

# Optimization of the instantaneous directivity in time-modulated linear arrays

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## Abstract

In this report, the reception of a signal avoiding undesired amplitude modulations of the received power is yielded by optimising the sequence of on-off pulses controlling the RF switches. More specifically, the 'switch-on instants' (i.e., the time-instants when the switches commute from the open to the short circuit state) are suitably optimised to keep constant as much as possible the maximum directivity throughout the modulation period while generating an average pattern with prescribed sidelobes and mainbeam width at the working frequency.

# Definizioni

## Array Factor

L'array factor di un array lineare di elementi uniformemente spazati è definito come:

$$AF(\theta) = \sum_{n=1}^N \alpha_n e^{j(n-1)kd \sin \theta} \quad \theta \in [-\pi/2; \pi/2] \quad (1)$$

dove  $\alpha_n$  è l'eccitazione complessa del  $n$ -esimo elemento dell'array e  $d$  è la distanza tra gli elementi.

La Time-Modulation applicata alla sintesi di array si realizza applicando degli interruttori ad alta velocità all'alimentazione degli elementi, che ne permettono l'accensione e lo spegnimento in determinati intervalli temporali, in tal caso l'array factor diventa

$$AF(\theta, t) = \sum_{n=1}^N \alpha_n U_n(t) e^{j(n-1)kd \sin \theta} \quad (2)$$

dove  $U_n(t)$  è la funzione periodica *time switching* di periodo  $T_p$  corrispondente all' $n$ -esimo elemento, generalmente ad impulso rettangolare:

$$U_n(t) = \begin{cases} 1, & t'_n \leq t \leq t''_n \\ 0, & \text{altrimenti} \end{cases} \quad (3)$$

dove  $0 \leq t'_n \leq t''_n \leq T_p$ .

## Normalized Array Factor

$$AF_n(\theta, t) = \frac{AF(\theta, t)}{\max_{\theta, t} \{|AF(\theta, t)|\}} \quad (4)$$

## Directivity

La direttività di un array lineare uniforme è definita come:

$$D(\theta, t) = \frac{\mathbf{J}(t)^t A(\theta) \mathbf{J}(t)}{\mathbf{J}(t)^t B \mathbf{J}(t)} \quad \theta \in [-\pi/2; \pi/2] \quad (5)$$

dove nell'equazione (5)  $\mathbf{J}(t)$  è il vettore delle eccitazioni complesse,  $A$  e  $B$  sono matrici di  $N \times N$  elementi. Gli elementi di queste matrici sono dati da:

$$\begin{aligned} a_{ij} &= \exp[jk(j-i)d \sin \theta] & i, j &= 1, \dots, N \\ b_{ij} &= \text{sinc}[k(j-i)d] & i, j &= 1, \dots, N \end{aligned} \quad (6)$$

dove  $k = \frac{2\pi}{\lambda}$  è il numero d'onda.

## Peak Directivity

Quando  $d = 0.5\lambda$ :

$$D_{max}(t) = \frac{\left| \sum_{n=1}^N a_n(t) \right|^2}{\sum_{n=1}^N |a_n(t)|^2} \quad (7)$$

-3 [dB] Main Beamwidth  $BW$

$$BW(t) = \theta_A(t) - \theta_B(t) \quad (8)$$

dove  $\theta_A(t)$  e  $\theta_B(t)$  sono le posizioni angolari tali per cui  $|AF(\theta, t)|_n = -3$  [dB].

### Sidelobe Level (SLL)

Il massimo livello dei lobi secondari di  $|AF(\theta, t)|_n$ .

### Sideband Level (SBL)

Il massimo livello della radiazione alle frequenze armoniche.

### Total Power and Sideband Radiation

La potenza totale da un array di elementi con eccitazioni sottoposte a Time-Modulation è calcolabile come:

$$P_{tot} = \sum_{n=0}^{N-1} \left\{ |\alpha_n|^2 \sum_{h=-\infty}^{\infty} u_{hn}^2 \right\} + 2 \sum_{m, n \in \Omega_{mn}} \left\{ \text{Re} \{ \alpha_m \alpha_n^* \} \text{sinc} [kd(m-n)] \sum_{h=-\infty}^{\infty} u_{hn} u_{hm} \right\} \quad (9)$$

dove  $u_{hn}$  rappresenta l' $h$ -esimo coefficiente della serie di Fourier associata alla forma d'onda utilizzata per modulare l' $n$ -esimo elemento, e  $\Omega_{mn}$  è l'insieme di indici che corrisponde alle coppie  $(m, n)$  non ripetute con  $m \neq n$ . La 9 si può tradurre anche in una sommatoria di infiniti termini rappresentanti la potenza associata alle varie frequenze, centrale ed armoniche:

$$P_{tot} = \sum_{h=-\infty}^{\infty} P_h \quad (10)$$

La Sideband Radiation rappresenta la quantità di potenza radiata dall'array alle frequenze spurie: sarà in seguito espressa in percentuale rispetto alla potenza totale radiata.

E' possibile ricavare la potenza associata alle frequenze spurie grazie alla seguente equazione:

$$P_{SR} = \sum_{n=1}^N \left\{ |\alpha_n|^2 \tau_n (1 - \tau_n) \right\} + \sum_{m, n \in \Omega_{mn}} \left\{ \text{Re} \{ \alpha_m \alpha_n^* \} \text{sinc} [k(z_m - z_n)] (\tau_{\{m, n\} MinVal} - \tau_m \tau_n) \right\} \quad (11)$$

dove  $\tau_n$  è la durata normalizzata rispetto al periodo di modulazione  $T_p$  dell'impulso associato al  $n$ -esimo elemento e

$$\tau_{\{m, n\} MinVal} = \begin{cases} \tau_n & \text{if } \tau_n \leq \tau_m \\ \tau_m & \text{altrimenti} \end{cases} \quad (12)$$

## Optimization Approaches

### PS-PSO, SBL Opt.

Il processo di ottimizzazione mediante algoritmo PSO agisce esclusivamente su shift temporale degli impulsi; l'ottimizzazione è applicata su metà elementi dell'array, l'altra metà viene considerata simmetrica.

Cost Function:

$$\Psi^{PSO} [\tau'_n(i_k)] = SBL^{act,(i_k)}$$

### PS-PSO, Directivity Opt.

Il processo di ottimizzazione mediante algoritmo PSO agisce esclusivamente su shift temporale degli impulsi; l'ottimizzazione è applicata su metà elementi dell'array, l'altra metà viene considerata simmetrica.

Cost Function:

$$\Psi^{PSO} [\tau'_n(i_k)] = \sum_{l=1}^L \left[ (D_l^{act,(i_k)} - D_{av}) \Delta t_l^{act,(i_k)} \right] / D_{av}$$

dove  $L$  è il numero di intervalli di variazione della direttività all'interno del periodo di modulazione,  $D_{av}$  è la direttività media sul periodo di modulazione,  $D_l$  è la direttività istantanea durante l'intervallo  $l$ -esimo e  $\Delta t_l$  è la durata del  $l$ -esimo intervallo normalizzata rispetto al periodo di modulazione  $T_p$ .

## TEST CASE 1.a - Dolph-Chebyshev -30dB

### Goal

Sintesi di un array con eccitazioni modulate nel dominio del tempo al fine di riprodurre un pattern di Dolph-Chebyshev alla frequenza centrale di lavoro e un pattern istantaneo con massima direttività costante.

### Test Case Description

- Number of Elements:  $N = 16$
- Elements Spacing:  $d = 0.5\lambda$
- Static Array Configuration:  $\alpha_n = 1, n = 0, \dots, N - 1$
- Pattern at Central Frequency: *Dolph – Chebyshev*,  $SLL = -30 \text{ dB}$
- Max Gain Pattern Direction :  $\theta^{max} = 90^\circ$

### Optimization Approach: PS-PSO, SBL Opt.

- Number of Variables:  $X = 8$
- Number of Particles:  $S = 10$
- Number of Iterations:  $I = 200$
- Inertial Weight: Linearly varying: *0.9 to 0.4*
- Swarm Initialization: *Random*

### Optimization Approach: PS-PSO, Directivity Opt.

- Number of Variables:  $X = 8$
- Number of Particles:  $S = 20$
- Number of Iterations:  $I = 100$
- Inertial Weight: Linearly varying: *0.9 to 0.4*
- Swarm Initialization: *Random*

