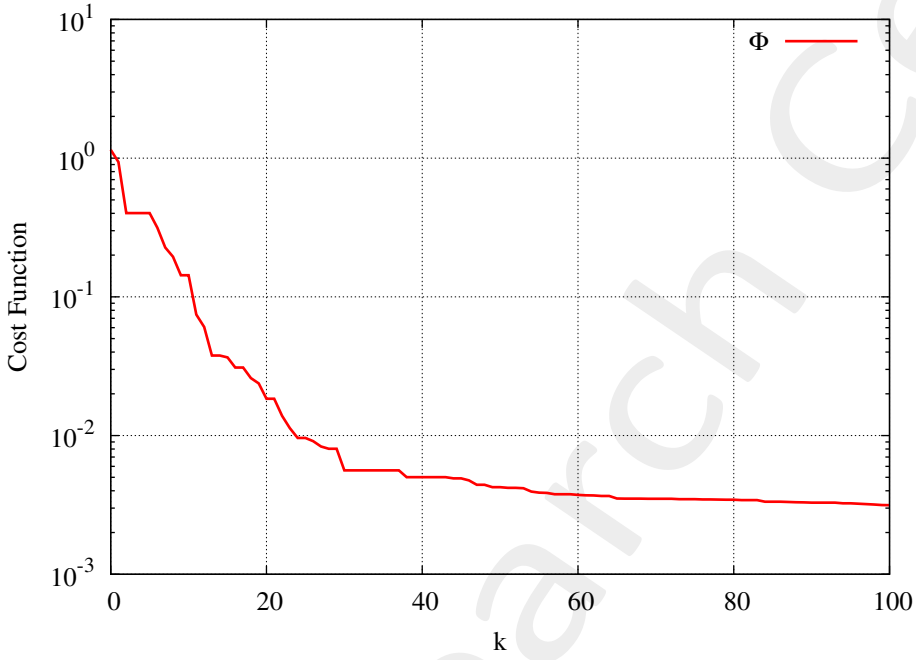

Figure 8.5.3: Fitness

Interval Pattern:

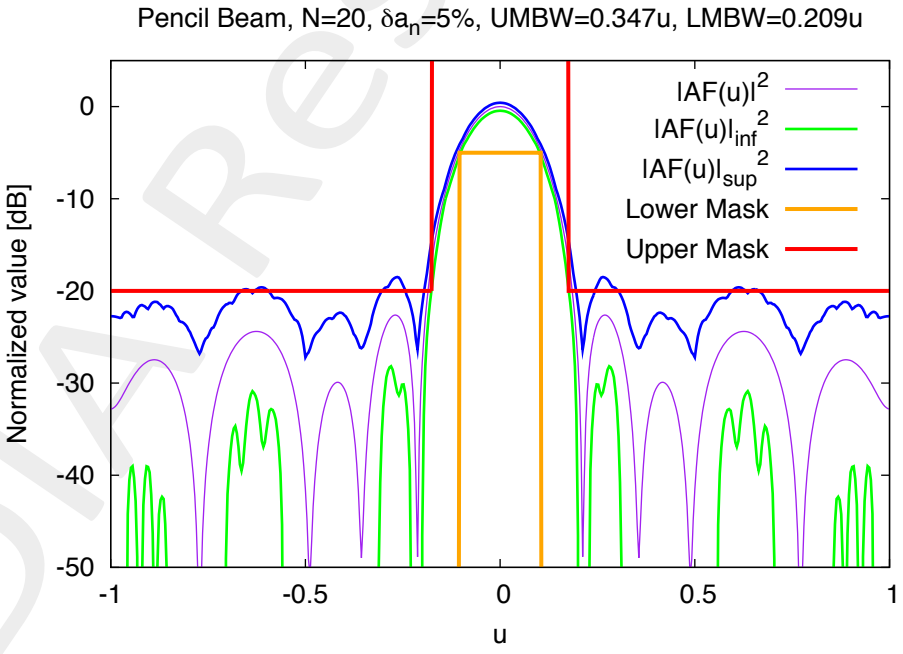

Figure 8.5.2: Optimal Interval Pattern

Parameter	inf	sup	nominal
$[D_{max}]$ [dB]	9.65	11.39	10.52
$[SLL]$ [dB]	-28.6	-18	-23
$[HPBW]$ [deg]	9.63°	11°	10.09°
$[\|E(u_{max})\|^2]$ [dB]	-0.44	0.42	0
Pattern Matching	//	//	0.051

Table 8.5.2: Interval Pattern Parameters

3.6 Test Case #6 - Pencil Beam -$\delta a_n = 10\%$

In this test case a mask constrained power synthesis is performed using Interval Analysis. The amplitude error is set to 10%. The mask is the same of the previous test cases (Test cases #4 and #5).

Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$
- Sample Points: 501

Test Case Parameters:

- Amplitude Error: 10%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 8.6.1: Max and Min excitations amplitudes values, for the PSO

PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 2
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 100
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Power Mask Constraints

- Upper Mask Main Beam Width: 0.347 u (20°)
- Lower Mask Main Beam Width: 0.209 u (12°)
- Side Lobe Level:−20 dB
- Lower Mask Height: −5 dB

