BCS-based inversion methods within a multifrequency framework

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Abstract

In this report, the multi-frequency Multi-Task Bayesian Compressive Sensing (MT-BCS) technique is compared with the Single-Task Bayesian Compressive Sensing one (ST-BCS). The comparison shows how the first method, which concurrently handle the multi-frequency data taking into account the relationship between the correlated inverse problems associated to different illumination frequencies, provides better results. Moreover, single-frequency and multi-frequency approaches have been investigated.

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Legenda

- SF-ST-BCS is the single-task Bayesian Compressive Sampling-based technique developed in [1] and working at a single frequency.
- MF-ST-BCS is the single-task Bayesian Compressive Sampling-based technique working at multiple frequencies.
- MF-MT-BCS is the multi-task Bayesian Compressive Sampling-based technique that exploits the correlation between multiple illumination frequencies.

1 Comparison with ST-BCS

1.1 Homogeneous Objects

1.1.1 Two Strips of Sides $l_1 = 0.16\lambda$, $l_2 = 0.50\lambda$

GOAL: show the performances of the multi-frequency MT - BCS when dealing with a sparse scatterer

- Number of frequencies F
- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$ (at the central frequency)
- D = 1296 (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$

•
$$2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$$

- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- N scelto in modo da essere vicino a #DOF: $N = 324 (18 \times 18)$

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$ (at the central frequency)
- $M \approx 2ka \rightarrow M = 27$

Sources:

- $V = 1 \ (\theta = 0^{\circ})$
- Amplitude: A = 1 (plane waves)
- Number of Frequencies: F = 11
- Frequency Range: $I_F = [150 Mhz : 450 MHz]$ Frequency Step: $S_F = [30 Mhz]$

Object:

- Two strips of sides $l_1 = 0.16\lambda$, $l_2 = 0.50\lambda$
- $\varepsilon_r = \in \{1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0\}$

• $\sigma = 0 \, [\text{S/m}]$

MT-BCS parameters:

- Gamma prior on noise variance parameters: $\beta_1 = 6.5 \times 10^{-1}$, $\beta_2 = 5.8 \times 10^{-2}$
- Convergenze parameter: $\tau = 1.0 \times 10^{-8}$



Two Homogeneous Strips of Sides $l_1 = 0.16\lambda$, $l_2 = 0.50\lambda$ - $\varepsilon_r = 1.5$ - BCS Reconstructions Comparison

Figure 35. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).



Two Homogeneous Strips of Sides $l_1=0.16\lambda,\ l_2=0.50\lambda$ - $\varepsilon_r=2.0$ - BCS Reconstructions Comparison

Figure 36. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).

(i)

(l)

(h)



Two Homogeneous Strips of Sides $l_1=0.16\lambda,\ l_2=0.50\lambda$ - $\varepsilon_r=3.0$ - BCS Reconstructions Comparison

Figure 37. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).



Two Homogeneous Strips of Sides $l_1=0.16\lambda,\ l_2=0.50\lambda$ - $\varepsilon_r=4.0$ - BCS Reconstructions Comparison

Figure 38. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).



Two Homogeneous Strips of Sides $l_1=0.16\lambda,\ l_2=0.50\lambda$ - $\varepsilon_r=5.0$ - BCS Reconstructions Comparison

Figure 39. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).



Two Homogeneous Strips of Sides $l_1 = 0.16\lambda$, $l_2 = 0.50\lambda$ - BCS Errors vs. ε_r Comparison

Figure 40. Behaviour of total error ξ_{tot} as a function of ε_r , for SNR = 50 [dB] (a), SNR = 20 [dB] (b), SNR = 15 [dB] (c), SNR = 10 [dB] (d) and SNR = 5 [dB] (e).



Two Homogeneous Strips of Sides $l_1 = 0.16\lambda$, $l_2 = 0.50\lambda$ - BCS Errors vs. SNR Comparison

Figure 41. Behaviour of total error ξ_{tot} as a function of SNR, for $\varepsilon_r = 1.5$ [dB] (a), $\varepsilon_r = 2.0$ [dB] (b), $\varepsilon_r = 3.0$ [dB] (c), $\varepsilon_r = 4.0$ [dB] (d) and $\varepsilon_r = 5.0$ [dB] (e).