Tennant 2008 - Dolph-Chebyshev Pattern, SLL=-30 dB

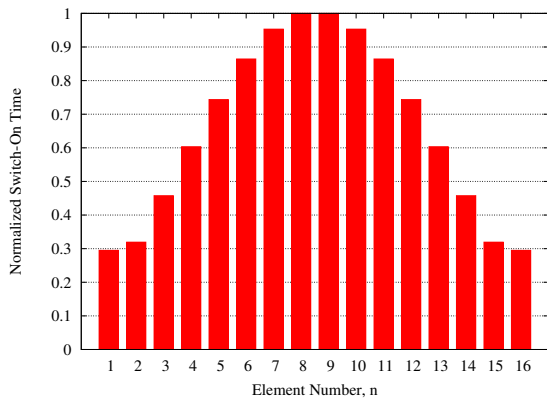


Fig.1 - Pulse Sequence

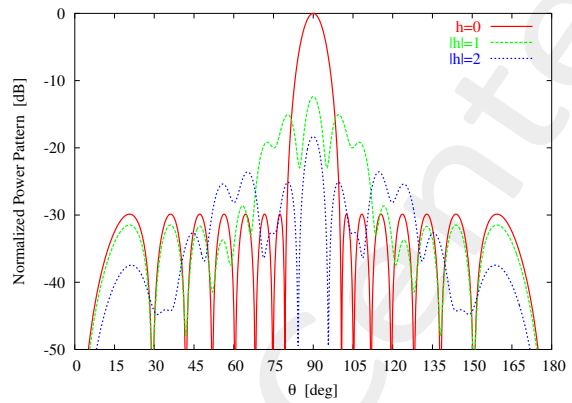


Fig.2 - Patterns

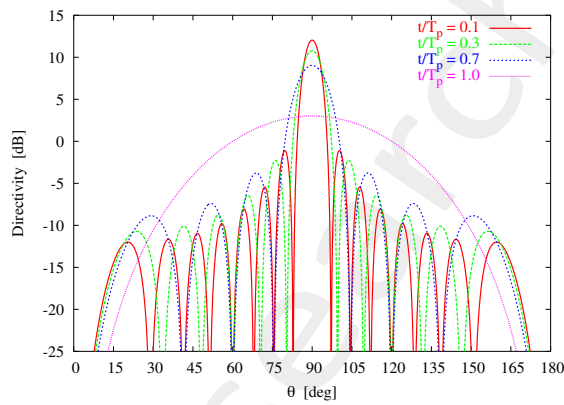


Fig.3 - Directivity

PS-PSO, SBL Opt. - Tennant 2008 - Dolph-Chebyshev Pattern, SLL=-30 dB

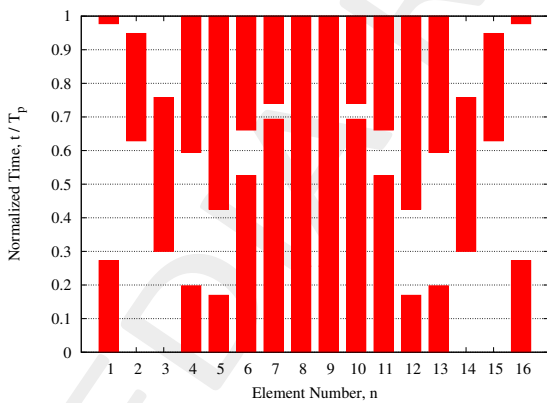


Fig.4 - Pulse Sequence

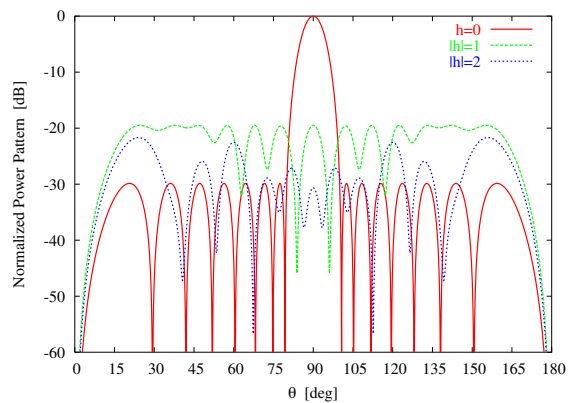


Fig.5 - Patterns

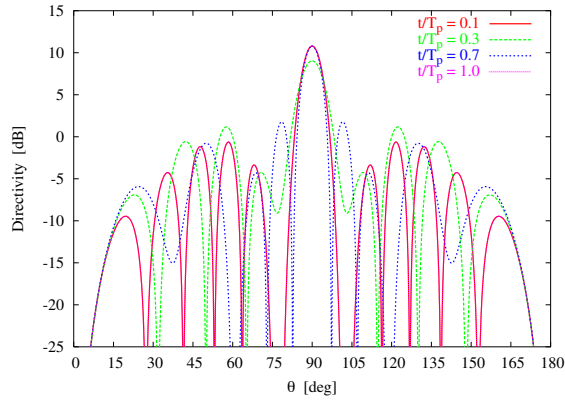


Fig.6 - Directivity

PS-PSO, Directivity Opt. - Tennant 2008 - Dolph-Chebyshev Pattern, SLL=-30 dB

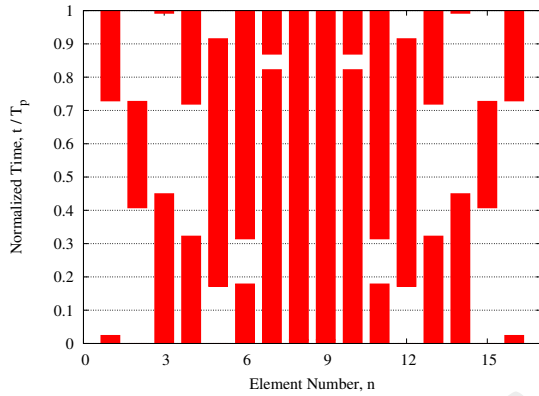


Fig.7 - Pulse Sequence

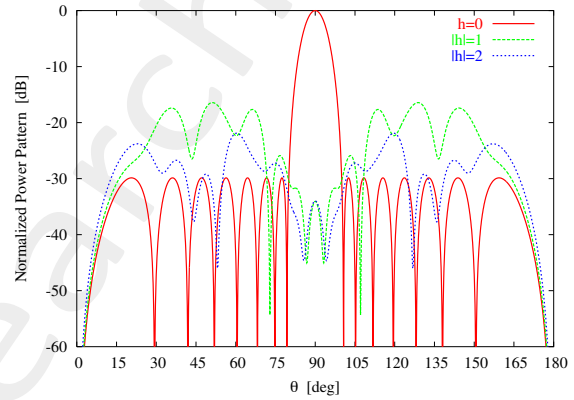


Fig.8 - Patterns

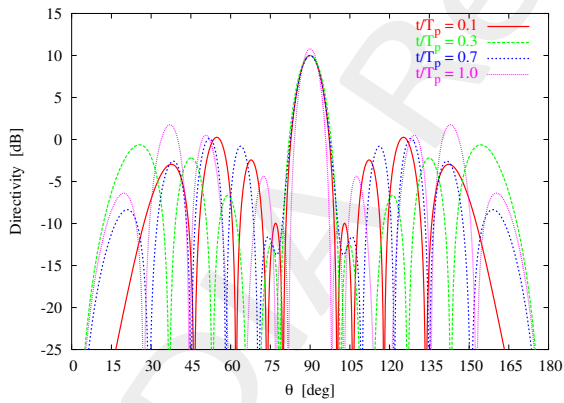


Fig.9 - Directivity

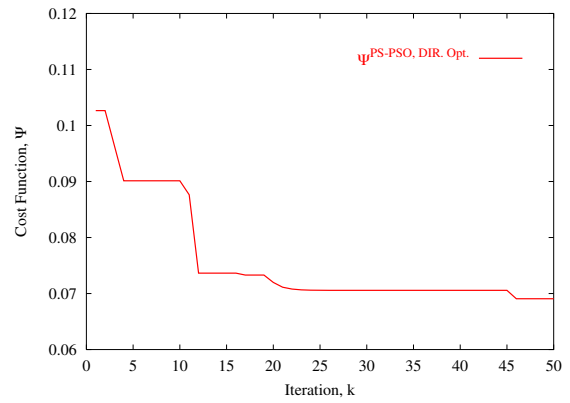
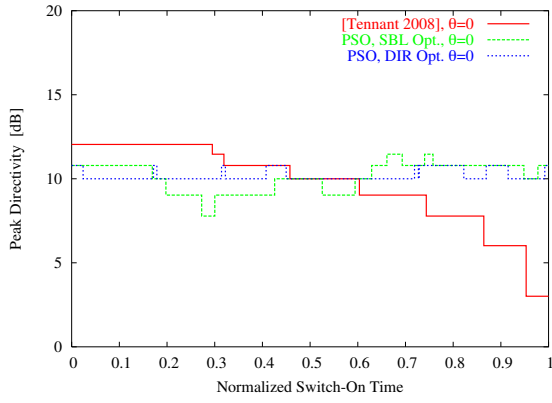
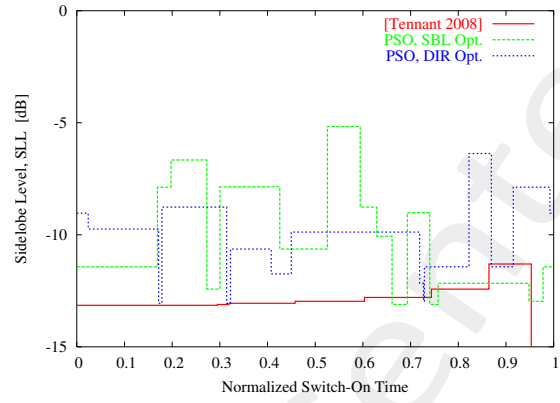


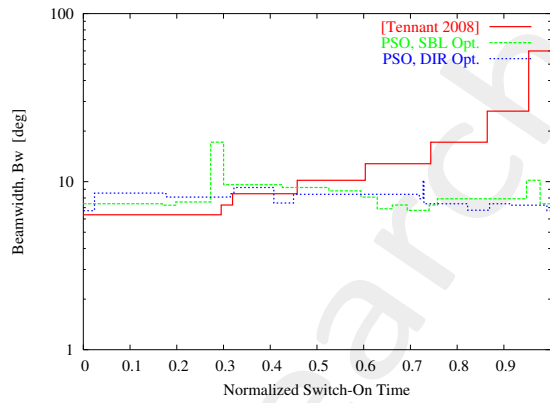
Fig.10 - Fitness



**Fig.11 - Peak Directivity**



**Fig.12 - Sidelobe Level**



**Fig.13 - Beamwidth**

	$SLL_{av}$ [dB]	SBL [dB]	$BW_{av}$ [°]	$P_{SR}$ [%]
[Tennant 2008]	-29.86	-12.39	7.95	24.17
PS - PSO, SBL Opt.	-29.86	-19.50	7.95	24.17
PS - PSO, DIR Opt.	-29.86	-16.43	7.95	24.17

**Tab.1 - Average Pattern Parameters: Sidelobe Level ( $SLL$ ), Sideband Level ( $SBL$ ), -3 dB Beamwidth ( $BW$ ), Sideband Radiation ( $P_{SR}$ )**



	$av^* \{D_{max}(t)\}$	$av \{D_{max}(t)\}$	$\sigma^2 \{D_{max}(t)\}$	$Min \{D_{max}(t)\}$	$Max \{D_{max}(t)\}$
[Tennant 2008]	9.66	8.76	8.17	3.01	12.04
<i>PS – PSO, SBL Opt.</i>	10.12	10.12	1.00	7.78	11.46
<i>PS – PSO, DIR Opt.</i>	10.19	10.42	0.16	10	10.79

**Tab.2 - Instant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of Peak Directivity ( $D_{max}$ ) in [dB]**

	$av^* \{SLL(t)\}$	$av \{SLL(t)\}$	$\sigma^2 \{SLL(t)\}$	$Min \{SLL(t)\}$	$Max \{SLL(t)\}$
[Tennant 2008]	–	–	–	$-\infty$	-11.30
<i>PS – PSO, SBL Opt.</i>	-10.06	-10.18	5.92	-13.11	-5.17
<i>PS – PSO, DIR Opt.</i>	-9.79	-10.48	3.83	-13.06	-6.38

**Tab.3 - Instant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of Sidelobe Level ( $SLL$ ) in [dB]**

	$av^* \{BW(t)\}$	$av \{BW(t)\}$	$\sigma^2 \{BW(t)\}$	$Min \{BW(t)\}$	$Max \{BW(t)\}$
[Tennant 2008]	13.72	18.55	281.55	6.35	59.90
<i>PS – PSO, SBL Opt.</i>	8.38	8.58	6.25	6.75	17.16
<i>PS – PSO, DIR Opt.</i>	8.05	7.93	0.91	6.72	10.19

**Tab.4 - Instant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of -3 dB Beamwidth ( $BW$ ) in [deg]**

## TEST CASE 1.b - Dolph-Chebyshev -20dB

### Goal

Sintesi di un array con eccitazioni modulate nel dominio del tempo al fine di riprodurre un pattern di Dolph-Chebyshev alla frequenza centrale di lavoro e un pattern istantaneo con massima direttività costante.

### Analogies and Differences wrt Previous Cases

**A:** Geometria dell'array, numero di elementi, algoritmo di ottimizzazione (PSO), funzione di costo implementata, configurazione statica dell'array.