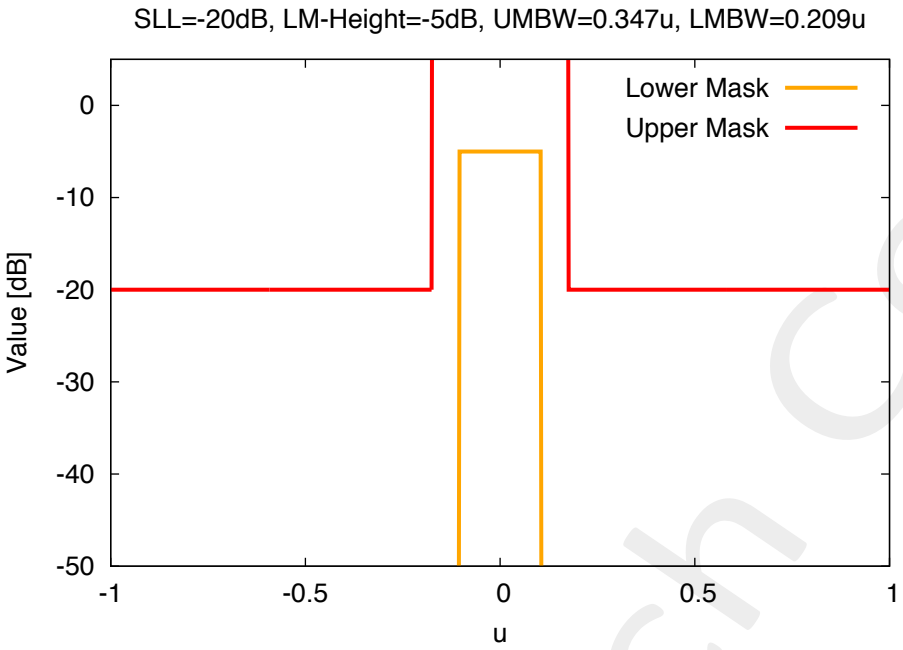

Figure 8.6.1: Power Synthesis Mask

Optimal Particle:

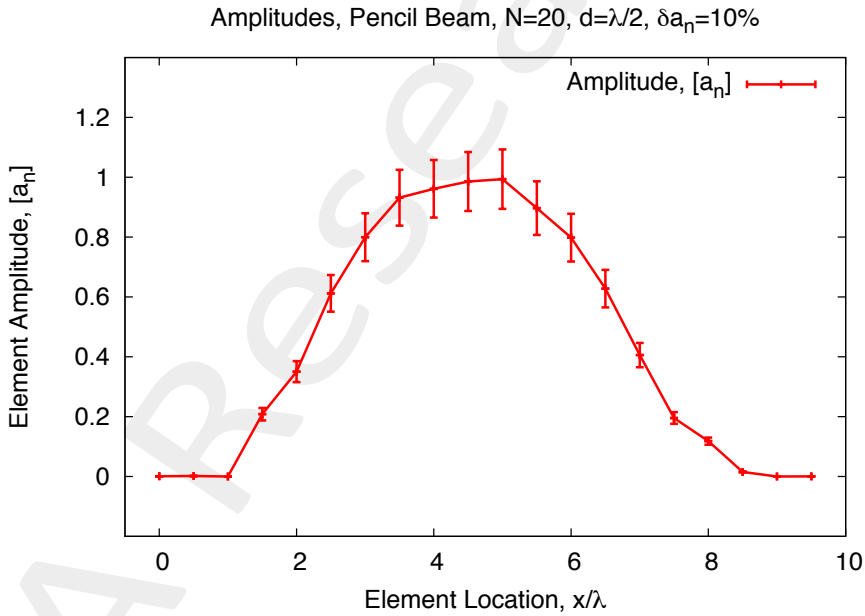

Figure 8.6.2: Optimal particle Amplitudes

Fitness:

- Performed Iterations: 100
- Fitness Finale : 2.6×10^{-2}

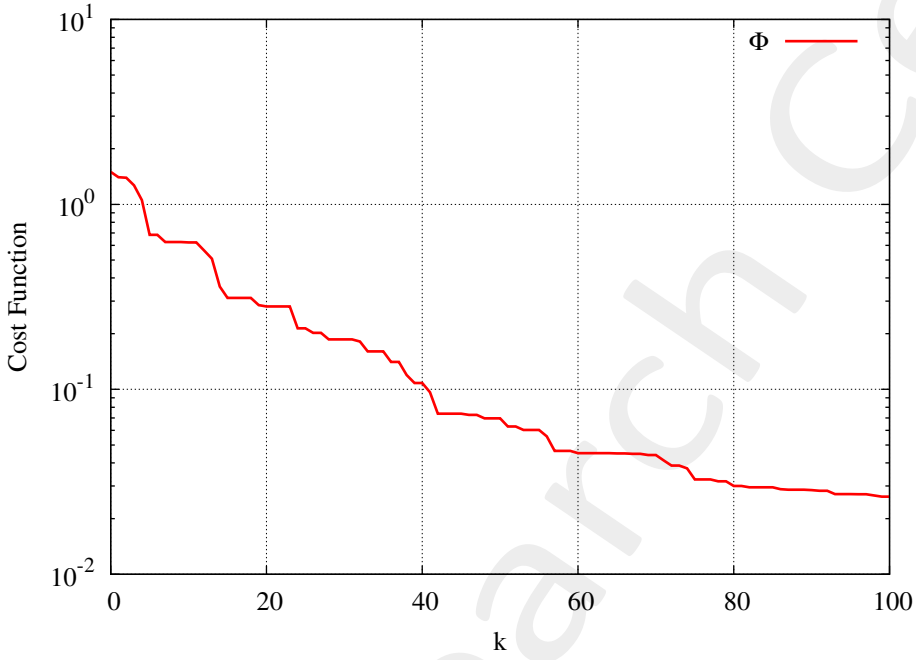

Figure 8.6.3: Fitness

Interval Pattern:

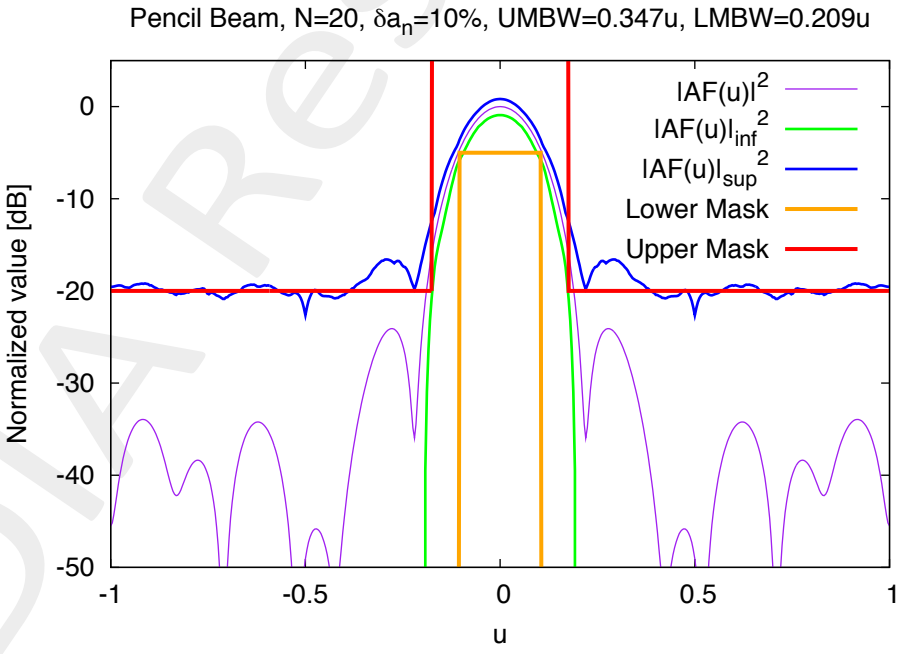

Figure 8.6.2: Optimal Interval Pattern

Parameter	inf	sup	nominal
$[D_{max}]$ [dB]	8.8	12.3	10.5
$[SLL]$ [dB]	$-\infty$	-15.6	-24
$[HPBW]$ [deg]	9.63°	12.4°	10.09°
$[\lvert E(u_{max})\rvert^2]$ [dB]	-0.91	0.82	0
Pattern Matching	//	//	0.1022

Table 8.6.2: Interval Pattern Parameters

3.7 Test Case #7 - Pencil Beam -$\delta a_n = 1\%$

In this test case a mask constrained power synthesis is performed using Interval Analysis. The amplitude error is set to 1%. The lower mask is 2 dB higher with respect of the Test Case #1's mask.

Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$
- Sample Points: 501

Test Case Parameters:

- Amplitude Error: 1%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 8.7.1: Max and Min excitations amplitudes values, for the PSO

PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 3
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 100
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Power Mask Constraints

- Upper Mask Main Beam Width: 0.347 u (20°)
- Lower Mask Main Beam Width: 0.174 u (10°)
- Side Lobe Level: -20 dB
- Lower Mask Height: -3 dB

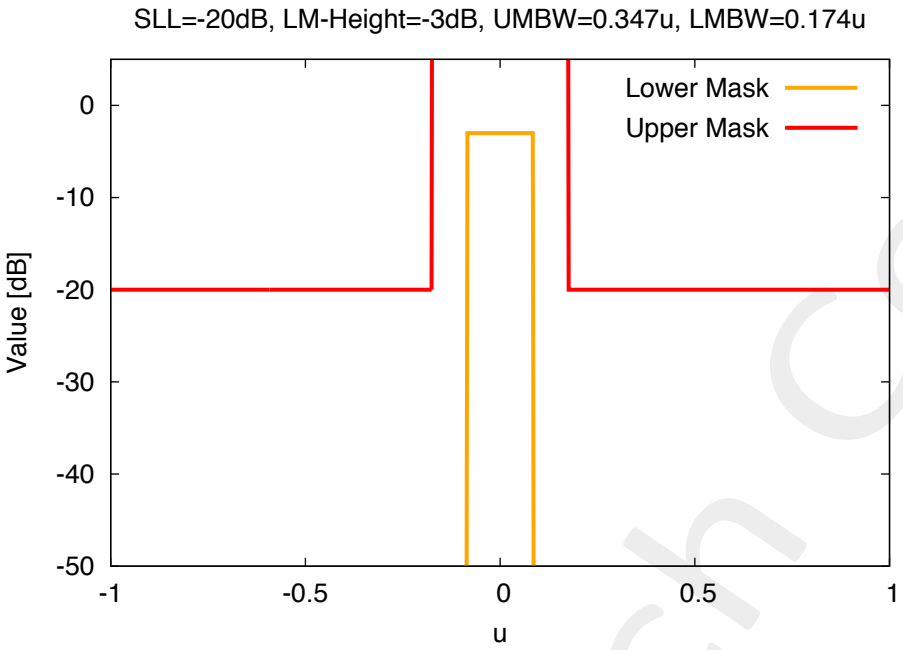

Figure 8.7.1: Power Synthesis Mask

Optimal Particle:

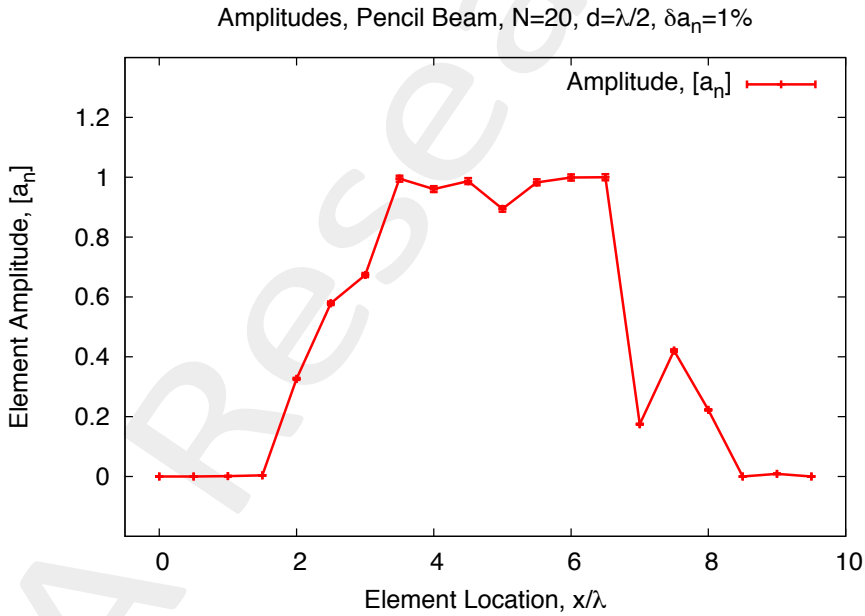

Figure 8.7.2: Optimal particle Amplitudes

Fitness:

- Performed Iterations: 100
- Fitness Finale : 6.6×10^{-4}

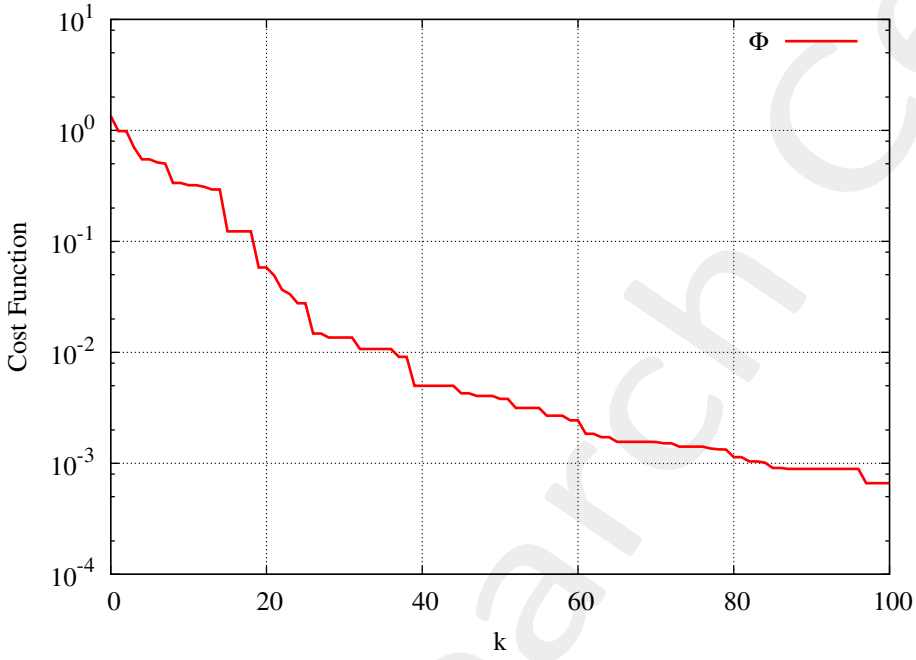

Figure 8.7.3: Fitness

Interval Pattern:

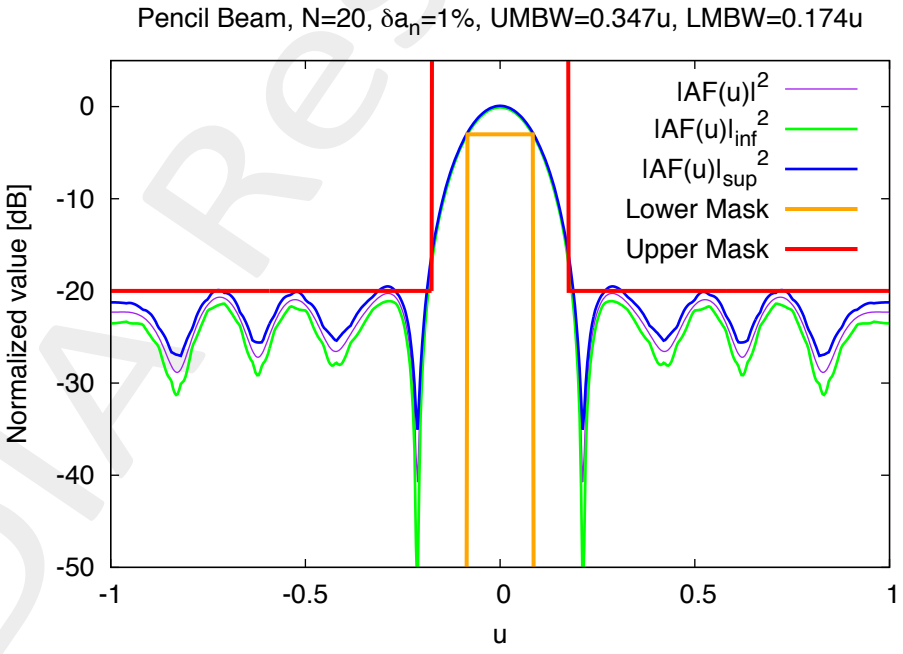

Figure 8.7.2: Optimal Interval Pattern

Parameter	inf	sup	nominal
$[D_{max}]$ [dB]	10.2	10.5	10.3
$[SLL]$ [dB]	-21.1	-19.4	-20
$[HPBW]$ [deg]	10.09°	10.09°	10.09°
$[\|E(u_{max})\|^2]$ [dB]	-0.087	0.086	0
Pattern Matching	//	//	0.011

Table 8.7.2: Interval Pattern Parameters

3.8 Test Case #8 - Pencil Beam - $\delta a_n = 5\%$

In this test case a mask constrained power synthesis is performed using Interval Analysis. The amplitude error is set to 5%. The mask is the same of the previous test case (Test Case #7).

Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$
- Sample Points: 501

Test Case Parameters:

- Amplitude Error: 5%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 8.8.1: Max and Min excitations amplitudes values, for the PSO

PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 3
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 100
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Power Mask Constraints

- Upper Mask Main Beam Width: 0.347 u (20°)
- Lower Mask Main Beam Width: 0.174 u (10°)
- Side Lobe Level: -20 dB
- Lower Mask Height: -3 dB

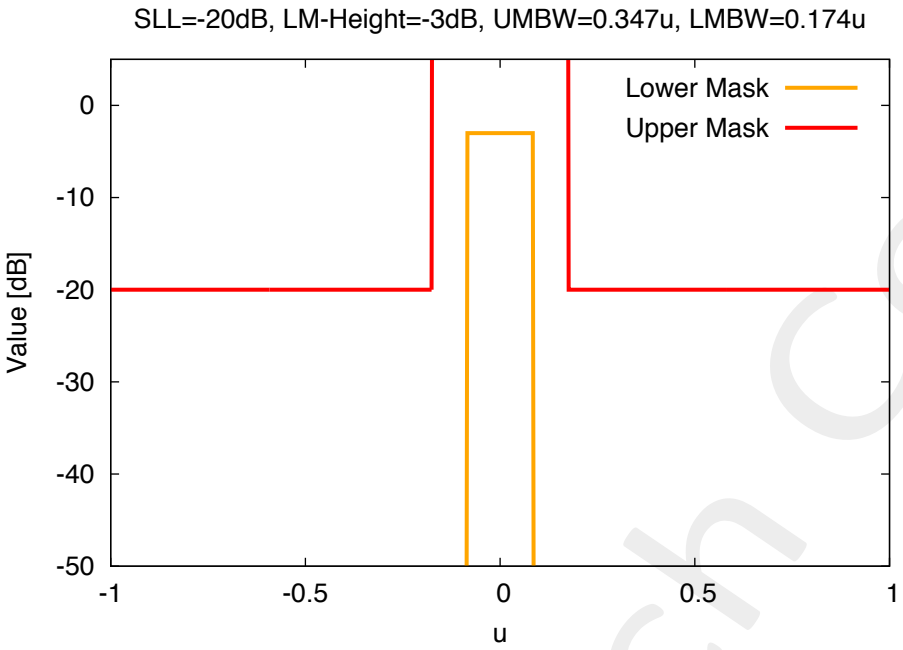

Figure 8.8.1: Power Synthesis Mask

Optimal Particle:

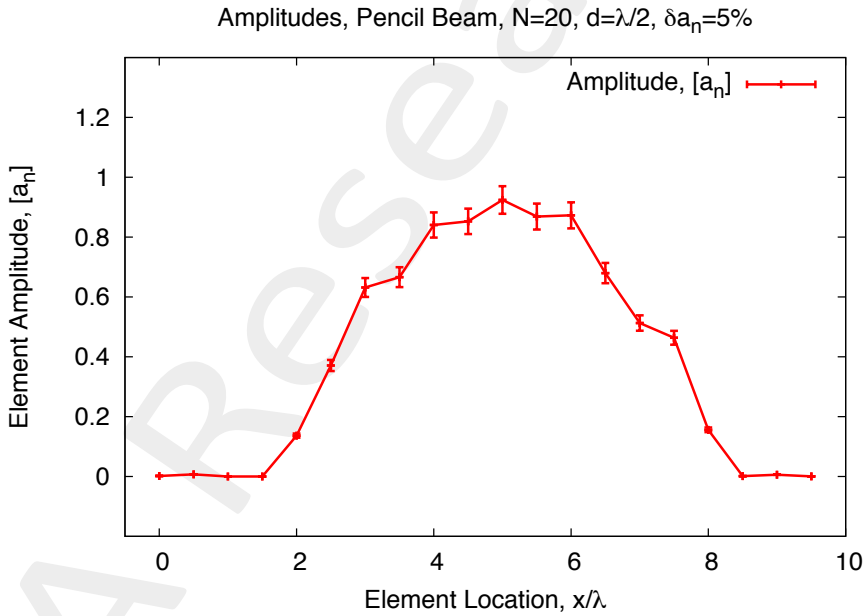

Figure 8.8.2: Optimal particle Amplitudes

Fitness:

- Performed Iterations: 100
- Fitness Finale : 8.5×10^{-3}

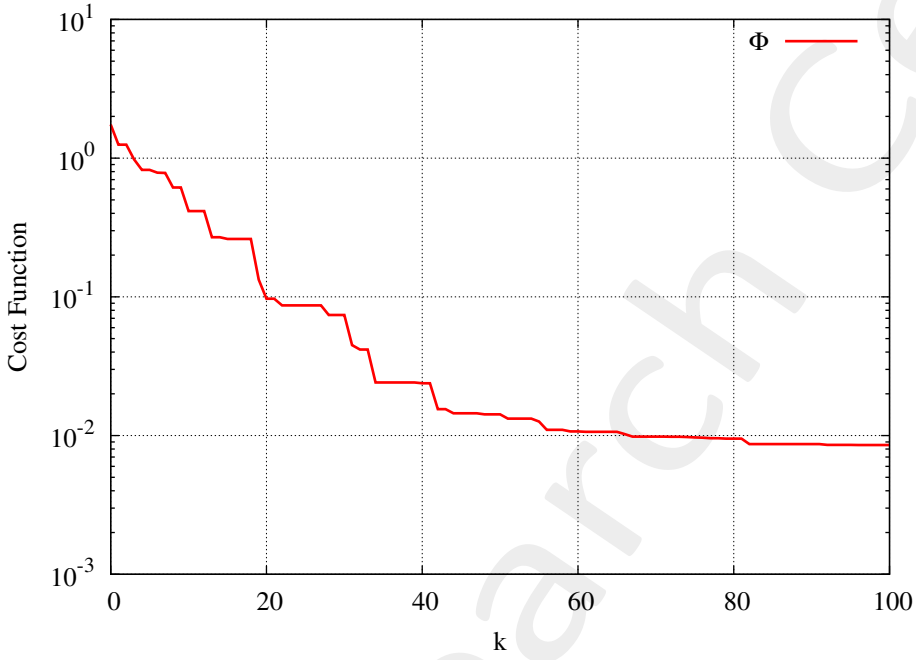

Figure 8.8.3: Fitness

Interval Pattern:

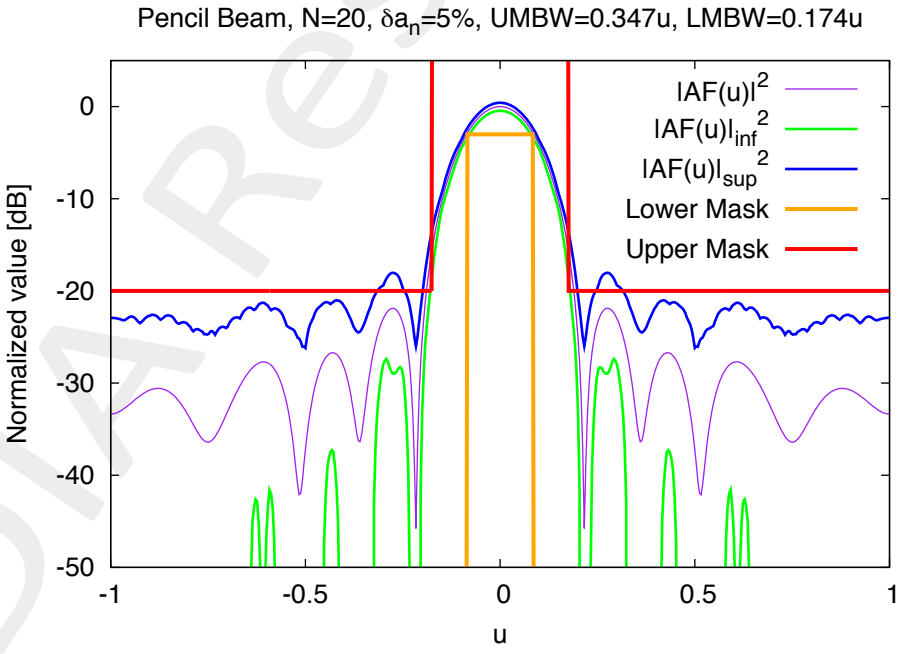

Figure 8.8.2: Optimal Interval Pattern

Parameter	inf	sup	nominal
$[D_{max}]$ [dB]	9.5	11.3	10.4
$[SLL]$ [dB]	-27.8	-17.5	-22
$[HPBW]$ [deg]	10.09°	11.47°	10.09°
$[\|E(u_{max})\|^2]$ [dB]	-0.44	0.42	0
Pattern Matching	//	//	0.051

Table 8.8.2: Interval Pattern Parameters

3.9 Test Case #9 - Pencil Beam - $\delta a_n = 10\%$

In this test case a mask constrained power synthesis is performed using Interval Analysis. The amplitude error is set to 10%. The mask is the same of the previous test cases (Test Cases #7 and #8).

Geometry:

- Number of Elements: $N = 20$
- Element Spacing: $d = \frac{\lambda}{2}$
- Sample Points: 501

Test Case Parameters:

- Amplitude Error: 10%
- Phase Error: $\delta\varphi_n = 0.0$ rad

Max Amplitude Value	1.0
Min Amplitude Value	0.0

Table 8.9.1: Max and Min excitations amplitudes values, for the PSO

PSO Parameters:

- Unknown Number: 20
- Swarm Dimension: 20
- Random Seed: 3
- Fitness Tolerance: 1×10^{-100}
- Max Iterations Number: 100
- Inertial Weight: 0.4
- c1: 2.0
- c2: 2.0

Power Mask Constraints

- Upper Mask Main Beam Width: 0.347 u (20°)
- Lower Mask Main Beam Width: 0.174 u (10°)
- Side Lobe Level: -20 dB
- Lower Mask Height: -3 dB

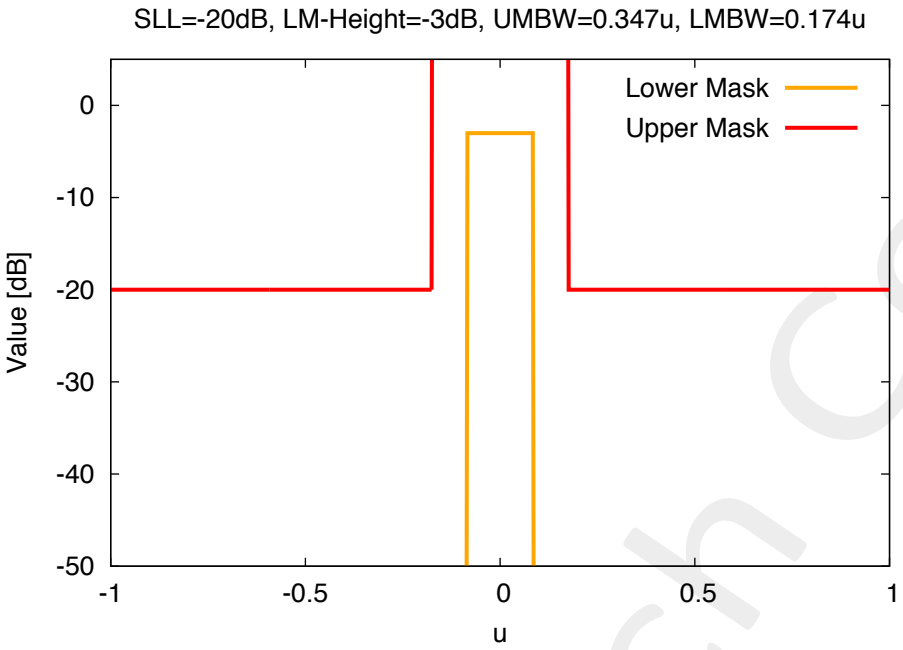

Figure 8.9.1: Power Synthesis Mask

Optimal Particle:

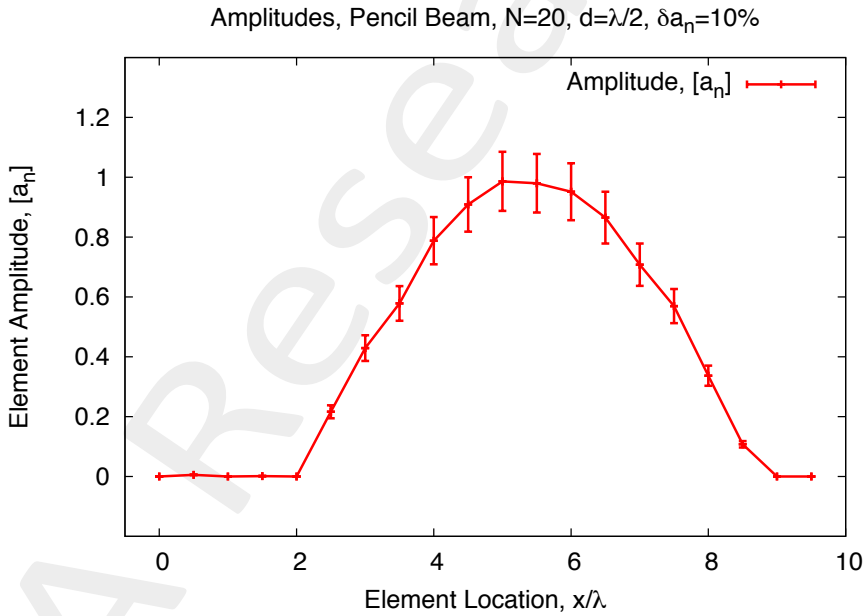

Figure 8.9.2: Optimal particle Amplitudes

Fitness:

- Performed Iterations: 100
- Fitness Finale : 4.8×10^{-2}

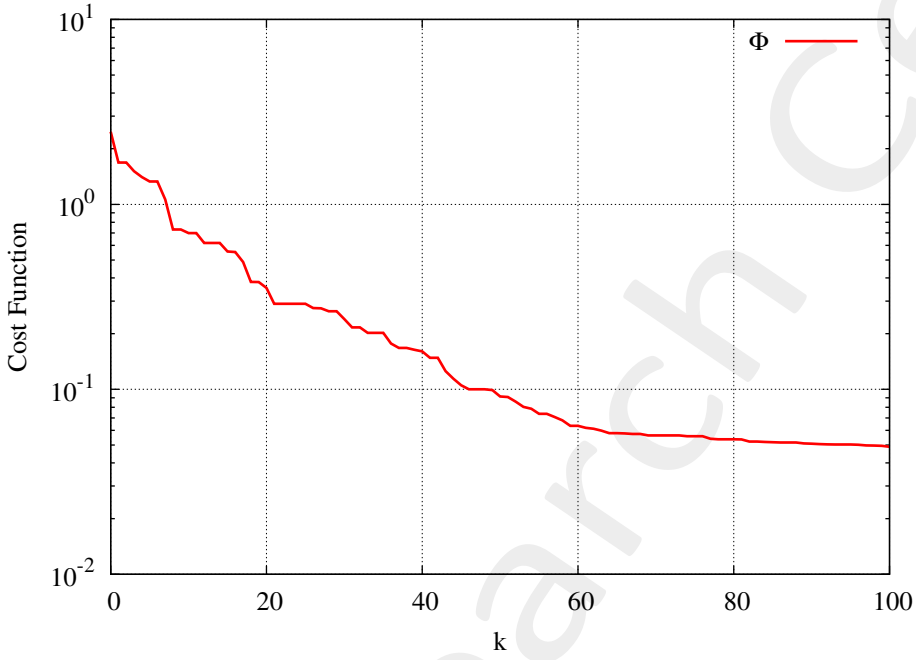

Figure 8.9.3: Fitness

Interval Pattern:

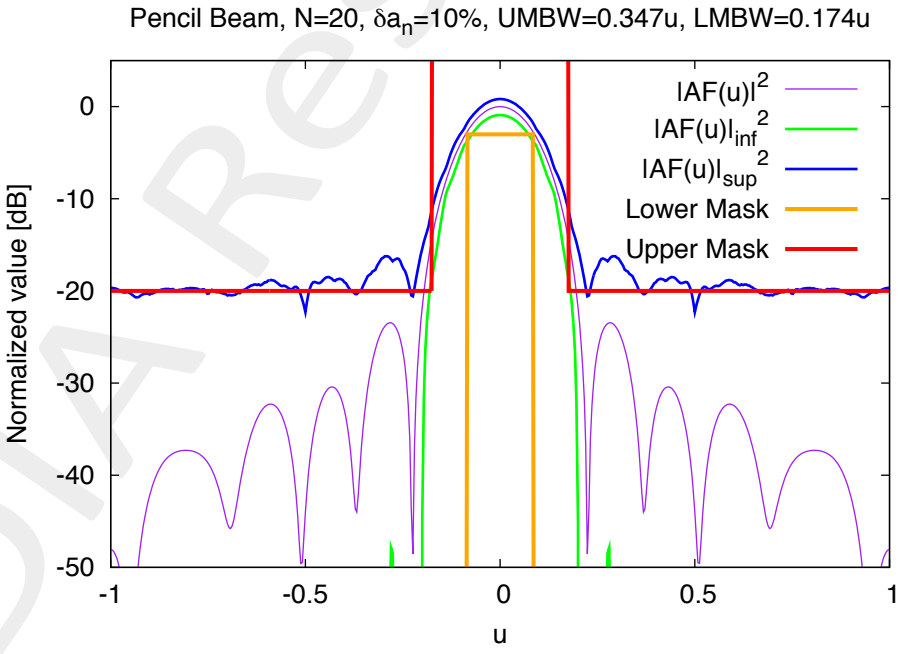

Figure 8.9.2: Optimal Interval Pattern

Parameter	inf	sup	nominal
$[D_{max}]$ [dB]	8.6	12.1	10.3
$[SLL]$ [dB]	-48.6	-15.3	-22.5
$[HPBW]$ [deg]	10.09°	13.32°	10.55°
$[\|E(u_{max})\|^2]$ [dB]	-0.91	0.82	0
Pattern Matching	//	//	0.1062