1.1.2 Eight Pixels of Side $l = 0.16\lambda$

GOAL: show the performances of the multi-frequency MT - BCS when dealing with a sparse scatterer

- Number of frequencies F
- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$ (at the central frequency)
- D = 1296 (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- N scelto in modo da essere vicino a #DOF: $N = 324 (18 \times 18)$

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$ (at the central frequency)
- $M \approx 2ka \rightarrow M = 27$

Sources:

- $V = 1 \ (\theta = 0^{\circ})$
- Amplitude: A = 1 (plane waves)
- Number of Frequencies: F = 11
- Frequency Range: $I_F = [150 Mhz : 450 MHz]$ Frequency Step: $S_F = [30 Mhz]$

Object:

- Eight square cylinders of side $l = 0.16\lambda$
- $\varepsilon_r = \in \{1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0\}$
- $\sigma = 0 [S/m]$

MT-BCS parameters:

- Gamma prior on noise variance parameters: $\beta_1 = 6.5 \times 10^{-1}$, $\beta_2 = 5.8 \times 10^{-2}$
- Convergenze parameter: $\tau = 1.0 \times 10^{-8}$



Eight Homogeneous Pixels of Side $l = 0.16\lambda$ - $\varepsilon_r = 1.5$ - BCS Reconstructions Comparison

Figure 42. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).

1.5 ٥ ۶ 0.5 0.6 Re[r(x,y)] У, 0 0.4 -0.5 0.2 -1 -1.5 0 0.5 1.5 -1.5 -1 -0.5 0 x/λ 1 (a)MF - MT - BCSSF - ST - BCSMF - ST - BCS1.5 1.5 1.5 0.5 0.9 0.45 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0.4 0.35 0.5 0.5 0.8 0.5 Re[r(x,y)] 0.3 Re[r(x,y)] Re[τ(x,y)] 0.6 0.25 Ś 0 Ř 0 Ķ 0 0.2 0.4 -0.5 -0.5 -0.5 ----0.15 0.1 -1 0.2 -1 -1 0.05 0 -1.5 -1.5 0 -1.5 0 0 x/λ 0 x/λ. 0 x/λ 1.5 0.5 1.5 -0.5 0.5 1.5 -0.5 0.5 -1.5 -1 -0.5 1 -1.5 -1 1 -1.5 -1 1 (b)(c)(d)1.5 0.9 1.5 1.5 0.5 0.45 0.8 0.9 0.8 0.7 0.6 0.5 0.4 0.3 1 1 0.4 0.7 0.35 0.6 0.5 0.5 0.5 0.3 Re[r(x,y)] 0.5 Re[r(x,y)] Re[r(x,y)] \$ 0 ₹ Ř 0.25 0 0 0.4 0.2 0.3 -0.5 -0.5 -0.5 0.15 ---0.2 0.2 0.1 -1 -1 -1 0.1 0.1 0.05 0 0 0 -1.5 -1.5 -1.5 0 x/λ 0 x/λ 0.5 0 x/λ 0.5 1.5 -0.5 0.5 1.5 -1.5 -1 -0.5 1.5 -1 -0.5 -1.5 -1.5 1 -1 1 1 (f)(e)(g)1.5 1.5 1.5 0.45 0.9 0.8 0.4 0.35 0.7 0.5 0.6 0.5 0.8 0.5 0.3 Re[r(x,y)] Re[τ(x,y)] Re[r(x,y)] 0.5 0.25 Ś 0 Ķ 0.6 Ϋ́, 0 0.4 0.2 0.4 0.3 -0.5 -0.5 -0.5 0.15 0.2 0.1 -1 -1 0.2 -1 0.05 0.1 --0 -1.5 0 -1.5 0 -1.5 0 x/λ 0 x/λ 1.5 -1.5 -1 -0.5 0.5 1 1.5 -1.5 -1 -0.5 0 χ/λ 0.5 1 1.5 -1.5 -1 -0.5 0.5 1 (h)(i)(l)

Eight Homogeneous Pixels of Side $l = 0.16\lambda$ - $\varepsilon_r = 2.0$ - BCS Reconstructions Comparison

Figure 43. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).



Eight Homogeneous Pixels of Side $l = 0.16\lambda$ - $\varepsilon_r = 3.0$ - BCS Reconstructions Comparison

Figure 44. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).