**D:** Configurazione degli impulsi di eccitazione degli elementi (durate degli impulsi).

### Test Case Description

- Number of Elements:  $N = 16$
- Elements Spacing:  $d = 0.5\lambda$
- Static Array Configuration:  $\alpha_n = 1, n = 0, \dots, N - 1$
- Pattern at Central Frequency: *Dolph - Chebyshev*,  $SLL = -20 \text{ dB}$
- Max Gain Pattern Direction :  $\theta^{max} = 90^\circ$

### Optimization Approach: PS-PSO, Directivity Opt.

- Number of Variables:  $X = 8$
- Number of Particles:  $S = 20$
- Number of Iterations:  $I = 100$
- Inertial Weight: Linearly varying:  $0.9 \text{ to } 0.4$
- Swarm Initialization: *Random*

### Dolph-Chebyshev Pattern, SLL=-20 dB

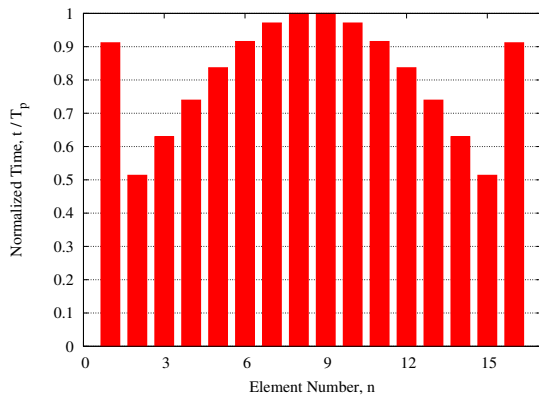


Fig.14 - Pulse Sequence

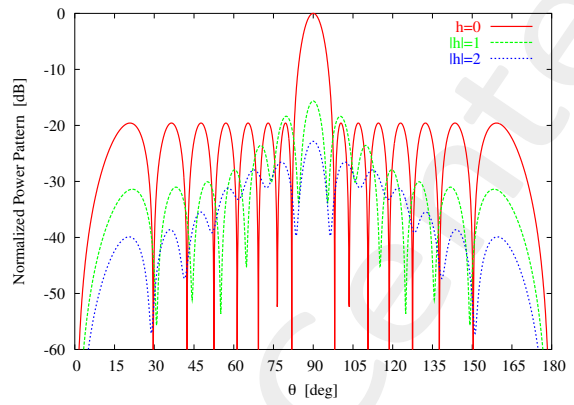


Fig.15 - Patterns

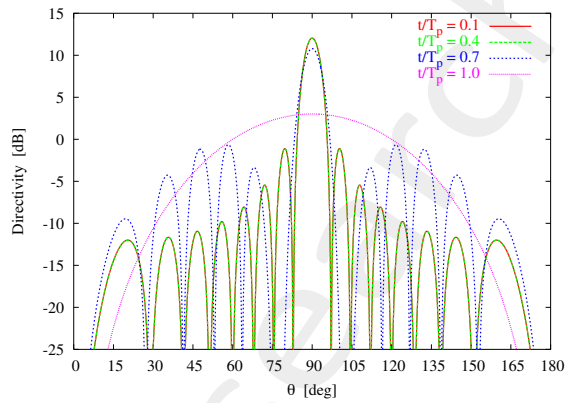


Fig.16 - Directivity

### PS-PSO, Directivity Opt. - Dolph-Chebyshev Pattern, SLL=-20 dB

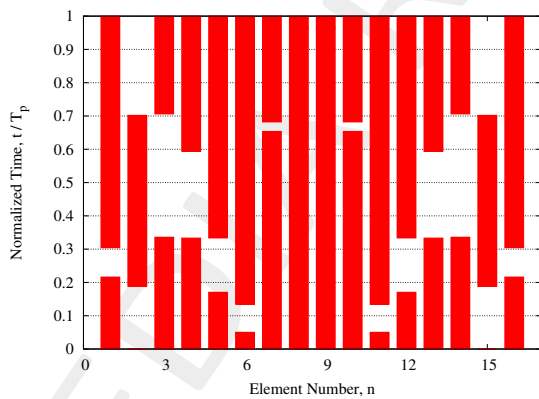


Fig.17 - Pulse Sequence

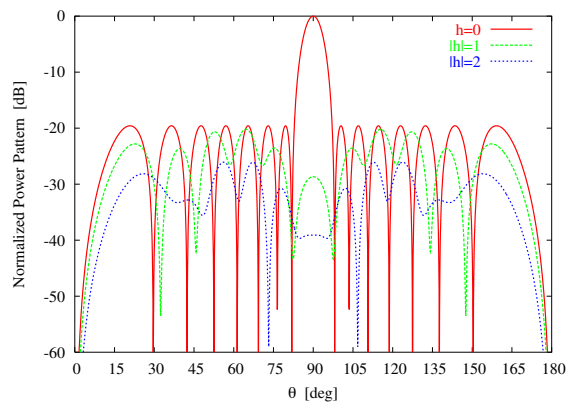


Fig.18 - Patterns

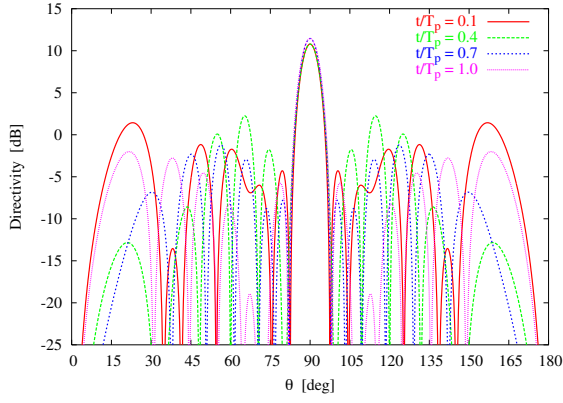


Fig.19 - Directivity

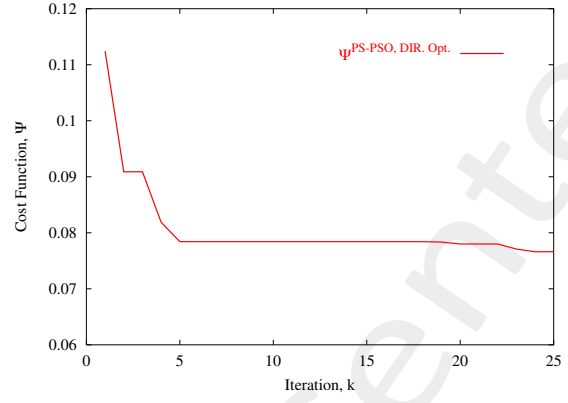


Fig.20 - Fitness

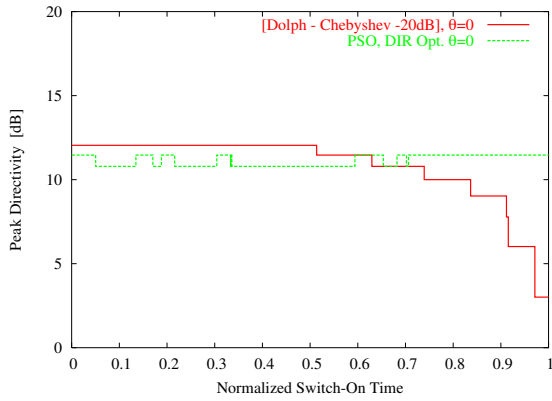


Fig.21 - Peak Directivity

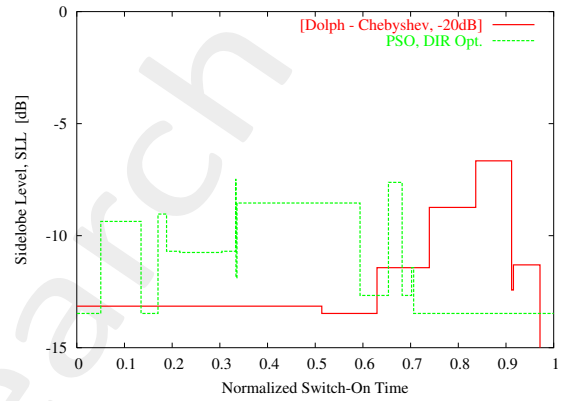


Fig.22 - Sidelobe Level

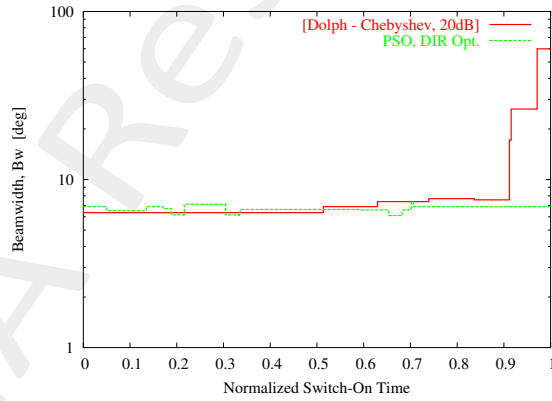


Fig.23 - Beamwidth

	$SLL_{av}$ [dB]	$SBL$ [dB]	$BW_{av}$ [°]	$P_{SR}$ [%]
<i>Dolph, SLL = -20 dB</i>	-19.57	-15.64	6.71	15.32
<i>PS - PSO, DIR Opt.</i>	-19.57	-20.18	6.71	15.31

Tab.5 - Average Pattern Parameters: Sidelobe Level ( $SLL$ ), Sideband Level ( $SBL$ ), -3 dB Beamwidth ( $BW$ ), Sideband Radiation ( $P_{SR}$ )

	$av^* \{D_{max}(t)\}$	$av \{D_{max}(t)\}$	$\sigma^2 \{D_{max}(t)\}$	$Min \{D_{max}(t)\}$	$Max \{D_{max}(t)\}$
<i>Dolph, SLL = -20 dB</i>	10.80	8.76	8.17	3.01	12.04
<i>PS - PSO, DIR Opt.</i>	11.13	11.15	0.11	10.79	11.46

**Tab.6 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of Peak Directivity ( $D_{max}$ ) in [dB]**

	$av^* \{SLL(t)\}$	$av \{SLL(t)\}$	$\sigma^2 \{SLL(t)\}$	$Min \{SLL(t)\}$	$Max \{SLL(t)\}$
<i>Dolph, SLL = -20 dB</i>	-	-	-	$-\infty$	-
<i>PS - PSO, DIR Opt.</i>	-11.13	-10.88	4.08	-13.47	-7.49

**Tab.7 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of Sidelobe Level ( $SLL$ ) in [dB]**

	$av^* \{BW(t)\}$	$av \{BW(t)\}$	$\sigma^2 \{BW(t)\}$	$Min \{BW(t)\}$	$Max \{BW(t)\}$
<i>Dolph, SLL = -20 dB</i>	9.44	17.40	301.10	6.35	59.90
<i>PS - PSO, DIR Opt.</i>	6.72	6.61	0.14	6.11	7.39

**Tab.8 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of -3 dB Beamwidth ( $BW$ ) in [deg]**

## TEST CASE 1.c - Dolph-Chebyshev -25dB

### Goal

Sintesi di un array con eccitazioni modulate nel dominio del tempo al fine di riprodurre un pattern di Dolph-Chebyshev alla frequenza centrale di lavoro e un pattern istantaneo con massima direttività costante.

### Analogies and Differences wrt Previous Cases

**A:** Geometria dell'array, numero di elementi, algoritmo di ottimizzazione (PSO), funzione di costo implementata, configurazione statica dell'array.