Table 8.9.2: Interval Pattern Parameters

References

[1] P. Rocca, N. Anselmi, and A. Massa, "Optimal synthesis of robust beamformer weights exploiting interval analysis and convex optimization," IEEE Trans. Antennas Propag., vol. 62, no. 7, pp. 3603-3612, Jul. 2014.

[2] L. Manica, N. Anselmi, P. Rocca, and A. Massa, "Robust mask-constrained linear array synthesis through an interval-based particle swarm optimisation," IET Microwaves, Antennas and Propagation, vol. 7, no. 12, pp. 976-984, Sep. 2013.

[3] N. Anselmi, L. Manica, P. Rocca, and A. Massa, "Tolerance analysis of antenna arrays through interval arithmetic," IEEE Transactions on Antennas and Propagation, vol. 61, no. 11, pp. 5496-5507, Nov. 2013.

[4] P. Rocca, L. Manica, N. Anselmi, and A. Massa, "Analysis of the pattern tolerances in linear arrays with arbitrary amplitude errors," IEEE Antennas Wireless Propag. Lett., vol. 12, pp. 639-642, 2013.

[5] T. Moriyama, L. Poli, N. Anselmi, M. Salucci, and P. Rocca, "Real array pattern tolerances from amplitude excitation errors," IEICE Electronics Express, vol. 11, no. 17, pp. 1-8, Sep. 2014.

[6] L. Manica, P. Rocca, N. Anselmi, and A. Massa, "On the synthesis of reliable linear arrays through interval arithmetic," IEEE International Symposium on Antennas Propag. (APS/URSI 2013), Orlando, Florida, USA, Jul. 7-12, 2013.

[7] L. Manica, P. Rocca, G. Oliveri, and A. Massa, "Designing radiating systems through interval analysis tools," IEEE International Symposium on Antennas Propag. (APS/URSI 2013), Orlando, Florida, USA, Jul. 7-12, 2013.

[8] M. Carlin, N. Anselmi, L. Manica, P. Rocca, and A. Massa, "Exploiting interval arithmetic for predicting real arrays performances - The linear case," IEEE International Symposium on Antennas Propag. (APS/URSI 2013), Orlando, Florida, USA, Jul. 7-12, 2013.

[9] P. Rocca, L. Manica, and A. Massa, "Interval-based analysis of pattern distortions in reflector antennas with bump-like surface deformations," IET Microwaves, Antennas and Propagation, vol. 8, no. 15, pp. 1277-1285, Dec. 2014.

[10] P. Rocca, N. Anselmi, and A. Massa, "Interval Arithmetic for pattern tolerance analysis of parabolic reflectors," IEEE Trans. Antennas Propag., vol. 62, no. 10, pp. 4952-4960, Oct. 2014.

[11] P. Rocca, M. Benedetti, M. Donelli, D. Franceschini, and A. Massa, "Evolutionary optimization as applied to inverse problems," Inverse Problems - 25 th Year Special Issue of Inverse Problems, Invited Topical Review, vol. 25, pp. 1-41, Dec. 2009.

[12] P. Rocca, G. Oliveri, and A. Massa, "Differential Evolution as applied to electromagnetics," IEEE Antennas Propag. Mag., vol. 53, no. 1, pp. 38-49, Feb. 2011.

[13] G. Oliveri, P. Rocca, and A. Massa, "Reliable diagnosis of large linear arrays - A Bayesian Compressive Sensing approach," IEEE Trans. Antennas Propag., vol. 60, no. 10, pp. 4627-4636, Oct. 2012.

[14] F. Viani, G. Oliveri, and A. Massa, "Compressive sensing pattern matching techniques for synthesizing planar sparse arrays," IEEE Trans. Antennas Propag., vol. 61, no. 9, pp. 4577-4587, Sept. 2013.

[15] G. Oliveri, F. Viani, and A. Massa, "Synthesis of linear multi-beam arrays through hierarchical ADS-based interleaving," IET Microw. Antennas Propag., vol. 8, no. 10, pp. 794-808, Jul. 2014.

[16] G. Oliveri, P. Rocca, and A. Massa, "Interleaved linear arrays with difference sets," Electronics Letters, vol. 46, no. 5, pp. 323-324, Mar. 2010.

[17] P. Rocca, L. Manica, and A. Massa, "An effective excitation matching method for the synthesis of optimal compromises between sum and difference patterns in planar arrays," Progress in Electromagnetic Research B, vol. 3, pp. 115-130, 2008.

[18] P. Rocca, L. Manica, and A. Massa, "Directivity optimization in planar sub-arrayed monopulse antenna," Progress in Electromagnetic Research L, vol. 4, pp. 1-7, 2008.

[19] L. Poli, P. Rocca, G. Oliveri, and A. Massa, "Failure correction in time-modulated linear arrays," IET Radar, Sonar & Navigation, vol. 8, no. 3, pp. 195-201, 2014.

[20] L. Poli, P. Rocca, G. Oliveri, and A. Massa, "Harmonic beamforming in time-modulated linear arrays," IEEE Trans. Antennas Propag., vol. 59, no. 7, pp. 2538-2545, Jul. 2011.