Eight Homogeneous Pixels of Side $l = 0.16\lambda$ - $\varepsilon_r = 4.0$ - BCS Reconstructions Comparison

Figure 45. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).



Eight Homogeneous Pixels of Side $l = 0.16\lambda$ - $\varepsilon_r = 5.0$ - BCS Reconstructions Comparison

Figure 46. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).



Eight Homogeneous Pixels of Side $l = 0.16\lambda$ - BCS Errors vs. ε_r Comparison

Figure 47. Behaviour of total error ξ_{tot} as a function of ε_r , for SNR = 50 [dB] (a), SNR = 20 [dB] (b), SNR = 15 [dB] (c), SNR = 10 [dB] (d) and SNR = 5 [dB] (e).



Eight Homogeneous Pixels of Side $l = 0.16\lambda$ - BCS Errors vs. SNR Comparison

Figure 48. Behaviour of total error ξ_{tot} as a function of SNR, for $\varepsilon_r = 1.5$ [dB] (a), $\varepsilon_r = 2.0$ [dB] (b), $\varepsilon_r = 3.0$ [dB] (c), $\varepsilon_r = 4.0$ [dB] (d) and $\varepsilon_r = 5.0$ [dB] (e).

1.1.3 Rectangle of Sides $l_1 = 0.66\lambda$, $l_2 = 0.33\lambda$

GOAL: show the performances of the multi-frequency MT - BCS when dealing with a sparse scatterer

- Number of frequencies F
- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$ (at the central frequency)
- D = 1296 (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- N scelto in modo da essere vicino a $\#DOF: N = 324 (18 \times 18)$

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$ (at the central frequency)
- $M \approx 2ka \rightarrow M = 27$

Sources:

- $V = 1 \ (\theta = 0^{\circ})$
- Amplitude: A = 1 (plane waves)
- Number of Frequencies: F = 11
- Frequency Range: $I_F = [150 Mhz : 450 MHz]$ Frequency Step: $S_F = [30 Mhz]$

Object:

- Rectangle of sides $l_1 = 0.33\lambda$, $l_2 = 0.66\lambda$
- $\varepsilon_r = \in \{1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0\}$
- $\sigma = 0 [S/m]$

BCS parameters:

- Gamma prior on noise variance parameters: $\beta_1 = 6.5 \times 10^{-1}$, $\beta_2 = 5.8 \times 10^{-2}$
- Convergenze parameter: $\tau = 1.0 \times 10^{-8}$

1.5 0.5 0.4 0.5 Re[r(x,y)] 0.3 y, 0 0.2 -0.5 0.1 -1 -1.5 0 0 x/λ 0.5 1 1.5 -1.5 -1 -0.5 $\overset{(a)}{SF-ST-BCS}$ MF - MT - BCSMF - ST - BCS1.5 1.5 1.5 0.5 0.45 0.6 0.5 1 0.4 0.5 0.35 0.4 0.5 0.5 0.5 0.00 0.3 0.25 Re[r(x,y)] Re[r(x,y)] 0.4 Re[r(x,y)] 0.3 \$ O Ś 0 Š 0 0.3 0.2 ---0.2 -0.5 -0.5 -0.5 0.15 0.2 0.1 -1 -1 0.1 -1 0.1 0.05 -1.5 0 -1.5 0 -1.5 0 -0.5 0 0.5 1.5 -0.5 0 x/λ 0.5 1.5 -0.5 0 x/λ 0.5 1.5 -1.5 -1 1 -1.5 -1 1 -1.5 -1 1 x/λ (b)(c)(d)1.5 1.5 1.5 0.5 0.8 0.45 0.7 0.7 0.6 0.5 0.4 0.3 0.4 0.6 0.35 0.5 0.5 0.5 0.4 0.3 0.5 0.3 Re[r(x,y)] Re[r(x,y)] Re[r(x,y)] 0.25 Ś 0 Ķ 0 Ķ, 0 0.2 -0.5 -0.5 -0.5 ---0.15 0.2 0.2 0.1 0.05 -1 -1 -1 0.1 0.1 ---1.5 0 0 -1.5 0 -1.5 . Ο χ/λ. . Ο x/λ -1.5 -1 -0.5 0.5 1 1.5 -1.5 -1 -0.5 0 x/λ 0.5 1 1.5 -1.5 -1 -0.5 0.5 1 1.5 (f)(e)(g)1.5 1.5 1.5 0.6 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.7 0.5 1 1 0.6 0.5 0.5 0.5 0.4 0.5 Re[r(x,y)] Re[r(x,y)] Re[r(x,y)] 0.3 0.4 Ϋ́ 0 y,y ٨Ņ 0 0 0.3 0.2 -0.5 -0.5 -0.5 0.2 -1 0.1 -1 -1 0.1 0.1 0 0 0 -1.5 -1.5 -1.5 -1.5 -1 -0.5 0 0.5 1.5 -1 -0.5 0 x/λ 0.5 1 1.5 -1.5 -1 -0.5 0 0.5 x/λ 1 1.5 1 -1.5 x/λ. (h)(i)(l)