**D:** Configurazione degli impulsi di eccitazione degli elementi (durate degli impulsi).

### Test Case Description

- Number of Elements:  $N = 16$
- Elements Spacing:  $d = 0.5\lambda$
- Static Array Configuration:  $\alpha_n = 1, n = 0, \dots, N - 1$
- Pattern at Central Frequency: *Dolph - Chebyshev*,  $SLL = -25 \text{ dB}$
- Max Gain Pattern Direction :  $\theta^{max} = 90^\circ$

### Optimization Approach: PS-PSO, Directivity Opt.

- Number of Variables:  $X = 8$
- Number of Particles:  $S = 20$
- Number of Iterations:  $I = 100$
- Inertial Weight: Linearly varying:  $0.9 \text{ to } 0.4$
- Swarm Initialization: *Random*

Dolph-Chebyshev Pattern, SLL=-25 dB

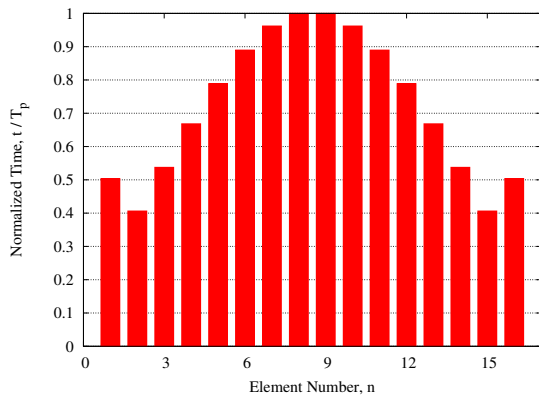


Fig.24 - Pulse Sequence

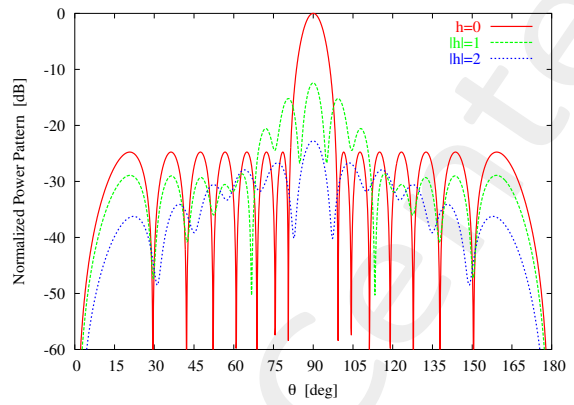


Fig.25 - Patterns

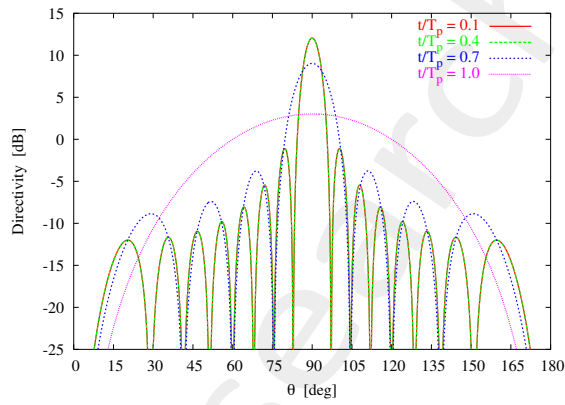


Fig.26 - Directivity

PS-PSO, Directivity Opt. - Dolph-Chebyshev Pattern, SLL=-25 dB

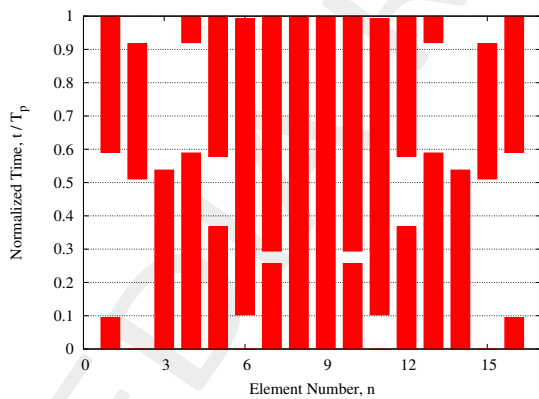


Fig.27 - Pulse Sequence

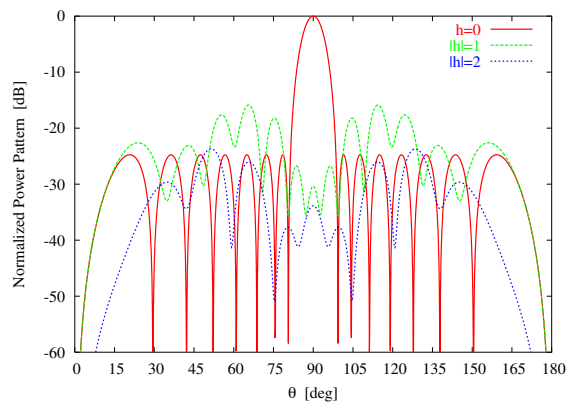


Fig.28 - Patterns

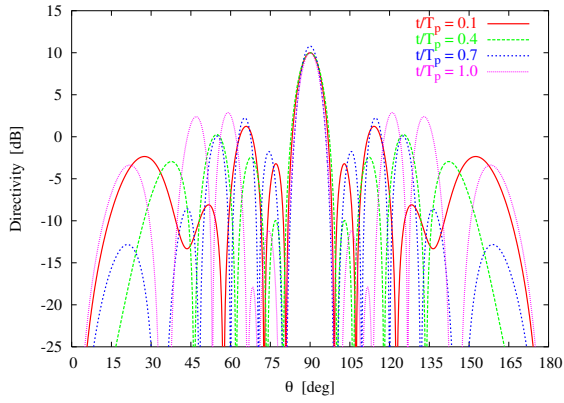


Fig.29 - Directivity

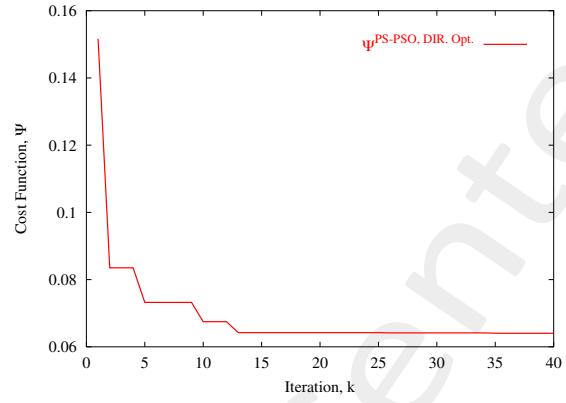


Fig.30 - Fitness

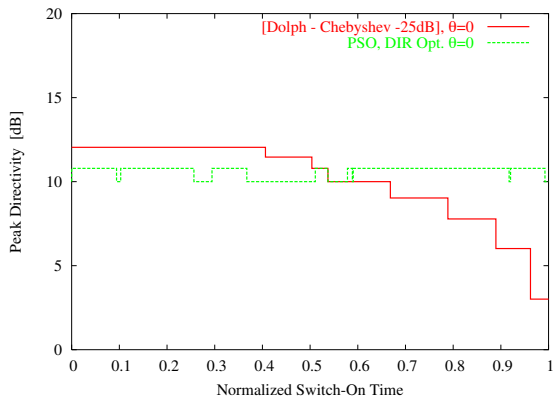


Fig.31 - Peak Directivity

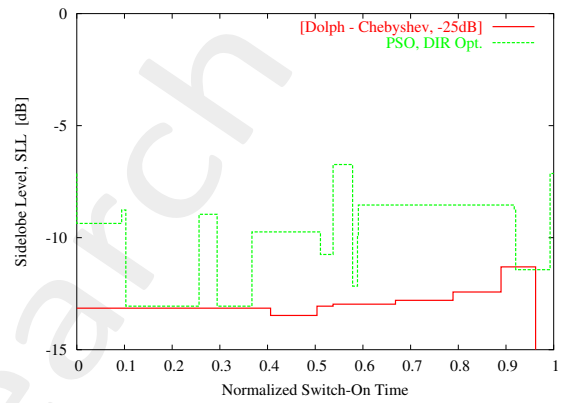


Fig.32 - Sidelobe Level

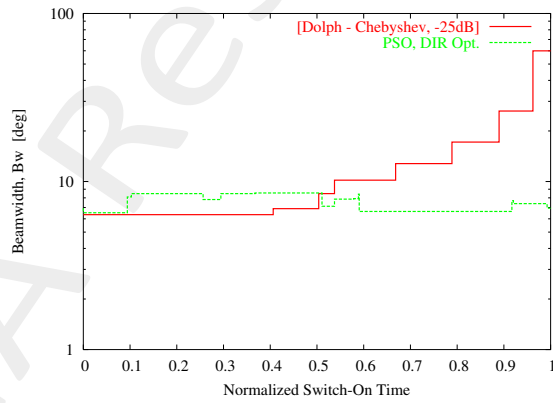


Fig.33 - Beamwidth

	$SLL_{av}$ [dB]	$SBL$ [dB]	$BW_{av}$ [°]	$P_{SR}$ [%]
<i>Dolph, SLL = -25 dB</i>	-24.76	-12.42	7.37	21.93
<i>PS - PSO, DIR Opt.</i>	-24.76	-15.86	7.36	21.93

Tab.9 - Average Pattern Parameters: Sidelobe Level ( $SLL$ ), Sideband Level ( $SBL$ ), -3 dB Beamwidth ( $BW$ ), Sideband Radiation ( $P_{SR}$ )



	$av^* \{D_{max}(t)\}$	$av \{D_{max}(t)\}$	$\sigma^2 \{D_{max}(t)\}$	$Min \{D_{max}(t)\}$	$Max \{D_{max}(t)\}$
<i>Dolph, SLL = -25 dB</i>	10.10	8.77	8.1	3.01	12.04
<i>PS - PSO, DIR Opt.</i>	10.60	10.37	0.16	10	10.79

**Tab.10 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of Peak Directivity ( $D_{max}$ ) in [dB]**

	$av^* \{SLL(t)\}$	$av \{SLL(t)\}$	$\sigma^2 \{SLL(t)\}$	$Min \{SLL(t)\}$	$Max \{SLL(t)\}$
<i>Dolph, SLL = -25 dB</i>	-	-	-	$-\infty$	-
<i>PS - PSO, DIR Opt.</i>	-10.05	-9.70	3.90	-13.06	-6.74

**Tab.11 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of Sidelobe Level ( $SLL$ ) in [dB]**

	$av^* \{BW(t)\}$	$av \{BW(t)\}$	$\sigma^2 \{BW(t)\}$	$Min \{BW(t)\}$	$Max \{BW(t)\}$
<i>Dolph, SLL = -25 dB</i>	12.32	18.55	282.60	6.35	59.90
<i>PS - PSO, DIR Opt.</i>	7.12	7.65	0.45	6.52	8.55

**Tab.12 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of -3 dB Beamwidth ( $BW$ ) in [deg]**

## TEST CASE 1.d - Dolph-Chebyshev -35dB

### Goal

Sintesi di un array con eccitazioni modulate nel dominio del tempo al fine di riprodurre un pattern di Dolph-Chebyshev alla frequenza centrale di lavoro e un pattern istantaneo con massima direttività costante.

### Analogies and Differences wrt Previous Cases

**A:** Geometria dell'array, numero di elementi, algoritmo di ottimizzazione (PSO), funzione di costo implementata, configurazione statica dell'array.

