Reconfigurable thinning for the maximization of the SINR in adaptive linear arrays

L. Poli, P. Rocca, M. Salucci, A. Massa

Abstract

In this report, an easily-reconfigurable and low-complexity antenna architecture is considered where a set of radio-frequency switches is exploited to either connect or disconnect the array elements for controlling the radiation pattern and generating deep nulls along the directions-of-arrival of the undesired signals. The mathematical formulation together with a preliminary validation of the adaptive nulling strategy are proposed. Part I Mathematical Formulation

Mathematical Formulation

Multiple Interferences at the Central Frequency - SINR Maximization

Consider a linear array of N isotropic elements equally spaced along the x axis: the desired signal received by the n-th element of the antenna array can be defined as

$$S_n^d(t) = p_d(t)e^{j\beta_n^d} \quad n = 1, ..., N$$
 (1)

where $\beta_n^d = (2\pi/\lambda)(u_d x_n)$, $u_d = \sin \theta_d \cos \phi_d$, x_n is the distance between the *n*-th element and the center of the array e θ_d , ϕ_d are the polar coordinates defining the direction of arrival (DOA) of the desired signal characterized by envelope $p_d(t)$. Assuming that one or more (I) interfering signals can be received by the antenna at the same angular frequency ω_d of the desired signal, it is possible to evaluate the contribution of each interference at the *n*-th element:

$$S_{n}^{i}(t) = p_{i}(t)e^{j\beta_{n}^{i}} \left\{ \begin{array}{c} n = 1, ..., N\\ i = 1, ..., I \end{array} \right.$$
(2)

where $\beta_n^i = (2\pi/\lambda)(u_i x_n)$, $u_i = \sin \theta_i \cos \phi_i \in \theta_i$, ϕ_i are the polar coordinates defining the direction of arrival (DOA) of the *i*-th interfering signal characterized by envelope $p_i(t)$. Moreover, let assume the presence of the noise modelled with an additive gaussian process with power \wp_n .

Hence, the coefficients of the covariance matrix $(N \times N)$ of the desired signal Φ_d are

$$\Phi_d^{mn} = E\left\{S_m^{d*}(t)S_n^d(t)\right\} \quad m, n = 1, ..., N$$
(3)

Similarly, it is possible to write the coefficients of the covariance matrix Φ_i of the *i*-th interfering signal (i = 1, ..., I) as

$$\Phi_i^{mn} = E\left\{S_m^{i*}(t)S_n^i(t)\right\} \quad m, n = 1, ..., N$$
(4)

while the covariance matrix of the noise is defined

$$\Phi_n = p_n 1^N \tag{5}$$

where 1^N is an identity matrix with dimension $N \times N$.

Let us write the covariance matrix of the undesired signal with the form

$$\Phi_u = \sum_{i=1}^{I} \Phi_i + \Phi_n \tag{6}$$

The power of the undesired signal received at the central frequency is

$$\wp_u = \frac{1}{2} \underline{W}^{T*} \Phi_u \underline{W} \tag{7}$$

where \underline{W} is defined as

$$\underline{W} = \left\{ \alpha_n e^{j\gamma_n}, \ n = 1, ..., N \right\}$$
(8)

where α_n amplitude excitation coefficients of the *n*-th element and γ_n is the phase excitation coefficient of the *n*-th element of the array. Using a thinning technique, the possible solutions of α_n are just two values: $\alpha_n \in \Upsilon, \ n = 1, ..., N$, where $\Upsilon = [\{0\}, \{1\}]$. We consider $\gamma_n = 0, \ n = 1, ..., N$.

The power contribution of the desired signal at the receiver is

$$\wp_d = \frac{1}{2} p_d^2(t) \left| W^T \underline{U}(\theta_d, \phi_d) \right|^2 \tag{9}$$

where

$$\underline{U}(\theta_d, \phi_d) = \left\{ e^{j\beta_n^d}, \ n = 1, ..., N \right\}$$
(10)

Considering (7) and (9) the SINR (Signal to Interferience plus Noise Ratio) can be defined as:

$$\Psi(\underline{G}) \triangleq \frac{\wp_d}{\wp_u} = \frac{p_d^2(t) \left| W^T \underline{U}(\theta_d, \phi_d) \right|^2}{\underline{W}^{T*} \Phi_u \underline{W}}$$
(11)

Since Φ_u and $p_d^2(t)$ are not directly misurable, (11) is not useful. But, it is possible to reformulate the SINR maximization problem through the following cost function

$$f(\underline{G}) = \frac{\left|W^T \underline{U}(\theta_d, \phi_d)\right|^2}{\underline{W}^{T*} \Phi_t \underline{W}}$$
(12)

where $\Phi_t = \Phi_d + \sum_{i=1}^{I} \Phi_i + \Phi_n$ is a quantity that can be measured at the receiver.

Part II

Preliminary Results - Unconstrained Directivity Cases

TEST CASE 1 - 40 Elements - Fixed Scenario, Single Interference

Goal

Maximization of the SINR using genetic algorithms (GA) to determine the optimal thinned array configuration, considering a static scenario with a single interference.