Homogeneous Rectangle of Sides $l_1 = 0.66\lambda$, $l_2 = 0.33\lambda$ - $\varepsilon_r = 1.5$ - BCS Reconstructions Comparison

Figure 56. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).

1.5 0.5 Re[r(x,y)] 0.6 ξ O 0.4 -0.5 0.2 -1 -1.5 0 x/λ 0.5 1 1.5 -1.5 -1 -0.5 $\overset{(a)}{SF-ST-BCS}$ MF - MT - BCSMF - ST - BCS1.5 1.5 1.5 0.9 0.8 1 0.7 0.6 0.5 0.5 0.5 1.5 0.5 0.4 Re[r(x,y)] Re[t(x,y)] Re[r(x,y)] 0.8 \$ O Ś 0 Š 0 0.6 0.3 -0.5 -0.5 -0.5 0.4 0.2 0.5 -1 -1 -1 0.2 0.1 -1.5 0 -1.5 -1.5 0 0 0 0.5 1.5 -0.5 0 x/λ 0.5 1.5 -0.5 0 x/λ 0.5 1.5 -1.5 -1 -0.5 1 -1.5 -1 1 -1.5 -1 1 x/λ (b)(d)(c)1.5 1.5 1.5 0.9 1.6 2.5 1 1.4 1.2 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 0.5 0.5 0.5 Re[r(x,y)] 1 0.8 0.6 0.4 Re[r(x,y)] Re[r(x,y)] Ķ, 1.5 Ś 0 0 Ň, 0 -0.5 -0.5 -0.5 -1 -1 0.5 -1 0.2 ---1.5 0 -1.5 -1.5 0 . Ο χ/λ. 0 x/λ -1.5 -1 -0.5 0.5 1 1.5 -1.5 -1 -0.5 0 x/λ 0.5 1 1.5 -1.5 -1 -0.5 0.5 1 1.5 (f)(g)(e)1.5 1.5 1.5 0.9 1 1 1 1.6 1.4 1.2 0.8 0.7 0.6 0.5 0.4 0.5 0.5 0.5 Re[r(x,y)] Re[r(x,y)] Re[r(x,y)] 1.5 Ś 0 y,y 0 ٨Ņ 1 0 0.8 -0.5 0.6 0.4 -0.5 -0.5 0.3 0.2 -1 -1 0.5 -1 0.1 0 0.2 -1.5 0 -1.5 0 -1.5 0 0.5 x/λ. 1.5 -1.5 -1 -0.5 0 0.5 1.5 -1 -0.5 0 x/λ 0.5 1 1.5 -1.5 -1 -0.5 1 1 -1.5 x/λ. (h)(i)(l)

Homogeneous Rectangle of Sides $l_1 = 0.66\lambda$, $l_2 = 0.33\lambda$ - $\varepsilon_r = 2.0$ - BCS Reconstructions Comparison

Figure 57. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).