**D:** Configurazione degli impulsi di eccitazione degli elementi (durate degli impulsi).

### Test Case Description

- Number of Elements:  $N = 16$
- Elements Spacing:  $d = 0.5\lambda$
- Static Array Configuration:  $\alpha_n = 1, n = 0, \dots, N - 1$
- Pattern at Central Frequency: *Dolph - Chebyshev*,  $SLL = -35 \text{ dB}$
- Max Gain Pattern Direction :  $\theta^{max} = 90^\circ$

### Optimization Approach: PS-PSO, Directivity Opt.

- Number of Variables:  $X = 8$
- Number of Particles:  $S = 20$
- Number of Iterations:  $I = 100$
- Inertial Weight: Linearly varying:  $0.9 \text{ to } 0.4$
- Swarm Initialization: *Random*

Dolph-Chebyshev Pattern, SLL=-35 dB

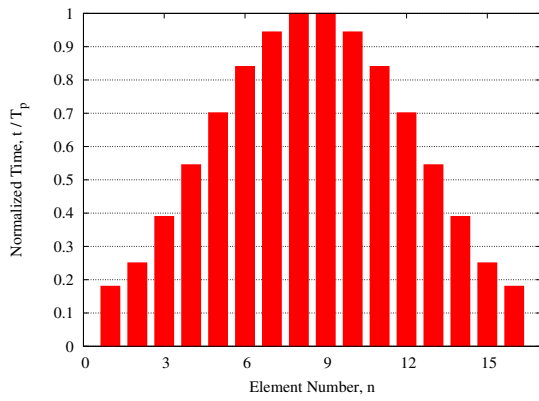


Fig.34 - Pulse Sequence

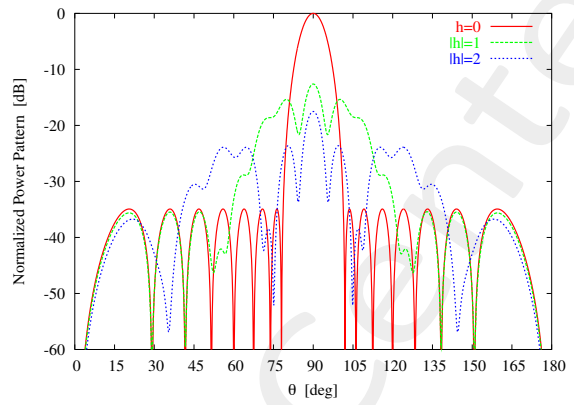


Fig.35 - Patterns

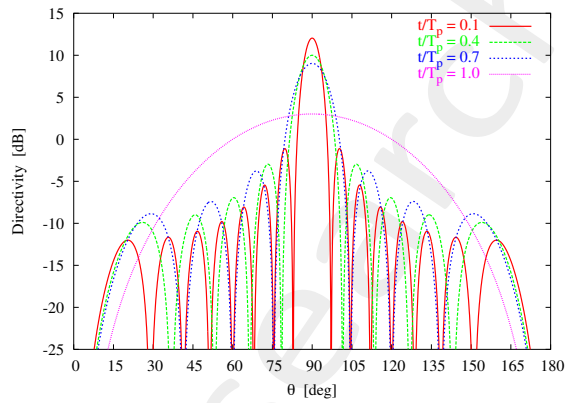


Fig.36 - Directivity

PS-PSO, Directivity Opt. - Dolph-Chebyshev Pattern, SLL=-35 dB

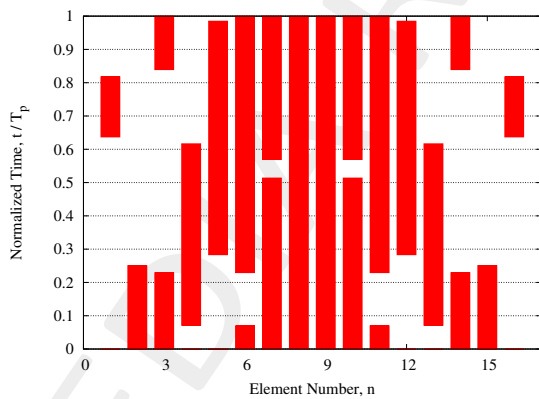


Fig.37 - Pulse Sequence

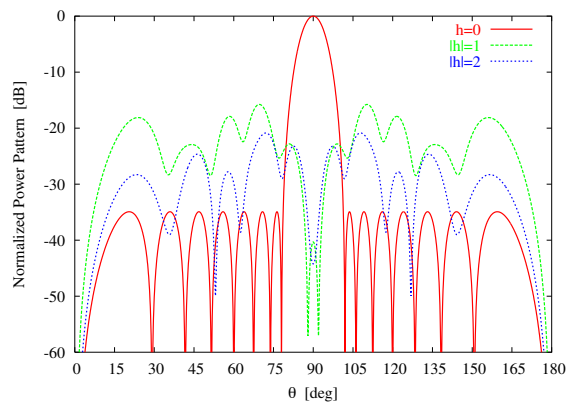
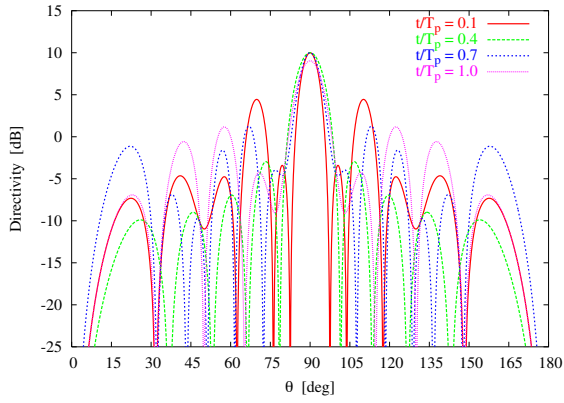
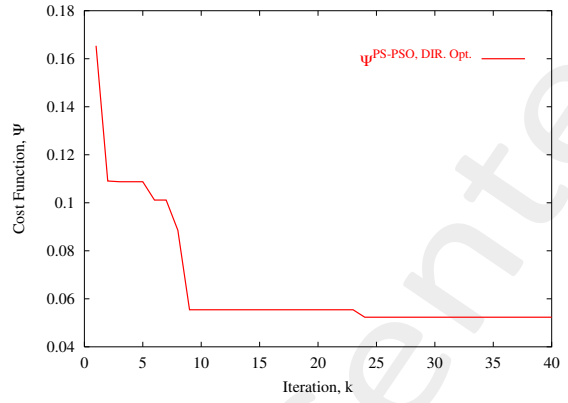


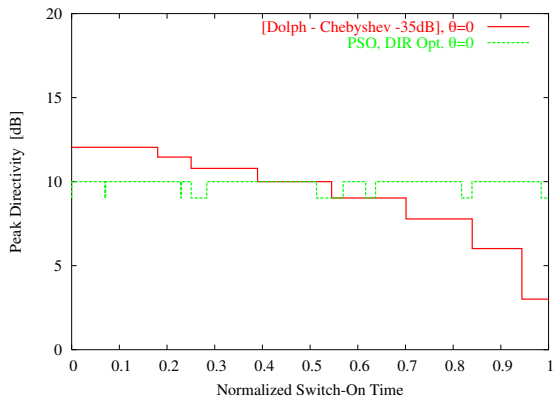
Fig.38 - Patterns



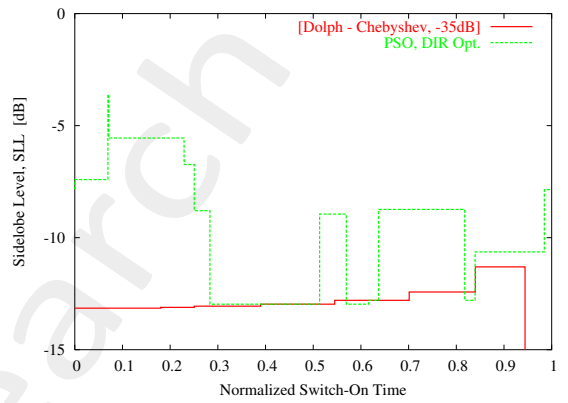
**Fig.39 - Directivity**



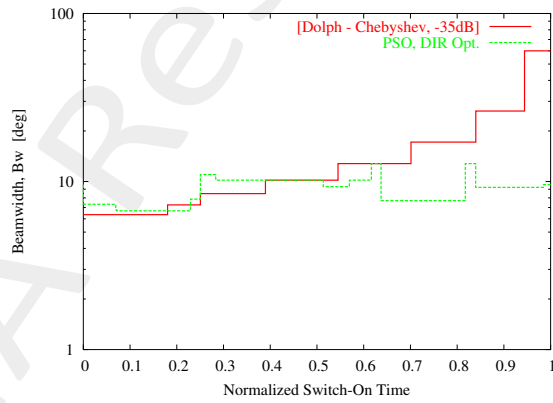
**Fig.40 - Fitness**



**Fig.41 - Peak Directivity**



**Fig.42 - Sidelobe Level**



**Fig.43 - Beamwidth**

	$SLL_{av}$ [dB]	$SBL$ [dB]	$BW_{av}$ [°]	$P_{SR}$ [%]
<i>Dolph, SLL = -35 dB</i>	-34.92	-12.61	8.48	25.11
<i>PS - PSO, DIR Opt.</i>	-34.92	-15.76	8.48	25.11

**Tab.13 - Average Pattern Parameters: Sidelobe Level ( $SLL$ ), Sideband Level ( $SBL$ ), -3 dB Beamwidth ( $BW$ ), Sideband Radiation ( $P_{SR}$ )**

	$av^* \{D_{max}(t)\}$	$av \{D_{max}(t)\}$	$\sigma^2 \{D_{max}(t)\}$	$Min \{D_{max}(t)\}$	$Max \{D_{max}(t)\}$
<i>Dolph, SLL = -35 dB</i>	9.31	8.77	8.17	3.01	12.04
<i>PS - PSO, DIR Opt.</i>	9.85	9.48	0.23	9.03	10

**Tab.14 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of Peak Directivity ( $D_{max}$ ) in [dB]**

	$av^* \{SLL(t)\}$	$av \{SLL(t)\}$	$\sigma^2 \{SLL(t)\}$	$Min \{SLL(t)\}$	$Max \{SLL(t)\}$
<i>Dolph, SLL = -35 dB</i>	-	-	-	$-\infty$	-
<i>PS - PSO, DIR Opt.</i>	-9.71	-8.95	8.01	-12.96	-3.62

**Tab.15 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of Sidelobe Level ( $SLL$ ) in [dB]**

	$av^* \{BW(t)\}$	$av \{BW(t)\}$	$\sigma^2 \{BW(t)\}$	$Min \{BW(t)\}$	$Max \{BW(t)\}$
<i>Dolph, SLL = -35 dB</i>	14.88	18.55	281.55	6.35	59.90
<i>PS - PSO, DIR Opt.</i>	8.87	9.22	3.63	6.69	12.78

**Tab.16 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of -3 dB Beamwidth ( $BW$ ) in [deg]**

## TEST CASE 1.e - Dolph-Chebyshev -40dB

### Goal

Sintesi di un array con eccitazioni modulate nel dominio del tempo al fine di riprodurre un pattern di Dolph-Chebyshev alla frequenza centrale di lavoro e un pattern istantaneo con massima direttività costante.

### Analogies and Differences wrt Previous Cases

**A:** Geometria dell'array, numero di elementi, algoritmo di ottimizzazione (PSO), funzione di costo implementata, configurazione statica dell'array.