Test Case Description

- Number of Elements N = 40
- Elements Spacing: $d = 0.5\lambda$
- Max Gain Pattern Direction : $\theta^d = 90^\circ, \ \phi^d = 90^\circ$
- Desired Signal Power: $0 \, dB$
- Interference Power: $30 \, dB$
- Noise Power: $-30 \, dB$
- Number of Interferences: $N_I = 1$

- Number of Variables: $X = 40 \ (\alpha_n, n = 1, ..., N)$
- Population: 40
- Crossover Probability: 0.9
- Mutation Probability: 0.01
- Number of Generations: 50

GA - Single Interference: $\theta^i_1=90^\circ,\,\phi^i_1=42^\circ$



Fig.1 - Thinning Configuration







		$AF(\theta_1^i,\phi_1^i)$	Nr.ActiveElements	$SINR\left[dB ight]$
1	GA	-59.58	25	29.42

Tab.1 - GA Simulation Results Analysis

TEST CASE 2 - 40 Elements - Fixed Scenario, Double Interference

Goal

Maximization of the SINR using genetic algorithms (GA) to determine the optimal thinned array configuration, considering a static scenario with a double interference.

Test Case Description

- Number of Elements N = 40
- Elements Spacing: $d = 0.5\lambda$
- Max Gain Pattern Direction : $\theta^d=90^\circ,\,\phi^d=90^\circ$
- Desired Signal Power: 0 dB
- Interference Power: $30 \, dB$
- Noise Power: $-30 \, dB$
- Number of Interferences: $N_I = 2$
- Interference Direction Of Arrival: $\theta_1^i = 90^\circ$, $\phi_1^i = 42^\circ$, $\theta_2^i = 90^\circ$, $\phi_2^i = 113^\circ$

- Number of Variables: $X = 40 \ (\alpha_n, n = 1, ..., N)$
- Population: 40
- Crossover Probability: 0.9
- Mutation Probability: 0.01
- Number of Generations: 100

GA - Double Intereference: $\theta_1^i = 90^\circ, \ \phi_1^i = 42^\circ, \ \theta_2^i = 90^\circ, \ \phi_2^i = 113^\circ$



Fig.5 - Thinning Configuration







	$AF(\theta_1^i, \phi_1^i)$	$AF(\theta_2^i, \phi_2^i)$	Nr.ActiveElements	$SINR\left[dB ight]$
GA	-62.49	-55.32	32	24.52

Tab.2 - GA Simulation Results Analysis

TEST CASE 3 - 40 Elements - Fixed Scenario, Triple Interference

Goal

Maximization of the SINR using genetic algorithms (GA) to determine the optimal thinned array configuration, considering a static scenario with a triple interference.

Test Case Description

- Number of Elements N = 40
- Elements Spacing: $d = 0.5\lambda$
- Max Gain Pattern Direction : $\theta^d=90^\circ,\,\phi^d=90^\circ$
- Desired Signal Power: $0 \, dB$
- Interference Power: $30 \, dB$
- Noise Power: $-30 \, dB$
- Number of Interferences: $N_I = 3$
- Interference Direction Of Arrival: $\theta_1^i = 90^\circ$, $\phi_1^i = 42^\circ$, $\theta_2^i = 90^\circ$, $\phi_2^i = 113^\circ$, $\theta_3^i = 90^\circ$, $\phi_3^i = 175^\circ$

- Number of Variables: $X = 40 \ (\alpha_n, n = 1, ..., N)$
- Population: 40
- Crossover Probability: 0.9
- Mutation Probability: 0.01
- Number of Generations: 100

GA - Triple Interference: $\theta_1^i = 90^\circ$, $\phi_1^i = 42^\circ$, $\theta_2^i = 90^\circ$, $\phi_2^i = 113^\circ$, $\theta_3^i = 90^\circ$, $\phi_3^i = 175^\circ$



Fig.9 - Thinning Configuration



	$AF(\theta_1^i,\phi_1^i)$	$\overline{AF}(\theta_2^i,\phi_2^i)$	$AF(heta_3^i,\phi_3^i)$	Nr.ActiveElements	$SINR\left[dB ight]$
GA	-49.21	-64.83	-67.19	34	19.02

Tab.3 - GA Simulation Results Analysis

TEST CASE 4 - 40 Elements - Time-Varying Scenario

Goal

Maximization of the SINR using genetic algorithms (GA) to determine the optimal thinned array configuration, considering a time-varying scenario.

Test Case Description

- Number of Elements N = 40
- Elements Spacing: $d = 0.5\lambda$
- Max Gain Pattern Direction : $\theta^d = 90^\circ, \ \phi^d = 90^\circ$
- Desired Signal Power: $0 \, dB$
- Interference Power: $30 \, dB$
- Noise Power: $-30 \, dB$
- Number of Interferences: variable
- Timesteps: T = 900

- Number of Variables: $X = 40 \ (\alpha_n, n = 1, ..., N)$
- Population: 40
- Crossover Probability: 0.9
- Mutation Probability: 0.01
- Number of Generations (for each iteration of the time-varying scenario): 200

GA - Time-Varying Scenario







	$Min\left\{SINR\right\}\left[dB\right]$	$Max\left\{SINR\right\}\left[dB ight]$	Average $\{SINR\}$ $[dB]$	$Variance \{SINR\} [dB]$
GA	-30.00	46.02	23.38	351.85

Tab.4 - SINR statistics

	$Min\left\{N_{ON}\right\}$	$Max\left\{N_{ON} ight\}$	Average $\{N_{ON}\}$	$Variance \{N_{ON}\}$
GA	4	40	31.10	41.46

Tab.5 - Number of Active Elements (N_{ON}) statistics

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