1.5 0.5 Re[r(x,y)] ξ O 1 -0.5 0.5 -1 -1.5 0 x/λ 0.5 1 1.5 -1.5 -0.5 $\overset{(a)}{SF-ST-BCS}$ MF - MT - BCSMF - ST - BCS1.5 1.5 1.5 2.5 0.5 0.5 0.5 0.8 Re[r(x,y)] 2 Re[r(x,y)] Re[r(x,y)] 0.6 \$ O Ś 0 Š 0 1.5 1.5 0.4 -0.5 -0.5 -0.5 -1 -1 -1 0.2 0.5 0.5 0 -1.5 0 -1.5 0 -1.5 -0.5 0 х/λ 0.5 1.5 -0.5 0 x/λ 0.5 1.5 0 x/λ 0.5 1.5 -1.5 -1 1 -1.5 -1 1 -1.5 -1 -0.5 1 (b)(c)(d)1.5 1.5 1.5 3.5 3 з 2.5 1.4 0.5 0.5 0.5 2.5 2 1.5 Be[t(x'x)] 1.2 Re[r(x,y)] Re[r(x,y)] Ķ, 2 Ś 0 0 Ň, 0 1.5 0.8 -0.5 -0.5 -0.5 0.6 0.4 0.2 0 -1 -1 -1 0.5 0.5 --0 -1.5 0 -1.5 -1.5 . Ο χ/λ. . Ο x/λ 0 x/λ -1.5 -1 -0.5 0.5 1 1.5 -1.5 -1 -0.5 0.5 1 1.5 -1.5 -1 -0.5 0.5 1 1.5 (f)(e)(g)1.5 1.5 1.5 1 1 3.5 3 2.5 2 1.5 0.5 0.5 0.5 1.2 1.5 Re[r(x,y)] Re[r(x,y)] Re[r(x,y)] Ś 0 Ś_0 ٨Ņ 0 0.8 0.6 -0.5 -0.5 -0.5 0.4 0.5 1 -1 -1 -1 0.5 0.2 0 -1.5 0 -1.5 0 -1.5 0 0.5 x/λ. -1.5 -1 -0.5 0 0.5 1 1.5 -1.5 -1 -0.5 0 x/λ 0.5 1 1.5 -1.5 -1 -0.5 1 1.5 x/λ. (h)(i)(l)

Homogeneous Rectangle of Sides $l_1 = 0.66\lambda$, $l_2 = 0.33\lambda$ - $\varepsilon_r = 3.0$ - BCS Reconstructions Comparison

Figure 58. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).



Homogeneous Rectangle of Sides $l_1 = 0.66\lambda$, $l_2 = 0.33\lambda$ - BCS Errors vs. ε_r Comparison

Figure 59. Behaviour of total error ξ_{tot} as a function of ε_r , for SNR = 50 [dB] (a), SNR = 20 [dB] (b), SNR = 15 [dB] (c), SNR = 10 [dB] (d) and SNR = 5 [dB] (e).



Homogeneous Rectangle of Sides $l_1 = 0.66\lambda$, $l_2 = 0.33\lambda$ - BCS Errors vs. SNR Comparison

Figure 60. Behaviour of total error ξ_{tot} as a function of SNR, for $\varepsilon_r = 1.5$ [dB] (a), $\varepsilon_r = 2.0$ [dB] (b), $\varepsilon_r = 3.0$ [dB] (c), $\varepsilon_r = 4.0$ [dB] (d) and $\varepsilon_r = 5.0$ [dB] (e).

1.1.4 Rectangle of Sides $l_1 = 0.66\lambda$, $l_2 = 0.33\lambda$ and Square of Side $l_3 = 0.33\lambda$

GOAL: show the performances of the multi-frequency MT - BCS when dealing with a sparse scatterer

- Number of frequencies F
- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$ (at the central frequency)
- D = 1296 (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- N scelto in modo da essere vicino a #DOF: $N = 324 (18 \times 18)$

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$ (at the central frequency)
- $M \approx 2ka \rightarrow M = 27$

Sources:

- $V = 1 \ (\theta = 0^{\circ})$
- Amplitude: A = 1 (plane waves)
- Number of Frequencies: F = 11
- Frequency Range: $I_F = [150 Mhz : 450 MHz]$ Frequency Step: $S_F = [30 Mhz]$

Object:

- Rectangle of sides $l_1^{obj_1} = 0.33\lambda$, $l_2^{obj_1} = 0.66\lambda$; Square of sides $l^{obj_2} = 0.33\lambda$
- $\varepsilon_r = \in \{1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0\}$
- $\sigma = 0 \, [\mathrm{S/m}]$

BCS parameters:

- Gamma prior on noise variance parameters: $\beta_1 = 6.5 \times 10^{-1}$, $\beta_2 = 5.8 \times 10^{-2}$
- Convergenze parameter: $\tau = 1.0 \times 10^{-8}$

Homogeneous Rectangle of Sides $l_1^{obj_1} = 0.66\lambda$, $l_2^{obj_1} = 0.33\lambda$ and Square of Side $l^{obj_2} = 0.33\lambda$ - $\varepsilon_r = 1.5$ - BCS Reconstructions Comparison



Figure 61. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).