**D:** Configurazione degli impulsi di eccitazione degli elementi (durate degli impulsi).

### Test Case Description

- Number of Elements:  $N = 16$
- Elements Spacing:  $d = 0.5\lambda$
- Static Array Configuration:  $\alpha_n = 1, n = 0, \dots, N - 1$
- Pattern at Central Frequency: *Dolph - Chebyshev*,  $SLL = -40 \text{ dB}$
- Max Gain Pattern Direction :  $\theta^{max} = 90^\circ$

### Optimization Approach: PS-PSO, Directivity Opt.

- Number of Variables:  $X = 8$
- Number of Particles:  $S = 20$
- Number of Iterations:  $I = 100$
- Inertial Weight: Linearly varying:  $0.9 \text{ to } 0.4$
- Swarm Initialization: *Random*

### Dolph-Chebyshev Pattern, SLL=-40 dB

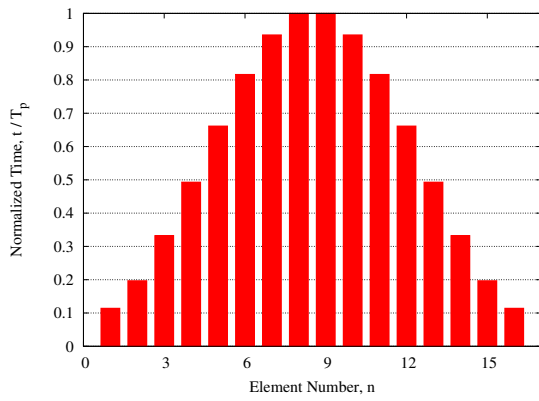


Fig.44 - Pulse Sequence

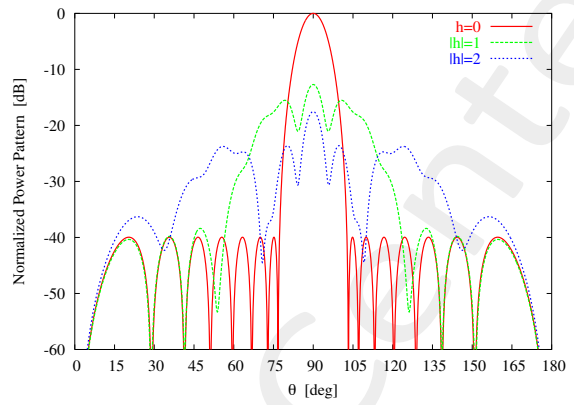


Fig.45 - Patterns

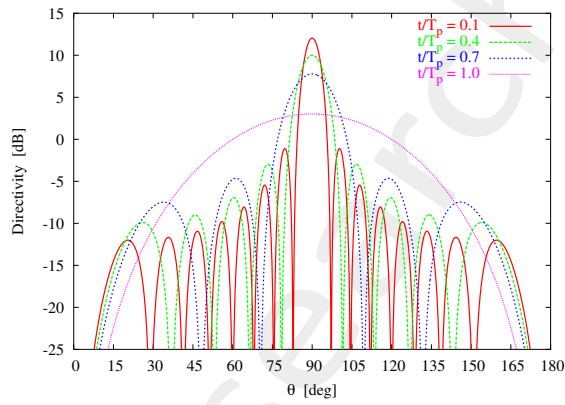


Fig.46 - Directivity

### PS-PSO, Directivity Opt. - Dolph-Chebyshev Pattern, SLL=-40 dB

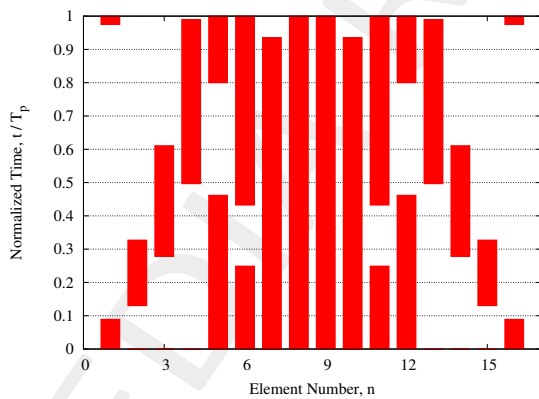


Fig.47 - Pulse Sequence

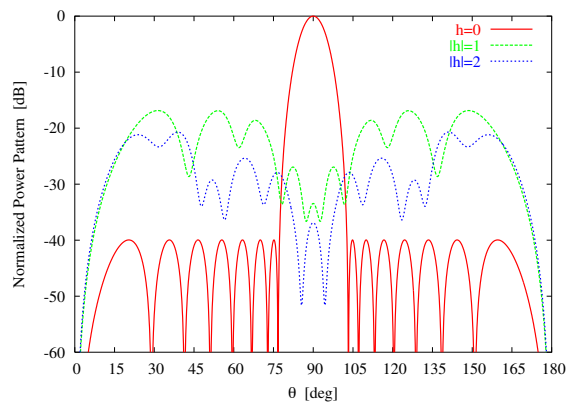


Fig.48 - Patterns

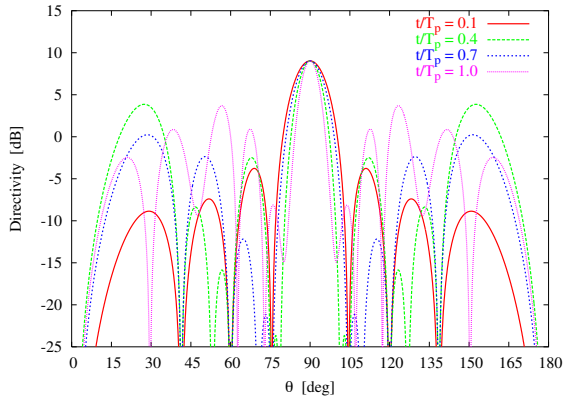


Fig.49 - Directivity

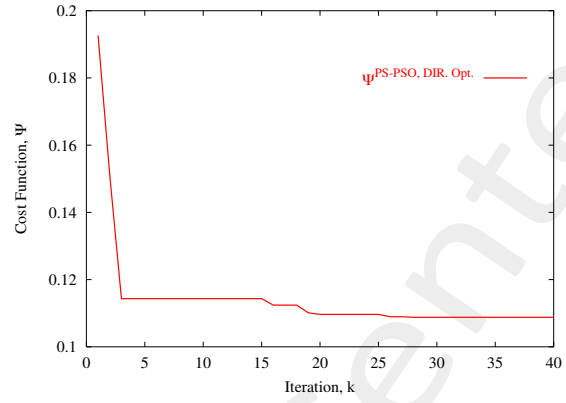


Fig.50 - Fitness

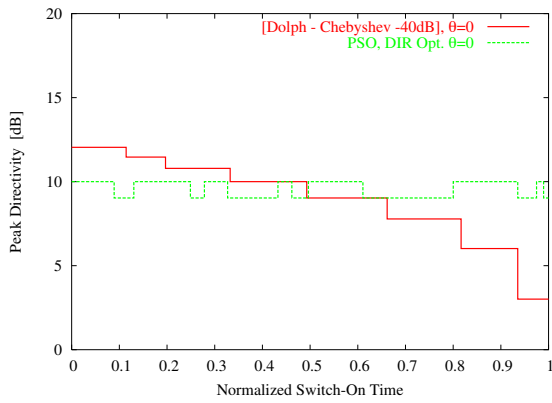


Fig.51 - Peak Directivity

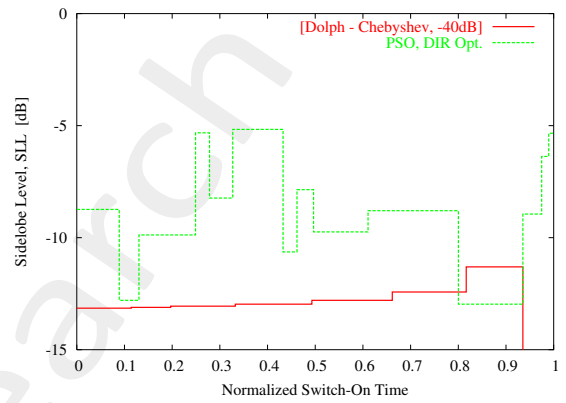


Fig.52 - Sidelobe Level

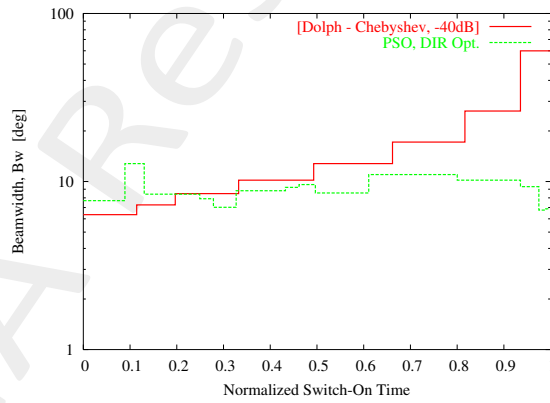


Fig.53 - Beamwidth

	$SLL_{av}$ [dB]	$SBL$ [dB]	$BW_{av}$ [°]	$P_{SR}$ [%]
<i>Dolph, SLL = -40 dB</i>	-39.96	-12.70	8.97	25.61
<i>PS - PSO, DIR Opt.</i>	-39.96	-16.86	8.97	25.61

Tab.17 - Average Pattern Parameters: Sidelobe Level ( $SLL$ ), Sideband Level ( $SBL$ ), -3 dB Beamwidth ( $BW$ ), Sideband Radiation ( $P_{SR}$ )



	$av^* \{D_{max}(t)\}$	$av \{D_{max}(t)\}$	$\sigma^2 \{D_{max}(t)\}$	$Min \{D_{max}(t)\}$	$Max \{D_{max}(t)\}$
<i>Dolph, SLL = -40 dB</i>	9.03	8.77	8.17	3.01	12.04
<i>PS - PSO, DIR Opt.</i>	9.56	9.51	0.23	9.03	10

**Tab.18 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of Peak Directivity ( $D_{max}$ ) in [dB]**

	$av^* \{SLL(t)\}$	$av \{SLL(t)\}$	$\sigma^2 \{SLL(t)\}$	$Min \{SLL(t)\}$	$Max \{SLL(t)\}$
<i>Dolph, SLL = -40 dB</i>	-	-	-	$-\infty$	-
<i>PS - PSO, DIR Opt.</i>	-9.20	-8.63	5.91	-12.97	-5.17

**Tab.19 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of Sidelobe Level ( $SLL$ ) in [dB]**

	$av^* \{BW(t)\}$	$av \{BW(t)\}$	$\sigma^2 \{BW(t)\}$	$Min \{BW(t)\}$	$Max \{BW(t)\}$
<i>Dolph, SLL = -40 dB</i>	15.92	18.55	281.55	6.35	59.90
<i>PS - PSO, DIR Opt.</i>	9.29	8.87	2.64	6.76	12.78

**Tab.20 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of -3 dB Beamwidth ( $BW$ ) in [deg]**

## TEST CASE 1.f - Dolph-Chebyshev -45dB

### Goal

Sintesi di un array con eccitazioni modulate nel dominio del tempo al fine di riprodurre un pattern di Dolph-Chebyshev alla frequenza centrale di lavoro e un pattern istantaneo con massima direttività costante.

### Analogies and Differences wrt Previous Cases

**A:** Geometria dell'array, numero di elementi, algoritmo di ottimizzazione (PSO), funzione di costo implementata, configurazione statica dell'array.