Homogeneous Rectangle of Sides $l_1^{obj_1} = 0.66\lambda$, $l_2^{obj_1} = 0.33\lambda$ and Square of Side $l^{obj_2} = 0.33\lambda$ - $\varepsilon_r = 2.0$ - BCS Reconstructions Comparison



Figure 62. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).

Homogeneous Rectangle of Sides $l_1^{obj_1} = 0.66\lambda$, $l_2^{obj_1} = 0.33\lambda$ and Square of Side $l^{obj_2} = 0.33\lambda$ - $\varepsilon_r = 3.0$ - BCS Reconstructions Comparison



Figure 63. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).



Homogeneous Rectangle of Sides $l_1^{obj_1} = 0.66\lambda$, $l_2^{obj_1} = 0.33\lambda$ and Square of Side $l^{obj_2} = 0.33\lambda$ - BCS Errors vs. ε_r Comparison

Figure 64. Behaviour of total error ξ_{tot} as a function of ε_r , for SNR = 50 [dB] (a), SNR = 20 [dB] (b), SNR = 15 [dB] (c), SNR = 10 [dB] (d) and SNR = 5 [dB] (e).



Homogeneous Rectangle of Sides $l_1^{obj_1} = 0.66\lambda$, $l_2^{obj_1} = 0.33\lambda$ and Square of Side $l^{obj_2} = 0.33\lambda$ - BCS Errors vs. SNR Comparison

Figure 65. Behaviour of total error ξ_{tot} as a function of SNR, for $\varepsilon_r = 1.5$ [dB] (a), $\varepsilon_r = 2.0$ [dB] (b), $\varepsilon_r = 3.0$ [dB] (c), $\varepsilon_r = 4.0$ [dB] (d) and $\varepsilon_r = 5.0$ [dB] (e).

1.2 Non-Homogeneous Objects

1.2.1 Two Strips of Sides $l_1 = 0.16\lambda$, $l_2 = 0.50\lambda$

GOAL: show the performances of the multi-frequency MT - BCS when dealing with a sparse scatterer

- Number of frequencies F
- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$ (at the central frequency)
- D = 1296 (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- N scelto in modo da essere vicino a #DOF: N = 324 (18 \times 18)

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$ (at the central frequency)
- $M \approx 2ka \rightarrow M = 27$

Sources:

- $V = 1 \ (\theta = 0^{\circ})$
- Amplitude: A = 1 (plane waves)
- Number of Frequencies: F = 11
- Frequency Range: $I_F = [150 Mhz : 450 MHz]$ Frequency Step: $S_F = [30 Mhz]$

Object:

- Two strips of sides $l_1 = 0.16\lambda$, $l_2 = 0.50\lambda$
- $\varepsilon_r^{obj_1} \in \{1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0\}, \varepsilon_r^{obj_2} = 1.6$
- $\sigma = 0$ [S/m]

BCS parameters:

- Gamma prior on noise variance parameter: $\beta_1 = 6.5 \times 10^{-1}$, $\beta_2 = 5.8 \times 10^{-2}$
- Convergenze parameter: $\tau = 1.0 \times 10^{-8}$

Two Non-Homogeneous Strips of Sides $l_1 = 0.16\lambda$, $l_2 = 0.50\lambda$ - $\varepsilon_r = 1.5$ - BCS Reconstructions Comparison



Figure 66. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).

Two Non-Homogeneous Strips of Sides $l_1 = 0.16\lambda$, $l_2 = 0.50\lambda$ - $\varepsilon_r = 2.0$ - BCS Reconstructions Comparison



Figure 67. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).

Two Non-Homogeneous Strips of Sides $l_1 = 0.16\lambda$, $l_2 = 0.50\lambda$ - $\varepsilon_r = 3.0$ - BCS Reconstructions Comparison



Figure 68. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).