**D:** Configurazione degli impulsi di eccitazione degli elementi (durate degli impulsi).

### Test Case Description

- Number of Elements:  $N = 16$
- Elements Spacing:  $d = 0.5\lambda$
- Static Array Configuration:  $\alpha_n = 1$ ,  $n = 0, \dots, N - 1$
- Pattern at Central Frequency: *Dolph - Chebyshev*,  $SLL = -45 \text{ dB}$
- Max Gain Pattern Direction :  $\theta^{max} = 90^\circ$

### Optimization Approach: PS-PSO, Directivity Opt.

- Number of Variables:  $X = 8$
- Number of Particles:  $S = 20$
- Number of Iterations:  $I = 100$
- Inertial Weight: Linearly varying:  $0.9 \text{ to } 0.4$
- Swarm Initialization: *Random*

Dolph-Chebyshev Pattern, SLL=-45 dB

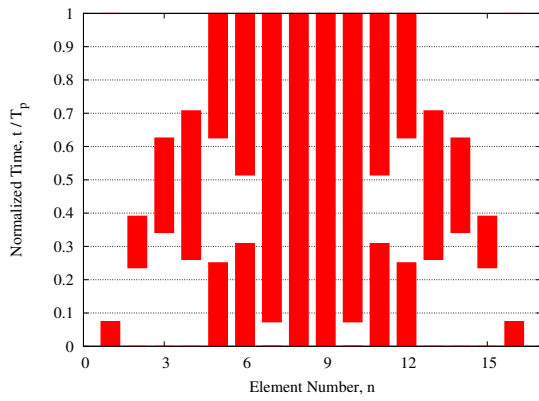


Fig.54 - Pulse Sequence

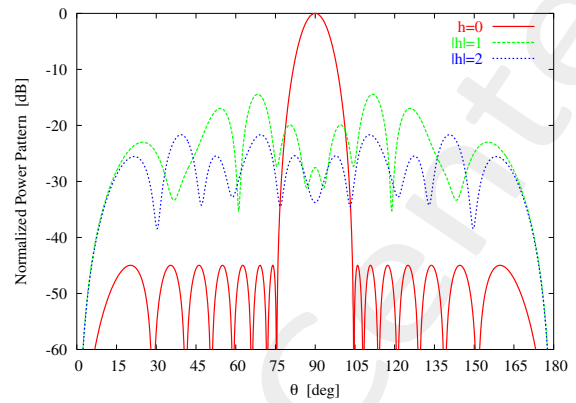


Fig.55 - Patterns

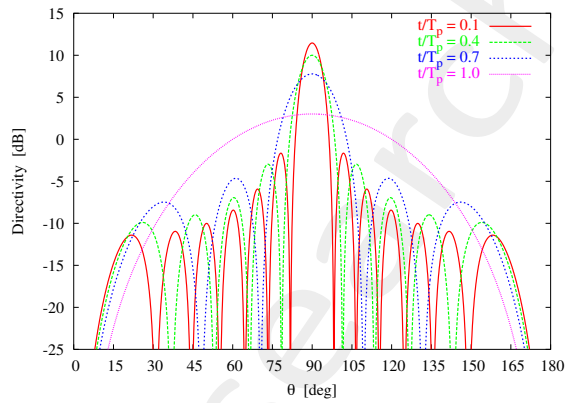


Fig.56 - Directivity

PS-PSO, Directivity Opt. - Dolph-Chebyshev Pattern, SLL=-45 dB

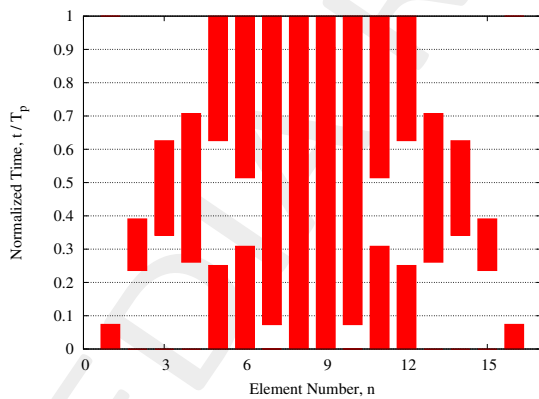


Fig.57 - Pulse Sequence

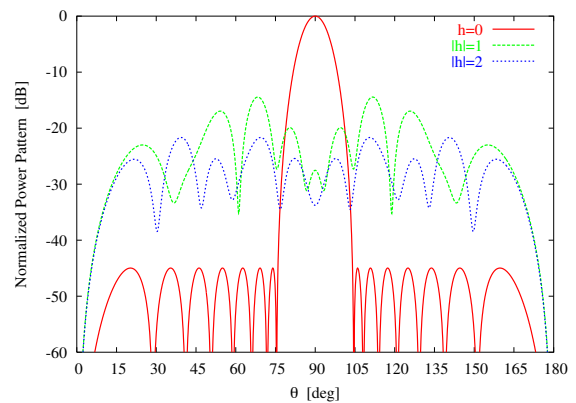
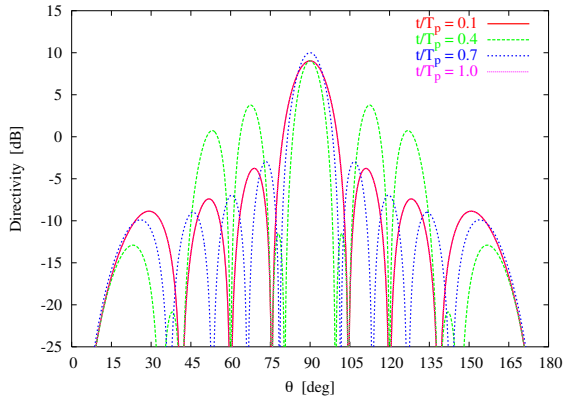
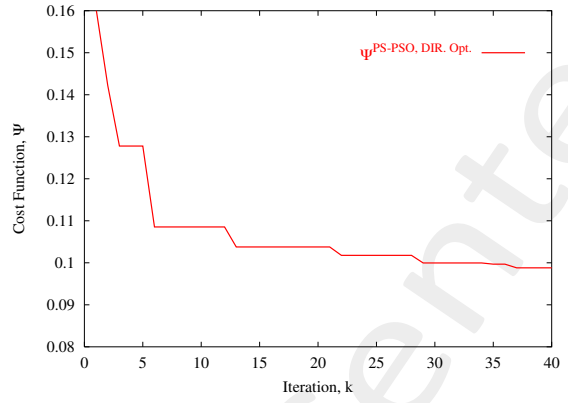


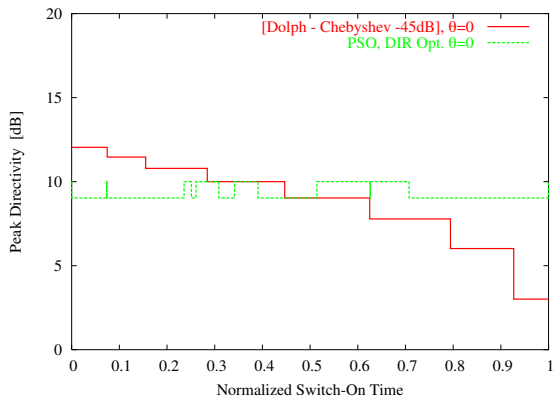
Fig.58 - Patterns



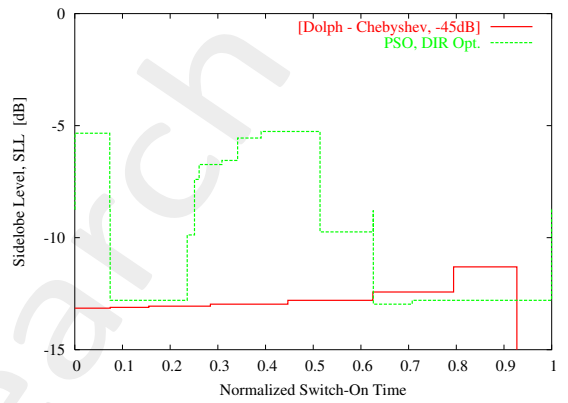
**Fig.59 - Directivity**



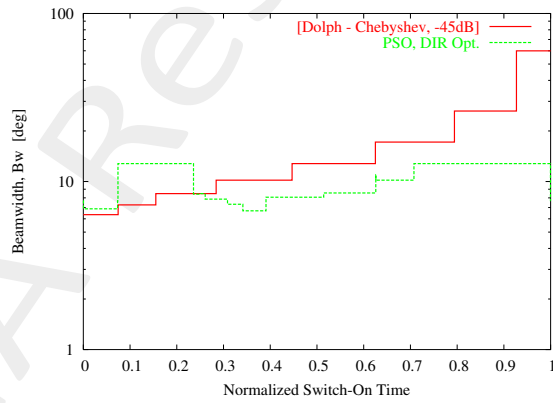
**Fig.60 - Fitness**



**Fig.61 - Peak Directivity**



**Fig.62 - Sidelobe Level**



**Fig.63 - Beamwidth**

	$SLL_{av}$ [dB]	$SBL$ [dB]	$BW_{av}$ [°]	$P_{SR}$ [%]
<i>Dolph, SLL = -45 dB</i>	-44.97	-12.72	9.24	25.91
<i>PS - PSO, DIR Opt.</i>	-44.97	-14.44	9.24	25.91

**Tab.21 - Average Pattern Parameters: Sidelobe Level ( $SLL$ ), Sideband Level ( $SBL$ ), -3 dB Beamwidth ( $BW$ ), Sideband Radiation ( $P_{SR}$ )**

	$av^* \{D_{max}(t)\}$	$av \{D_{max}(t)\}$	$\sigma^2 \{D_{max}(t)\}$	$Min \{D_{max}(t)\}$	$Max \{D_{max}(t)\}$
<i>Dolph, SLL = -45 dB</i>	8.78	8.77	8.16	3.01	12.04
<i>PS - PSO, DIR Opt.</i>	9.33	9.55	0.23	9.03	10

**Tab.22 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of Peak Directivity ( $D_{max}$ ) in [dB]**

	$av^* \{SLL(t)\}$	$av \{SLL(t)\}$	$\sigma^2 \{SLL(t)\}$	$Min \{SLL(t)\}$	$Max \{SLL(t)\}$
<i>Dolph, SLL = -45 dB</i>	-	-	-	$-\infty$	-
<i>PS - PSO, DIR Opt.</i>	-10.04	-8.67	6.50	-12.97	-5.26

**Tab.23 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of Sidelobe Level ( $SLL$ ) in [dB]**

	$av^* \{BW(t)\}$	$av \{BW(t)\}$	$\sigma^2 \{BW(t)\}$	$Min \{BW(t)\}$	$Max \{BW(t)\}$
<i>Dolph, SLL = -45 dB</i>	16.85	18.55	281.55	6.35	59.90
<i>PS - PSO, DIR Opt.</i>	10.25	8.80	3.62	6.69	12.78

**Tab.24 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of -3 dB Beamwidth ( $BW$ ) in [deg]**

## TEST CASE 1.g - Dolph-Chebyshev -50dB

### Goal

Sintesi di un array con eccitazioni modulate nel dominio del tempo al fine di riprodurre un pattern di Dolph-Chebyshev alla frequenza centrale di lavoro e un pattern istantaneo con massima direttività costante.