Two Non-Homogeneous Strips of Sides $l_1 = 0.16\lambda$, $l_2 = 0.50\lambda$ - $\varepsilon_r = 4.0$ - BCS Reconstructions Comparison



Figure 69. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).

Two Non-Homogeneous Strips of Sides $l_1 = 0.16\lambda$, $l_2 = 0.50\lambda$ - $\varepsilon_r = 5.0$ - BCS Reconstructions Comparison



Figure 70. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).



Two Non-Homogeneous Strips of Sides $l_1 = 0.16\lambda$, $l_2 = 0.50\lambda$ - BCS Errors vs. ε_r Comparison

Figure 71. Behaviour of total error ξ_{tot} as a function of ε_r , for SNR = 50 [dB] (a), SNR = 20 [dB] (b), SNR = 15 [dB] (c), SNR = 10 [dB] (d) and SNR = 5 [dB] (e).



Two Non-Homogeneous Strips of Sides $l_1 = 0.16\lambda$, $l_2 = 0.50\lambda$ - BCS Errors vs. SNR Comparison

Figure 72. Behaviour of total error ξ_{tot} as a function of SNR, for $\varepsilon_r = 1.5$ [dB] (a), $\varepsilon_r = 2.0$ [dB] (b), $\varepsilon_r = 3.0$ [dB] (c), $\varepsilon_r = 4.0$ [dB] (d) and $\varepsilon_r = 5.0$ [dB] (e).

1.2.2 Rectangle of Sides $l_1 = 0.66\lambda$, $l_2 = 0.33\lambda$ and Square of Side $l_3 = 0.33\lambda$

GOAL: show the performances of the multi-frequency MT - BCS when dealing with a sparse scatterer

- Number of frequencies F
- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$ (at the central frequency)
- D = 1296 (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- N scelto in modo da essere vicino a #DOF: $N = 324 (18 \times 18)$

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$ (at the central frequency)
- $M \approx 2ka \rightarrow M = 27$

Sources:

- $V = 1 \ (\theta = 0^{\circ})$
- Amplitude: A = 1 (plane waves)
- Number of Frequencies: F = 11
- Frequency Range: $I_F = [150 Mhz : 450 MHz]$ Frequency Step: $S_F = [30 Mhz]$

Object:

- Rectangle of sides $l_1^{obj_1} = 0.33\lambda$, $l_2^{obj_1} = 0.66\lambda$; Square of sides $l^{obj_2} = 0.33\lambda$
- $\varepsilon_r^{obj_1} = 1.9, \, \varepsilon_r^{obj_2} \in \{1.5, \, 2.0, \, 2.5, \, 3.0, \, 3.5, \, 4.0, \, 4.5, \, 5.0\}$
- $\sigma = 0 \, [\mathrm{S/m}]$

MT-BCS parameters:

- Gamma prior on noise variance parameters: $\beta_1 = 6.5 \times 10^{-1}$, $\beta_2 = 5.8 \times 10^{-2}$
- Convergenze parameter: $\tau = 1.0 \times 10^{-8}$

Homogeneous Rectangle of Sides $l_1^{obj_1} = 0.66\lambda$, $l_2^{obj_1} = 0.33\lambda$ and Square of Side $l^{obj_2} = 0.33\lambda$ - $\varepsilon_r = 1.5$ - BCS Reconstructions Comparison



Figure 80. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).

Homogeneous Rectangle of Sides $l_1^{obj_1} = 0.66\lambda$, $l_2^{obj_1} = 0.33\lambda$ and Square of Side $l^{obj_2} = 0.33\lambda$ - $\varepsilon_r = 2.0$ - BCS Reconstructions Comparison



Figure 81. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).

Homogeneous Rectangle of Sides $l_1^{obj_1} = 0.66\lambda$, $l_2^{obj_1} = 0.33\lambda$ and Square of Side $l^{obj_2} = 0.33\lambda$ - $\varepsilon_r = 3.0$ - BCS Reconstructions Comparison



Figure 82. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).

Homogeneous Rectangle of Sides $l_1^{obj_1} = 0.66\lambda$, $l_2^{obj_1} = 0.33\lambda$ and Square of Side $l^{obj_2} = 0.33\lambda$ - $\varepsilon_r = 4.0$ - BCS Reconstructions Comparison



Figure 83. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).