### Analogies and Differences wrt Previous Cases

**A:** Geometria dell'array, numero di elementi, algoritmo di ottimizzazione (PSO), funzione di costo implementata, configurazione statica dell'array.

**D:** Configurazione degli impulsi di eccitazione degli elementi (durate degli impulsi).

### Test Case Description

- Number of Elements:  $N = 16$
- Elements Spacing:  $d = 0.5\lambda$
- Static Array Configuration:  $\alpha_n = 1, n = 0, \dots, N - 1$
- Pattern at Central Frequency: *Dolph - Chebyshev*,  $SLL = -50 \text{ dB}$
- Max Gain Pattern Direction :  $\theta^{max} = 90^\circ$

### Optimization Approach: PS-PSO, Directivity Opt.

- Number of Variables:  $X = 8$
- Number of Particles:  $S = 20$
- Number of Iterations:  $I = 100$
- Inertial Weight: Linearly varying:  $0.9 \text{ to } 0.4$
- Swarm Initialization: *Random*

Dolph-Chebyshev Pattern, SLL=-50 dB

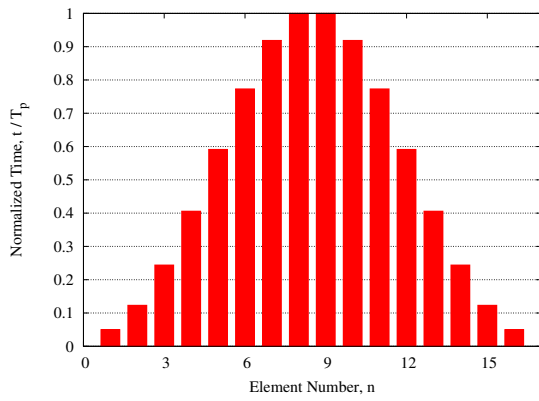


Fig.64 - Pulse Sequence

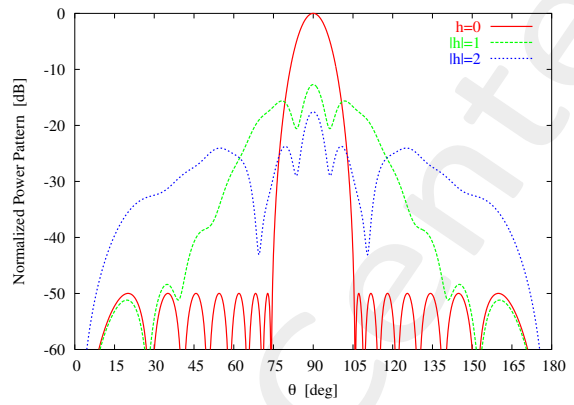


Fig.65 - Patterns

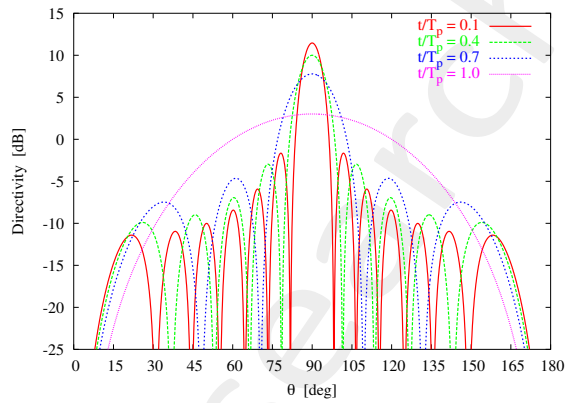


Fig.66 - Directivity

PS-PSO, Directivity Opt. - Dolph-Chebyshev Pattern, SLL=-50 dB

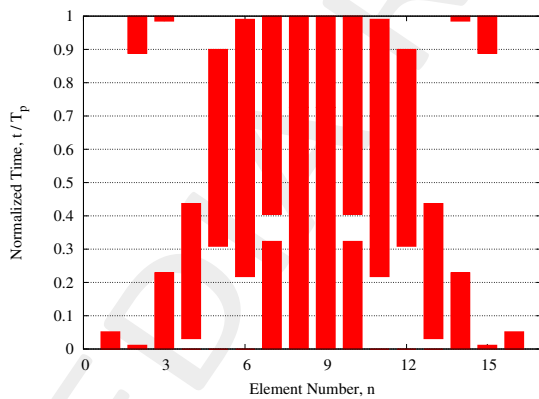


Fig.67 - Pulse Sequence

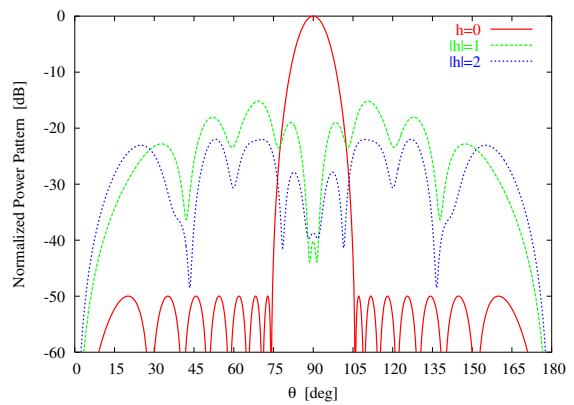


Fig.68 - Patterns

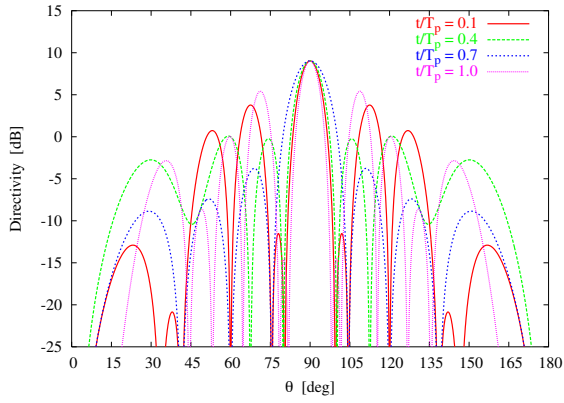


Fig.69 - Directivity

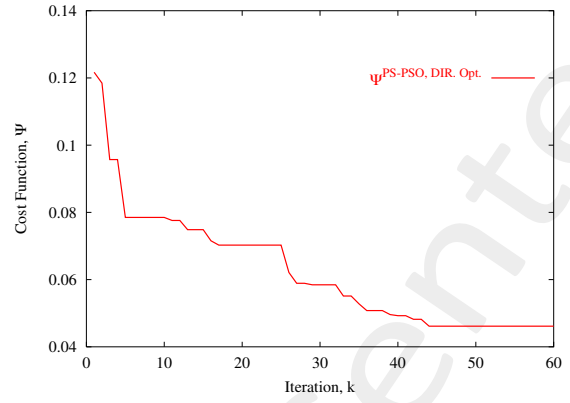


Fig.70 - Patterns

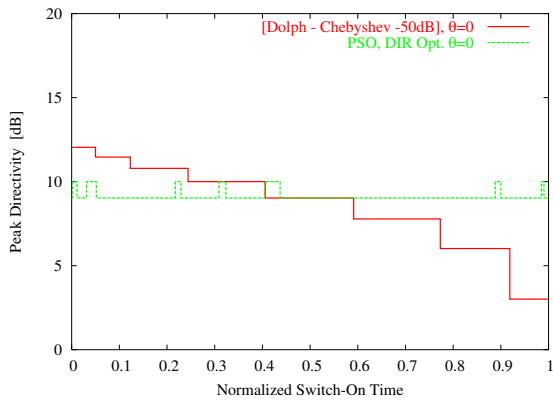


Fig.71 - Peak Directivity

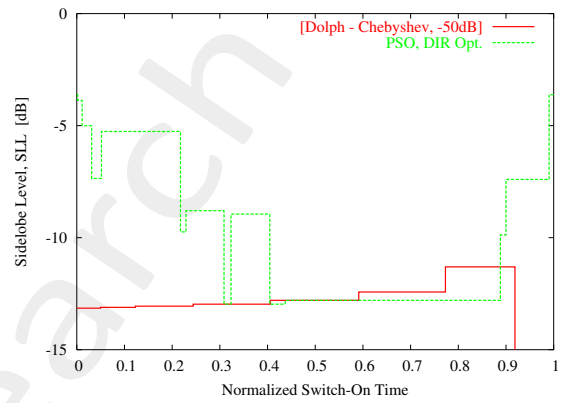


Fig.72 - Sidelobe Level

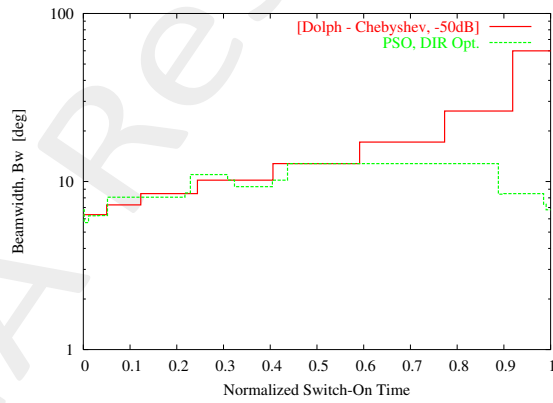


Fig.73 - Beamwidth

	$SLL_{av}$ [dB]	$SBL$ [dB]	$BW_{av}$ [°]	$P_{SR}$ [%]
<i>Dolph, SLL = -50 dB</i>	-49.96	-12.70	9.84	26.12
<i>PS - PSO, DIR Opt.</i>	-49.96	-15.18	9.84	26.12

Tab.25 - Average Pattern Parameters: Sidelobe Level ( $SLL$ ), Sideband Level ( $SBL$ ), -3 dB Beamwidth ( $BW$ ), Sideband Radiation ( $P_{SR}$ )



	$av^* \{D_{max}(t)\}$	$av \{D_{max}(t)\}$	$\sigma^2 \{D_{max}(t)\}$	$Min \{D_{max}(t)\}$	$Max \{D_{max}(t)\}$
<i>Dolph, SLL = -50 dB</i>	9.13	9.48	8.17	9.03	10
<i>PS - PSO, DIR Opt.</i>	9.14	9.48	0.23	9.03	10

**Tab.26 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of Peak Directivity ( $D_{max}$ ) in [dB]**

	$av^* \{SLL(t)\}$	$av \{SLL(t)\}$	$\sigma^2 \{SLL(t)\}$	$Min \{SLL(t)\}$	$Max \{SLL(t)\}$
<i>Dolph, SLL = -50 dB</i>	-	-	-	$-\infty$	-
<i>PS - PSO, DIR Opt.</i>	-9.91	-7.97	10.24	-12.97	-3.62

**Tab.27 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of Sidelobe Level ( $SLL$ ) in [dB]**

	$av^* \{BW(t)\}$	$av \{BW(t)\}$	$\sigma^2 \{BW(t)\}$	$Min \{BW(t)\}$	$Max \{BW(t)\}$
<i>Dolph, SLL = -50 dB</i>	10.56	8.41	3.74	5.70	12.78
<i>PS - PSO, DIR Opt.</i>	10.56	8.41	3.74	5.70	12.78

**Tab.28 - Istant Pattern Parameters: Weighted Average ( $av^*$ ), Average ( $av$ ), Variance ( $\sigma^2$ ), Minimum ( $Min$ ) and Maximum ( $Max$ ) of -3 dB Beamwidth ( $BW$ ) in [deg]**

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