Homogeneous Rectangle of Sides $l_1^{obj_1} = 0.66\lambda$, $l_2^{obj_1} = 0.33\lambda$ and Square of Side $l^{obj_2} = 0.33\lambda$ - $\varepsilon_r = 5.0$ - BCS Reconstructions Comparison



Figure 84. Actual object (a), MF - MT - BCS reconstructed object (b)(e)(h), SF - ST - BCS(c)(f)(i) and MF - ST - BCS (d)(g)(l) for SNR = 50 [dB] (b)(c)(d), SNR = 10 [dB] (e)(f)(g) and SNR = 5 [dB] (h)(i)(l).



Homogeneous Rectangle of Sides $l_1^{obj_1} = 0.66\lambda$, $l_2^{obj_1} = 0.33\lambda$ and Square of Side $l^{obj_2} = 0.33\lambda$ - BCS Errors vs. ε_r Comparison

Figure 85. Behaviour of total error ξ_{tot} as a function of ε_r , for SNR = 50 [dB] (a), SNR = 20 [dB] (b), SNR = 15 [dB] (c), SNR = 10 [dB] (d) and SNR = 5 [dB] (e).



Homogeneous Rectangle of Sides $l_1^{obj_1} = 0.66\lambda$, $l_2^{obj_1} = 0.33\lambda$ and Square of Side $l^{obj_2} = 0.33\lambda$ - BCS Errors vs. SNR Comparison

Figure 86. Behaviour of total error ξ_{tot} as a function of SNR, for $\varepsilon_r = 1.5$ [dB] (a), $\varepsilon_r = 2.0$ [dB] (b), $\varepsilon_r = 3.0$ [dB] (c), $\varepsilon_r = 4.0$ [dB] (d) and $\varepsilon_r = 5.0$ [dB] (e).

1.3 Statistical Analysis - Square Cylinders of Side $l = 0.16\lambda$

GOAL: show the statistical performances of the multi-frequency MT - BCS when dealing with sparse scatterers

- Number of frequencies F
- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$ (at the central frequency)
- D = 1296 (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- N = 324

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$ (at the central frequency)
- $M \approx 2ka \rightarrow M = 27$

Sources:

- $V \approx 2ka \rightarrow V = 27$
- Amplitude A = 1 (plane waves)
- Number of Frequencies: F = 11
- Frequency Range: $I_F = [150 Mhz : 450 MHz]$ Frequency Step: $S_F = [30 Mhz]$

Object:

- $S \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ Square cylinders of side $\frac{\lambda}{6} = 0.16667$
- $\varepsilon_r = 2.0$
- $\sigma = 0 [S/m]$

MT-BCS parameters:

- Gamma prior on noise variance parameters: $\beta_1 = 6.5 \times 10^{-1}$, $\beta_2 = 5.8 \times 10^{-2}$
- Convergenze parameter: $\tau = 1.0 \times 10^{-8}$

Statistical Analysis:

• K = 14 random seeds used for each case





Figure 87. Statistical analysis $[K = 14, \varepsilon_r = 2.0]$ - Behaviour of mean, maximum and minimum of the error figures as a function of S (sparsity factor) of the total error ξ_{tot} (a), internal error ξ_{int} (b) and external error ξ_{ext} (c).





Figure 88. Statistical analysis $[K = 14, \varepsilon_r = 2.0]$ - Behaviour of mean, maximum and minimum of the error figures as a function of S (sparsity factor) of the total error ξ_{tot} (a), internal error ξ_{int} (b) and external error ξ_{ext} (c).

Statistical Analysis - Error Figures - BCS Comparison - SNR = 10 [dB]



Figure 89. Statistical analysis $[K = 14, \varepsilon_r = 2.0]$ - Behaviour of mean, maximum and minimum of the error figures as a function of S (sparsity factor) of the total error ξ_{tot} (a), internal error ξ_{int} (b) and external error ξ_{ext} (c).

Statistical Analysis - Error Figures - BCS Comparison - SNR = 5 [dB]



Figure 90. Statistical analysis $[K = 14, \varepsilon_r = 2.0]$ - Behaviour of mean, maximum and minimum of the error figures as a function of S (sparsity factor) of the total error ξ_{tot} (a), internal error ξ_{int} (b) and external error ξ_{ext} (c).

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